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EXTENSIONS OF THE COMPLEX JACOBI ITERATION TO SIMULATE PHOTONIC WAVELENGTH SCALE COMPONENTS

Peter Vandersteegen*, Peter Bienstman and Roel Baets

*UGent-IMEC, Department of Information Technology
St.-Pietersnieuwstraat 41, 9000 Gent, Belgium
e-mail: Peter.Vandersteegen@intec.UGent.be
web page: http://photonics.intec.UGent.be

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Abstract. *We have extended the complex Jacobi iteration to simulate nonlinear optical components. The size of these components is comparable to the used wavelength (μm). In this paper we will give an overview of the technique and several simulation results. The algorithm uses a central difference analogue of the Helmholtz equation. The used nonlinearity is the third order Kerr effect, which is modelled as an instantaneous intensity-dependent refractive index.*

1 INTRODUCTION

In present day optical fiber networks routing of light requires a conversion from light to electrical signal and back. This botteneck can be alleviated by doing all signal processing in the optical domain. The only way to achieve all optical signal processing, light switching by light, is by using fast non linear optical components. One promissing nonlinear effect is the third order Kerr effect. This small instanteneous nonlinearity (time scale: fs) changes the refractive index of a material based on the local intensity ($n = n + n_2|E|^2$). Recent fabrication progress results in high confinement of light which could allow the exploitation of this effect. However, an accurate integration of Maxwell's laws is required to effectively investigate promising components.

Modelling of these components can be done either in the time domain or in the frequency domain. In the time domain this means an integration of Maxwell's laws. A very popular method is Finite Differences in Time Domain (FDTD, [3]) . This method has the advantage of giving the time evolution of the electromagnetic field. In the frequency domain, the problem reduces to numerical integration of the Helmholtz equation. The found fields are invarient in time, save a phase factor. One method is non linear Eigenmode Expansion (EME, [4]). This method is able to simulate a structure fast and rigorously. It however imposes some restrictions to the structure.

We propose the use of the complex Jacobi iteration (CJ, [1] and [2]). This method solves the central difference equivalent of the Helmholtz equation. The fields are only calculated for points located on an equidistent grid.

$$\begin{aligned} \Delta e_{i,j} + (\frac{2\pi}{\lambda_0} n_{i,j})^2 e_{i,j} &= 0 \\ e_{i,j} &= e(i\Delta x, j\Delta y) \\ \Delta e_{i,j} &= \delta_x^2 e_{i,j} + \delta_y^2 e_{i,j} \\ \delta_x e_{i,j} &= \frac{e_{i+1,j} - e_{i-1,j}}{2\Delta x} \end{aligned} \tag{1}$$

(2)

The wavelength in vacuum is λ_0 and the discretization step is Δx.

In this paper we will briefly outline the simulation method used. We will use this method to simulate two non trivial optical components. The first component is a cavity enclosed by 2 mirrors, [6]. In this section we will also address the problems which occur when using finite differences compared to the actual analytical derivatives. The second component will be a so called 'vertical grating coupler', [7]. We will discuss the influence of the nonlinearity. In order to determine convergence we have used an alternative convergence criterium.

2 Complex Jacobi iteration

The complex Jacobi iteration calculates the finite difference analogue of the Helmholtz equation. Each iteration step the fields are refined based on previous values. This iteration process finishes when sufficient convergence has been achieved. We will discus both the linear algorithm and its nonlinear extension in this section. The field values have to satisfy:

$$\Delta e_{i,j} + (\frac{2\pi}{\lambda_0} n_{i,j})^2 e_{i,j} = 0 \tag{3}$$

The refractive index can be locally dependent on the local intensity:

$$n_{i,j} = n_{i,j,0} + n_{i,j,2}|E|^2 \tag{4}$$

The complex Jacobi iteration uses a Dirichlet boundary condition. This means the fields of the simulation box's boundaries have to been known. These fields however are usually the unknowns. Using sources and absorbing boundary conditions can alleviate this restriction. This has been described in [2], a small overview will be given in the next subsection.

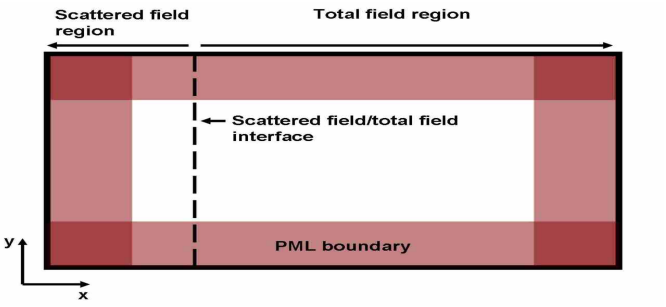
The complex Jacobi iteration refines the field amplitudes with a two step update equation until desired accuracy has been achieved. The additional update equation for the

refractive index allows the simulation of Kerr based components.

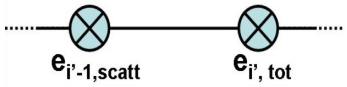
$$\begin{aligned} e_{i,j}^{n+\frac{1}{2}} &= e_{i,j}^{n} + \left(2\alpha_1(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2})\right)^{-1}\left((\delta_x^2+\delta_y^2+(\frac{2\pi}{\lambda}n_{i,j})^2)e_{i,j}^{n}\right) \\ e_{i,j}^{n+1} &= e_{i,j}^{n+\frac{1}{2}} + \left(2\alpha_2(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2})\right)^{-1}\left((\delta_x^2+\delta_y^2+(\frac{2\pi}{\lambda}n_{i,j})^2)e_{i,j}^{n+\frac{1}{2}}\right) \\ n_{i,j}^{n+1} &= n_{i,j,0} + n_{i,j,2}|e_{i,j}|^2 \end{aligned} \tag{5}$$

The superscript n is the iteration step. The position dependent refractive index is$n_{i,j}$ Using the time dependent phase factor $\exp^{(-j\omega t)}$ results in the following optimal constants for this iteration scheme: $\alpha_1 = \sqrt{3} - 1j$ and $\alpha_2 = -\alpha_1^*$. This derivation has been described in [1].

2.1 Sources and absorbing boundary conditions



(a) Simulation box with 'total field/scattered field' as field source and PMLs as absorbing boundary conditions.



(b) Interface at index i' between scattered field - total field.

Figure 1: A general simulation box with sources and absorbing boundary conditions.

We use sources to circumvent the restrictions imposed by the Dirichlet boundary conditions. The dipole source is one possible source. Another source formalism is the total field/scattered field. This formalism is a well known formalism in FDTD, [3]. With this formalism, we are able to inject a right propagating field profile in figure 1. Reflections which might occur in the total field region can travel back to the left. These left propagating waves can pass the transparent TFSF-interface. In the left 'scattered field' region we only find the reflected field. A TFSF interface placed at index i', figure 1(b), can be implemented with following equations: -The j-index has been omitted for simplicity.-

$$e_{i'-1,scatt}^{n} = \left(2\alpha(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2})\right)^{-1}\left(\Delta e_{i',scatt}^{n} - \frac{e_{source,i'}}{\Delta x^2}\right) \tag{6}$$

$$e_{i',tot}^{n} = \left(2\alpha(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2})\right)^{-1}\left(\Delta e_{i',scatt}^{n} + \frac{e_{source,i'-1}}{\Delta x^2}\right) \tag{7}$$

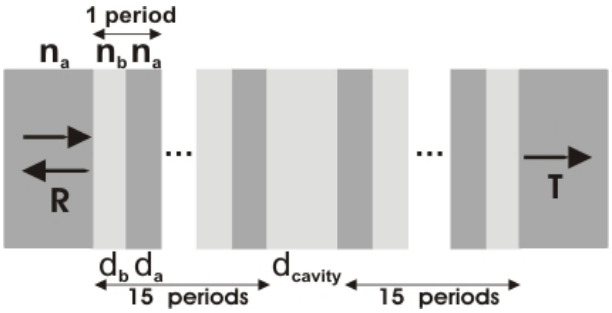
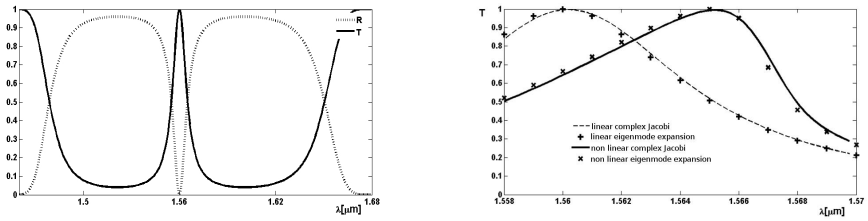


Figure 2: Two Bragg mirrors encapsulate a cavity. ($\lambda_{resonance} = 1.56\mu m$)



(a) Power Tranmission and Reflection for the linear structure

(b) Comparison between Eigenmode Expansion en complex Jacobi iteration, both linear as non-linear.

Figure 3: Power Transmission and Reflection of 2

Elimination of spurious reflections at the boundaries of the finite simulation box can be done by perfectly matched layers (PML). We have choosen a PML based on complex coordinate stretching. This results in the easiest implementation for the complex Jacobi iteration. The 'j-index' has again been omitted for simplicity.

$$\delta_x e_{i,j} \quad = \quad \frac{1}{s_{x,i,j}} \frac{e_{i+1,j} - e_{i+1,j}}{2\Delta x} \tag{8}$$

Regions where $s_x = 1.0$ can be considered as normal space. Absorption occurs in regions where $\Re(s_x) = 1.0$ and $\Im(s_x) > 0.0$. Reflection is limited at an interface between $s_x = 1.0$, $\Im(s_x) > 0.0$ and $\Re(s_x) = 1.0$, $\Im(s_x) > 0.0$.

3 Cavity enclosed by 2 Bragg mirros

In this section we will discuss the reflection and transmission of a nonlinear cavity enclosed by 2 Bragg mirrors, as seen in figure 2. We will validate our simulation results by comparing with nonlinear eigenmode expansion. We will identify the biggest influence of the numerical errors caused by central differences.

Figure 3 (a) shows the reflection and transmission of the linear structure. The two

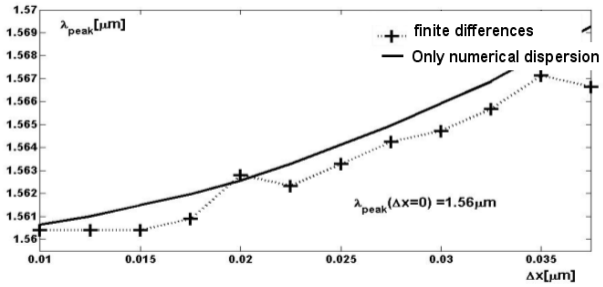


Figure 4: Shift of resonance peak of figure 3(b) in function of discretization step.

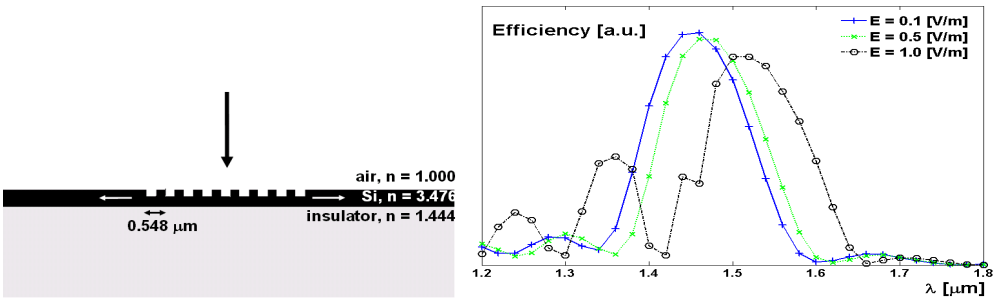
Bragg reflectors at left and right side are wavelength dependent mirrors. Transmission can however occur at the resonance wavelength of $\lambda = 1.56\mu m$. This resonance is created by the cavity. Without this cavity, we would see a photonic band gap from approximately $\lambda = 1.5\mu m$ to $1.62\mu m$.

A nonlinear cavity of $n_2 = 5.10^{-3}\frac{m^2}{V^2}$ and incoming plane wave with field amplitude $e = 1\frac{V}{m}$ causes the shift of the resonance peak seen in figure 3(b). Figure 3(b) also illustrates the good agreement between non linear eigenmode expansion and non linear complex Jacobi iteration. The advantage of CJ is its flexibility. EME is faster than CJ in the case of non linearity located in the cavity. In realistic cases, the non linearity will also be present in other parts of the simulation area. In this general case, the EME speed will lower significantly. From this point of view CJ is a much more flexible method. Our method has been shown to converge for a refractive index shift up to $\Delta n = n_2|E|^2 \approx 0.6$. This is higher than the highest shift possible with EME.

3.1 Influence of numerical dispersion

Figure 4 shows the shift of the resonance peak for a *linear structure* in function of the discretization step. This demonstrates the dependency of a solution of equation 2 in function of the discretization step. A comparison between the continuous Helmholtz equation and the finite difference Helmholtz equation gives us 2 causes for this effect: stair-casing and numerical dispersion. Stair casing means we approximate the continuous refractive index profile $n(x, y)$ with $n_{i,j}$, the refractive index located on the grid. Numerical dispersion changes the dispersion relationship in function of the discretization step. The dispersion relationship is the relationship between between frequency and wavelength for a plane wave satisfying equation 2 in homogeneous space.

$$\begin{aligned} \frac{4\sin^2(\frac{\frac{2\pi}{\lambda_0}n_{eff}\Delta x}{2})}{\Delta x^2} &= \frac{\omega^2 n}{c^2} \\ n_{eff} &= \frac{\arcsin(\frac{\pi n \Delta x}{\lambda_0})}{\frac{\pi \Delta x}{\lambda_0}} \end{aligned} \tag{9}$$



(a) (b)

Figure 5: Vertical injection of light in 2 horizontal waveguides. Injection is symmetrical.

The refractive index n in equation 9 is used in equation 2. The effective refractive index n_{eff} is the refractive index which actually is 'felt' by a plane wave. Small Δx result in $n_{eff} \approx n$.

The resonance peak of the linear structure of figure 3(a) can be calculated analytically. This allows us to investigate the influence of numerical dispersion by replacing the refractive indices (n_a, n_b) with the effective refractive indices ($n_{a,eff}$, $n_{b,eff}$) of equation 9. Figure 4 then gives an indication what the influence is of numerical dispersion. We therefore may conclude that the influence of numerical dispersion is much bigger than the influence of stair casing. This also means that an adaptive grid to minimize stair casing will not give a more accurate result for this structure.

4 Vertical Grating coupler

In this section we will discuss the simulation results of the vertical grating coupler, as sketched in figure 5. Optical power is vertically injected in the structure. The setup has been made completely symmetrical. Increasing the power will cause a shift in the central peak, figure 5(b).

The used refractive indices correspond with the refractive indices found in a Silicon on Insulator material system. (SOI) The non linearity used in the central material is $n_2 = 0.1\frac{m^2}{V^2}$. The used discretization step in both directions is $\Delta x = \Delta y = 0.0137\mu m$.

We use the change of the efficiency, figure 5(b), as convergence criterion. Convergence is achieved when the change of the efficiency is below a certain value -for a certain amplitude and wavelength-. This convergence criterion is used instead of calculating $e_{i,j}^{n+1} - e_{i,j}^{n}$. For a certain convergence criterion value we see that convergence is met for 10000 iteration steps for $E = 0.1\frac{V}{m}$. The increased amplitude $E = 1.0\frac{V}{m}$ requires 60000 iteration steps to reach the same convergence requirement.

Non linear eigenmode expansion is not suited to simulate this structure. Eigenmode expansion is not efficient for injection of fields in a direction perpendicular on the waveguides.

5 CONCLUSIONS

Our extensions to the recently introduced complex Jacobi iteration allows the numerical integration of the Helmholtz equation with Kerr-based materials.

Our proposed extension to the complex Jacobi method is very well suited for structures where the non linearity is present in a large portion of the simulation domain.

We have used this technique to simulate 2 non trivial optical components. Good agreement with non linear eigenmode expansion has been demonstrated for the first structure.

6 ACKNOWLEDGEMENTS

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