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Cache Code Math: Mathematics Lesson Plans: Fractions

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Abstract

This document entails three 5th grade mathematics lesson plans. The lessons are intended to be implemented in conjunction with the computer lab activities found in "Cache Code Math: Fractions, Functions, & For-Loops." The mathematics focus of these lessons is multiplying fractions, which is paired with the computer coding concepts of functions and for-loops.

Acknowledgment

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Cache Code Math Fractions Unit

Fifth-Grade Math Routines

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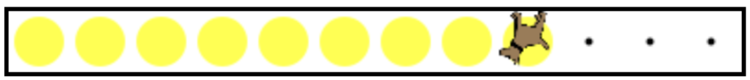
#1 Math Routine: Karel Cleans Up Tennis Balls

Go Math! Lesson: This math routine fits with Chapter 7 lessons after *Lesson 7.1: Find Part of a Group.* The routines could be implemented with any lesson in Chapter 7 as a review. There are 5 equations in this math routine, which is a suggested sequence. These equations can be split into 5- to 15-minute activities over 2-3 days. For example, the first three equations can be grouped as an activity for one mini-lesson, then the next two equations as a second mini-lesson, or each as a mini-lesson for a second and third mini-lesson.

The math routine uses visuals from CodeHS:



These visuals are tied to representations in *Go Math!*:

- A key goal for this routine is to help students interpret the product *(a/b) x q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a x q* divided by *b* (5.NF.4).
- Another goal for this routine is for students to engage in an anchor idea of computer science: algorithmic design and abstraction. In math, students can model the fractional part of a group using a Code HS program and connect an equation to the modeling of the fractional part of a group. In computer science, students can discuss the concept of repeats by using and modifying the "for-loop" within the code. Using a "for-loop" also helps students engage in abstraction by making a task more efficient with the use of the "for-loop."

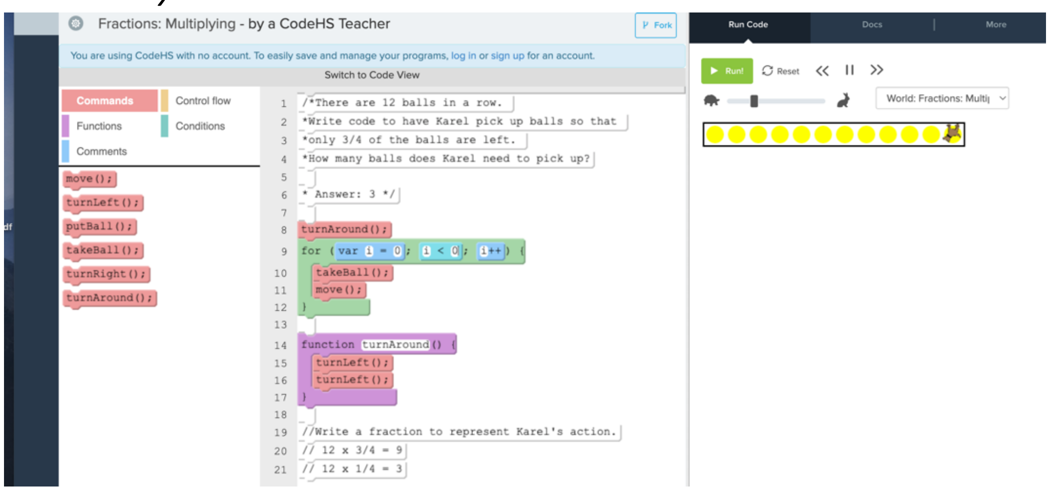
Use a Think-Pair-Share format for each equation: Think - silent individual think time; Pair - share their solution and why they think it is that solution with their partner; and Share - share solutions and mathematical thinking as a whole class. This is followed by Run, Write, and Discuss.

You all code with Karel the Dog in the computer lab. Tell me about Karel (elicit responses). *I don't know if you know this, but Karel loves playing catch with tennis balls. Karel is trained to pick them up and move them when instructed to clean up.*

As the computer scientist, you need to instruct Karel to clean up a specific amount of tennis balls in CodeHS. Read the code. What does it say?

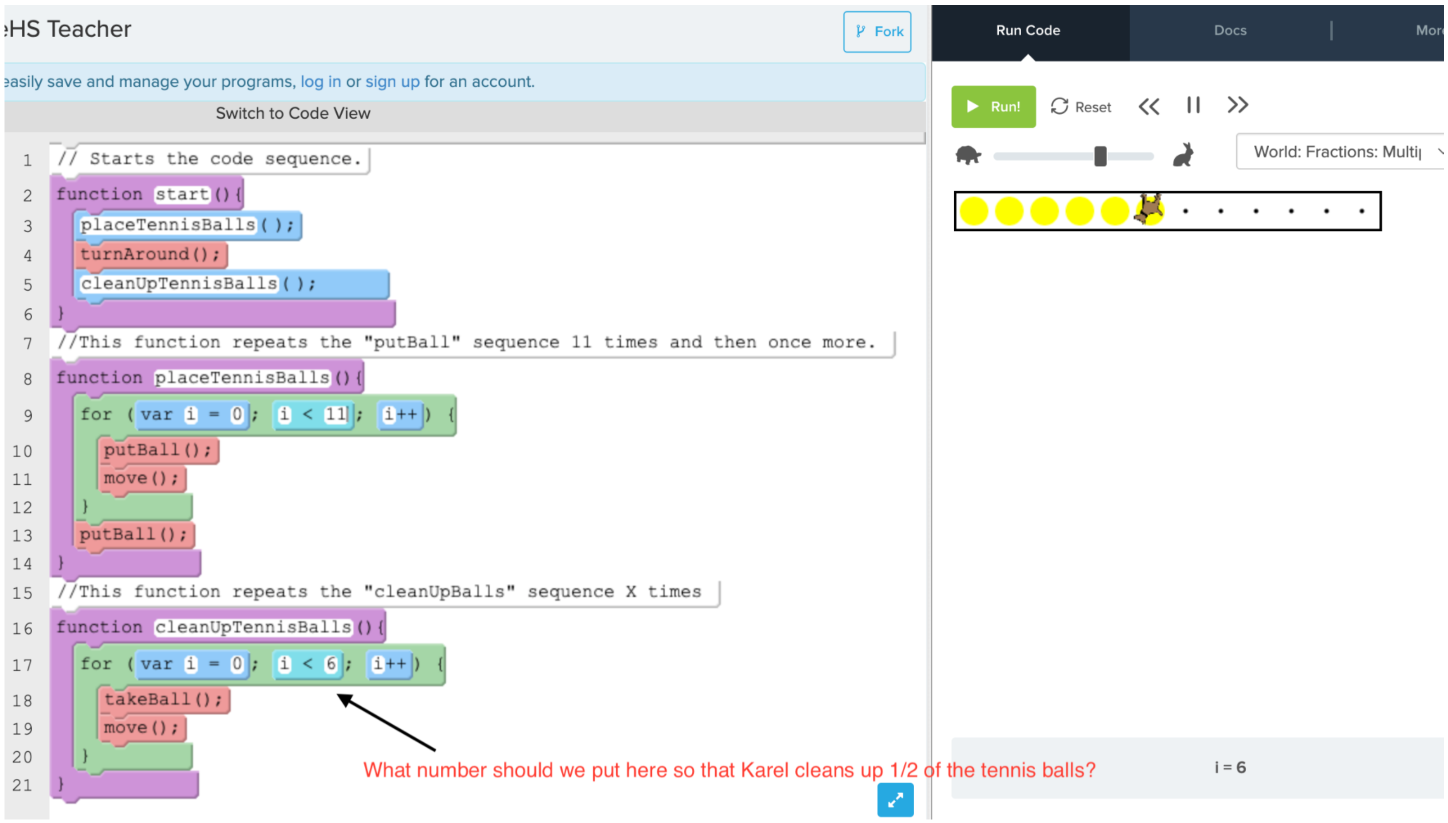
(The code tells Karel to take the tennis ball and we can repeat it using the "for-loop"; the i or index tells Karel how many times to pick up a tennis ball and move)

Yes, the "for-loop" has this letter i, which you call the index in the computer lab, and the i lets us instruct Karel to pick up a specified number of tennis balls. The code has Karel repeat the "takeBall" and "move" commands for however many times we instruct.



Equation #1: ½ x 12 = 6

Think: *Karel has 12 tennis balls. What number should we put [here] so that Karel cleans up ½ of the tennis balls? Visualize what Karel will do.*



Pair and Share: *Explain your reasoning.*

(Possible responses: We put 6 so Karel picks up 6 balls, which is ½ of the balls. There will be ½ left.)

Run: *Let's run the program. Does your own visualization match what Karel did?*

Write: *We see the code that tells Karel what to do. On your whiteboard, write a math equation, using fractions, that represents what Karel did. Remember, we want to write an equation about Karel cleaning up ½ of the tennis balls.*

(Possible responses: 1/2 x 12 = 6; students might suggest 12-6=6, remind them to represent the actions in terms of cleaning up 1/2 of the balls. Students might erroneously suggest 12-6=½, and go back to the context to make sense of the equation. Also, emphasize "6 is what part of 12?" to elicit that ½ of 12 is 6.)

Discuss:

What does the denominator of the fraction factor represent?

(number of equal groups in the whole; we have 2 equal groups of 6)

What about the numerator?

(number of those groups we want Karel to clean up; we instructed 1 of those groups)

What does the product mean in this situation?

(the number of balls in the group that Karel needs to pick up, the i in our "for-loop" code in the "cleanUpTennisBalls" function)

Teacher writes the equation ½ x 12 = 6 near the Karel visualization. Run the program again: *Does this equation represent the fractional amount of tennis balls that Karel is cleaning up? How do you know?*

Equation #2: ¼ x 12 = 3

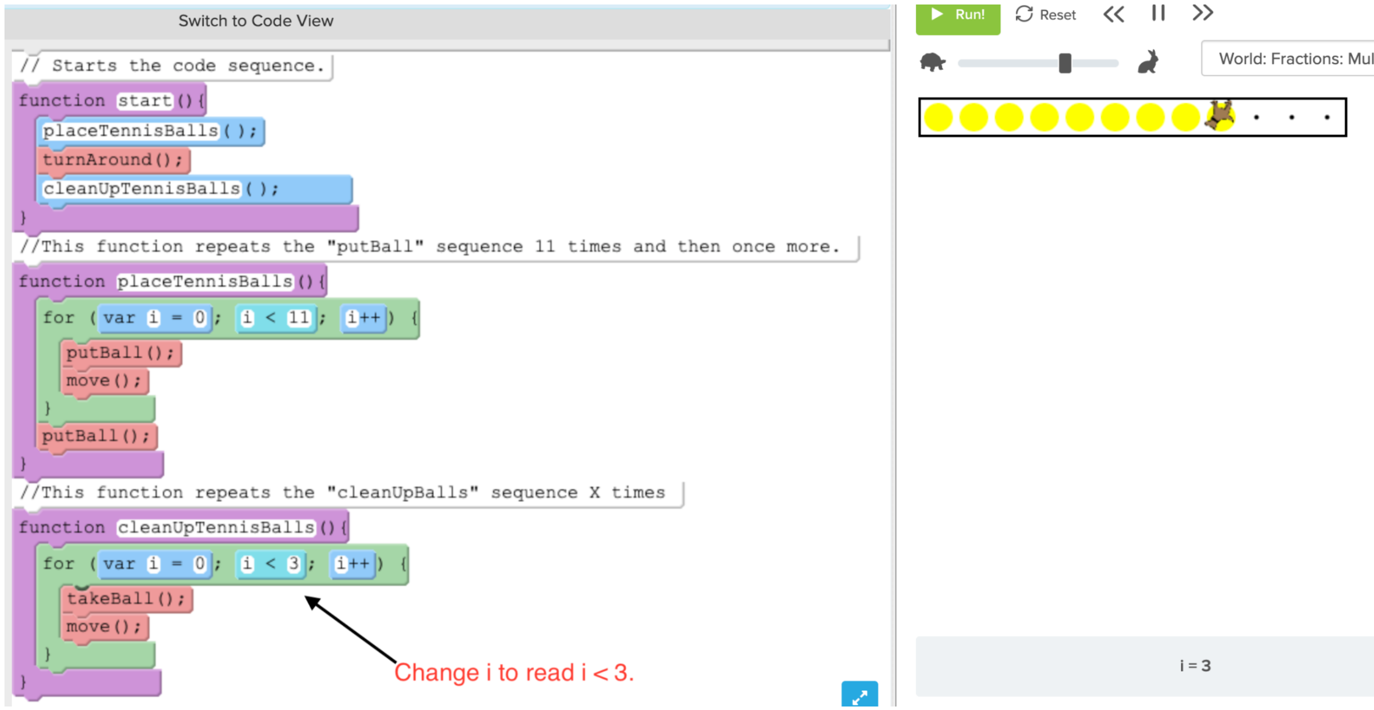
Think: *What number do we put [here] so Karel cleans up ¼ of the tennis balls? Visualize what Karel will do.*

Pair and Share: *Explain your reasoning.*

(Possible responses: We put 3 so Karel picks up 3 balls, which is ¼ of the balls. There will be ¾ left.)

Run: *Let's run the program. Does your own visualization match what Karel did?*

(Change *i* in the "cleanUpTennisBalls" function to read *i<3.*)



Write: *We see the code that tells Karel what to do. On your whiteboard, write a math equation that represents what Karel did. Remember that your equation should have the fractional amount of the tennis balls Karel cleaned up.*

(Possible responses: ¼ x 12 = 3; students might suggest 12-3=9, remind them to represent the actions in terms of cleaning up ¼ of the balls.)

Discuss:

What does the denominator of the fraction factor represent?

(number of equal groups in the whole; we have 4 equal groups of 3)

What about the numerator?

(number of those groups we want Karel to clean up; we instructed 1 of those groups)

What does the product mean in this situation?

(the number of balls in the group that Karel needs to pick up, the i in our "for-loop" code in the "cleanUpTennisBalls" function)

Teacher writes the equation ¼ x 12 = 3 near the Karel visualization and runs the program again: *Does this equation represent the fractional amount of tennis balls that Karel is cleaning up? How do you know?*

Equation #3: 2/3 x 12 = 8

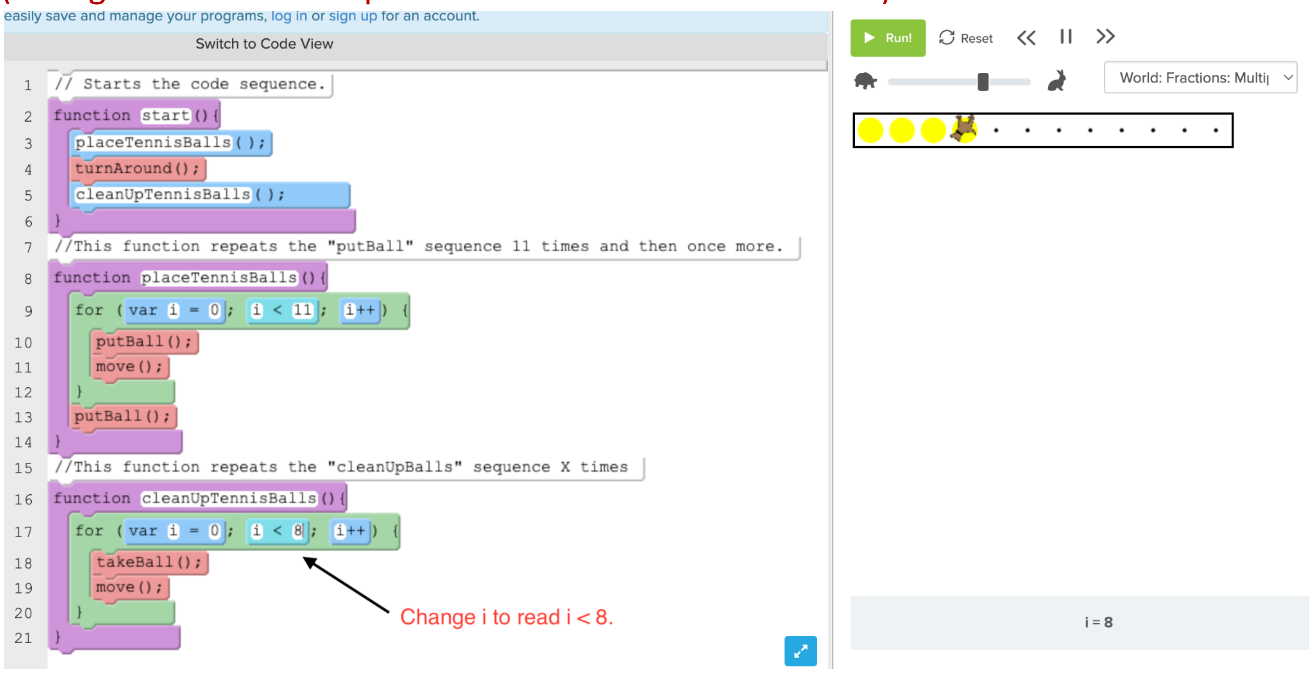
Think: *What number do we put [here] so Karel cleans up 2/3 of the tennis balls? Visualize what Karel will do.*

Pair and Share: *Explain your reasoning.*

(Possible responses: We put 8 so Karel picks up 8 balls, which is 2/3 of the balls. There will be 1/3 left.)

Run: *Let's run the program. Does your own visualization match what Karel did?*

(Change *i* in the "cleanUpTennisBalls" function to read *i<8.*)



Write: *We see the code that tells Karel what to do. Write a math equation that represents what Karel did.*
(Possible responses: 2/3 x 12 = 8; students might suggest 12-8=4, remind them to represent the actions in terms of cleaning up 2/3 of the tennis balls.)

Discuss:

What does the denominator of the fraction factor represent?
(number of equal groups in the whole; we have 3 equal groups of 4)

What about the numerator?
(number of those groups we want Karel to clean up; we instructed 2 of those groups)

What does the product mean in this situation?
(the number of tennis balls in the group that Karel needs to pick up, the i in our "for-loop" code in the "cleanUpTennisBalls" function)

Teacher writes the equation 2/3 x 12 = 8 near the Karel visualization and runs the program again: *Does this equation represent the fractional amount of tennis balls that Karel is cleaning up? How do you know?*

Equation #4: 3/4 x 12 = 9

Think: *What number do we put [here] so Karel cleans up ¾ of the tennis balls? Visualize what Karel will do.*

Pair and Share: *Explain your reasoning.*
(Possible responses: We put 9 so Karel picks up 9 tennis balls, which is 3/4 of the balls. There will be 1/4 left.)

Run: *Let's run the program. Does your own visualization match what Karel did?*
(Change *i* in the "cleanUpTennisBalls" function to read *i<9.*)

Write: *We see the code that tells Karel what to do. Write a math equation that represents what Karel did.*
(Possible responses: 3/4 x 12 = 9; students might suggest 12-9=3, remind them to represent the actions in terms of cleaning up 3/4 of the tennis balls.)

Discuss:

What does the denominator of the fraction factor represent?
(number of equal groups in the whole; we have 4 equal groups of 3)

What about the numerator?

(number of those groups we want Karel to clean up; we instructed 3 of those groups)

What does the product mean in this situation?

(the number of tennis balls in the group that Karel needs to pick up, the i in our “for-loop” code in the “cleanUpTennisBalls” function)

Teacher writes the equation 3/4 x 12 = 9 near the Karel visualization and runs the program again: *Does this equation represent the fractional amount of tennis balls that Karel is cleaning up? How do you know?*

Equation #5: 3/6 x 12 = 6

Think: *What number do we put [here] so Karel cleans up 3/6 of the tennis balls? Visualize what Karel will do.*

Pair and Share: *Explain your reasoning.*

(Possible responses: We put 6 so Karel picks up 6 balls, which is 3/6 (or 1/2) of the tennis balls. There will be 3/6 (or 1/2) left.)

Run: *Let’s run the program. Does your own visualization match what Karel did?*

(Change *i* in the “cleanUpTennisBalls” function to read *i<6.*)

Write: *We see the code that tells Karel what to do. Write a math equation that represents what Karel did.*

(Possible responses: 3/6 x 12 = 6; students might suggest 12-6=6, remind them to represent the actions in terms of cleaning up 1/2 of the balls.)

Discuss:

What does the denominator of the fraction factor represent?

(number of equal groups in the whole; we have 6 equal groups of 2)

What about the numerator?

(number of those groups we want Karel to clean up; we instructed 3 of those groups)

What does the product mean in this situation?

(the number of tennis balls in the group that Karel needs to pick up, the i in our “for-loop” code in the “CleanUpTennisBalls” function)

Teacher writes the equation 3/6 x 12 = 6 near the Karel visualization and runs the program again: *Does this equation represent the fractional amount of tennis balls that Karel is cleaning up? How do you know?*
**Optional: short discussion on equivalent fractions here (3/6 = 1/2).*
Compare 3/6 x 12 = 6 to 1/2 x 12 =6.

Discuss: *What did you learn by modeling the fractional parts of groups with Karel and representing the actions with an equation?*

Teacher statement after discussion: *Today we learned about creating an equation to represent a fractional part of a group. In computer science, this representation is called abstraction. You will practice abstraction in the computer lab to model fractional groups like we did today. You also saw “for-loops” in today’s lesson. You will use “for-loops” in the computer lab to make the coding more efficient.*

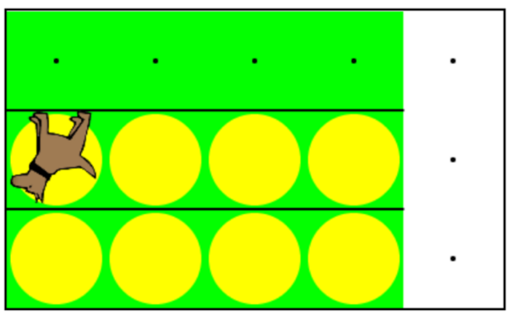
CS Glossary:
Abstraction - Reducing complexity by focusing on important elements of a problem or situation
For-Loop - repeats a specific piece of code a set number of times
Index - a number value that tells the for-loop how many times the code in the for-loop will repeat

#2 Math Routine: Karel Shows the Product! What's the Equation?

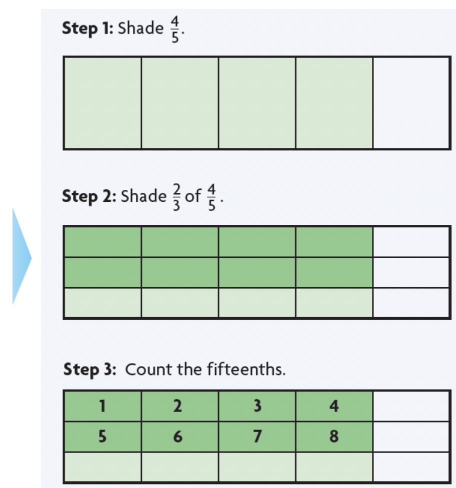
Go Math! Lesson: This math routine fits with *Lesson 7.4: Multiply Fractions* and could be implemented with any lesson after 7.4 as a review. In this routine, Karel shows a product by putting down a specified number of tennis balls (which represent the numerator of the product). Students will need to pick apart the model to figure out what the multiplication equation represents, but this discussion will be more focused on the size of the product as a result of a fraction times a fraction (a part of a part).

This lesson provides background knowledge for the more in-depth #3 Math Routine.

The math routine will use visuals from CodeHS:



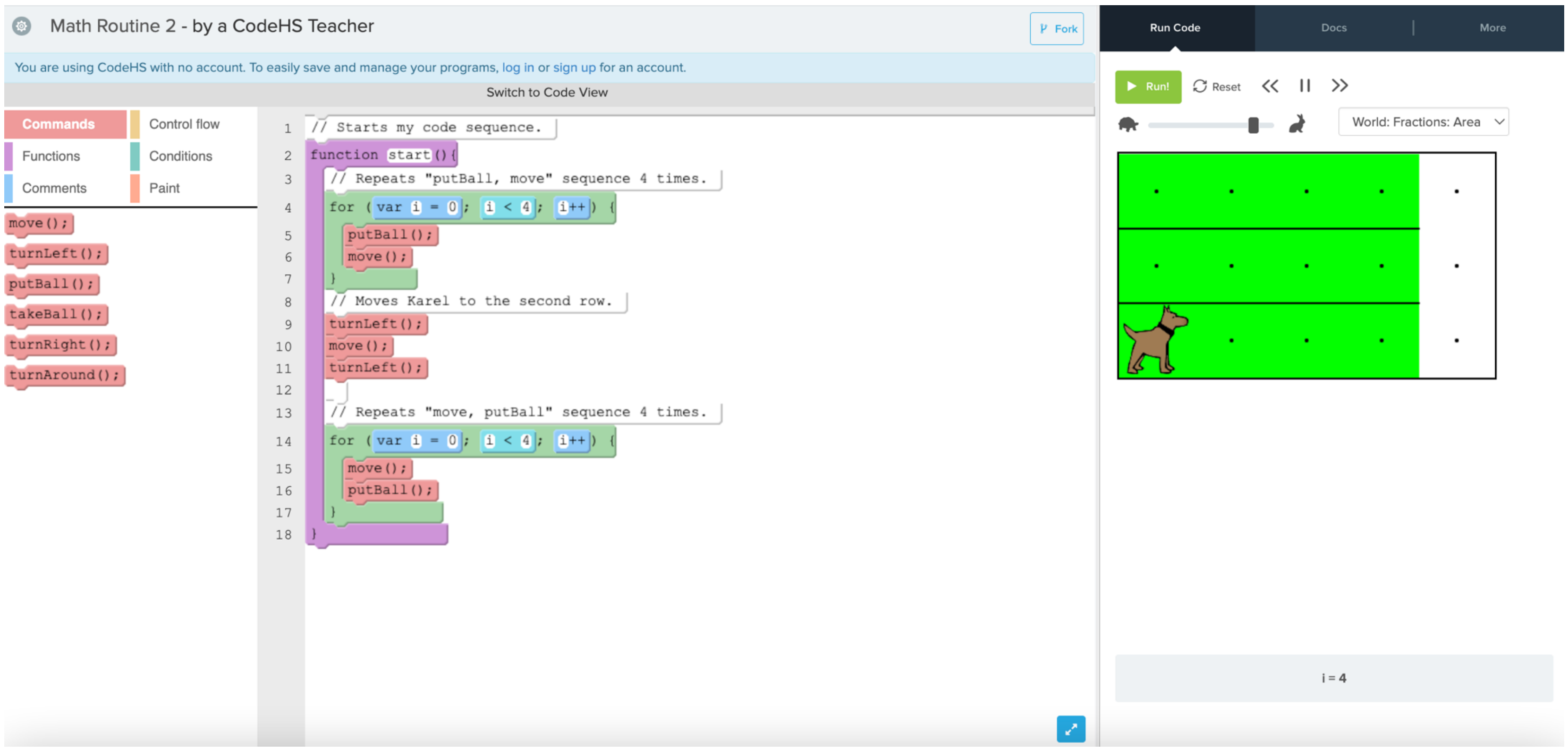
These visuals are tied to representations in *GoMath!*:



- The key goals for the "Karel Shows the Product" routine (#2 and #3) are to help students interpret multiplication as scaling (5.NF.5) and use visual fraction models to represent multiplication of fractions (5.NF.6).

- Another key goal for this routine is to provide another kind of area model to support students' understanding of multiplying a fraction by a fraction. In these visualizations from CodeHS, students use an area model for the first factor and Karel's tennis balls for the second factor, helping students see that they are finding a fraction of the shaded part, not the whole.
- A computer science goal for this routine is for students to engage in an anchor idea of algorithmic design. In math, students can model the fractional part of a group using a Code HS program and connect an equation to the modeling of the fractional part of a group. In computer science, students can read and discuss the changing "for-loop" that instructs Karel to clean up a fractional part of the tennis balls.
- Students will need a whiteboard and marker or paper and pencil to record the equation and circle parts of the equation.
- Optional Materials: In addition to the Karel-the-Dog CodeHS Sandbox, students will need a rectangular piece of paper divided into fifths with premade lines. Students will partition the rectangle into fifths by folding on the lines and then folding or shading into thirds (without the use of lines). If you choose to use this material, guide students to make connections among the paper-folding manipulative, the dynamic pictorial representation from CodeHS, and the equation on the whiteboard (both personal and teacher board).

I'm going to show an [activity from CodeHS]() (show). What do you notice about the visual?



Yes, there are 3 rows and 5 columns in the workspace. There is 4/5 of the workspace shaded green. We know that 4/5 will be a factor in our activities.

Now read the code. What do you think Karel is going to do with the tennis balls according to the code?

You are noticing the "for-loops," and each of those has Karel put a tennis ball down and move. You noticed the 4 in that line of code. The i or index lets us instruct Karel to put down a specified number of tennis balls.

We are going to have Karel shade 2/3 of 4/5 using the tennis balls. The number of tennis balls Karel puts down will actually tell us the numerator of the product of 2/3 x 4/5.

(<u>Optional:</u> Before we run the program, let's use our rectangular paper to represent this CodeHS visual. First fold your paper into fifths ("hamburger" orientation), using the lines. Then, fold your paper into thirds in the other direction ("hot dog" orientation). Do you see the connections between your folding paper and the Karel visual?)

Write the equation **2/3 x 4/5 = ?** on the board near the Karel image. Have students write the equation on their whiteboards.

Run: *Let's watch!* (run the program) *What did you notice?*
(Possible responses: Karel puts down 4 tennis balls, moves off the green to go around the wall, then puts down 4 more tennis balls; Karel puts down 8 tennis balls total.)
Students might not tell you the product of 2/3 x 4/5.

Ask and Write: *I told you that the number of tennis balls Karel puts down will tell us the numerator of the product of 2/3 x 4/5. What is the product? What does it mean in the image? Write your idea on your whiteboard. (Optional: Can you represent this with your paper?)*
(The product is 8/15. There are 8 tennis balls and 15 places to put the tennis balls so it is 8 balls out of 15 spaces, or 8/15. In other words, 8/15 is 2/3 of 4/5.)

Think-Pair-Share

Write: *Write the correct equation* **2/3 x 4/5 = 8/15** *on your whiteboards.*

Think: *What do you notice about the product? Is 8/15 larger or smaller than 2/3? What about 4/5? Why do you think the product is smaller than either of the factors?*

Pair and Share: *Explain your reasoning. Use the CodeHS image and your whiteboard (and your paper) to show your partner what you are thinking.*

Discuss: *I'm going to run the program again. Read this equation* (2/3 x 4/5 = 8/15 or "two-thirds of four-fifths is eight-fifteenths") *and watch Karel model it.*

Does this equation represent the CodeHS model?

Does counting the number of tennis balls tell you the product of 2/3 x 4/5?

What does the product mean in this CodeHS situation with Karel's tennis balls?

The product is what part of the whole park?

Teacher statement after discussion: *Today we learned to explain how our parts of our equations represent specific actions in a CodeHS model. We programmed Karel to put down a specified number of tennis balls, which gave us our product. We learned that a fraction multiplied by a fraction leads to a smaller product because it is a part of a part. You will practice abstraction in the computer lab to model fractional groups like we did today. You also saw "for-loops" in today's lesson. You will use "for-loops" in the computer lab to make the coding more efficient.*

#3 Math Routine: Karel Shows the Product! Why is the Product Getting Smaller?

In this routine, Karel shows a product by putting down a specified number of tennis balls (which represent the product). Students will discuss why the product gets smaller and how the representation shows that multiplying a fraction by a fraction is about a part of a part. They will compare and discuss yesterday's equation (2/3 x 4/5 = 8/15) with today's equation (1/3 x 4/5 = 4/15). Students will again need a whiteboard.

Equation #1: 1/3 x 4/5 = 4/15

1/3 x 4/5 = 4/15

Teacher codes and runs the program so Karel only puts down 4 tennis balls.

> **Think:** *Karel shows the product. The product is 4/15. What is the equation that Karel modeled for you?*
> **Write:** *Write your equation on your whiteboard. Check that it represents what Karel did and results in a product of 4/15.*
> **Pair and Share:** *Explain your reasoning.* (Possible responses: it is 1/3 of 4/5; 1/3 x 4/5 = 4/15)
> **Run:** *Let's run the program. Check your thinking.*
> **Discuss:** *What does the product mean in this situation? What does the 4 in 4/15 represent? What does the 15 in 4/15 represent? How did we get to that product of 4/15?* (Possible responses: students may jump ahead to standard algorithm; bring attention back to modeling)
> **Compare and Discuss:** *Let's look at yesterday's equation and today's equation.* (Write 2/3 x 4/5 = 8/15 and 1/3 x 4/5 = 4/15 on the board near the Karel CodeHS representation.) *What do you notice about the products? What do you notice about the representations?*
> **Statement:** *When we multiply, the product gets smaller because it is a part of a part.*

Equation #2: 3/3 x 4/5 = 12/15

3/3 x 4/5 = 12/15

Teacher runs the program. Karel puts 12 tennis balls (3 rows of tennis balls).

Think: K*arel shows the product. The product is 12/15. What is the equation that Karel modeled for you? (Optional: You can look at the model and use your folding paper.)*

Write: *Write your equation on your whiteboard. Check that it represents what Karel did and results in a product of 12/15.*

Pair and Share: *Explain your reasoning.*

Run: *Let's run the program. Check your thinking.*

Discuss: *What does the product mean in this situation? What does the 12 in 12/15 represent? What does the 15 in 12/15 represent? How did we get to that product of 12/15?*

(This is a great opportunity to discuss equivalence, that 3/3 = 1 and 12/15 = ⅘. This is also a great space to discuss the Identity Property - the product of 1 and any number is that number. In this case, 3/3 is equal to 1 and the Identity Property applies. At first, you might miss it if you don't see that ⅘ and 12/15 are equivalent fractions.)

Discuss: *What did you learn by modeling the fractional parts of groups with Karel and representing the actions with an equation?*

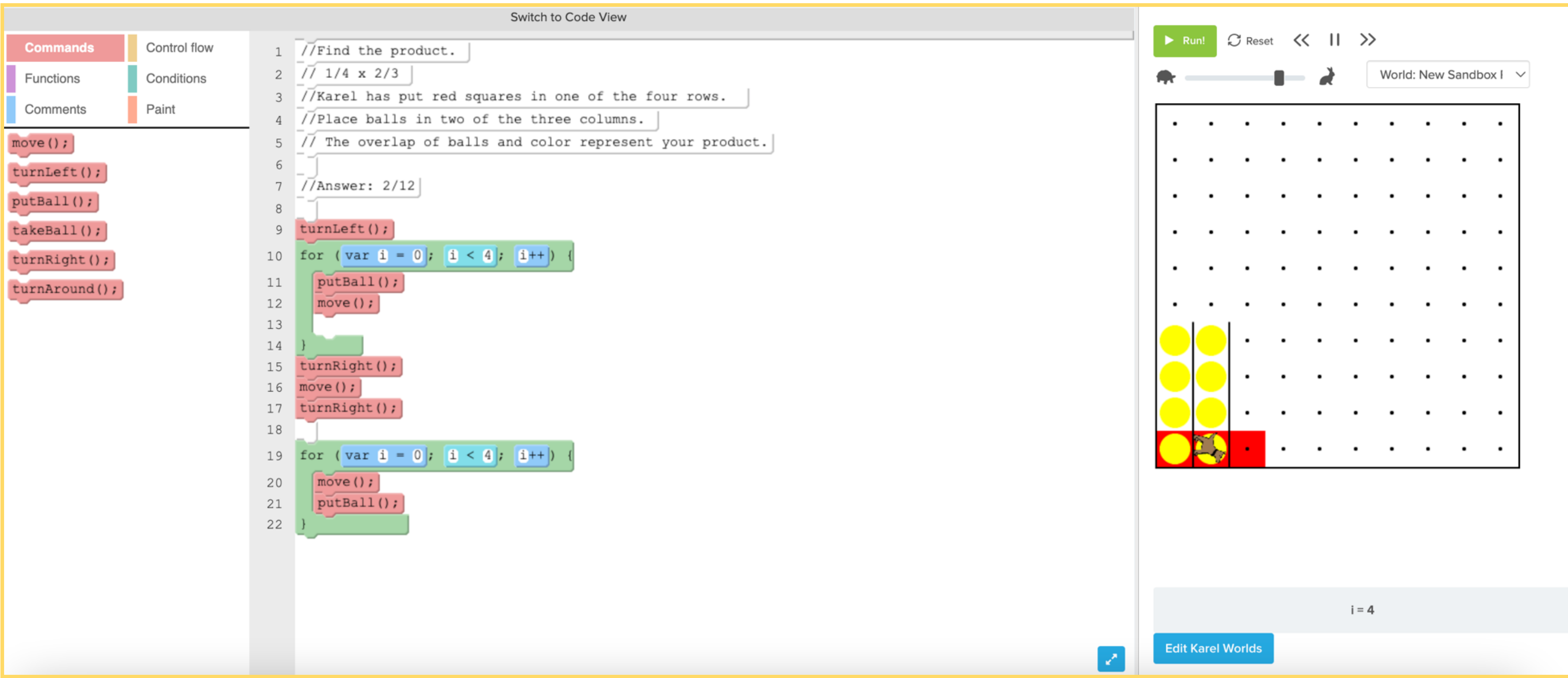
Teacher statement after discussion:

Today we learned about creating an equation to represent a fraction being multiplied by another fraction. In computer science, this representation is called abstraction. As we did today, you will practice abstraction in the computer lab to model multiplying fractions. You also saw "for-loops" in today's lesson. In computer science, this is part of algorithm design. You will use "for-loops" in the computer lab to make the coding more efficient by writing out a sequence of steps for Karel to repeat.

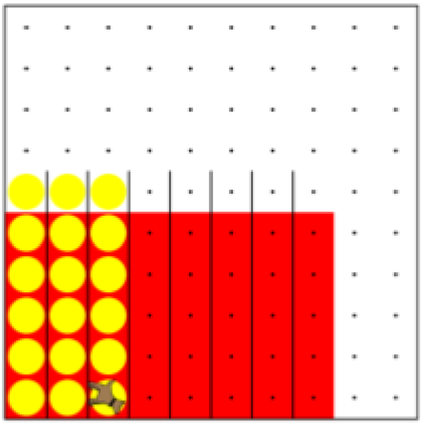
OPTIONAL EXTENSION: Math Routine: Karel Shows the Product! Why is the Product Getting Smaller?

This Sandbox is similar to Math Routines #2 and 3 but Karel is programmed to place balls in the overlap area as well as the empty space. There is also an expanded work area instead of isolating the view to a single problem.

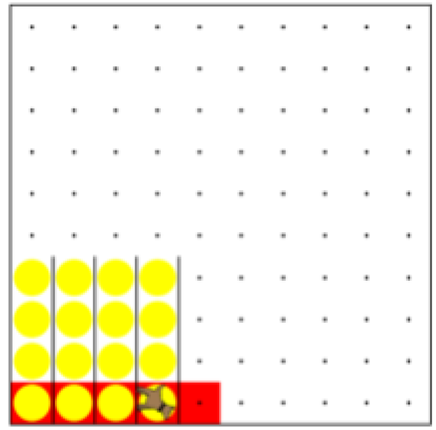
Shown below is 1/4 x 2/3 = 2/12. This activity could emphasize the "whole" in the part-whole relationship. Discussions could center around denominators – for example, why is the denominator in this problem 12?



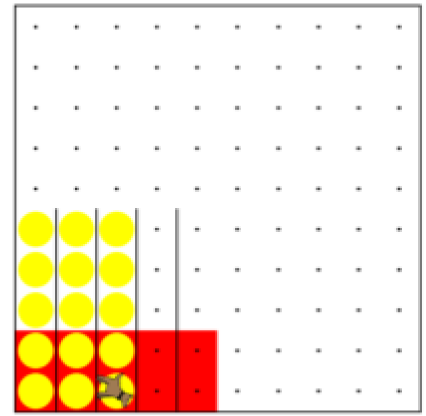
Optional images for teacher use:



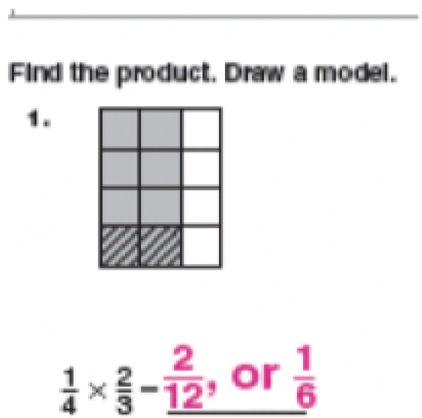
5/6 x 3/8 = 15/48

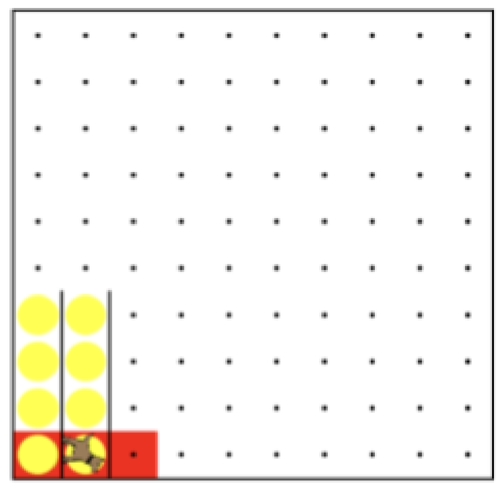


1/4 x 4/5 = 4/20



2/5 x 3/5 = 6/25





Go Math! and CodeHS models

NOTES about anchor ideas or cross-cutting concepts:

Math Class: understanding what an equation/algorithm represents
Computer Lab: abstraction & algorithmic design (writing a procedure for Karel’s actions)

What is the purpose of an equation/algorithm?
What is the purpose of a code?
What is the purpose of a for-loop or a function?

In math, the algorithm hides all the conceptual stuff. Helps us do efficient calculations, but there’s a whole lot of meaning behind an equation or an algorithm.