

# 2-D Direction of Arrival Estimation Combining UCA-RARE and MUSIC for Uniform Circular Arrays Subject to Mutual Coupling

Roald Goossens

Supervisor(s): Hendrik Rogier

**Abstract**—A new algorithm for 2-D direction-of-arrival estimation of azimuth and elevation angle is presented for uniform circular arrays by combining UCA-RARE and MUSIC in the presence of the mutual coupling. To describe mutual coupling in a uniform circular array we make use of the symmetry of the array and expand the open-circuit voltages into a limited number of spherical modes. The UCA-RARE technique is then applied to estimate the azimuth angle independent from the elevation angle. Next for each azimuth angle we estimate the corresponding elevation angles. Some illustrative examples are given to validate our approach.

**Keywords**—Mutual coupling, UCA, MUSIC

## I. INTRODUCTION

ESTIMATING the directions of arrival (DOAs) of plane waves impinging on antenna arrays is an important issue in mobile communication systems. In recent years several algorithms were proposed to estimate DOAs [1]. However the performance of these algorithms is often affected by mutual coupling between the antenna elements in the array. Especially in uniform circular arrays (UCAs) mutual coupling can be significant. In [2] it is proven that the open-circuit voltage of each element in the UCA can be described with a limited number of parameters by means of a spherical mode expansion. Another challenge is the simultaneous estimation of azimuth and elevation angles  $(\varphi, \theta)$ . UCA-ESPRIT [1] is a useful technique for 2-D angle estimation, but the difficulty to compensate for mutual coupling is a disadvantage. Other methods which are compatible with mutual coupling effects are mostly restricted to a one-dimensional search over  $\varphi$  which is numerically easy to implement. The extension to 2D angle estimation is computationally expensive. The aim is to develop a method which overcomes these problems.

In this paper we combine two algorithms, being UCA-RARE [3] and MUSIC [4]. In [3] a new eigenstructure-based estimation method (UCA-RARE) is developed and first of all we combine this algorithm with a phase-mode expansion for the mutual coupling effects, allowing to estimate the azimuth parameters of the source directions  $\varphi$  decoupled from the elevation angles  $\theta$  in the presence of mutual coupling. Given the knowledge of the azimuth angles, we estimate the elevation parameters by performing a one-dimensional search over the MUSIC-spectrum. For constructing the MUSIC-spectrum we rely on the spherical mode expansion, which describes mutual coupling in the uniform circular array. In section II the combination of the two search algorithms is explained. In section III some illustrative results are presented, proving that the simultaneous

estimation of azimuth and elevation parameters is realized by combining the proposed algorithms.

## II. COMBINED ALGORITHM

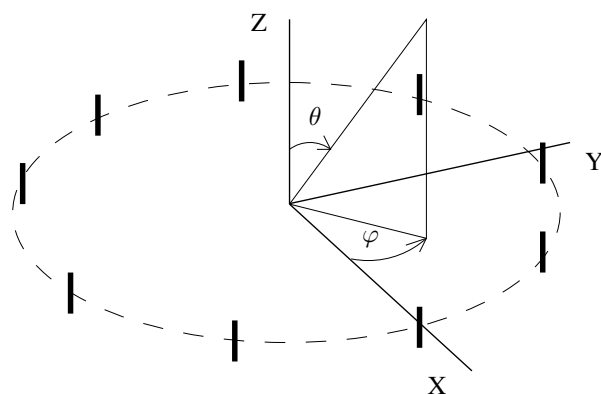


Fig. 1. uniform circular array composed of nine antenna elements.

### A. Estimating azimuth parameters: UCA-RARE

When plane waves impinge on a uniform circular array, voltages are induced over the antenna elements. In figure 1 the antenna array configuration is shown. The basis for direction of arrival estimation is the construction of the MUSIC spectrum [4]

$$f_{MUSIC}(\theta, \varphi) = 1/\mathbf{a}^H(\theta, \varphi) E_N E_N^H \mathbf{a}(\theta, \varphi) \quad (1)$$

$\mathbf{a}(\theta, \varphi)$  is the array manifold of the UCA and contains the voltages which are induced over the antenna elements when a plane wave impinges on the array at DOA  $(\theta, \varphi)$ . By relying on the symmetry in the UCA it is possible to separate the azimuth and elevation angle dependency in the array manifold. This allows to obtain estimates for the azimuth angles independent from the elevation angle by searching for roots in a modified MUSIC function.

### B. Estimating elevation parameters: MUSIC

The UCA-RARE algorithm, a rank reduction algorithm, delivers us estimates for the azimuth angles. To obtain estimates for the elevation angle we sample the MUSIC spectrum along the estimated azimuth angles, obtained by UCA-RARE, as a function of the elevation angle. To construct the array manifold

R. Goossens is with the department of Information Technology, Ghent University (UGent), Gent, Belgium. E-mail: Roald.Goossens@intec.UGent.be .

we rely on the spherical mode expansion of the array manifold

$$a_1(\varphi, \theta) = \sin \theta \sum_{m=-M}^{+M} \sum_{n=|m|}^N b_{mn} P_n^{|m|}(\cos \theta) e^{jm\varphi}, \quad (2)$$

where  $M$  and  $N$  are restricted by the dimension of the array compared to the wavelength. This spherical mode expansion describes the mutual coupling in a rigorous manner. Describing the array manifold in full detail by a limited number of coefficients results in a numerically simple 1-D search for peaks over the elevation angle.

### III. RESULTS

The algorithm to estimate DOAs in two dimensions is tested on a UCA of nine dipole antennas tuned to 900 MHz. The array elements are distributed uniformly over a circle with diameter  $d = l(\approx \frac{\lambda}{2})$ .

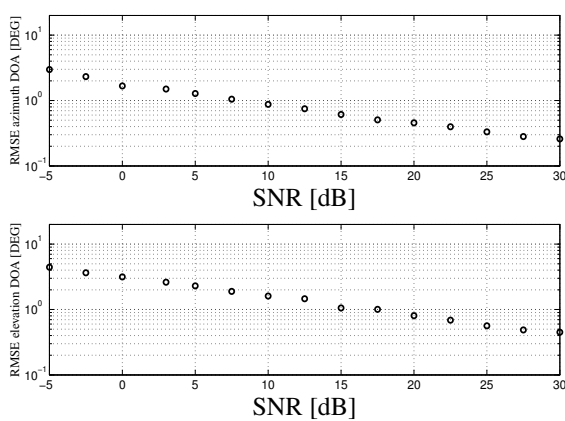


Fig. 2. RMSE's of azimuth and elevation angles versus SNR.

To demonstrate the combined algorithm UCA-RARE + MUSIC, consider three uncorrelated sources emitting 10000 bit pseudo-random bit sequences. The signals are received in the presence of additive white Gaussian noise and the different DOAs are  $\{\varphi_1 = 50^\circ, \theta_1 = 50^\circ\}$ ,  $\{\varphi_2 = 135^\circ, \theta_2 = 145^\circ\}$  and  $\{\varphi_3 = 280^\circ, \theta_3 = 90^\circ\}$ . In figure 2 the Root-mean squared error (RMSE) of the azimuth and elevation DOA estimates are plotted versus the SNR for an ensemble consisting of 500 implementations. It is clear that the combined algorithm provides good estimates of the DOAs.

A special case is the situation where two plane waves impinge at the same azimuth angle e.g. two sources impinge at  $\{\varphi_1 = 50^\circ, \theta = 45^\circ\}$  and  $\{\varphi_1 = 50^\circ, \theta = 80^\circ\}$  at SNR level 30dB. UCA-RARE provides the estimation for the azimuth angle; in figure 3 the MUSIC spectrum is shown as a function of the elevation angle, at  $\hat{\varphi} = 50.0143^\circ$ . It is clear that two peaks can be distinguished at  $\theta = 48^\circ$  and  $\theta = 79^\circ$ . The symmetry of the antenna array implies that it isn't possible to distinguish between directions  $\theta$  and  $\pi - \theta$  (four peaks are actually observed). The antenna elements are dipole antennas and they cannot detect plane waves that impinge at the poles. This property is expressed by peaks in the MUSIC function at  $\theta = 0, \pi$ .

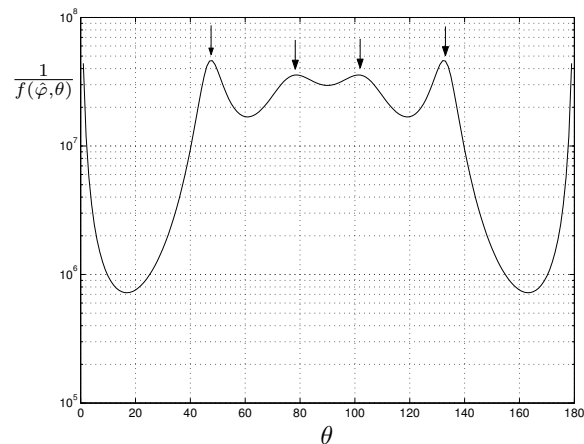


Fig. 3. The inverse MUSIC spectrum versus the elevation angle.

### IV. CONCLUSIONS

A new method for combined azimuth and elevation DOA estimation in UCAs is presented. In a first step an estimation for the azimuth angles is performed by UCA-RARE which decouples the estimation of azimuth angles from the estimation of elevation angles. The estimated azimuth angles enable us to perform a 1D search over the MUSIC spectrum and in order to determine the elevation angles. Innovating in the combination of these two methods UCA-RARE + MUSIC is the fact that mutual coupling is fully taken into account by means of a limited number of parameters, given the expansion of the open-circuit voltages into spherical modes.

### ACKNOWLEDGMENTS

H. Rogier is a Postdoctoral Research Fellow of the FWO-V. His research was supported by a grant of the DWTC/SSTC, MOTION project.

### REFERENCES

- [1] C.P. Mathews and M.D. Zoltowski, "Eigenstructure techniques for 2-D angle estimation with uniform circular arrays", IEEE trans. on signal processing, sept. 1994, vol. 42, issue 9 Page(s):2395 - 2407.
- [2] H. Rogier and E. Bonek, "Analytical spherical-mode based compensation of mutual coupling in uniform circular arrays for direction-of-arrival estimation", Int. Journal of Electr. and Comm., p. in press, 2005.
- [3] M. Pesavento and J.F. Böhme, "Direction of arrival estimation in uniform circular arrays composed of directional elements", Sensor Array and Multichannel Signal Processing Workshop Proceedings, 4-6 Aug. 2002 Page(s):503 - 507.
- [4] R.O. Schmidt, "Multiple emitter location and signal parameter estimation", IEEE Trans. Antenn. Propagat., vol. 34 March 1986, page(s): 276-280.
- [5] H. Rogier and E. Bonek, "A model for describing mutual coupling in uniform circular arrays and its application to DOA estimation", IEEE Antennas and Propagation Society Symposium, June 2004, Vol. 2 Page(s):1251 - 1254.