# Distributed control of battery energy storage systems in distribution networks for voltage regulation at transmission–distribution network interconnection points

**DOI:**
10.1016/j.conengprac.2021.104988

Control Engineering Practice

# Distributed Control of Battery Energy Storage Systems in Distribution Networks for Voltage Regulation at Transmission-Distribution Network Interconnection Points

--Manuscript Draft--

| | |
|---|---|
| **Manuscript Number:** | CONENGPRAC-D-21-00296R1 |
| **Article Type:** | VSI: Smart technologies for net-zero emi |
| **Keywords:** | Distributed algorithms, online convex optimisation, energy storage systems, voltage control, distribution networks |
| **Corresponding Author:** | Tianqiao Zhao<br>Brookhaven National Laboratory<br>UNITED STATES |
| **First Author:** | Tianqiao Zhao |
| **Order of Authors:** | Tianqiao Zhao |
| | Alessandra Parisio |
| | Jovica V. Milanovic |
| **Abstract:** | This paper describes a control framework that enables distributed battery energy storage systems (BESS) connected to distribution networks (DNs) to track voltage setpoints requested by the transmission system operator (TSO) at specific interconnection points in an optimal and coordinated manner. The control design is based on an optimisation problem whose objective is to minimise the real-time voltage-tracking mismatch while satisfying local physical network constraints. A novel agent-based control scheme adopting an online convex optimisation (OCO) framework is developed and solved in a distributed fashion to guarantee the solution's scalability and the service provision within the required time. The BESS agents under the proposed control framework automatically adapt to the time-varying network conditions so as to track the required voltage setpoints whilst fulfilling the technical operating requirements of the local network. The designed OCO-based framework addresses the existing conflict between the accuracy and optimality of the solution and the communication and computational efficiency. The convergence to the optimal solution is demonstrated. Several case studies are performed to corroborate the analytical results and assess the performance of the proposed approach. |

# Distributed Control of Battery Energy Storage Systems in Distribution Networks for Voltage Regulation at Transmission-Distribution Network Interconnection Points

Tianqiao Zhao, Alessandra Parisio, *Senior Member, IEEE*, Jovica V. Milanović, *Fellow, IEEE*[1]

*Department of Electrical and Electronic Engineering, The University of Manchester, M13 9PL, Manchester, UK.*

**Abstract**

This paper describes a control framework that enables distributed battery energy storage systems (BESS) connected to distribution networks (DNs) to track voltage setpoints requested by the transmission system operator (TSO) at specific interconnection points in an optimal and coordinated manner. The control design is based on an optimisation problem whose objective is to minimise the real-time voltage-tracking mismatch while satisfying local physical network constraints. A novel agent-based control scheme adopting an online convex optimisation (OCO) framework is developed and solved in a distributed fashion to guarantee the solution's scalability and the service provision within the required time. The BESS agents under the proposed control framework automatically adapt to the time-varying network conditions so as to track the required voltage setpoints whilst fulfilling the technical operating requirements of the local network. The designed OCO-based framework addresses the existing conflict between the accuracy and optimality of the solution and the communication and computational efficiency. The convergence to the optimal solution is demonstrated. Several case studies are performed to corroborate the analytical results and assess the performance of the proposed approach.

*Keywords:* Distributed algorithms, online convex optimisation, energy storage systems, voltage control, distribution networks

## 1. Introduction

The increasing deployment of utility-level renewable generation in transmission networks (TNs) and distributed energy resources (DERs) in distribution networks (DNs) can potentially result in significant benefits and increased flexibility of operation to the whole network. It can, at the same time, result in unexpected challenges to voltage regulation, particularly in DNs [1]. These challenges call for the development of control schemes that facilitate the coordination between transmission (TSOs) and distribution system operators (DSOs) to efficiently utilise new DER capabilities. In this context, emerging technical solutions, particularly focusing on voltage support at DN, are being investigated by different system operators worldwide [2, 3]. In current practice, control setpoints for DERs in DNs are determined by the TSO in a centralised manner, considering droop-based control and voltage measurements at the interconnection points (see, e.g., [4, 5]). For instance, studies conducted by the UK TSO, National Grid (NG) ESO, found that to support frequency and voltage recovery in particular in power systems with renewable generation, additional dynamic voltage support will be re-

[1]This work was partially supported by the EC H2020 'Border management of variable renewable energies and storage units enabling a translational Wholesale market (CROSSBOW)' project.

quired to replace that which is currently provided by synchronous generation at the interconnection points. NG ESO is thus running the Power Potential project [5], which is the first world trial to test the delivery of dynamic voltage support from different types of DERs embedded at various voltage levels, including storage assets, whose at least 90% of response is to be provided in 2s in order to be effective.

To deliver advanced voltage support DERs must be able to: *i*) dynamically adapt to changes resulting from system voltage changes; *ii)* respond to the TSO instructions/requested settings within 2s (see, e.g., [5]); *iii)* satisfy the first two points while maintaining communication efficiency. These requirements bring new control challenges to the coordination of a potentially large number of various distributed assets in DNs and the capability to dynamically respond to the time-varying network conditions [6]. BESS are suitable for delivering such improved voltage support and are one of the most promising flexibility providers in future power grids [7, 8]. Therefore, the focus of this study is on BESS and on devising a scalable control scheme which can optimally and dynamically coordinate an arbitrary number of BESS located at DN. It coordinates BESS to track the time-varying voltage profiles specified by the TSO at one or more TN-DN interconnection points while satisfying the DN constraints. This would allow maximising the support that BESS located at the DN can provide to both TN and DN and facilitate the TSO task of operating a more complex and flexible power network.

Various techniques have been adopted to address the voltage control problem caused by an increased level of renewable generation, especially photovoltaic (PV) power generation, in DNs [9, 10, 11, 12]. The majority of these have some notable deficiencies, such as: *i)* using the OLTC transformer increases the stress on transformers due to the continuous change of the tap [9]; *ii)* the PV curtailment could reduce the efficiency and revenues of PV generation. In addition, due to the high R/X (resistance/reactance) ratio in DN, the use of reactive power compensation from PV generation would not be effective enough [10]. The high R/X ratio however makes DNs more sensitive to the active power and therefore, fast-response storage devices are technically capable to support network voltage by regulating active power and compensating the reactive power via an inverter [13]. The available control approaches for voltage regulation in DNs can be classified into three main categories: centralised [14], decentralised [15] and distributed [10, 16, 17, 18]. The reader is referred to [19] for a detailed review.

Centralised approaches are computationally prohibitive when the number of devices to coordinate is large, and highly rely on expensive communication networks. The decentralised approaches, which rely only on locally available information, suffer from instability and sub-optimality issues [20]. In contrary, distributed approaches combine the advantages of the two aforementioned ones as they are significantly more scalable than centralised approaches and result in a far more optimal solution than decentralised approaches. They can efficiently coordinate distributed devices utilising information exchange among neighbouring devices through a much simpler communication network than in centralised approaches [17]. Nonetheless, the available distributed approaches, e.g., [8, 16, 17, 18, 21], do not address the aforementioned control challenges and are not suitable for coordinating a large number of BESS to provide voltage support to the TN in real time. Authors in [17] propose a distributed approach that coordinates DERs to regulate voltage profiles, whilst in [21] a distributed approach to the accelerated voltage regulation, which requires the same $R/X$ ratio, is developed. Authors in [8] explore distributed voltage control using active and reactive power from distributed generators, which however has a limited capability of being implemented online.

Note that the TN-DN interaction and TSO requirements are not considered in the aforementioned studies. Although there are some existing studies [8, 16, 17, 21] dealing with voltage control from DERs, just a few of them have both online and distributed implementation capabilities and none of them consider dynamic voltage services, the TN-DN interaction and have such as real-time capabilities, which makes the proposed solution truly scalable and flexible, and able to optimally coordinate an arbitrary number of BESS located at distribution networks without the need of any central entity and preserving privacy. Although a very few studies attempted to design distributed methods, their focus is on regulating voltages only at distribution levels, ignoring the requirement of dynamic services and of the transmission system. Furthermore, the proposed control algorithm is non-iterative, thus providing real-time capabilities, differently from the algorithms described in the aforementioned studies, which are iterative, thus without any guarantee to converge to an applicable solution within the required time. The novelty of our contribution is twofold: i) novel control algorithm design, since it combines online convex optimisation (OCO) and distributed algorithms in a dynamic and uncertain environment; ii) novel application area, since it integrates the dynamic voltage support requirements and the instruction from system operators into the control design. Furthermore, the proposed solution gives the aggregator or the system operator the possibility to tune the performance so as to give priority to the voltage tracking performance or to the BESS operational costs and lifetime, according to the specific needs and preferences.

*Main contribution*

In order to address the challenges described above, in this paper a fully distributed and non-iterative control approach is proposed, which is able to optimally coordinate an arbitrary number of BESS in a DN to regulate the voltage at specific TN-DN interconnection points instructed by the TSO, while accounting for both the time-varying grid conditions and the local network constraints, as well as saving communication and computational costs.

We aim to give a first original contribution to solve the open challenge of devising an effective control solution to the need of provision of dynamic voltage support from DERs, and distributed storage in particular, which also consider the TN-DN interaction. The TSO requirements are expressed as time-varying voltage setpoints at specific TN-DN interconnection points, which need to be tracked. A network-wide optimisation problem for BESS located at DN level is firstly formulated. It aims to track time-varying voltage setpoints specified by the TSO. The optimisation problem is then reformulated in a dynamic setting using an OCO framework [22] and solved in a distributed fashion, allowing for a real-time and scalable implementation. Please note that, although we focus on the time scale of interest to dynamic voltage services, the proposed framework can be applied at any time scale and sampling period of interest. The control setpoints to the BESS are calculated based on the past and current measurements (e.g., voltage, active and reactive power) and adapted to the varying network conditions, while contributing to tracking the TSO voltage setpoints in real time with an efficient communication resource usage. Comparing with the iterative algorithms, the proposed approach solves the problem sequentially following the OCO framework that regards the optimization as a process. Only one iteration is performed for each time, and the associated updates are applied directly to track voltage references, resulting in a faster response time. The effectiveness of the proposed scheme is illustrated on case studies using the IEEE 123-bus test feeder. Although the control design is based on a linearised power flow model, a fully AC power flow model is used to assess the proposed framework in the simulation environment.

## 2. TN-DN Voltage Control Problem

Traditionally TNs and DNs are operated independently. However, DNs with the increasing penetration of DER technologies are becoming more active and providing more flexibility and control options to operate the network and support the TN operation. For instance, a proper coordinated control of DER in DNs could provide flexible and local voltage support to the bulk system more efficiently and at a lower cost compared with installing VAR components in the TN [6]. BESS are one of the most promising devices entitled to offer this support through a local control in DNs and an enhanced control in TN [5]. This section formulates a voltage control problem, by which BESS are controlled to keep DN voltage within acceptable ranges, while the enhanced control should provide a dynamic voltage support to the TNs and therefore guarantee that the voltage setpoints at relevant TN-DN interconnection points satisfactorily track the profiles provided by the TSO. Table 1 defines the parameters and variables used in this paper.

Table 1: Variables and parameters

| | |
|---|---|
| $\mathcal{N}$ | set of nodes |
| $\mathcal{N}^c$ | set of TN-DN interconnection points |
| $\mathcal{E}$ | set of line segments |
| $r_{ij}$ ($x_{ij}$) | resistance and reactance of each line $(i,j) \in \mathcal{E}$ |
| $M$ | incidence matrix of the graph modelling the power network |
| $v_i/v_c$ | voltage magnitude at the $i$th node/ $c$th interconnection point |
| $\mathbf{v}_{\min}$ ($\mathbf{v}_{\max}$) | the vector of minimum (maximum) voltage limits |
| $\mathbf{v}^{\text{set}}(t)$ | the time-varying voltage references at time $t$, $\forall n \in \mathcal{N}$ |
| $p_i$ ($q_i$) | active (reactive) power injection at the $i$th node |
| $p_j^b$ ($q_j^b$) | active (reactive) power provided by an inverter of a battery unit connected to the $j$th node |
| $p_j^g$ ($q_j^g$) | active (reactive) power injected by an inverter of a renewable generating unit at the $j$th node |
| $p_j^c$ ($q_j^c$) | active (reactive) power absorbed by a load at the $j$th node |
| $P_{ij}$ ($Q_{ij}$) | active (reactive) power flow from $i$th node to $j$th node. |
| $\eta^+$ ($\eta^-$) | charging (discharging) efficiency |
| $p_i^{b,\min}$ ($p_i^{b,\max}$) | minimum (maximum) active power limit of a battery at the $i$th node |
| $\text{SoC}_i^{b,\min}$ ($\text{SoC}_i^{b,\max}$) | State-of-Charge (SoC) limits |
| $s_j^b$ | apparent power |
| $s_{\max}$ | ratio of the apparent power capacity to the real power capacity of the inverter |
| $\kappa$, $\epsilon$ | positive stepsizes |
| $f_t$ | objective function |
| $g_t$ | coupled inequality constraints |
| $\mathcal{T}$ | set of service time horizon |
| $\mathcal{Z}$ | local feasible set of the optimisation problem |
| $\Pi$ | projection operator |
| $\omega$, $\gamma$ | weight for voltage tracking and BESS operational costs |
| $\alpha$ | designed algorithm parameter |

### 2.1. Distribution network modelling

The set of DN nodes is denoted by $\mathcal{N}$, $\mathcal{N} := \{0,\dots,N\}$, which comprises $N+1$ nodes. The set of TN-DN interconnection points is $\mathcal{N}^c \subseteq \mathcal{N}$ with $|\mathcal{N}^c| = N_c$. The voltage setpoints provided by the TSO at time $t$ at the interconnection point $c$ is denoted by $v_c^{\text{set}}(t) \forall c \in \mathcal{N}^c \subseteq \mathcal{N}$. $\mathbf{v} := [v_1,\dots,v_N]^T$ collects all measured voltage magnitudes. The voltage magnitude $v_0$ of a reference bus is assumed to be constant. Let $\mathcal{E} := \{(i,j), \forall i,j \in \mathcal{N}\}$, be the set of lines. The linearised DistFlow model in [23] is widely adopted to represent the distribution flow model. Line losses are neglected and almost flat voltages are assumed. Its accuracy has been numerically corroborated by several recent studies on voltage regulation in DN [15, 17]. For every $(i,j) \in \mathcal{E}$, one has

$$P_{ij} = \sum_{k\mathcal{N}_i} P_{jk} - p_j \tag{1a}$$

$$Q_{ij} = \sum_{k\mathcal{N}_i} Q_{jk} - q_j \tag{1b}$$

$$v_i - v_j = r_{ij}P_{ij} + x_{ij}Q_{ij} \tag{1c}$$

Let $p_j = p_j^g + p_j^b - p_j^c$ and $q_j = q_j^g + q_j^b - q_j^c$ denote total injected active and reactive power at the $j$th node, respectively. Since the proposed framework focusses on BESS, the only control variables are the BESS-related ones; the variables related to renewable generation, synchronous generators, loads are measured or estimated. The network model described in this section can be written compactly as

$$\mathbf{v} = \bar{\mathbf{c}} + f^b(\mathbf{p}^b(t), \mathbf{q}^b(t)) + f^g(\mathbf{p}^g(t), \mathbf{q}^g(t)) - f^c(\mathbf{p}^c(t), \mathbf{q}^c(t))$$

where $\bar{\mathbf{c}}$ is a constant n-dimensional vector denoting the voltage profile under no VAR support [24]. $f^b(\mathbf{p}^b(t), \mathbf{q}^b(t)) := R\mathbf{p}^b(t) + X\mathbf{q}^b(t)$, $f^c(\mathbf{p}^c(t), \mathbf{q}^c(t)) := R\mathbf{p}^c(t) + X\mathbf{q}^c(t)$ and $f^g(\mathbf{p}^g(t), \mathbf{q}^g(t)) := R\mathbf{p}^g(t) + X\mathbf{q}^g(t)$ where $\mathbf{p}^b$ ($\mathbf{p}^c$,$\mathbf{p}^g$) and $\mathbf{q}^b$ ($\mathbf{q}^c$,$\mathbf{q}^g$) are the vector of $p_j^b$ ($p_j^c$,$p_j^g$) and $q_j^b$ ($q_j^c$,$q_j^g$), respectively; $R := M^{-T}D_rM^{-1}$ where $D_r \in \mathbb{R}^{N\times N}$ is a diagonal matrix with $r_{ij}$ being the diagonal entry; and similarly for $X := M^{-T}D_xM^{-1}$. The matrices $R$ and $X$ are defined so as to be linked to physical parameters, i.e., resistances and reactances of the lines (through the incidence matrix) and to facilitate a fully distributed design. Note that, since BESS are the controllable units, (2) is rewritten as

$$\mathbf{v} = \bar{\mathbf{v}}(t) + f^b(\mathbf{p}^b(t), \mathbf{q}^b(t)), \tag{2}$$

where $\bar{\mathbf{c}}$, $f^g(\mathbf{p}^g(t), \mathbf{q}^g(t))$ and $f^c(\mathbf{p}^c(t), \mathbf{q}^c(t))$ are included in $\bar{\mathbf{v}}(t)$ for the sake of convenience. The time-varying $\bar{\mathbf{v}}(t)$ will be tackled by an OCO framework in Section 3.

*2.2. Problem formulation*

As illustrated in [6], the current practice to control energy storage devices, including BESS, is to use a droop-based controller. However, this decentralised approach fails to regulate the voltage to the desired setpoint since it is basically a proportional controller. Furthermore, the local voltage at some nodes in the DN could be violated, as shown in Section 4.

Note that $\mathbf{v}^{\text{set}}(t)$ comprises two parts: 1) $\mathbf{v}_c^{\text{set}}(t) = [v_1^{\text{set}}(t), \ldots, v_c^{\text{set}}(t), \ldots, v_{N_c}^{\text{set}}(t)]$, $\forall c \in \mathcal{N}^c$ is the vector of references instructed by the TN for the interconnection points ; 2) $\mathbf{v}_h^{\text{set}}(t)$ contains the reference values at each local node, i.e., $\mathbf{v}_h^{\text{set}}(t) = [\mathbf{v}_1^{\text{set}}(t), \ldots, \mathbf{v}_h^{\text{set}}(t), \ldots, \mathbf{v}_{N_h}^{\text{set}}(t),]$, where $h \in \mathcal{N} \backslash \mathcal{N}^c$ and $N_h$ are the number of nodes that are not TN-DN interconnection points. In order to capture the BESS charging/discharging modes, let $p_i^b(t) = p_i^{b,+}(t) + p_i^{b,-}(t)$ be continuous variables representing the charging and discharging power of the storage unit $i$ at time $t$. The objective is to minimise the deviation between the local DN voltage and the corresponding reference at relevant nodes and penalize the BESS power injection as below,

$$\min_{\mathbf{p}^b(t),\mathbf{q}^b(t)} \sum_{t=1}^{T} \Bigg( \frac{\gamma}{2} C_1(\mathbf{p}^b(t), \mathbf{q}^b(t)) + \frac{\omega}{2} \bigg( C_2(\mathbf{p}^b(t), \mathbf{q}^b(t)) \bigg) \Bigg) \tag{3a}$$

$$\text{s.t} \quad \mathbf{v}_{\min} \leq \bar{\mathbf{v}}(t) + R\mathbf{p}^b(t) + X\mathbf{q}^b(t) \leq \mathbf{v}_{\max} \tag{3b}$$

$$0 \leq p_i^{b,+}(t) \leq p_i^{b,\max}, \ -p_i^{b,\min} \leq -p_i^{b,-}(t) \leq 0 \tag{3c}$$

$$0 \leq \frac{p_i^{b,-}(t)}{p_i^{b,\min}} + \frac{p_i^{b,+}(t)}{p_i^{b,\max}} \leq 1, \tag{3d}$$

$$\text{SoC}_i^{b,\min} \leq \text{SoC}_i^b(t) \leq \text{SoC}_i^{b,\max} \tag{3e}$$

$$\text{SoC}_i^b(t) = \text{SoC}_i^b(t-1) + \eta^+ p_i^{b,+}(t) - \eta^- p_i^{b,-}(t) \tag{3f}$$

$$\sqrt{(p_i^b(t))^2 + (q_i^b(t))^2} \leq s_i^b, \ \text{for} \ \ i = 1, \ldots, N_b. \tag{3g}$$

The first term of the control objective (3a), i.e.,

$$C_1(\mathbf{p}^b(t), \mathbf{q}^b(t)) := \left\| \bar{\mathbf{v}} + R\mathbf{p}^b + X\mathbf{q}^b - \mathbf{v}^{\text{set}}(t) \right\|_2^2$$

is adopted to track the voltage references and regulate the voltage magnitudes at each node, while the second term, i.e., $C_2(\mathbf{p}^b(t), \mathbf{q}^b(t) = \frac{1}{2}\mathbf{p}^b(t)^T R \mathbf{p}^b(t) + \frac{1}{2}\mathbf{q}^b(t)^T X \mathbf{q}^b(t)$ represents a cost function associated with each battery unit, which includes $R$ and $X$ so as to dispatch the battery units based on the network parameters. The two weights, i.e., $\gamma > 0$ and $\omega > 0$, are designed to balance the voltage regulation and BESS power provision cost. Note that $C_1(\mathbf{p}^b(t), \mathbf{q}^b(t))$ is so that $\mathbf{p}^b$ and $\mathbf{q}^b$ can be updated only based on the local voltage mismatch, as shown in Section 3, which, in the simulation study, is calculated by using a fully AC power flow model. Equation (3b) ensures that the voltage magnitude at all nodes in the DN is within a pre-defined range. In the first term of (3a) the tracking objective is to be achieved only at the identified interconnection points $c \in \mathcal{N}^c$, while the voltage at the remaining nodes of the DN are not provided (they are set to zero) so as to minimise the voltage fluctuations. Therefore, at those remaining DN nodes the voltage is regulated so as to be within an acceptable range, as imposed by (3b). The BESS dynamics are described by (3f). Equation (3d) guarantees a feasible and realistic charging/discharging behaviour of each battery unit (3d) [25]. Equation (3g) represents the maximum apparent power constraint of the inverter of a battery unit, which is further linearised as

$$-s_j^b \leq \sin(\tau \frac{\pi}{\kappa}) q_j^b(t) + \cos(\tau \frac{\pi}{\kappa}) p_j^b(t) \leq s_j^b, \tag{4}$$

where $\tau = 1, \ldots, \kappa$. The accuracy loss is determined by the setting of $\kappa$; the loss is reduced to 1.5% if $\kappa = 8$ [14].

The problem defined in this section is formulated over a prediction horizon $T$. The potentially high complexity of the solution for a large number of BESS, in terms of computational and communication costs, and the unpredictability of the network conditions motivate the use of a dynamic online optimisation framework, such as OCO, and of a distributed approach. By doing so, the BESS can be controlled in real-time and so as to adapt to the time-varying network conditions.

## 3. Distributed TN-DN voltage control

The challenges described in the previous sections require a scalable and flexible control, which provides the optimal solution at a low communication and computational cost, thus accounting for the existing local communication and computational infrastructures limited resources. This section introduces a novel control approach to solve the problem defined in (2.2), which also meets the above requirements of a scalable and flexible control.

### 3.1. Online convex optimisation framework

In the OCO framework, a player plays a repeated game in a time horizon $T$ [22]. Denoting $t \in \mathcal{T}$ as the per time slot and $\mathcal{T}$ being the time slot set, a player selects an action $x_t$ from a convex set $\mathcal{X} \in \mathbb{R}^n$ and suffers a loss $f_t(x_t)$, where $f_t(\cdot)$: $\mathbb{R}^n \rightarrow \mathbb{R}$ is the loss function. To cope with dynamic operations, the traditional OCO setting, which considers a static environment, is adapted to an unknown and time-varying environment. In this modified optimisation problem, the player is a battery unit with the time-varying objective function of tracking setpoints/regulating voltage. The decision to be made is how to control the BESS outputs so as to minimise the voltage mismatch and regulate voltage in the whole DN. To make the OCO scheme compatible with the considered dynamic environment, an improved performance index, a *dynamic regret*, which compares the performance of the OCO scheme to the sequence of optimal solutions, is applied [26], i.e.,

$$R_T^d := \sum_{t\in\mathcal{T}} f_t(x_t) - \sum_{t\in\mathcal{T}} f_t(x_t^*), \tag{5}$$

where $x_t^*$ is the sequence of best dynamic decisions given as $x_t^* \in \arg\max_{x\in\mathcal{X}} f_t(x)$, s.t. $g_t(x) \leq 0$, $h_t(x) = 0$, $\forall t \in [0, T]$,. Thus the goal of the OCO algorithm is to generate decisions with a sub-linear regret as a function of $T$ [26], where $R_T^d = \mathcal{O}(\sqrt{T})$ and consequently, $\lim_{T\rightarrow\infty} \frac{R_T^d}{T} = 0$. This implies that the online algorithm asymptotically converges to the sequence of best dynamic schedules.

### 3.2. Distributed algorithm design

In order to solve the OCO algorithm in a distributed fashion, define $\mathbf{z}_t = [\mathbf{z}_{1,t}, \ldots, \mathbf{z}_{N,t}]^T$, with $\mathbf{z}_{i,t}$ collecting all the optimisation variables $\{p_i^b(t), q_i^b(t)\}$; $f_t(\mathbf{z}_t)$ represents the objective (3a); $g_t(\mathbf{z}_t) \leq 0$ includes the coupling inequality constraints (3b); $\mathcal{Z}_i$ is the set defined by the local constraints (3c) - (3g) for battery unit $i$, and $\mathcal{Z}$ collects all $\mathcal{Z}_i$. Define the modified Lagrangian of the problem defined in Section 2.2 per time-slot as

$$L_t(\mathbf{z}_t, \boldsymbol{\lambda}_t) = f_t(\mathbf{z}_t) + \boldsymbol{\lambda}_t^T g_t(\mathbf{z}_t) - \frac{\kappa\alpha}{2}\|\boldsymbol{\lambda}_t\|_2^2, \tag{6}$$

where $\boldsymbol{\lambda}_t = [\boldsymbol{\lambda}_{1,t}, \ldots, \boldsymbol{\lambda}_{N,t}]^T$ are the Lagrangian multipliers of $g_t(\mathbf{z}_t), \forall i \in \mathcal{N}$; $\kappa$ is a positive number to be decided later; $\alpha > 0$ is the stepsize of the designed algorithm. The difference between (6) and the traditional Lagrangian is the additional term $\frac{\kappa\alpha}{2}\|\boldsymbol{\lambda}_t\|_2^2$. Note that this term is added not only to prevent $\boldsymbol{\lambda}_t$ from being too large at the initial time but also to facilitate the design of the sub-linear regret function, which is required for ensuring the convergence property of the proposed OCO algorithm (see Theorem 3.1). Based on (6), an online saddle-point algorithm is introduced. The primal variable $\mathbf{z}_t$ is updated at $t+1$ by

$$\mathbf{z}_{t+1} \in \arg\min_{\mathbf{z}\in\mathcal{Z}} \nabla_{\mathbf{z}} L_t(\mathbf{z}_t, \boldsymbol{\lambda}_t)(\mathbf{z} - \mathbf{z}_t) + \frac{1}{2\alpha}\|\mathbf{z} - \mathbf{z}_t\|_2^2, \tag{7}$$

where $\nabla_{\mathbf{z}} L_t(\mathbf{z}_t, \boldsymbol{\lambda}_t)$ is the gradient of $L_t(\mathbf{z}_t, \boldsymbol{\lambda}_t)$ with respect to $\mathbf{z}$ at $\mathbf{z} = \mathbf{z}_t$. The dual variable $\boldsymbol{\lambda}_t$ is updated at $t+1$ by

$$\boldsymbol{\lambda}_{t+1} = \left[\boldsymbol{\lambda}_t + \alpha\nabla_{\boldsymbol{\lambda}} L_t(\mathbf{z}_t, \boldsymbol{\lambda}_t)\right]^+, \tag{8}$$

where $\nabla_{\boldsymbol{\lambda}} L_t(\mathbf{z}_t, \boldsymbol{\lambda}_t)$ is the gradient of $L_t(\mathbf{z}_t, \boldsymbol{\lambda}_t)$ with respect to $\boldsymbol{\lambda}$ at $\boldsymbol{\lambda} = \boldsymbol{\lambda}_t$; $[\cdot]^+ := \max(\cdot, 0)$, which guarantees the feasibility of the dual variable. Intuitively, each local controller can update the primal and dual variables at the next time step by applying (7) - (8) based on the current information and thus the injected/absorbed active (reactive) power at time $t$, which can be performed in real-time.

### 3.3. Distributed algorithm implementation

The proposed algorithm can be implemented in a distributed manner, where each local controller of a battery unit only utilises neighbouring information to obtain its optimal power injections. As illustrated in Fig. 1, the control signals to each battery unit are their active and reactive power setpoints for its inverter, calculated by the proposed distributed algorithm at each time step. Please note that the proposed solution is based on the information at the last

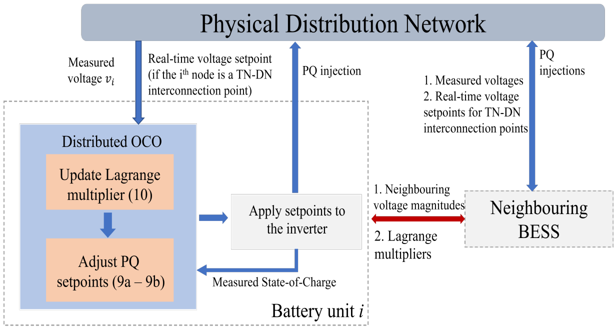



Figure 1: The proposed control framework

time step: the real-time data required by each battery unit i at each time step are the local voltage measurement, the real-time voltage setpoint, if i is an interconnection point, and the information from its neighbouring battery units. The SoC measurement of each battery unit is used to avoid battery over-charging/discharging. The one-step information requirement helps the solution providing results at a lower communication and computational cost.

The distributed design takes advantage from the sparsity of the inverse of the matrices $R$, $X$ and $X/R$, i.e., $H_r$, $H_x$, $H_{\frac{x}{r}}$, respectively, as in [27]. At each time $t$ the local controller of the battery unit $i$, $\forall i \in \mathcal{N}$, performs the following updates in order to calculate its power output at $t$

*3.3.1. Update primal variables: $v_{i,t}$, $p^b_{i,t}$, $q^b_{i,t}$:*

$$p^b_{i,t} = \Pi_{\mathcal{Z}_i}\{p^b_{i,t-1} - \alpha(\gamma(v_{i,t-1} - v^{\text{set}}_n) + \sum_{j=1}^{N} H_{r\,ij} R^p_j p^b_{j,t-1} - \lambda_{i,t}) \tag{9a}$$

$$q^b_{i,t} = \Pi_{\mathcal{Z}_i}\{q^b_{i,t-1} - \alpha(\gamma \sum_{j=1}^{N} H_{\frac{x}{r}\,ij}(v_{j,t-1} - v^{\text{set}}_n) + \sum_{j=1}^{N} H_{r\,ij} R^q_i q^b_{i,t-1} - \sum_{j=1}^{N} H_{\frac{x}{r}\,ij}\lambda_{j,t}), \tag{9b}$$

where (2) is formulated to obtain (9a) - (9a), i.e., $v_{i,t} = \bar{v} + \sum_{j=1}^{N} r_{ij} p^b_{j,t-1} + \sum_{j=1}^{N} x_{ij} q^b_{j,t-1}$; $v_{i,t}$ can be seen as an estimation of the local voltage based on (1). $\Pi_{\mathcal{Z}}$ is a projection operator onto $\mathcal{Z}_i$ [26].

*3.3.2. Update dual variables $\lambda_i$:*

$$\lambda_{i,t} = \left[\lambda_{i,t-1} + \alpha(p^b_{i,t-1} + \sum_{j=1}^{N} H_{\frac{x}{r}\,ij}(q^b_{j,t-1}) - \kappa\alpha\lambda_{i,t}\right]^+. \tag{10}$$

The calculation of setpoints at each time step can be concluded as

- Obtain the local bus voltage magnitude $v_{i,t-1}$
- Update the active and reactive power according to (9a) - (9b) with projection operations to ensure them in the feasible ranges.
- Exchange the active and reactive power with neighboring
- Update Lagrangian multipliers according to (10).

The implementation of the algorithm depends on the physical power network, i.e. $r_{ij}$ and $x_{ij}$. In order to obtain (9) - (10) the *scaled gradient* method following [28] is applied.

**Remark 3.1.** *As to the nodes without a storage unit, several approaches can be adopted:* i) *assuming the injection from such node to be equal to 0, $p^b_{i,t} = q^b_{i,t} = 0$;* ii) *including additional bounds on this node, such as $p^b_{i,min} = p^b_{i,max} = 0$;* iii) *as in [29], $R$ and $X$ can be decomposed according to the block decomposition for $\boldsymbol{v}$, i.e. $\boldsymbol{v} = [\boldsymbol{v}_0, \boldsymbol{v}_c, \boldsymbol{v}_l]^T$; $\boldsymbol{v}_c$ are the voltage magnitudes of nodes with controllable BESS defined by $Controllabe\ Nodes$; $\boldsymbol{v}_l$ are the voltage magnitudes of nodes with only uncontrollable renewable generators and loads defined by $Non-controllable\ Nodes$. With this decomposition, $R$ and $X$ can be rewritten as $R = [R_{cc},\ R_{cl}; R_{lc},\ R_{ll}]$ and $X = [X_{cc},\ X_{cl}; X_{lc},\ X_{ll}]$. Accordingly, one has for a subset of the grid nodes, i.e., the controllable nodes, $\boldsymbol{v}_c = \bar{\boldsymbol{v}}_c + R_{cc}\boldsymbol{p}^b + X_{cc}\boldsymbol{q}^b + R_{cl}\boldsymbol{p}^l + X_{cl}\boldsymbol{q}^l$. Note that the proposed distributed algorithm is compatible with any of the approaches described above. In the case study we adopt the first approach mentioned above for convenience.*

*3.4. Convergence analysis*

The convergence analysis adopts the notations defined in Section 3. The goal of the section is to show that the proposed distributed OCO algorithm can achieve a sublinear regret, i.e., $\mathrm{Reg}_T^d \leq \mathcal{O}(T)$, which implies that, as $T \to \infty$, $\frac{\mathrm{Reg}_T^d}{T} \leq \mathcal{O}(1) \to 0$, i.e., the performance of the sequence of $\mathbf{z}_t$ is no worse than the best solution $\mathbf{z}_t^*$. Before introducing the main results, the following assumptions are given to facilitate the convergence analysis.

*3.4.1. Assumptions*

A.1 For any $\mathbf{z}_t \in \mathcal{Z}$, the functions $f_t(\mathbf{z}_t)$ and $g_t(\mathbf{z}_t)$ have uniformly bounded gradients, i.e., $\left\|\triangledown f_t(\mathbf{z}_t)\right\|_2 \leq G$ and $\left\|\triangledown g_t(\mathbf{z}_t)\right\|_2 \leq G$ for some positive constant $G$.

A.2 The radius of $\mathcal{Z}$ is bounded, i.e., $\|\mathbf{z} - \mathbf{y}\| \leq R_z$, $\forall \mathbf{z}, \mathbf{y} \in \mathcal{Z}$.

A.3 All constraints $g_t(\mathbf{z}_t)$ are uniformly bounded, i.e., $\left\|g_t(\mathbf{z}_t)\right\|_2 \leq D$ for some positive constant $D$.

Note that the above assumptions are widely used in existing OCO-based studies [26]. Most of the objective functions used in distributed voltage regulation problems, including the one defined in the Problem (2.2), can easily satisfy these assumptions, e.g., network loss minimisation [27], and voltage control [24, 13].

The following Lemma is therefore given based on the modified Lagrangian (6).

**Lemma 3.1.** *Supposing Assumptions A.1 - A.3 are satisfied, for any $\boldsymbol{\lambda} \geq 0$ it holds that*

$$\sum_{t=1}^{T}(L_t(\boldsymbol{z}_t, \boldsymbol{\lambda}) - L_t(\boldsymbol{z}_t^*, \boldsymbol{\lambda}_t)) \leq$$
$$\frac{\alpha}{2}(1+N)G^2\left[T + \sum_{t=1}^{T}\|\boldsymbol{\lambda}_t\|_2^2\right] + \alpha T D^2$$
$$+ \kappa^2\alpha^3\sum_{t=1}^{T}\|\boldsymbol{\lambda}_t\|_2^2 + \frac{1}{2\alpha}(R^2 + 2R\Delta_z(T) + \|\boldsymbol{\lambda}\|_2^2), \tag{11}$$

*where $\Delta_z(T) := \sum_{t=2}^{T}\left\|\boldsymbol{z}_{t-1}^* - \boldsymbol{z}_t^*\right\|_2$ that is the drift of the best static solution $\{\boldsymbol{z}_t^*\}_{t=1}^T$.*

Using Lemma 3.1, the following theorem guarantees the sublinear regret.

**Theorem 3.1.** *Under Assumptions A.1 - A.3, if $\kappa = (N+1)G^2 + 1$, then the dynamic regret $Reg_T^d$ is bounded by*

$$Reg_T^d \leq \frac{1}{2\alpha}(R^2 + 2R\Delta_z(T)) + \frac{\alpha T}{2}[(2D^2 + (1+N)G^2)]. \tag{12}$$

*With $\alpha = \sqrt{\frac{\Delta_z(T)}{T}}$ the proposed algorithm yields a sublinear regret, i.e., $Reg_T^d \leq \mathcal{O}(\sqrt{T\Delta_z(T)})$.*

**Remark 3.2.** *The flexibility and scalability of distributed algorithms enable the proposed solution to different applications. In particular, it is not limited to the BESS but it can be applied to different DERs or other conventional systems that need a solution to be scalable and flexible (e.g., plug-and-play), by simply adding these components and the associated constraints as long as they maintain the convex property.*

## 4. Case study

In this section a simulation study is conducted to demonstrate the effectiveness of the proposed distributed voltage control approach. This simulation study was performed on a 2.4 GHz Intel Core I5 PC and the problem was formulated, coded and solved in the Matlab/Matpower environment. The IEEE 123-bus test feeder (4.16 kV level) [30] is considered. This section performs several studies to demonstrate the effectiveness of the proposed control approach through two distribution systems, a 33-bus test feeder (12.66 kV level) [31] and a 123-bus test feeder (4.16 kV level) [32], under static and dynamic operating conditions.

The initial voltage reference $v_n^{\mathrm{set}}$ is chosen to be 1 p.u. at each node and the minimum/maximum voltage deviations are set as to be $\pm 5\%$. Then the voltage references vary according to the current network conditions. A subset of nodes are randomly selected to install PV panels and BESS. The maximum apparent power capacity of the inverter is set to be a constant value, i.e., $s_j = s_{\max} p_j^{max}$, where $s_{\max}$ is 1.1. The SoC limits of BESS are 0.2 and 0.8, respectively. The algorithm parameters $\alpha$ and $\kappa$ are chosen based on Theorem 3.1. The sampled time interval is 0.1s and 1s for the static case and the dynamic case, respectively. Although the designed solution is based on the linearised power flow model of Section 2.2, the actual voltage magnitudes of the test system are obtained by solving the AC power flow in Matpower [33] and used to update the voltage values in (9a) and (9b). The $l_\infty$-norm of $\boldsymbol{v} - \boldsymbol{v}_n^{\text{set}}$ is introduced to to quantify the tracking performance.

Before implementing the proposed solution, we first illustrate the evaluation of regrets as in Fig. 2. The results shows our solution can effectively address the optimization problem in a online manner.

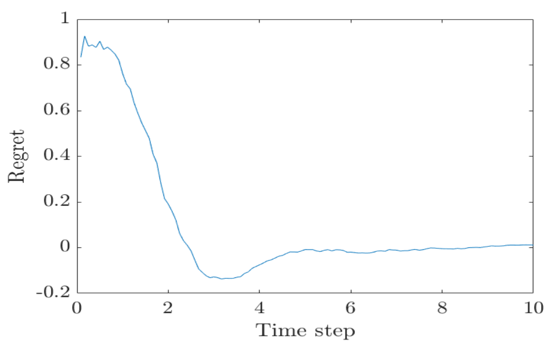



Figure 2: The evaluation of the regret function

## 4.1. Static operating condition

In this case, 33 ESS and 3 PV panels are installed in the modified 33-bus test feeder. The DN is supposed to be connected to a TN through Node 10. A case of multiple DN-TN interfaces and a subset of nodes installed ESS is proposed in Case 4.2. The distribution system operating condition is assumed to be static but TN can update the voltage set-point of the interface during the simulation process, where to clearly show the effectiveness these setpoints are supposed to be different for each ESS. The algorithm parameters $\alpha$ and $\kappa$ are chosen based on Theorem 3.1 and $\gamma$ is set to be 0.5 in this case. Fig. 3 gives the update of voltage magnitudes using the proposed solution. Fig. 4 shows that the voltage magnitude update of Node 10 is regulated to its set-point, i.e., $v_n^{\text{set}} = 1$ p.u. at the beginning. While TN updates the set-point during the system operation, for example 1.02 p.u., the proposed solution can still track the voltage following the instruction (see, e.g. Fig. 4).

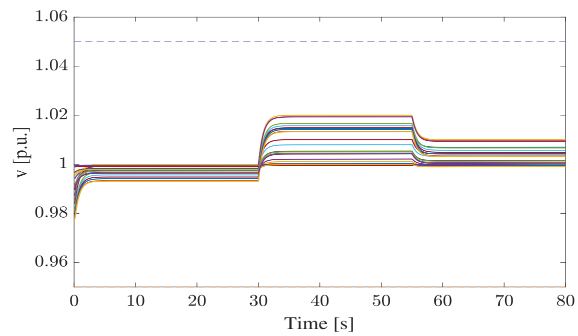



Figure 3: Voltage magnitude using the proposed solution

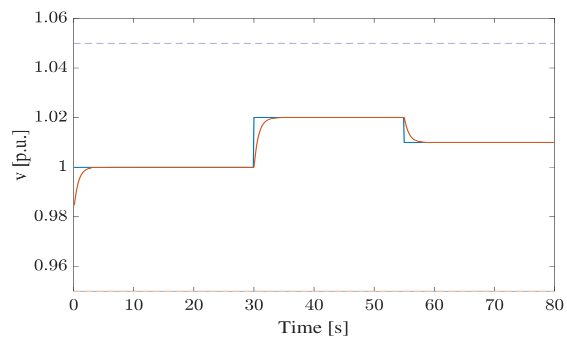



Figure 4: Voltage magnitude of Node 10

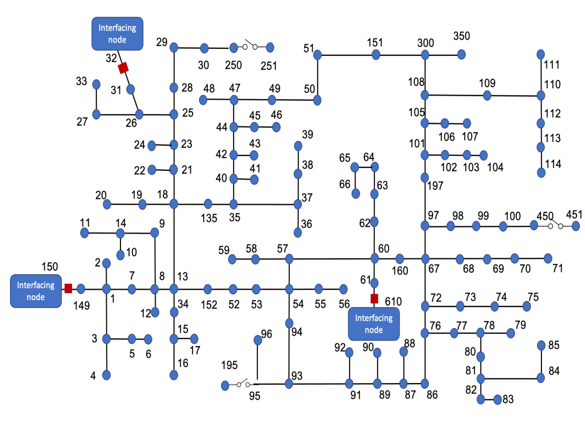



Figure 5: Modified IEEE 123-bus test feeder [30]

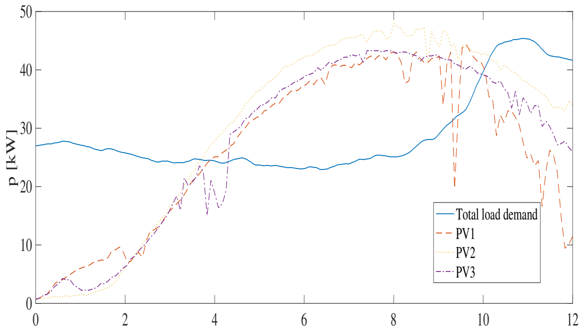



Figure 6: Load and PV profiles [34]

### 4.2. Dynamic operating condition

In this case study the performance of the proposed control framework are assessed in a dynamic setting, showing that the framework is suitable for an online implementation. A modified IEEE 123-bus test feeder is considered, where 100 residential BESS (1.2 kW rating each) and 3 commercial PV generators (60 kW rating each) are installed. Three TN-DN interconnection points are selected for implementing the TSO requirements and shown in Figure 5, i.e., Node 31, Node 60 and Node 150 [30]. The time-varying active power profiles of loads and PV plants are generated based on [34]; the unit is scaled up from $W$ to $kW$ to be consistent with the case study, as shown in Fig. 6. One hundred BESS are randomly located in the network, and the corresponding resistance and reactance matrices, denoted as $R_{cc}$ and $X_{cc}$ respectively, are obtained following the procedure in [35].

A comparison study is carried out to further demonstrate the improved performance of the proposed approach, where three different control approaches are tested: $i$) no voltage control; $ii$) a fully decentralised control [36], which can be seen as an advanced and improved version of the control techniques being tested by TSOs [4, 5]; $iii$) the proposed control approach. The resulting voltage profiles at each node of the network, including the selected interconnection points, are depicted in Figs. 7-9 based on the three different control approaches, where the dash lines are lower and upper voltage limits and the solid lines are voltage profiles at all network buses. It can be seen that both the benchmark case of no voltage control and the decentralised approach are not able to regulate the voltages at all nodes of the DN within acceptable ranges, especially during periods of high PV power generation, as shown in Fig. 7 and Fig. 8. The proposed control framework, on the other hand, can effectively regulate all the voltages within the acceptable range, see 9.

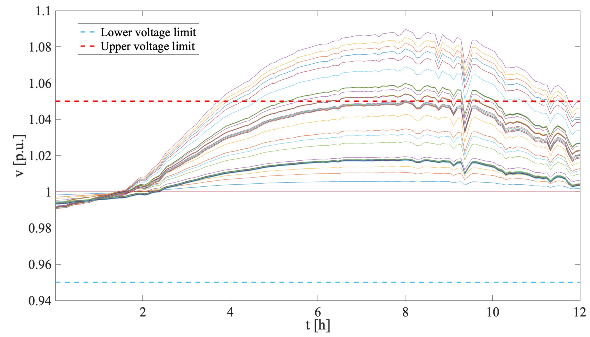



Figure 7: No voltage control

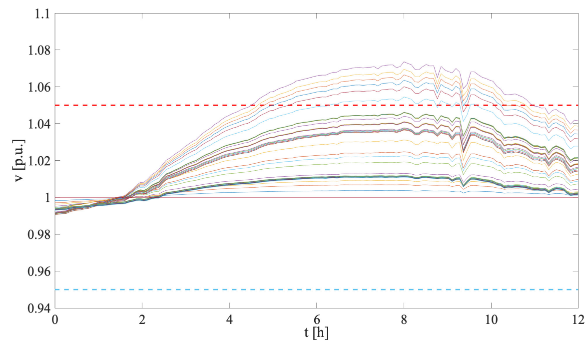



Figure 8: Decentralised solution [36]

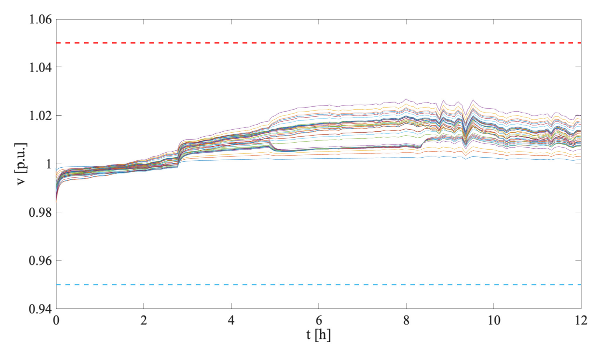



Figure 9: Proposed method (9) with $\gamma = 20$

In addition to this standard control objective, commonly considered in practice and in the literature,

Table 2: Comparative results

| | No control | Decentralised control [36] | Proposed method ($\gamma = 5$) | Proposed method ($\gamma = 20$) |
|---|---|---|---|---|
| $\left\| \boldsymbol{v} - \boldsymbol{v}_n^{\text{set}} \right\|_\infty$ | 0.0897 | 0.0751 | 0.0289 | 0.0097 |

the proposed approach is able to satisfactorily track the time-varying voltage references instructed by the TSO at the selected TN-DN interconnection points, as illustrated in Fig. 10. Fig. 10 shows the tracking performance at Node 60; the tracking performance at the other selected TN-DN connection nodes are equally satisfactory, yielding a maximum tracking error of $0.6e^{-}3$ at Node 31 and $0.8e^{-}3$ at Node 150, with a similar profile to track as in Fig. 10. Table 2 compares the performance resulting from the control approaches mentioned above. It can be seen that the proposed control framework significantly outperforms the other two approaches in terms of voltage regulation. Note that the proposed solution is distributed and each battery unit only needs to interact with its neighbours. By doing so the proposed control framework is scalable and the computational time to calculate BESS setpoints is very short (just 8 milliseconds in the discussed case study).

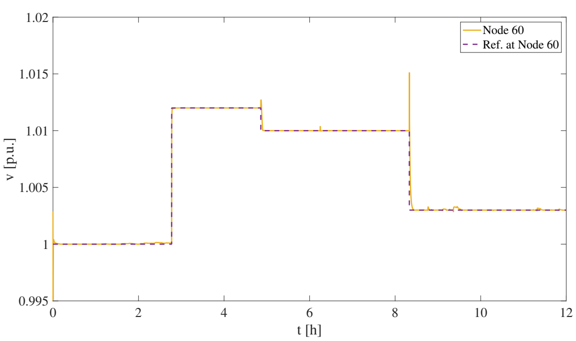
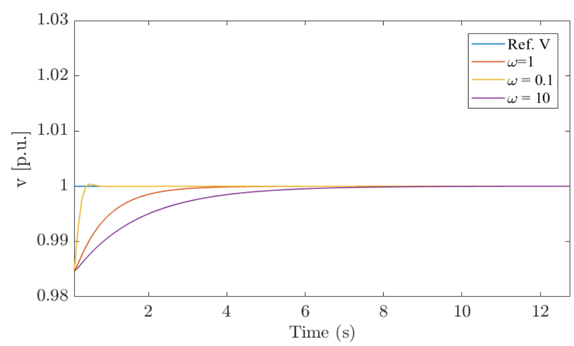
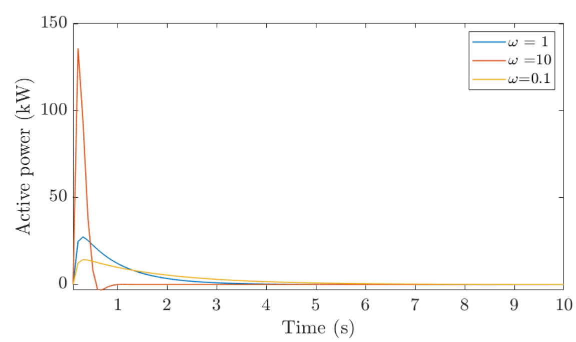



Figure 10: Voltage tracking performance at the selected TN-DN connection point Node 60

Furthermore, the performance of the proposed framework for voltage control can be tuned through the control parameter $\gamma$. The control objectives of the proposed framework are the voltage profile tracking of all nodes in $N^c$ and the minimisation of voltage fluctuation at the other nodes, as the voltage control tracking might affect the voltage profiles at other nodes. In the case studies presented here we considered 100 BESS distributed around the system, and the nodes with BESS have higher weights than the other nodes. As a result, increasing $\gamma$ can improve the voltage profile of all nodes in the network. Besides, larger values of $\gamma$ produce stronger (and more expensive) control actions but smaller voltage mismatches, in line with the formulation of the objective function in Section 2.2. It can be seen from Table 2 that a larger value of $\gamma$ (20) results in a smaller voltage deviation and hence tracking error with respect to the case with a smaller value of $\gamma$ (5).



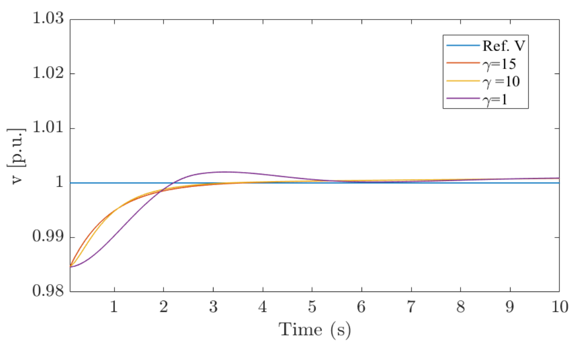

(a) The update of voltage profiles



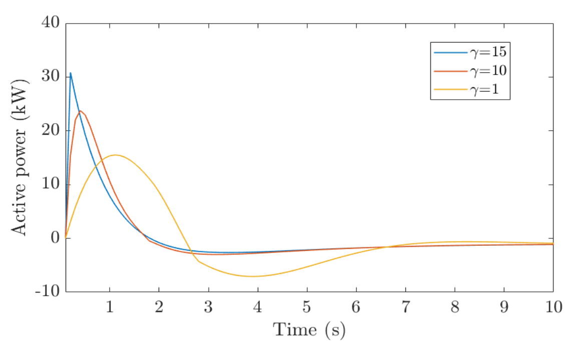

(b) The update of active power outputs

Figure 11: The comparative results when $\gamma$ is fixed

As shown in Table 2, a larger $\gamma$ gives priority to the tracking objective over the BESS operational costs, therefore BESS will be controlled to deliver a large amount of power in short time for tracking voltage setpoints. This may bring additional BESS operational costs and affect the battery life, as shown in Fig. 11 and affect the battery life. In this study, $\gamma$ is initialized as 1 and increased gradually. It is selected to achieve an acceptable tracking performance and still reduce the impact on battery life. The findings in Table II are further verified by Figs. 11 - 12, where the comparative results of different selection of $\omega$ and $\gamma$ are depicted respectively. In Fig. 11, $\gamma$ is fixed as 1 and $\omega$ is varied from 0.1 to 10, while in Fig. 12, $\omega$ is fixed as 1 and $\gamma$ is varied from 1 to 15. We can note that a larger $\omega$ results in delivering less BESS power (therefore smaller operational costs) but in a poorer tracking performance, whilst $\gamma$ results in an improved tracking performance but in larger power outputs, therefore higher operational costs. These parameters provide more choices to the system operators and they can be appropriately set according to the specific needs of the power system.




(a) The update of voltage profiles




(b) The update of active power outputs

Figure 12: The comparative results when $\omega$ is fixed

## 5. Conclusion

A distributed control scheme is developed for coordinating distributed BESS in DNs to provide real-time voltage regulation and satisfy the required voltage profiles specified by TSOs. An optimisation problem is formulated to schedule the operation of the BESS inverters for an efficient and real-time delivery of voltage support. The optimisation problem is solved in a distributed fashion using an OCO framework to reduce the computational and communication costs while adapting to the time-varying network conditions. The performance of the proposed approach is verified through case studies showing that it outperforms the current practice and the existing decentralised approaches. Further studies will include the analysis of the communication network design and the development of robust approaches to further compensate the uncertainty coming from renewable sources and load conditions.

## Appendix A. Proof of Lemma 3.1

Upper bounds are firstly introduced. Being $\mathbf{z}_{t+1}$ the optimal solution of (3), one has

$$(\mathbf{z}-\mathbf{z}_{t+1})^T(\alpha\nabla_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)+(\mathbf{z}-\mathbf{z}_{t+1}))\geq 0,\ \forall\mathbf{z}\in\mathcal{Z}, \tag{A.1}$$

which leads to the following upper bound,

$$\alpha(\mathbf{z}_t-\mathbf{z})^T\nabla_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\leq\frac{1}{2}\|\mathbf{z}-\mathbf{z}_t\|_2^2-\frac{1}{2}\|\mathbf{z}_{t+1}-\mathbf{z}_t\|_2^2 -\frac{1}{2}\|\mathbf{z}-\mathbf{z}_{t+1}\|_2^2+\alpha(\mathbf{z}_t-\mathbf{z}_{t+1})^T\nabla_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t), \tag{A.2}$$

where the inequality follows (A.1) and the fact that $(\mathbf{z}-\mathbf{z}_{t+1})^T(\mathbf{z}_{t+1}-\mathbf{z}_t)=\frac{1}{2}\|\mathbf{z}-\mathbf{z}_t\|_2^2-\frac{1}{2}\|\mathbf{z}_{t+1}-\mathbf{z}_t\|_2^2-\frac{1}{2}\|\mathbf{z}-\mathbf{z}_{t+1}\|_2^2$ is used. The last term in (A.2) can be bounded using Cauchy-Schwartz inequality and Young's inequality subsequently, i.e.,

$$\alpha(\mathbf{z}_t-\mathbf{z}_{t+1})^T\nabla_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\leq\frac{1}{2}\|\mathbf{z}_t-\mathbf{z}_{t+1}\|_2^2 +\frac{\alpha^2}{2}\|\nabla_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\|_2^2. \tag{A.3}$$

The last term in (A.3) is bounded by

$$\begin{aligned}\|\nabla_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\|_2^2&=\left\|\nabla f_t(\mathbf{z}_t)+\sum_{i=1}^N\boldsymbol{\lambda}_{i,t}\nabla g_{i,t}(\mathbf{z}_{i,t})\right\|_2^2\\ &\leq(1+N)\left[\|\nabla f_t(\mathbf{z}_t)\|_2^2+\sum_{i=1}^N\boldsymbol{\lambda}_{i,t}\|\nabla g_{i,t}(\mathbf{z}_{i,t})\|_2^2\right]\\ &\leq(1+N)G^2(1+\|\boldsymbol{\lambda}_t\|_2^2),\end{aligned} \tag{A.4}$$

where $g_{i,t}$ is the local decoupled constraint, and Assumptions A.1 is adopted to obtain the last inequalities. Likewise,

$$\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_{t+1}\|_2^2=\left\|\boldsymbol{\lambda}-[\boldsymbol{\lambda}_t+\alpha\triangledown_{\boldsymbol{\lambda}}L_t(\mathbf{z}_t,\boldsymbol{\lambda})]^+\right\|_2^2\le\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_t\|_2^2 \\ -2\alpha(\boldsymbol{\lambda}-\boldsymbol{\lambda}_{t+1})^T(\triangledown_{\boldsymbol{\lambda}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t))+\alpha^2\left\|\triangledown_{\boldsymbol{\lambda}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\right\|_2^2, \tag{A.5}$$

where the nonexpansive property of the projection operator is used. Similarly,

$$\left\|\triangledown_{\boldsymbol{\lambda}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\right\|_2^2=\left\|g_t(\mathbf{z}_t)-\kappa\alpha\boldsymbol{\lambda}_t\right\|_2^2\le 2D^2+2(\kappa\alpha)^2\|\boldsymbol{\lambda}_t\|_2^2, \tag{A.6}$$

where Assumptions A.3 is adopted to obtain the last inequalities.

Lemma 3.1 can be proved now. Noting that $\mathbf{z}_t^*\in\mathcal{Z}$, given $\boldsymbol{\lambda}_t$, due to the convex-concave property of $L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)$, one has

$$L_t(\mathbf{z}_t,\boldsymbol{\lambda})-L_t(\mathbf{z}_t^*,\boldsymbol{\lambda}_t) \\ \le\frac{1}{2\alpha}\Big(\left\|\mathbf{z}_t^*-\mathbf{z}_t\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2+\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_t\|_2^2 \\ -\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_{t+1}\|_2^2\Big)+\frac{\alpha}{2}\Big(\left\|\triangledown_{\boldsymbol{\lambda}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\right\|_2^2+\left\|\triangledown_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\right\|_2^2\Big), \tag{A.7}$$

where the inequality follows (A.2) and (A.5). Plugging (A.3) and (A.6) into (A.7) and summing up to (A.7) over $T$, one has

$$\sum_{t=1}^T\Big[L_t(\mathbf{z}_t,\boldsymbol{\lambda})-L_t(\mathbf{z}_t^*,\boldsymbol{\lambda}_t)\Big] \\ \le\frac{1}{2\alpha}\|\boldsymbol{\lambda}\|_2^2+\frac{1}{2\alpha}\sum_{t=1}^T\Big(\left\|\mathbf{z}_t^*-\mathbf{z}_t\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2\Big) \\ +\frac{\alpha}{2}\sum_{t=1}^T\Big(\left\|\triangledown_{\boldsymbol{\lambda}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\right\|_2^2+\left\|\triangledown_{\mathbf{z}}L_t(\mathbf{z}_t,\boldsymbol{\lambda}_t)\right\|_2^2\Big), \tag{A.8}$$

where the telescoping sum of $\sum_{t=1}^T(\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_t\|_2^2-\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_{t+1}\|_2^2)$ and the nonnegativity of $\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_{T+1}\|_2^2$ are used to obtain the inequality. In order to bound the second term in (A.8), note that $\|\mathbf{z}_t^*-\mathbf{z}_t\|_2^2-\left\|\mathbf{z}_t-\mathbf{z}_{t-1}^*\right\|_2^2=(\mathbf{z}_t^*-\mathbf{z}_{t-1}^*)^T(\mathbf{z}_t^*-2\mathbf{z}_t+\mathbf{z}_{t-1}^*)\le 2R\left\|\mathbf{z}_t^*-\mathbf{z}_{t-1}^*\right\|$, where $R$ is defined by using Assumption A.2, and hence

$$\left\|\mathbf{z}_t^*-\mathbf{z}_t\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2 \\ =\left\|\mathbf{z}_t^*-\mathbf{z}_t\right\|_2^2-\left\|\mathbf{z}_t-\mathbf{z}_{t-1}^*\right\|_2^2+\left\|\mathbf{z}_t-\mathbf{z}_{t-1}^*\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2 \\ \le 2R\left\|\mathbf{z}_t^*-\mathbf{z}_{t-1}^*\right\|+\left\|\mathbf{z}_t-\mathbf{z}_{t-1}^*\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2, \tag{A.9}$$

summing up over $T$ such as

$$\sum_{t=1}^T\Big(\left\|\mathbf{z}_t^*-\mathbf{z}_t\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2\Big)\le 2R\sum_{t=1}^T\left\|\mathbf{z}_t^*-\mathbf{z}_{t-1}^*\right\| \\ +\sum_{t=1}^T\Big(\left\|\mathbf{z}_t-\mathbf{z}_{t-1}^*\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2\Big)\le 2R\Delta_z(T)+R^2, \tag{A.10}$$

where the last inequality follows $\sum_{t=1}^T\Big(\left\|\mathbf{z}_t-\mathbf{z}_{t-1}^*\right\|_2^2-\left\|\mathbf{z}_t^*-\mathbf{z}_{t+1}\right\|_2^2\Big)=\|\mathbf{z}_0^*-\mathbf{z}_1\|-\|\mathbf{z}_T^*-\mathbf{z}_{T+1}\|$. Then plugging (A.2) - (A.6) and (A.8) into (A.7), and rearranging terms, the result in Lemma 3.1 is yielded.

## Appendix B. Proof of Theorem 3.1

Using the definition of the modified Lagrangian (6), for any $\boldsymbol{\lambda}\ge 0$, one has

$$\sum_{t=1}^T(L_t(\mathbf{z}_t,\boldsymbol{\lambda})-L_t(\mathbf{z}_t^*,\boldsymbol{\lambda}_t))=\sum_{t=1}^T(f_t(\mathbf{z}_t)-f_t(\mathbf{z}_t^*))+\sum_{t=1}^T\|\boldsymbol{\lambda}_t\| \\ +\sum_{i=1}^N\sum_{t=1}^T(\lambda_i g_{i,t}(\mathbf{z}_{i,t})-\lambda_{i,t}g_{i,t}(\mathbf{z}_{i,t}^*)-\frac{\kappa\alpha T}{2}\|\boldsymbol{\lambda}\|_2^2\ge\sum_{t=1}^T\|\boldsymbol{\lambda}_t\| \\ +\sum_{t=1}^T(f_t(\mathbf{z}_t)-f_t(\mathbf{z}_t^*))+\sum_{i=1}^N\sum_{t=1}^T\lambda_i g_{i,t}(\mathbf{z}_{i,t})-\frac{\kappa\alpha T}{2}\|\boldsymbol{\lambda}\|_2^2, \tag{B.1}$$

where the inequality follows $\mathbf{z}_t^*$ being a feasible solution and thus $g_{i,t}(\mathbf{z}_{i,t}^*)\le 0$ and $\lambda_{i,t}\ge 0$. Using the result of Lemma 3.1, it follows that

$$\sum_{t=1}^T(f_t(\mathbf{z}_t)-f_t(\mathbf{z}_t^*))+\sum_{i=1}^N\sum_{t=1}^T\lambda_i g_{i,t}(\mathbf{z}_{i,t})-\frac{\kappa\alpha T}{2}\|\boldsymbol{\lambda}\|_2^2 \\ \le\frac{1}{2\alpha}(R^2+2R\Delta_z(T)+\|\boldsymbol{\lambda}\|_2^2)+\frac{\alpha T}{2}[(2D^2+(1+N)G^2)], \tag{B.2}$$

where $\kappa=(1+N)G^2+1$ leads to the last inequality, i.e. $((1+N)G^2+2\kappa^2\alpha^2-\alpha)\le 0$. Rearranging terms in both sides of (B.2) one has

$$\sum_{t=1}^T(f_t(\mathbf{z}_t)-f_t(\mathbf{z}_t^*))+\sum_{i=1}^N\sum_{t=1}^T\lambda_i g_{i,t}(\mathbf{z}_{i,t}) \\ -(\frac{\kappa\alpha T}{2}+\frac{1}{2\alpha})\sum_{i=1}^N\lambda_i\le\frac{1}{2\alpha}(R^2+2R\Delta_z(T)) \\ +\frac{\alpha T}{2}[(2D^2+(1+N)G^2)]. \tag{B.3}$$

Note that maximising $\sum_{i=1}^{N}\sum_{t=1}^{T}\lambda_i g_{i,t}(\mathbf{z}_{i,t}) - (\frac{\kappa\alpha T}{2} + \frac{1}{2\alpha})\sum_{i=1}^{N}\lambda_i$ over $\lambda_i \geq 0$, i.e., $\lambda_i = \frac{[\sum_{t=1}^{T} g_{i,t}(\mathbf{z}_{i,t})]^+}{\kappa\alpha T + \frac{1}{\alpha}}$, $\forall i \in \mathcal{N}$, and substituting $\alpha = \sqrt{\frac{\Delta_z(T)}{T}}$ into (B.3) Theorem 3.1 is demonstrated.

## References


[1] N. Mahmud, A. Zahedi, Review of control strategies for voltage regulation of the smart distribution network with high penetration of renewable distributed generation, Renewable and Sustainable Energy Reviews 64 (2016) 582–595.

[2] O. Arnaud, M. Chapert, O. Carre, DSO TSO coordination needs induced by smart grids: the ongoing french project between RTE and enedis, CIRED-Open Access Proceedings Journal 2017 (1) (2017) 1048–1050.

[3] P. Li, Y. Wang, H. Ji, J. Zhao, G. Song, J. Wu, C. Wang, Operational flexibility of active distribution networks: Definition, quantified calculation and application, International Journal of Electrical Power & Energy Systems 119 (2020) 105872.

[4] Eirgrid, 2016 ds3 voltage control workstream plan (2016).
URL `http://www.eirgridgroup.com/site-files/library/EirGrid/DS3-Voltage-Control-Workstream-Plan-2016.pdf`

[5] NationalgridESO, Power potential DER technical requirements v2.5.3 (mar 2019).
URL `https://www.nationalgrideso.com/document/114901/download`

[6] I. M. Sanz, B. Stojkovska, A. Wilks, J. Horne, A. R. Ahmadi, T. Ustinova, Enhancing transmission and distribution system coordination and control in Great Britain using power services from distributed energy resources, in: 2018 IET Renewable Power Generation Conference, 2018.

[7] J. Krata, T. K. Saha, Real-time coordinated voltage support with battery energy storage in a distribution grid equipped with medium-scale PV generation, IEEE Transactions on Smart Grid 10 (3) (2018) 3486–3497.

[8] Y. Xie, L. Liu, Q. Wu, Q. Zhou, Robust model predictive control based voltage regulation method for a distribution system with renewable energy sources and energy storage systems, International Journal of Electrical Power & Energy Systems 118 (2020) 105749.

[9] Y. Wang, P. Zhang, W. Li, W. Xiao, A. Abdollahi, Online overvoltage prevention control of photovoltaic generators in microgrids, IEEE Transactions on Smart Grid 3 (4) (2012) 2071–2078.

[10] Y. Wang, K. Tan, X. Y. Peng, P. L. So, Coordinated control of distributed energy-storage systems for voltage regulation in distribution networks, IEEE transactions on Power Delivery 31 (3) (2015) 1132–1141.

[11] Z. Tang, D. J. Hill, T. Liu, Two-stage voltage control of subtransmission networks with high penetration of wind power, Control Engineering Practice 62 (2017) 1–10.

[12] M. Farina, A. Guagliardi, F. Mariani, C. Sandroni, R. Scattolini, Model predictive control of voltage profiles in mv networks with distributed generation, Control Engineering Practice 34 (2015) 18–29.

[13] M. Zeraati, M. E. H. Golshan, J. M. Guerrero, Distributed control of battery energy storage systems for voltage regulation in distribution networks with high PV penetration, IEEE Transactions on Smart Grid 9 (4) (2016) 3582–3593.

[14] R. A. Jabr, Linear decision rules for control of reactive power by distributed photovoltaic generators, IEEE Transactions on Power Systems 33 (2) (2017) 2165–2174.

[15] M. Farivar, L. Chen, S. Low, Equilibrium and dynamics of local voltage control in distribution systems, in: 52nd IEEE Conference on Decision and Control, IEEE, 2013, pp. 4329–4334.

[16] T. Xu, W. Wu, Accelerated admm-based fully distributed inverter-based volt/var control strategy for active distribution networks, IEEE Transactions on Industrial Informatics 16 (12) (2020) 7532–7543.

[17] H. J. Liu, W. Shi, H. Zhu, Distributed voltage control in distribution networks: Online and robust implementations, IEEE Transactions on Smart Grid 9 (6) (2017) 6106–6117.

[18] B. Fan, S. Guo, J. Peng, Q. Yang, W. Liu, L. Liu, A consensus-based algorithm for power sharing and voltage regulation in dc microgrids, IEEE Transactions on Industrial Informatics 16 (6) (2019) 3987–3996.

[19] K. E. Antoniadou-Plytaria, I. N. Kouveliotis-Lysikatos, P. S. Georgilakis, N. D. Hatziargyriou, Distributed and decentralized voltage control of smart distribution networks: Models, methods, and future research, IEEE Transactions on Smart Grid 8 (6) (2017) 2999–3008.

[20] S. Bolognani, R. Carli, G. Cavraro, S. Zampieri, On the need for communication for voltage regulation of power distribution grids, IEEE Transactions on Control of Network Systems 6 (3) (2019) 1111–1123.

[21] Z. Tang, D. J. Hill, T. Liu, Fast distributed reactive power control for voltage regulation in distribution networks, IEEE Transactions on Power Systems 34 (1) (2018) 802–805.

[22] S. Shalev-Shwartz, et al., Online learning and online convex optimization, Foundations and Trends® in Machine Learning 4 (2) (2012) 107–194.

[23] M. E. Baran, F. F. Wu, Optimal capacitor placement on radial distribution systems, IEEE Transactions on Power Delivery 4 (1) (1989) 725–734.

[24] H. Zhu, H. J. Liu, Fast local voltage control under limited reactive power: Optimality and stability analysis, IEEE Transactions on Power Systems 31 (5) (2015) 3794–3803.

[25] K. Worthmann, C. M. Kellett, P. Braun, L. Grüne, S. R. Weller, Distributed and decentralized control of residential energy systems incorporating battery storage, IEEE Transactions on Smart Grid 6 (4) (2015) 1914–1923. doi:10.1109/TSG.2015.2392081.

[26] T. Chen, Q. Ling, G. B. Giannakis, An online convex optimization approach to proactive network resource allocation, IEEE Transactions on Signal Processing 65 (24) (2017) 6350–6364.

[27] J. Li, Z. Xu, J. Zhao, C. Zhang, Distributed online voltage control in active distribution networks considering PV curtailment, IEEE Transactions on Industrial Informatics 15 (10) (2019) 5519–5530.

[28] G. Qu, N. Li, Optimal distributed feedback voltage control under limited reactive power, IEEE Transactions on Power Systems.

[29] S. Bolognani, S. Zampieri, A distributed control strategy for reactive power compensation in smart microgrids, IEEE Transactions on Automatic Control 58 (11) (2013) 2818–2833.

[30] P. Wirasanti, E. Ortjohann, Active distribution grid power flow analysis using asymmetrical hybrid technique, International Journal of Electrical and Computer Engineering 7 (4) (2017) 1738.

[31] W. H. Kersting, Radial distribution test feeders, in: 2001 IEEE Power Engineering Society Winter Meeting. Conference Proceedings (Cat. No.01CH37194), Vol. 2, 2001, pp. 908–912 vol.2. doi:10.1109/PESW.2001.916993.

[32] IEEE PES Distribution Test Feeders.
URL https://site.ieee.org/pes-testfeeders/

[33] R. D. Zimmerman, C. E. Murillo-Sanchez, R. J. Thomas, Matpower: Steady-state operations, planning, and analysis tools for power systems research and education, IEEE Transactions on Power Systems 26 (1) (2011) 12–19. doi:10.1109/TPWRS.2010.2051168.

[34] R. Pedersen, C. Sloth, G. B. Andresen, R. Wisniewski, DiSC: A simulation framework for distribution system voltage control, in: 2015 European Control Conference (ECC), IEEE, 2015, pp. 1056–1063.

[35] S. Bolognani, S. Zampieri, A distributed control strategy for reactive power compensation in smart microgrids, IEEE Transactions on Automatic Control 58 (11) (2013) 2818–2833. doi:10.1109/TAC.2013.2270317.

[36] P. N. Vovos, A. E. Kiprakis, A. R. Wallace, G. P. Harrison, Centralized and distributed voltage control: Impact on distributed generation penetration, IEEE Transactions on Power Systems 22 (1) (2007) 476–483.

## Response to Editor and Reviewers

We would like to thank the Reviewers and the Editor for their valuable comments and suggestions. All comments have been carefully addressed and the paper has been revised accordingly taking all suggestions into account. Once more, we appreciate the time and effort of Reviewers and Editor for improving the quality of the paper. Detailed responses to individual comments are provided below. References in the response letter refer to the revised manuscript and to additional references provided in this response letter.

### Editor

**Comments from reviewers were received, while it was agreed that the paper is interesting and has some publishable materials, some concerns were raised, primarily on the novelty, clarity and applicability which need to be strengthend significantly. Please address all the comments raised by the reviewers, and highlight the changes made in the revised paper. Please note that all accepted papers in the Control Engineering Practice need to demonstrate the practical applications, the paper needs to strengthen the application element in the revised version in particular.**

RE: We would like to thank the Reviewers and the Editor for their constructive comments. We carefully addressed all the comments in this revision to further clarify the contributions and the potential application to power systems. We refer the Editor in particular to our responses to the comments 1 and 2 of Reviewer 1 and our responses to the comments of Reviewer 4, which clarify the novelty and the technical aspects of our algorithm, which is distributed and not decentralized and does not make any strong assumptions and does not decouple the system. Our control algorithm has learning and real-time capabilities and provides an optimal solution still considering the strong coupling due to the network constraints and power balance. Furthermore, it addresses an open problem, which is being investigated by several Transmission System Operators worldwide and for which, to the best of our knowledge, there is not any effective control solution proposed yet. This aspect, along with the real-time capabilities of the proposed control algorithm and the fact that it is non-iterative, make the proposed solution highly likely applicable in practice. Furthermore, the proposed solution combines the benefits of online convex optimization (OCO) and distributed consensus-based algorithms, which have been verified experimentally and the experimental results have confirmed the theoretical findings. In this regards, we refer the Editor to our response to the last comment of Reviewer 5.

### Reviewer 1

**In this paper, a control framework is proposed to track voltage setpoints using distributed BESS. The topic is interesting. However, the innovation of this paper is unclear and many concerns need to be addressed for publication.**
**1. The control setpoints of the interconnection points and DERs are not adjusted frequently. The control command is issued in minute scale. Compared with the time scale of voltage control in DNs, the dynamic adjust of control setpoints can be ignored.**

RE: Thank you for raising this point. Indeed, these setpoints have not been traditionally adjusted frequently. However, with increased renewable generation capacity in the form of distributed energy resources (DER), which displaces conventional transmission connected plants, the potential value of DERs to provide network support, not only at a distribution but also at transmission level, will need to be explored [1]. There is a clear trend, worldwide, to reduce the time scales of grid operation in order to react more promptly to the time varying grid conditions, mainly due to the variability of renewable generation. TSOs are exploring more dynamic support services, including the voltage services [2] (in the revised manuscript). For instance, the aim of Power Potential project [5] (in the revised manuscript) run by the UK TSO, National Grid ESO (NG ESO), is to be the first world trial to test the delivery of dynamic voltage support from different types of DERs embedded at various voltage levels, including storage assets, whose at least 90% of response is to be provided in 2s in order to be effective. Studies conducted by NG ESO found that, in order to support frequency and voltage recovery, additional dynamic voltage support will be required to replace that which is currently provided by synchronous generation at the interconnection points.
(https://www.nationalgrideso.com/sites/eso/files/documents/SOF%20Report%20-%20Frequency%20and%20Voltage%20assessment.pdf).
Within the Power Potential project, it is being explored also a novel concept of reactive power market from distributed resources, which will create more flexibility enabling transmission and distribution networks to operate more efficiently [2]. With the concept of active distribution networks, authors in [3] investigated using DERs at distribution networks to improve voltage stability in transmission networks dynamically. The existing literature and launched projects by system operators show a need for dynamic and optimal operation of DERs at the distribution level. Thus, we aim at providing a technical solution for system operators to control a large number of storage assets for dynamical voltage services when needed, whose delivery time is just a few seconds. We propose a distributed approach for online voltage regulation and demonstrate its scalability and its dynamic tracking capability. The investigated services are provided by tracking the dynamic voltage setpoints of the interconnection points, which are calculated in real-time. Although there are some works that focus on distributed voltage control (e.g., [17], [20] and [27] in the revised manuscript), a few studies put efforts in coordinating multiple storage assets located at distribution networks for dynamic services, which are to be provided close to/in real-time, and the TSO-DSO interaction. Furthermore, the proposed solution gives the aggregator or the system operator the possibility to tune the performance so as to give priority to the voltage tracking performance or to the BESS operational costs and lifetime, according to the specific needs and preferences. We thus aim to provide a first original contribution to solve the open challenge of devising an effective control solution to the need of additional dynamic voltage support from DERs, and distributed storage in particular.
Please, note that the proposed framework can be applied at any time scale of interest; we focused on fast dynamic services, however the proposed solution works at any sampling period. The following context is added in this revision and shown below for Reviewer's convenience.

*Main Contribution, Section 1, Page 3, "Please note that, although we focus on the time scale of interest to dynamic voltage services, the proposed framework can be applied at any time scale and sampling period of interest."*

**2. In paragraph 5 of Introduction, the author summarized: "None of the existing frameworks are able to adapt the control response to dynamic changes in the grid conditions and to system voltage variations". The conclusion is not accurate. Many studies have proposed many methods to adapt the control response to system voltage variations resulting from DERs integration.**

RE: Thank you for pointing this out. Indeed, some of the existing studies put efforts in controlling DERs for system voltage regulation. However, we would like to emphasize that just a few of them have both online and distributed implementation capabilities and none of them consider dynamic voltage services, the TN-DN interaction and have such as real-time capabilities, which makes the proposed solution truly scalable and flexible, and able to optimally coordinate an arbitrary number of BESS located at distribution networks without the need of any central entity and preserving privacy. Please, note that the ability to coordinate the BESS *optimally* (and other performance criteria can be considered) is a benefit of the proposed solution, which cannot be achieved by decentralized approach, since they do not consider coupling constraints, thus the BESS location in the network and the network model. The focus is on BESS, but the proposed framework can be extended to DERs in general, included demand, which is the topic of our current work. Although a very few studies attempted to design distributed methods, their focus is on regulating voltages only at distribution levels, ignoring the requirement of dynamic services and of the transmission system (see [16], [17] and [21] in the manuscript). Furthermore, those algorithms are iterative (we refer the Reviewer to our response to the comment 3 in this regard). The novelty of our contribution is twofold: *i*) novel control algorithm design, since it combines online convex optimization and distributed algorithms in a dynamic and uncertain environment; *ii*) novel application area, since it integrates the dynamic voltage support requirements and the instruction from system operators into the control design.

We agree that our conclusive summary may be misleading and revise it in this revision. The revised text is shown below for Reviewer's convenience.

*Section I, Page 2, "Although there are some existing studies [8, 16, 17, 21] dealing with voltage control from DERs, just a few of them have both online and distributed implementation capabilities and none of them consider dynamic voltage services, the TN-DN interaction and have such as real-time capabilities, which makes the proposed solution truly scalable and flexible, and able to optimally coordinate an arbitrary number of BESS located at distribution networks without the need of any central entity and preserving privacy. Although a very few studies attempted to design distributed methods, their focus is on regulating voltages only at distribution levels, ignoring the requirement of dynamic services and of the transmission system. Furthermore, the proposed control algorithm is non-iterative, thus providing real-time capabilities, differently from the algorithms described in the aforementioned studies, which are iterative, thus without any guarantee to converge to an applicable solution within the required time. The novelty of our contribution is twofold: i) novel control algorithm design, since it combines online convex optimization (OCO)and distributed algorithms in a dynamic and uncertain environment; ii) novel application area, since it integrates the dynamic voltage support requirements and the instruction from system operators into the control design. Furthermore, the proposed solution gives the aggregator or the system operator the possibility to tune the performance so as to give priority to* *the voltage tracking performance or to the BESS operational costs and lifetime, according to the specific needs and preferences. "*

**3. In paragraph 6 of Introduction, the author introduced that the method in this paper is an non-iterative control approach. However, how to solve equation (6) in an non-iterative manner is not clear.**

RE: Thank you for this comment. The major difference between our non-iterative and other iterative algorithms is that the solution calculated at each iteration is directly applied without having to wait for the final convergence. We prove that the regret function is sublinear, thus the sequence of setpoints calculated at each point in time converges to the optimal sequence in few time steps. The more aspects of the system are dynamically observed and learned over time, the closer the solution gets to the optimal one. The proposed control algorithm has thus adaptive and learning capabilities, differently form the other few distributed algorithms existing in the literature for voltage control, which are iterative. Specifically, for those iterative algorithms, multiple sub-problems have to be solved iteratively at each time step before obtaining the final applicable solution for voltage control. Consequently, the response time provided by these iterative algorithms cannot always catch up with the fast variations of system conditions, which is the major barrier for an online application. In contrast, the proposed approach is based on the online convex optimization framework, which sees the optimization as a process and solves the problem sequentially. Therefore, the response time is faster than iterative algorithms, and the voltage control can be implemented in real-time.

To clarify this, we add the following discussions and implementation details to clarify how the algorithm works in the revised paper, copied below for Reviewer's convenience.

*Main Contribution, Section 1, Page 3: "Differently from the iterative algorithms, which requires solving several sub-problems and thus commit more computational and communication resources, the proposed approach is non-iterative and solves the problem sequentially following the OCO framework, which regards the optimization as a process. Only one iteration is performed at each point in time, and the obtained reactive and active power setpoints are applied directly to track voltage references, resulting in a faster response time. "*

*Section 3.3.2, Page 7, "The calculation of setpoints at each time step can be concluded as*

- *Obtain the local bus voltage magnitude* $v_{i,t-1}$
- *Update the active and reactive power according to (9a) – (9b) with projection operations to guarantee they are within the feasible ranges.*
- *Exchange the active and reactive power with neighboring storage systems*
- *Update Lagrangian multipliers according to (10)."*

**4. In the paragraph following equation (3), the author introduces that in the simulation study, p_b and q_b are calculated by using a fully AC power flow model. However, the power flow in equation (1) is linearized Distflow model. Why the fully AC power flow model is used in simulation, rather than the linearized Distflow model.**

RE: Thank you for this comment. The linearized Distflow model is used only in the algorithm design in order to formulate a convex problem and facilitate the convergence analysis and the calculations to be performed. The setpoints are calculated based on a linearized power flow model and also based on current voltage measurements. Although the use of a linearized power flow model has been proved to be efficient, it still brings some numerical inaccuracy. In order to further improve the control performance, we equip the control algorithm with learning capabilities following a dynamic OCO framework. In order to more realistically assess the control performance, however, the simulation environment needs to be as close as possible to the real one, therefore the simulations of the power system use the fully AC power flow model. The calculated power setpoints are applied to the simulated power system, whose response is based on these power setpoints, but also on the AC power flow. We can thus have a more realistic picture of the response of the real system to the application of the setpoints calculated by the proposed algorithm.

**5. In equation (6), the local constraints (3c) - (3g) are not contained.**

RE: Thank you for this comment. The major difficulty in the design of a distributed algorithm is handling the coupling system-wide constraints (Eq. 3b in Problem 3), so the description of the algorithm focuses mainly on this aspect. The local constraints for each agent, 3c-3g, define the local set $Z$ (as indicated in the first column of page 6 in the revised manuscript), which is what is used in the description of the algorithm. Therefore, only coupled constraints are included in (6), whilst the local constraints are addressed using the projection operations in (9a) – (9b).

**6. The solution method in equation (8) brings calculation errors, which should be carefully demonstrated in the simulation part.**

RE: Thank you for the suggestion. Equation (8) is part of the algorithm design and the dual variables are updated by performing the simple operations of derivation and max, as indicated in this equation. As explained in our response to the comment 3, in this revision, we add the evaluation of the regret function yielded in simulations, which measures the performance or accuracy of an OCO-based algorithm (it basically gives an estimation of the "error" in the solution, how much the obtained solution deviates from the optimal one). As shown in Figure 1 below (Figure 2 in the revised manuscript), the regret, thus the error, converges to almost zero after just a few time steps; after that, we can conclude our solutions are as good as the optimal ones in hindsight. Theorem 3.1 and its proof guarantee that this is achieved by any application of the proposed algorithm, since it is proven that the regret function is sublinear (i.e., it learns and produces a smaller error as time grows).

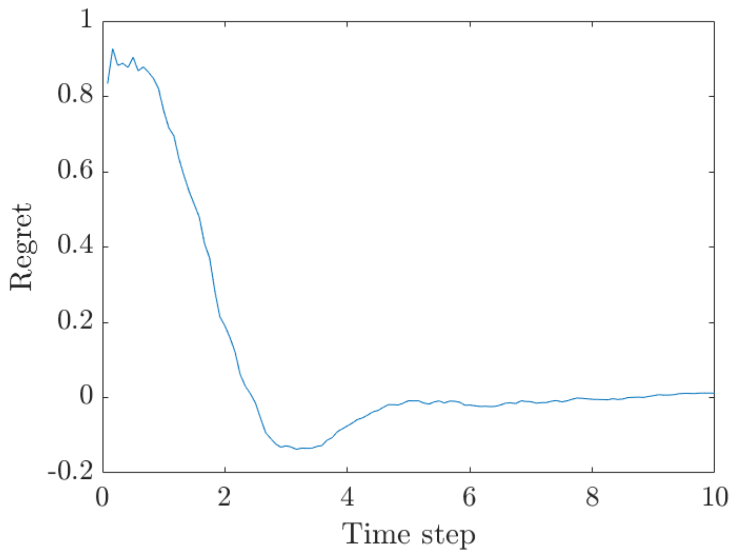


Figure 1. The evolution of the regret function.

**Again, we thank the Reviewer for the careful review and insights. We hope our revision addresses all the constructive comments.**

---

**Reviewer 2**

---

**This paper proposes a decentralized solution for voltage regulation at Transmission and Distribution interconnection points. The authors formulate an optimal power flow problem with voltage-tracking objective and then use a modified OCO framework to solve the problem in a decentralized manner to select the BESS setpoints. This is a very interesting and timely problem. The paper is well written.**

Thank you very much for your positive comment.

**1. Why is the paper focused on BESS systems and generally on DERs? Besides the obvious operation constraints, I don't envision major differences in the methodology.**

RE: Thank you for raising this point. The methodology would indeed be applicable generally to DERs. In this paper, we mainly focus on BESS because of their fast response time, for their flexibility and capability to compensate for the variability of renewable generation, as well as because it is expected that a very large number of them will be more and more available at distribution networks (please, note that electric vehicles can be regarded, in this context, as mobile storage devices which could be handled by the proposed framework because of its plug and play feature, therefore because of its capability to handle storage systems connecting and disconnecting at any point in time). In general, as the number of DERs increases and displaces conventional transmission connected plants, several projects have been launched by system operators, e.g., National Grid, UK, to explore their potential value to provide network support, not only at a distribution but also at transmission level. In particular, the fast-varying renewable generation is at a time scale that is not consistent with conventional voltage control using on-load tap changers, step voltage regulators, and shunt capacitors. To address this, we propose an online and distributed solution for BESS systems. However, the proposed solution is not limited to the BESS but it can be applied to different DERs or other conventional systems by simply adding these components and the associated constraints as long as they maintain the convex property. We are currently extending our framework to unlock the benefit afforded by a combination of diversified DERs.

In this revision, we add the following remark to clarify the capability of handling different DERs.

*Remark 3.2, Section 3.4, Page 8, "The flexibility and scalability of distributed algorithms enable the proposed solution to different applications. In particular, it is not limited to the BESS but it can be applied to different DERs or other conventional systems that need a solution to be scalable and flexible (e.g., plug-and-play), by simply adding these components and the associated constraints as long as they maintain the convex property."*

**2. Can you explain the reasoning behind the multiple T&D interconnection points of the 123 bus test system? To the best of my knowledge, DNs are rarely (if even) operate with multiple TN connection for fault protection reasons.**

RE: Thank you for the comment. New grid concepts for grid operation, e.g., microgrid, bring a change in distribution system operation and topological network structure [4]. Traditionally these interconnection points have existed for the purpose of delivery of energy to a distribution network, however, because of the growth of generation connected at distribution level, increasingly they are also being required to export power onto the transmission network. This may lead to additional investment in transmission infrastructure. Therefore, we include more interconnection points in a distribution system to cover the new structure in emerging power systems and attempt to demonstrate the capability of the proposed solution in this new setting. More interconnection points need to be considered as satisfying one might create problems for other points. The tracking problem in such new setting is complicated, which has not been fully addressed and needs more investigation. This paper provides a contribution to this challenge.

**3. What is the reason behind using \Delta(u) as a performance indicator? It shows deviation from the base (uncontrolled) but higher deviation doesn't really mean better or worse. The l_\inty norm on the voltage deviation from setpoint is enough in my opinion.**

RE: Thank you for the useful suggestion. Indeed, the $l_\infty$ norm would be enough for verifying the tracking performance. We initially wanted to define a performance indicator for the tracking performance and one for the voltage deviation. However, we agree on the point raised by the Reviewer and we consider only the infinity norm as performance indicator, where all the network nodes are included. Please note that this performance indicator includes all the nodes in the network, thus it assesses both the tracking performance at the TN-DN interconnection points and the voltage deviation in the other nodes of the network. The paper has been revised accordingly.

**4. Can you discuss the best selection of \gamma for the algorithm? While the impact is shown on the performance (Table 2), it vaguely mentions a trade-off between performance and BESS size.**

RE: Thank you for the question. The two weights, i.e., $\gamma > 0$ and $\omega > 0$, are designed to balance the voltage regulation and BESS power provision cost. A larger value of $\gamma$ will give priority to the tracking objective over the BESS operational costs, therefore BESS will be controlled to deliver a large amount of power for tracking voltage setpoints. This may bring additional BESS operational costs and affect the battery life. A lower value of $\gamma$ will penalize the tracking objective and aim at limiting the BESS operational costs and their power outputs as a consequence, thus preserving the battery itself. In the simulation study, $\gamma$ is initialized to 1 and increased gradually. It is selected to achieve an acceptable tracking performance and still reduce the impact on battery life.

We add the Figures 11 and 12 (page 12 in the revised manuscript), which show the impact of the two tuning parameters on the power outputs, and the following text in this revision, which is shown below for the Reviewer's convenience.

*Section 4.2, Page 11: "As shown in Table II, a larger $\gamma$ gives priority to the tracking objective over the BESS operational costs, therefore BESS will be controlled to deliver a large amount of power in short time for tracking voltage setpoints. This may bring additional BESS operational costs and affect the battery life, as shown in Fig. 11, and affect the battery life. In this study, $\gamma$ is initialized to 1 and increased gradually. It is selected to achieve an acceptable tracking performance and still reduce the impact on battery life. The findings in Table II are further verified by Figs. 11 - 12, where the voltage profiles and the power outputs obtained by setting different values of $\omega$ and $\gamma$ are depicted, respectively. In Fig. 11, $\gamma$ is fixed to 1 and $\omega$ is varied from 0.1 to 10, while in Fig. 12, $\omega$ is fixed to 1 and $\gamma$ is varied from 1 to 15. We can note that a larger $\omega$ results in delivering less BESS power (therefore smaller operational costs) but in a poorer tracking performance, whilst a larger $\gamma$ results in an improved tracking performance but in larger power outputs, therefore higher operational costs. These parameters provide more choices to the system operators, and they can be appropriately set according to the specific needs of the power system."*

**5. In the definition of equation (2) in page 4, you define (2) as a rewrite of (2).**

RE: We apologize for this typo. This mistake has been amended, and meanwhile, we have carefully proofread the paper to avoid similar mistakes and typos.

**6. In page 4, the deviation of v_h^{set} should be [v_1^{set}(t), ..., v_h^{set}(t), ..., v_{N_h}^{set}(t)]**

RE: We apologize for this mistake. It has been addressed in this revision.

**7. Figure 1 is hard to read.**

RE: Thank you for pointing this out. We have redrawn Figure 1 for better readability.

**8. In the Figures 2-3, what is the time index unit? Can you convert to SI (s/m/h)?**

RE: Thank you for the comment. The time scale setting does not affect the performance of the proposed solution, which can be applied with different time scales. The unit of time in this study is the second, which is added in this revision.

**Again, we thank the reviewer for the careful review and insights. We hope our revision addresses all the constructive comments.**

**Reviewer 4**

**The paper presents a distributed control approach for battery storage to ensure voltage regulation at TN-DN interconnection points. The paper is fairly well written and formulation of the problem has been presented in a systematic manner.**

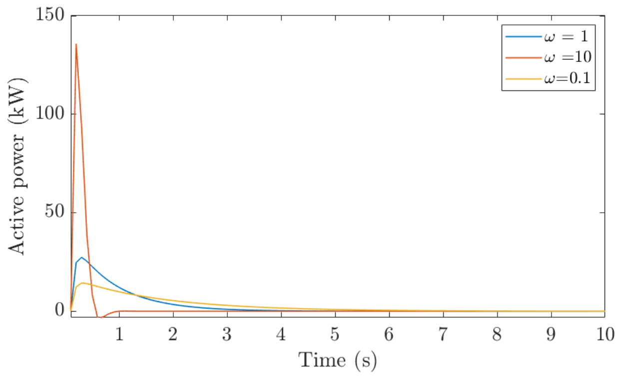
**1. In equation (3a), the author presents the weight parameter omega applied to the battery costs however no further mention of this is made in section 4, could the author clarify on the value chosen with this parameter and how variations of parameter gamma affect the battery costs if applied**.

RE: Thank you for raising this point. The tuning parameters $\omega$ and $\gamma$ are introduced to provide the storage aggregators or the system operators with the possibility to tune the control performance according to specific operational concerns. A larger value of $\gamma$ will give priority to the tracking objective over the BESS operational costs, therefore BESS will be controlled to deliver a large amount of power for tracking voltage setpoints. This may bring additional BESS operational costs and affect the battery life. A lower value of $\gamma$ will penalize the tracking objective and aim at limiting the BESS operational costs and their power outputs as a consequence, thus preserving the battery itself. In the simulation study, $\gamma$ is initialized to 1 and increased gradually. It is selected to achieve an acceptable tracking performance and still reduce the impact on battery life. To better illustrate their impacts on the performance, we add the Figures 11 and 12 (page 12) to the revised manuscript, as well as the following text:

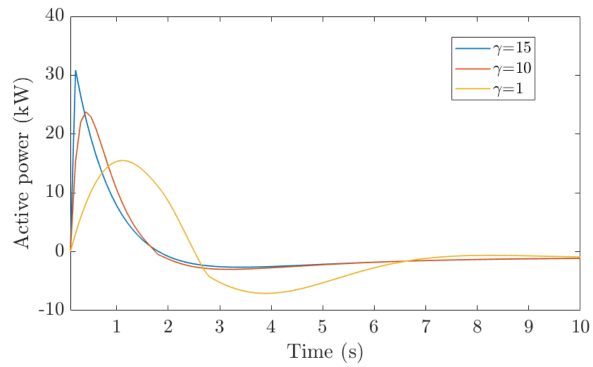
*Section 4.2, Page 11: "As shown in Table II, a larger $\gamma$ gives priority to the tracking objective over the BESS operational costs, therefore BESS will be controlled to deliver a large amount of power in short time for tracking voltage setpoints. This may bring additional BESS operational costs and affect the battery life, as shown in Fig. 11, and affect the battery life. In this study, $\gamma$ is initialized to 1 and increased gradually. It is selected to achieve an acceptable tracking performance and still reduce the impact on battery life. The findings in Table II are further verified by Figures 11 - 12, where the voltage profiles and the power outputs obtained by setting different values of $\omega$ and $\gamma$ are depicted, respectively. In Fig. 11, $\gamma$ is fixed to 1 and $\omega$ is varied from 0.1 to 10, while in Fig. 12, $\omega$ is fixed to 1 and $\gamma$ is varied from 1 to 15. We can note that a larger $\omega$ results in delivering less BESS power (therefore smaller operational costs) but it also leads to a poorer tracking performance, whilst a larger $\gamma$ results in an improved tracking performance but in larger power outputs, therefore higher operational costs. These parameters provide more options to the system operators and they can be appropriately set according to the specific needs of the power system."*

**2. The author has presented the algorithm performance in section 4, however, the effect of the control requirement to the battery power has not been outlined, the author could consider presenting the effects on the battery active and reactive power injections to further enhance self further analysis of the viability of the solution presented.**

RE: Thank you for this comment. In the revised manuscript, we added the following figures (Fig. 11(b) and Fig. 12 (b) in the manuscript, page 12) to illustrate the impact of the application of the proposed control algorithm on power injections, highlighting the impact of the selection of the tuning parameters on the performance. We also added appropriate narrative to comment these results, as included in our response to the Reviewer's comment 1.




(a) Fixed $\gamma$




(b) Fixed $\omega$

Figure 3. Active power outputs for different values of $\gamma$ and $\omega$

**3. Could the author comment on the physical practically of the solution presented given the need to install battery units on over 80% of the distribution nodes.**

RE: Thank you for the question. The proposed solution does not require such amounts of BESS installed in the distribution nodes, and no assumption on the number of BESS is needed. However, it is expected that the future distribution networks will have a large number of units as controllable devices for voltage regulation [5]. There is indeed a need for coordinating a large number of BESS in a fast and scalable manner. Therefore, we considered a large number of BESS in the discussed case studies to show that the proposed solution is scalable and can handle an arbitrary number of BESS. However, as the Reviewer correctly pointed out, a large number of (small size) BESS would be needed provide an effective response; there are still not a significant number of storage devices at distribution networks, but this number is expected to increase significantly in the near future. We can think of the electric vehicles as well, which could be integrated in the proposed methodology, because of its plug and play capability and the fact that electric vehicles can be regarded as mobile storage devices in this context.

**Again, we thank the reviewer for the careful review and insights. We hope our revision addresses all the constructive comments.**

**Reviewer 5**

**Decentralized control for large scale linear systems have been intensively researched in the literature, the paper reformulates the problem within the agent-based framework to enable battery storage systems to be connected to distribution networks for voltage tracking demanded by transmission system operators. A convex optimization function is designed to derive the optimal charging/discharging power from battery storage systems. The paper is not well written, seems written in a rush with quite a few flaws and errors, hence it is not easy to follow the logic. There are a few strong assumptions in the proposed approach, hence making the optimization problem easy to solve, no different from decentralised control of traditional linear systems with weakly coupled subsystems. Note that the journal only accepts papers with practical elements, the paper needs to validate the proposed methods either in the laboratory experiments (e.g. at least using real-time simulators) or demonstrate the practicality of the proposed approach on one way or another.**

Re: Thank you for the comments. We would like to point out that the proposed algorithm is not decentralized but distributed and that the made assumptions are not strong, nor decouple the system. The assumptions made are relevant to the control design and to the proofs, but do not make any assumptions on the structure or layout of the real power systems. These assumptions are standard within the OCO framework, and online optimization has been validated against empirical results, however in different fields, and in particular compressed sensing of a dynamic scene, traffic surveillance, tracking self-exciting point processes and network behavior in the Enron email corpus. All the experimental results support the core theoretical findings [7].
The coupling we consider is strong and not weak (e.g., the power balance constraint is considered and each calculated solution guarantees its satisfaction), which is the main reason for resorting to distributed and not decentralized algorithms (as we illustrate in page 2, where we discuss the various control approaches to voltage regulation in distribution networks). We address an open challenge. In fact, with increased renewable generation capacity in the form of distributed energy resources (DER), which displaces conventional transmission connected plants, the potential value of DERs to provide network support, not only at a distribution but also at transmission level, will need to be explored [1]. There is a clear trend, worldwide, to reduce the time scales of grid operation in order to react more promptly to the time varying grid conditions, mainly due to the variability of renewable generation. TSOs are exploring more dynamic support services, including the voltage services [2] (in the revised manuscript). For instance, the aim of Power Potential project [5] (in the revised manuscript) run by the UK TSO, National Grid ESO (NG ESO), is to be the first world trial to test the delivery of dynamic voltage support from different types of distributed energy resources (DER) embedded at various voltage levels, including storage assets, whose at least 90% of response is to be provided in 2s in order to be effective. Studies conducted by NG ESO found that to support frequency and voltage recovery, additional dynamic voltage support will be required to replace that which is currently provided by synchronous generation at the interconnection points.
(https://www.nationalgrideso.com/sites/eso/files/documents/SOF%20Report%20-%20Frequency%20and%20Voltage%20assessment.pdf). The existing literature and launched projects by system operators show a need for a control solution to manage dynamic and optimal operation of DERs at the distribution level, which thus is not yet available.
Therefore, we aim at providing a technical solution to an open problem focusing on BESS, for controlling a large number of storage assets to provide dynamical voltage services when needed.

We propose a distributed approach for online voltage regulation and demonstrate its scalability and its dynamic tracking capability. Some of the existing studies put efforts in controlling DERs for system voltage regulation. However, we would like to emphasize that just a few of them have both online and distributed implementation capabilities and none of them consider dynamic voltage services, the TN-DN interaction and have such as real-time capabilities, which makes the proposed solution truly scalable and flexible, and able to optimally coordinate an arbitrary number of BESS located at distribution networks without the need of any central entity and preserving privacy. Furthermore, the proposed solution gives the aggregator or the system operator the possibility to tune the performance so as to give priority to the voltage tracking performance or to the BESS operational costs and lifetime, according to the specific needs and preferences. Please, note that the ability to coordinate the BESS *optimally* (and other performance criteria can be considered) is a benefit of the proposed solution, which cannot be achieved by the existing decentralized approach, since they do not consider coupling constraints, thus the BESS location in the network and the network model. The focus is on BESS, but the proposed framework can be extended to DERs in general, included demand, which is the topic of our current work. Although a very few studies attempted to design distributed methods for voltage regulation, their focus is on regulating voltages only at distribution levels, ignoring the requirement of dynamic services and of the transmission system (see [16], [17], [21], [27] in the manuscript). Furthermore, most of the existing decentralized/distributed algorithms are iterative, whilst our proposed one is non-iterative. The major difference between our non-iterative and other iterative algorithms is that the solution calculated at each iteration is directly applied without having to wait for the final convergence. We prove that the regret function is sublinear, thus the sequence of setpoints calculated at each point in time converges to the optimal sequence in few time steps (just 4 or 5 in our case study, as shown in Figure 2, page 9). The more aspects of the system are dynamically observed and learned over time, the closer the solution gets to the optimal one. The proposed control algorithm has thus adaptive and learning capabilities, differently form the other few distributed algorithms existing in the literature for voltage control, which are iterative. Specifically, for those iterative algorithms, multiple sub-problems have to be solved iteratively at each time step before obtaining the final applicable solution for voltage control. Consequently, the response time provided by these iterative algorithms cannot always catch up with the fast variations of system conditions, which is the major barrier for an online application. In contrast, the proposed approach is based on the online convex optimization framework, which sees the optimization as a process and solves the problem sequentially. Therefore, the response time is faster than iterative algorithms, and our proposed solution can be implemented in real-time.
The novelty of our contribution is thus twofold: *i*) novel control algorithm design, since it combines online convex optimization and distributed algorithms in a dynamic and uncertain environment; *ii*) novel application area, since it integrates the dynamic voltage support requirements and the instruction from system operators into the control design.

To clarify this, we add the following discussions and implementation details in the revised paper, copied below for Reviewer's convenience.

*Section I, Page 2, "Although there are some existing studies dealing with voltage control from DERs, just a few of them have both online and distributed implementation capabilities and none of them consider dynamic voltage services, the TN-DN interaction and have such as real-time capabilities, which makes the proposed solution truly scalable and flexible, and able to optimally* *coordinate an arbitrary number of BESS located at distribution networks without the need of any central entity and preserving privacy. Although a very few studies attempted to design distributed methods, their focus is on regulating voltages only at distribution levels, ignoring the requirement of dynamic services and of the transmission system. Furthermore, the proposed control algorithm is non-iterative, thus providing real-time capabilities, differently from the algorithms described in the aforementioned studies, which are iterative, thus without any guarantee to converge to an applicable solution within the required time. The novelty of our contribution is twofold: i) novel control algorithm design, since it combines online convex optimization and distributed algorithms in a dynamic and uncertain environment; ii) novel application area, since it integrates the dynamic voltage support requirements and the instruction from system operators into the control design. Furthermore, the proposed solution gives the aggregator or the system operator the possibility to tune the performance so as to give priority to the voltage tracking performance or to the BESS operational costs and lifetime, according to the specific needs and preferences. "*

*Main Contribution, Section I, Page 3: "Differently from the iterative algorithms, which requires solving several sub-problems and thus commit more computational and communication resources, the proposed approach is non-iterative and solves the problem sequentially following the OCO framework, which regards the optimization as a process. Only one iteration is performed at each point in time, and the obtained reactive and active power setpoints are applied directly to track voltage references, resulting in a faster response time. "*

We also carefully proofread the manuscript to amend typos.

**Apart from the above general comments, specific comments are given below:**
**1. Table 1 is far from completion, quite a few key parameters are not defined in this table, like \gamma in (3a), \p_i^{b,-}, \p_i^{b,+ } in (3c) and (3d), \lambda in (6), \alpha in (7), \H_r, \H_x, \H_{x/r} in (9a), etc… This makes the paper quite difficult to follow.**

RE: Thank you for raising this point. These algorithm parameters were introduced when describing the algorithm design, since they are related to it. We agree with the reviewer that these parameters should also be included in Table 1 for the reader's convenience, and we added them in this revision.

**2. (1c) is wrong, and it does not make sense at all. The paper seems failed to use the correct linearized model. The second P_{ij} should be a mapping of the reactive power, not the active power. But overall, they should not be the P_{ij} and Q_{ij}, rather, these two quantities should be affine mappings. Details could refer to ref [1] listed at the end of the comments.**

RE: We thanks the Reviewer and apologize for the mistake. This is a typo that should be $Q_{ij}$. It has been amended in this revision. The Linearized Distflow model used in this paper has been widely adopted in the study of voltage control (see [15] [16] [20] [21] [24] [27] [28] in the manuscript), particularly in distributed voltage regulation. Therefore, this work utilizes this linearized model in the control design based on recent studies. Please, note that the linearized Distflow model is only used in the algorithm design in order to formulate a convex problem and facilitate the convergence analysis and the calculations to be performed. In order to more realistically assess the control performance, however, the simulation environment needs to be as close as possible to the real one, therefore the simulations of the power system use the fully AC power flow model. The calculated power setpoints are applied to the simulated power system, whose response is based on these power setpoints, but also on the AC power flow. We can thus have a more realistic picture of the response of the real system to the application of the setpoints calculated by the proposed algorithm.

**3. There are quite a few design parameters, e.g. in (3a), (6), (9a), and theorem 3.1 only gives a very generic framework to define the conditions of a couple parameters in order to make the system converge, how these parameters affect the performance are largely ignored, though only in the simulation section the choice of \gamma in (9a) is discussed. The simulation section needs to be significantly strengthened.**

RE: Thank you raising this point. Please, note that there are only two tuning parameters (the other parameters are the network resistances and reactances and the BESS related ones, i.e., efficiencies and capacities), which can be set by the system operators according to specific needs and concerns. More importantly, those parameters do not have any impact on the convergence properties of the proposed algorithm (there is not any values of $\gamma$ and $\omega$ that make the algorithm solution deviate from the optimal one). In order to better show the convergence properties and the impact of the choice of the tuning parameters on the control performance, we extended the simulation section. We included the analysis of the regret function to show that it approaches zero after just 4 or 5 time steps, thus converging to the optimal solution in very short time, as depicted in the added Figure 2 in the revised manuscript, which is copied here for the Reviewer's convenience.

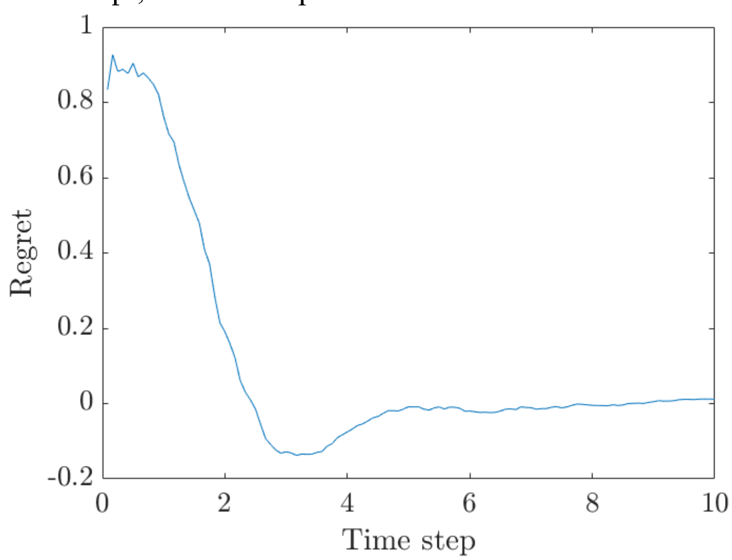
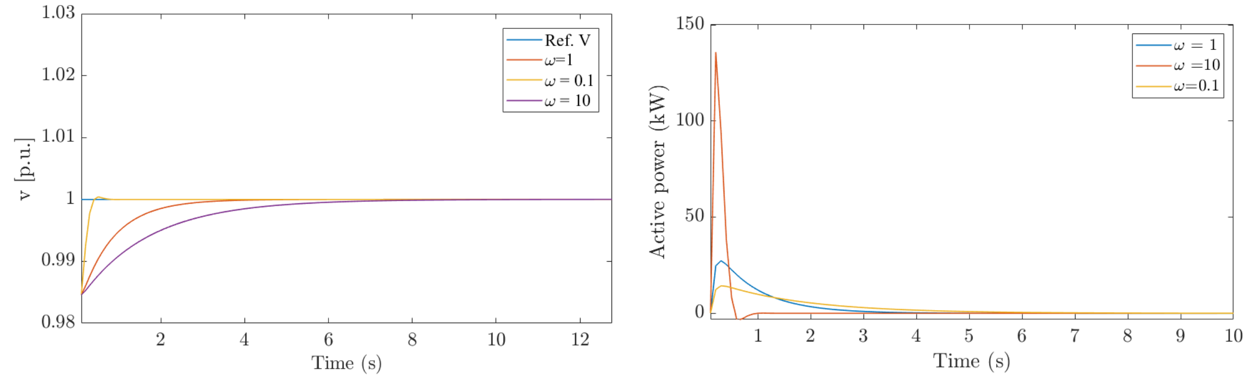


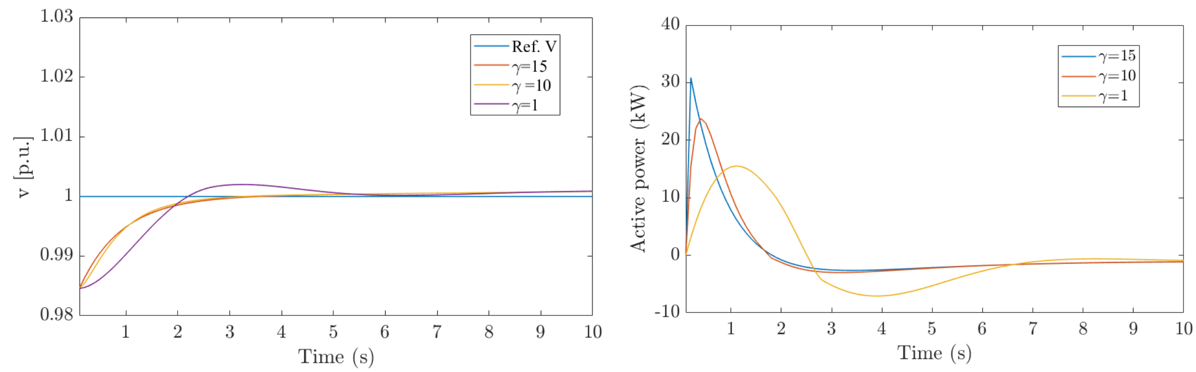

Figure 1. The evolution of the regret function.

In the revised manuscript we also analyzed the impact of the parameters $\gamma$ and $\omega$ on the control performance. The tuning parameters $\omega$ and $\gamma$ are introduced to provide the storage aggregators or the system operators with the possibility to tune the control performance according to specific operational concerns. A larger value of $\gamma$ will give priority to the tracking objective over the BESS operational costs, therefore BESS will be controlled to deliver a large amount of power for tracking voltage setpoints. This may bring additional BESS operational costs and affect the battery life. A lower value of $\gamma$ will penalize the tracking objective and aim at limiting the BESS operational costs and their power outputs as a consequence, thus preserving the battery itself. In the simulation study, $\gamma$ is initialized to 1 and increased gradually. It is selected to achieve an acceptable tracking performance and still reduce the impact on battery life. To better illustrate their impacts on the performance, we add the Figs. 11 and 12 (page 12 in the revision) to the revised manuscript, which we include below.




(a) Voltage profiles (b) Active power outputs

Figure 4. Comparative results for fixed $\gamma$.




(a) Voltage profiles (b) Active power outputs

Figure 5. Comparative results for fixed $\omega$.

We included also the following text:

*Section 4.2, Page 11: "As shown in Table II, a larger $\gamma$ gives priority to the tracking objective over the BESS operational costs, therefore BESS will be controlled to deliver a large amount of power in short time for tracking voltage setpoints. This may bring additional BESS operational costs and affect the battery life, as shown in Fig. 11, and affect the battery life. In this study, $\gamma$ is initialized to 1 and increased gradually. It is selected to achieve an acceptable tracking performance and still reduce the impact on battery life. The findings in Table II are further verified by Figures 11 - 12, where the voltage profiles and the power outputs obtained by setting different values of $\omega$ and $\gamma$ are depicted, respectively. In Fig. 11, $\gamma$ is fixed to 1 and $\omega$ is varied from 0.1 to 10, while in Fig. 12, $\omega$ is fixed to 1 and $\gamma$ is varied from 1 to 15. We can note that a larger $\omega$ results in delivering less BESS power (therefore smaller operational costs) but it leads to a poorer tracking* *performance, whilst a larger $\gamma$ results in an improved tracking performance but in larger power outputs, therefore higher operational costs. These parameters provide more options to the system operators and they can be appropriately set according to the specific needs of the power system."*

**4. Remark 3.1 gives a very strong assumptions, making the subsystems almost decoupled.**

RE: Thank you for the comment. Please, note that Remark 3.1 does not make any assumptions on the network structure or decouple the subsystem, but it just explains how nodes without any storage units can be modeled and integrated in the proposed control framework. It just provides the reader with several alternative options when applying the proposed solution. We assume that the comment of the Reviewer might refer to the second option, where $R$ and $X$ can be decomposed following the reference [29] (in the revised manuscript). However, we would like to highlight that this does not decouple the subsystem as well. This method, rather than decoupling the system, just uses the block decomposition approach to rearrange the elements in $R$ and $X$ based on the node controllability since the control solution is only developed for the node with controllable devices (BESS). It just provides an equivalent mathematical description of the same system, without any decoupling or any assumption on the coupling between the sub-systems. Furthermore, it should be noted that the proposed solution utilizes the first option without decomposing $R$ and $X$. We take advantage of the sparse nature of the inverse of $R$ and $X$ to design the solution without decoupling the system.

**5. In addition to strengthen the simulation section, the practicality of the proposed approach should be strengthened in order that the paper is acceptable in this journal - the journal only primarily accepts papers with practical applications.**

RE: Thank you for the suggestion. Indeed, a real implementation would be ideal, however an experimental study for such a large-scale system would be too hard to arrange, since such a number of BESS is not still not available. Furthermore, because of the pandemic, laboratories were closed most of the time and it was not possible to plan for any experimental work. The possibility of performing hardware in the loop experiments is currently being explored. We would like to emphasize that the proposed solution combines the benefits of online convex optimization (OCO) and distributed consensus-based algorithms. However in different application areas, both these algorithms have been verified experimentally, e.g., [6] for online optimization and [7] for voltage regulation. The experimental results have confirmed the theoretical findings. We strove to make our simulation environment as accurate as possible (see, for instance, our response to the Reviewer's comment 2). These aspects above, along with the real-time capabilities of the proposed control algorithm and the fact that it is non-iterative, make the proposed solution highly likely applicable in practice.
The implementation in practical systems would be one of the key directions in our future works.

**Again, we thank the reviewer for the careful review and insights. We hope our revision addresses all the constructive comments.**

# References


[1] "Power Potential," National Grid ESO, [Online]. Available: https://www.nationalgrideso.com/document/195911/download.

[2] C. C. a. B. Stojkovksa, "The Power Potential Project: Trialling the Procurement of Reactive Power Services from Distribution-Connected Assets," in *15th International Conference on the European Energy Market (EEM)*, 2018.

[3] a. T. S. a. C. Giorgos Prionistis, "Voltage stability support offered by active distribution networks," *Electric Power Systems Research,* vol. 190, p. 106728, 2021.

[4] P. a. O. E. Wirasanti, "Active Distribution Grid Power Flow Analysis using Asymmetrical Hybrid Technique," *International Journal of Electrical and Computer Engineering,* vol. 7, no. 4, p. 1738, 2017.

[5] a. D. J. H. a. T. L. Z. Tang, "Distributed Coordinated Reactive Power Control for Voltage Regulation in Distribution Networks," *IEEE Transactions on Smart Grid,* vol. 12, no. 1, pp. 312-323, 2021.

[6] E. C. H. a. R. M. Willett, "Online Convex Optimization in Dynamic Environments," *IEEE Journal of Selected Topics in Signal Processing,* vol. 9, no. 4, pp. 647-662, 2015.

[7] N. H. a. Y. O. K. Hirata and H. Akutsu and A. Ohori, "Decentralized Voltage Regulation for PV Generation Plants Using Real-Time Pricing Strategy," *IEEE Transactions on Industrial Electronics,* vol. 64, no. 6, pp. 5222-5232, 2017.

[8] I. M. a. S. B. a. W. A. a. H. J. a. A. A. a. U. T. Sanz, "Enhancing transmission and distribution system coordination and control in Great Britain using power services from distributed energy resources," in *The 7th International Conference on Renewable Power Generation*, 2018.

[9] P. N. a. P. Jena, "Adaptive Protection Scheme for Microgrid With Multiple Point of Common Couplings," *IEEE Systems Journal,* 2020.

[10] M. E. B. a. F. F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Transactions on Power Delivery,* vol. 4, no. 1, pp. 725-734, 1989.

**Declaration of interests**

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: