Fictitious play: two viewpoints and two versions

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I will be focusing on non-cooperative two-player games in strategic form. An important type of player behavior in iterated games of this type is described by so-called *fictitious play*. In this context, the players *act myopically*, in the sense that they are only interested in maximizing their immediate expected pay-off.

There are two viewpoints, which differ in the interpretation given to mixed strategies.

Using a randomization device

In the first viewpoint, a player faces the same opponent iteratively, and both players can play mixed strategies using a randomization device.

In the classical version of fictitious play, the players use a *precise Dirichlet model (PDM)*. This means that a Dirichlet distribution over the strategy simplex of their opponent is used to represent their own beliefs about the mixed strategy their opponent is going to play. Maximizing their expected payoff then corresponds to *playing a best reply to the expected strategy of their opponent* under this PDM. After observing the pure strategy played by his opponent, the player adapts his beliefs by modifying the (parameters of the) PDM previously used.

The parameters of a PDM correspond to the expected strategy and to the strength of their current belief. The choice for initial values of these parameters is problematic, for example, the initial choice of expected strategy will always be arbitrary.

To remove the problem of choosing an initial strategy, fictitious play can be modified by letting the players use an *imprecise Dirichlet model (IDM)*; [4]. This means that they use a set of Dirichlet distributions instead of one. Initially the set can be chosen such that all strategies are expected under one of the distributions in the set. To maximize his expected pay-off, *the player chooses a strategy from the set of best replies to the expected strategies*. Each time a game is played, the player updates all the distributions forming his IDM.

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Using a population of players

In the *population dynamics viewpoint*, players can only play pure strategies and are faced with a whole population of possible opponents; [4]. All the players in the population, whose strategies are fixed, play among themselves in some sort of competition. A new player enters the competition each iteration; [3].

Classically, a PDM is again used by the new player to choose his strategy upon entering the competition. The PDM represents the beliefs of the outside world about the relative frequencies of players in the population playing a certain pure strategy. The new entrant will play a best reply to the mixed strategy that corresponds to the expectation of these frequencies. After observing the pure strategy chosen by the last new entrant, the outside world updates its PDM for later entrants.

Unless the outside world has observations starting from the first entrant to the competition, the use of an IDM can also remove the arbitrariness of the choice of initial expected frequencies in this case.

Convergence results

The interesting convergence results of Fudenberg and Kreps [1] for the PDM-version remain valid in a slightly modified form for the IDM-version; [4]. These results guarantee that if convergence occurs, it is to an equilibrium. They hold for both viewpoints, which are mathematically equivalent.

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