SMALL NUMBERS AND NUMERICAL INTERACTION:
THE BUILDING BLOCKS OF EARLY NUMERICAL COMPETENCIES?

Anneliesje Ceulemans

UNIVERSITEIT
GENT

FACULTEIT PSYCHOLOGIE EN
PEDAGOGISCHE WETENSCHAPPEN

Small numbers and numerical interaction: The building blocks of early numerical competencies?

Anneliesje Ceulemans

Promotor: Prof. dr. Anne Desoete
Copromotor: Prof. dr. Geert Crombez

Proefschrift ingediend tot het behalen van de academische graad
van Doctor in de Pedagogische Wetenschappen

2014

DANKWOORD/ACKNOWLEDGEMENTS

"As we express our gratitude, we must never forget that
the highest appreciation is not to utter words, but to live by them."
(J.F. Kennedy)

Zes jaar geleden werd mij de keuze gegeven tussen verlenging van mijn stage-ervaring of een nieuw avontuur aan de universiteit. Het doctoraatsproefschrift dat u nu in uw handen heeft, getuigt duidelijk van het feit dat ik koos voor de nieuwe uitdaging en tevens laatstgenoemde optie. Al hoop ik dat ik de voorbije jaren mijn dankbaarheid reeds voldoende heb getoond tussendoor; doe ik alvast hieronder nog een officiële poging om iedereen te bedanken die bijgedragen heeft tot dit eindproduct.

Allereerst bedankt, Petra, om aan mij te denken als kandidaat-assistent. Dank prof. dr. Roeyers om me deze kans te geven en mij deel te laten uitmaken van de onderzoeksgroep ontwikkelingsstoornissen. U introduceerde mij in het JOnG!-project wat een belangrijke aanzet was voor het hier gepresenteerde doctoraatsonderzoek.

Beste promotor en copromotor, graag wil ik jullie bedanken voor jullie begeleiding. Prof. dr. Annemie Desoete, bedankt om steeds (in mij) te blijven geloven en letterlijk te zeggen dat alles goed zou komen en dat ik mocht genieten van een menselijk zacht promotorschap. Het was een op zijn minst boeiend en bewogen traject, waar iemand als u welkom was. Prof. dr. Geert Crombez, bedankt voor het zorgzame oog waarmee u over mijn schouder mijn traject hebt mee gevolgd. Uw bereidheid, ook als vakgroepvoorzitter, om deze werkplek zo aangenaam mogelijk te maken en uw feedback om kwaliteitsvolle manuscripten af te kunnen leveren, werden enorm geapprecieerd. Ook aan alle andere leden van mijn begeleidingscommissie richt ik een welgemeende dank voor alle feedback, met in het bijzonder Erik Van Loosbroek. Uw overlijden kwam onverwacht. Het gaat u goed … Tom Loeys, ik zou bijna durven zeggen dat statistiek fijn werd met u erbij, maar zover ga ik niet. U maakte het daarentegen wel haalbaar dankzij zinvolle feedback en hulp. Uw aanspreekbaarheid en begrip voor alle vragen, maakten dat ik bij u terecht kon. Ook al kon statistiek niet steeds een antwoord bieden (toch niet zoals soms gehoopt), was het nooit verloren moeite. Ik wens u het beste in uw carrière.

Aan het secretariaat: Wouter, Annick en Sylvie, voor jullie hulp bij administratieve en technische beslommeringen worden jullie ook hier in de bloemetjes gezet. Sylvie in het bijzonder, bedankt voor je vrolijke aanstekelijkheid en gesprekjes tussendoor! Aan de kuisploeg: dank om tussen de 'rommel' de bureaus net te houden. Het FSA: bedankt, Leen en de laatste jaren vooral Anja, voor de goede samenwerking.

Graag geef ik vervolgens een plaatsje in dit dankwoord aan alle kinderen, jongeren en hun ouders die hebben deelgenomen aan het huidige doctoraatsonderzoek. Ik heb jullie immers lastig gevallen met soms te lang durende computertaken of vragenlijsten, waarvoor mijn excuses maar ook oprechte dank voor jullie inzet. Vooral jullie kleintjes: het was fijn jullie te zien zoeken naar stickers of balletjes of jullie te zien spelen en jullie te mogen opvolgen vier jaar lang. Ik heb genoten van jullie speelse lach, kindse eerlijkheid, soms oprechte koppigheid en het gewoon 'kind' zijn. Alle ouders, bedankt voor het steeds hartelijke contact. Veel van de voldoening dank ik aan jullie.

Het zou bedrog zijn dit doctoraatsonderzoek af te doen als een eenmansproject, want achter JOnG! ging een team van enthousiaste medewerkers schuil aan wie ik veel erkenning en respect toedraag. Eerst en vooral bedankt aan wie, hetzij kort, hebben mogen proeven van dit samenwerkingsverband: alle thesisstudenten voor en achter de schermen. Bedankt voor jullie extra hulp tijdens het 'draaien' van onze onderzoeksdagen of het codeerwerk achteraf. Jullie inzet en enthousiasme was lonend! Het officiële JOnG!-team kende zijn verloop doorheen de jaren, maar een woord van dank aan de kern. Dank aan de promotoren en Valérie Carette om jullie schouders onder dit project te zetten en het organiseren van vruchtbare vergaderingen en leerrijke studiedagen. Bedankt om onze wederzijdse interdisciplinaire inspanningen te erkennen. Speciale dank aan het onderzoeksteam, met in het bijzonder Cécile en Nancy voor de aangename samenwerking en het samen wachten op ouders en gezellige babbels tussendoor. Sofie, Sara, Nikita en Valerie, het was een eer om samen onderzoek te doen en naast gevloekt te hebben, ook veel gelachen te hebben (ook tijdens cinemabezoekjes en *sleepovers*!). Sofie, stel het goed in Israël met kleine Eden. Het is mooi om te zien dat alle JOnG!'ere collega's moeder werden. Die baby-cohorte was ons wel degelijk op het lijf geschreven :)

En het houdt niet op… Valerie, je was niet alleen een fantastische JOnG!-collega, maar samen met Daisy ook een dierbaar bureaugenootje tot op heden… Wat hebben we het fijn gehad op 130.017 de voorbije jaren. We konden het hebben over alles en niets, gaande van dagelijkse verzuchtingen tot vele speculaties over onze ongeboren kindjes. Alle vragen en wensen die we hadden, zijn nu ingevuld want *once and for all*: het is het beste wat ons ooit overkwam… Bedankt dat we alles met elkaar konden delen over onze kleine kindjes en soms (te) grote zorgen… maar toch ook vooral elkaars geluk en plezier. Dank je ook, Valerie, om via jou beter kennis te kunnen maken met Sara en Chloè tijdens leuke lunchbreaks of *afterwork* drinks, etentjes, spelletjes- of knutselavonden. Sara, het was spannend jou je doctoraat te weten afleggen en ontspannend je te horen genieten van de voorbereidingen voor je trouw (waaraan wij zelfs plezier beleefd hebben, al was het [voor jou toch] in het geniep :)) en nog meer fantastisch om je grote dag van dichtbij te mogen meemaken. Chloè, my dear, gelukkig mocht je nog langer in onze bureau blijven. Vanaf de eerste dag heb je een voetafdruk achter gelaten. Merci om gezonde slaatjes te bestellen via je mama en bedankt voor de (restjes van) dessertjes om te compenseren en alle liefs tussendoor. Valerie en Chloè, succes nog met jullie doctoraat!

Daisy, *last but not least*, jou kan ik niet genoeg bedanken, om naast teamgenootje, ook mijn *partner-in-pregnancy* te zijn (naast nog zoveel meer, maar geef toe, dit was toch wel erg bijzonder). Het heeft ons alleen nog maar dichter bij elkaar gebracht… Niet alleen zijn wij niet uit elkaars leven weg te denken, ook onze partners en kindjes maken deel uit van wat ik versta onder echte *quality time …* Dank je voor alle steun en vriendschap en zoveel meer dat ik niet in genoeg woorden kan vatten. Het was een hobbelige weg die we hebben afgelegd om tot dit eindproduct te komen, maar achteraan bij jou op de brommer, was het eens zo aangenaam. Nog even… en een nieuw hoofdstuk begint voor ons beiden. Alvast een welgemeende proficiat en succes! Ik kan alleen hopen dat we elkaar ook in de toekomst nog tegenkomen op professioneel vlak. Met niemand lukte het samenwerken immers op zo eenzelfde golflengte als met jou en dat hebben we in de definitieve eindspurt tot het indienen nog maar eens bewezen.

Van onze bureau de gang door, voorbij de keuken, tot aan bureau 130.046… een weg die jij en ik frequent aflegden naar de rest van ons ‘team Leerstoornissen’. Op de weg kwamen we vroeger nog Frauke en Stefanie tegen, nu enkel Jannes en Annemie.

Het is steeds een team geweest om trots op te zijn dat met hart en ziel een zo goed mogelijk onderwijs wilde organiseren, waarin we oog en oor hadden voor elkaar. Samenwerken was daarom steeds een fijne ervaring, net als de onderzoeksmeeting in het toen wel zeer koude Londen, maar ook het erg warme Cyprus in de winter dan nog wel. Af en toe mocht het eens iets meer zijn dan een zelf op te warmen croissant in een oven die er niet is, na een nacht in een te klein bed (al had ik daar geen last van)... nietwaar, Frauke en Stefanie? Ook jij, Lies, hebt deel uitgemaakt van dit team, al was het omdat je me verving. Bedankt om dit zo nauwgezet te doen! Je oprechte interesse voor de reden van de vervanging deed mij plezier en binnenkort weet jij er zelf alles van ;)

Overigens een algemene dank aan alle vroegere en huidige leden van de onderzoeksgroep ontwikkelingsstoornissen met wie we in totaal toch een aantal uren en dagen in lokaal 130.047 hebben gesleten. Velen heb ik weten komen, maar ook velen weer zien gaan. Met een vleugje nostalgie denk ik terug aan de prille start van dit traject toen we met veel samen zijn gestart in oktober 2008. Het lijkt enerzijds al zo lang geleden maar soms ook weer niet. Lieselot, Stefanie en Tinneke, wat hadden veel te ontdekken toen we hier pas waren, maar ik ben blij dat ik dit de eerste jaren met jullie heb kunnen en mogen doen. Het was fijn om jullie collega te zijn en met spijt in het hart heb ik jullie zien vertrekken. Inge, ik zal ook nooit mijn eerste congres vergeten. Je was super gezelschap en de associatie met “Billy Jean” blijft nog steeds (jij weet waarom :)).

Tinneke, het is alweer 6 jaar geleden dat we samen werden gezet in bureau 130.022. Het werd al vrij snel duidelijk dat je geen collega was met wie ik bevriend was. Je werd daarentegen een vriendin met wie ik naast een bureau nog zoveel meer deelde. De combinatie werk en privé was ons op het lijf geschreven: lange werkdagen met traditionele lunch om 13u en veelvuldig mailverkeer of *insomnia*-smsjes, maar ook (film)momentjes vanuit de zetel mét dekentje bij slecht weer of een ijsje/cocktail in de hand bij mooier weer… Het aanvankelijke plan mij te leren zwemmen alvorens je jouw doctoraat zou neerleggen, is mislukt. Toch is het dankzij jou dat ik tijdens de eerste vier jaar van dit traject het hoofd boven water heb kunnen houden. Dat we geen tekst en uitleg hoefden, maar elkaar al begrepen zonder woorden, heeft hier een hand in gehad. Ik mis je uiteraard op het werk, maar gelukkig hoef ik dat niet te doen daarbuiten. Bedankt voor je warme vriendschap sinds 1 oktober 2008 en tot een volgende date…

Aansluitend hierop een welgemeende dank aan al mijn vrienden om het leven meer te maken dan werk alleen en mij te herinneren aan wat belangrijk is: vriendschap.

Katia, nog een paar jaar en we kunnen onze ’20-jarige’ vriendschap vieren. Dat toont enerzijds aan dat we (wat) ouder worden, maar anderzijds getuigt het vooral van de mooie weg die wij al samen hebben afgelegd doorheen het leven. Je bent een prachtexemplaar, altijd daar voor een goede babbel en een lief gebaar. Bedankt om de laatste jaren de doctoraatsverhalen er met plezier als gespreksonderwerp bij te nemen.

Jente, onze wegen kruisten elkaar wat later, maar ook wij hebben al geschiedenis geschreven. Het blijft fijn om weten dat geen onderwerp te moeilijk of te zwaar is om aan te kaarten en met je ervaring op de unief, ken je ook als geen ander dit verhaal hier. Het was een opluchting te zien dat jij elders al snel je draai vond. Je doet dat super goed!

Lien en Jan, voor jullie zijn er geen woorden. Bedankt voor de leuke afleidingen en de lach die jullie steeds op onze gezichten toveren. Jullie zijn fantastisch en niet alleen voor ons. Ook voor Ameya, zijn jullie samen met Jente, de beste meter die ze kon hebben. Wij tellen samen met jullie af tot de geboorte van jullie wondertje en hopen zoveel en meer terug te kunnen doen voor jullie en het ongetwijfeld, naast Ameya, mooiste meisje :-) Lien, wat ik hier neerleg, staat jou te wachten en wat ga jij schitteren!

Pieter, Helena en Zoë, ook wij leerden elkaar kennen tijdens onze studies en ik ben blij dat wij elkaar nog steeds frequent tegenkomen op onze weg door het leven. Meer Julie’s house, terrasjes, lunchbreaks, uitstapjes, weekendjes, … ook na dit, aub! “*Den Astrid*”, ook aan jullie heb ik veel afleiding te danken de voorbije jaren en zelfs ver daarvoor. Bij jullie is het volledig ontspannen en daar zijn vrienden voor, toch?

Jonie en Danielle, ook jullie weten wat bergen werk verzetten is. Danielle, wij hebben dan niet meer samen gewerkt of geschreven de laatste jaren, maar toch was je nooit echt ver, zelfs niet op congres in China. Bedankt om ons ‘onderdak’ te verlenen :) Jonie, het combineren van de AILO-(stage) met ons werk was geen lachertje, maar het is ons toch maar gelukt! Proficiat ook met jouw (of beter jullie) nieuwe “prestatie”: Paulo.

Liesbeth en Alicia ook jullie kennen de knepen van het vak en zijn mij met glans voorgegaan. Jullie waren een voorbeeld – waarbij ik dacht: “Dit lukt mij nooit” – met elk jullie visie op de toekomst waarvan ik hoop dat die jullie alleen maar breed toelacht.

Liesbeth, het was fijn jouw *paranimf* te zijn op jouw verdediging en merci voor dat bonnetje om dit ‘werkje’ na te lezen: Ik heb beslist je er toch maar van te sparen :). Alicia, wanneer ik dit hier neerschrijf in september zullen jullie volop aan het aftellen zijn. Je droomjob, een nieuw huis en op de koop toe nog iemand erbij… Dik verdiend!

Verder is de steun van mijn familie mij zeker niet ontgaan, erg bedankt daarvoor. Moemoe en vava, net zoals bij enkele andere levensmijlpalen mis ik jullie en had ik jullie graag trots gemaakt bij het afsluiten van dit hoofdstuk. Moeke en vake, bedankt om altijd zo fier als een gieter te zijn op mij. Ik vermoed dat dit nu niet anders is. Weet echter dat ik dit nooit bereikt had zonder de kansen die jullie mij gaven. Jullie hebben mij een veilige thuishaven gegeven die ik misschien nooit gekend had en eerlijk… dat *werkethos* is ook iets dat jullie mij hebben meegegeven als ik denk aan onze kindertijd met bijvoorbeeld ons moeke druk in de weer voor haar klasje tot ’s avonds laat: herkenbaar? Gelukkig was het ook en plek om gewoon te spelen samen met Mieke en steeds een thuis om U tegen te zeggen, al heb ik al even mijn eigen huis en tuin in Gent.

Het is een huisje van vier dezelfde, maar wie even verder kijkt, ziet hoe bijzonder. Niet alleen wordt elk huisje meer en meer gesierd door eigen kenmerken, elk huisje wordt ook bewoond door stuk voor stuk mensen die ons eigen huis een ‘thuis’ maken. De voorbije jaren hebben we zoveel lief en gelukkig nog maar zo weinig leed mogen delen: Wolf, Aiko, Ameya, Marielou en Alice zijn daar het mooiste bewijs van. Zoveel herinneringen die ik reeds koester van de voorbije vier jaren: geen zomer is in te denken zonder jullie in de tuin of geen winter om bij elkaar op de koffie of de thee te gaan. Wannes en Nathalie, het is fijn te weten dat jullie naast ons wonen en Lieselotte, ook al woon je weldra misschien in Brussel, je blijft ‘onze’ buur. Maar het begon eigenlijk aan zijde van Hannele en PJ, wat maakt dat wij jullie des te meer zullen missen de komende jaren. Bedankt voor jullie begrip, luisterend oor, ontspannende babbeltjes en avondjes. Jammer dat jullie er niet bij kunnen zijn op de voorziene dag van mijn verdediging, maar een verslag komt dan jullie richting uit en wie weet in levende lijve, daar in Guatemala!

En helaas, voor wie mijn epistel toch graag helemaal leest: *the best is yet to come* (wat ben ik stiekem ook blij, dat dit op pagina 7[1] van dit dankwoord zal beginnen).

---

[1] Wie de link legt, lach; voor anderen: gewoon ‘beroepsmisvorming’ met een zwak voor bepaalde cijfers

Bij wie ik (nu bijna 10 jaar!) jaar en dag werkelijk thuiskom, dat ben jij, Tiemen. Dat ik van je hou, hoef ik niet meer te schrijven. Dat ik je niet genoeg kan bedanken, wil ik wel nog een plaats geven. Ondanks het hoge *workaholic* gehalte (laten we het bij naam noemen) dat ik de voorbije jaren aan de dag heb gelegd, bleef jij steeds mijn alomtegenwoordige steun en toeverlaat[2]. Gelukkig had je al enige referentie van tijdens onze studies en kon je hiermee om. Dankzij jou waren de voorbije zes jaren tot hiertoe de mooiste van mijn leven, het was immers in deze periode dat we samen ons huisje met tuintje kochten en dit bezegelden met een prachtige huwelijksviering van ons '7 jaar en 7 maanden' samenzijn. Wisten wij veel toen we hoopten op '*Win for Life*' dat het geluk iets veel groter voor ons in petto had dat zou staan voor grenzeloze liefde: Ameya. Ik dacht dat er niemand in de wereld bestond die ik zo graag kon zien als jouw papa. Maar het leven leert je steeds dingen bij en jij leerde mij, vanaf de eerste seconde dat ik je zag, dat liefde inderdaad geen grenzen kent… Eén blik naar jou en ik kan de wereld weer aan... Jij bent mijn allergrootste trots en het liefste en mooiste meisje dat ik ken… Ik mag alleen maar hopen dat jij niet teveel last hebt gehad van mama's stress (papa kan daar wel tegen ;)) het voorbije jaar en anders hoop ik dit vanaf nu weer goed te maken… Ik kan je alleszins op het hart drukken, dat als er één iets was dat voorrang kreeg, dat jij dat was. Geen doctoraat of welke andere tijdsbesteding ook, kan dat in de weg staan. Tiemen en Ameya, mijn alles en meer, willen jullie bij het lezen van het einde van dit lange dankwoord, samen met mij de bladzijde naar een nieuw hoofdstuk omslaan?

"I may not have gone where I intended to go,
but I think I have ended up where I needed to be"
(D. Adams)

Annelies(je) Usha Ceulemans,
September 2014

---

[2] Ik wil in de zijlijn van dit epistel ook nog An en Phil bedanken om mij een extra 'thuis' te geven bij hen de voorbije jaren. Het is altijd fijn te weten dat er extra mensen paraat staan voor raad en daad of babysit :) Dankzij jullie konden wij af en toe genieten van nog wat extra ontspanningstijd tussendoor. Dikke merci!

CONTENTS

Chapter 1 *General introduction* 1

Chapter 2 *Enumeration of small and large numerosities in adolescents with mathematical learning disorders* 45

Chapter 3 *Small number discrimination in early human development: The case of one versus three* 69

Chapter 4 *The sense of number sense: The predictive value of number discrimination in infancy and toddlerhood for numerical competencies in kindergarten* 85

Chapter 5 *A guided walk into numerical competencies in kindergarten: The predictive value of numerical mother-child interaction in toddlers* 111

Chapter 6 *General discussion* 161

Nederlandstalige samenvatting 203

CHAPTER 1

GENERAL INTRODUCTION

In this chapter, the theoretical background of this doctoral research is outlined. In light of gaining insight in typical and atypical mathematical development, there is a growing interest in *number sense* at early age as a predictor of later achievement. Although number sense has already been studied in kindergarten, early risk detection might be accelerated even more through the exploration of budding number sense from infancy on. In addition, attention can be given to the parental influence at this early age. As yet, research on these topics is still in its infancy. Along with the general aims of the current research, an overview of the chapters included in this dissertation is given.

Numbers are a pervasive feature of our everyday life and numerosity itself is one of the most abstract dimensions of our environment (Rousselle & Noël, 2007, 2008). Unlike color, shape, or texture, numerosity is not a property of individual objects, but a property of collections, which might include extremely different entities. Nonetheless, the necessity to manipulate quantities and numbers pervades numerous everyday life activities (Kaufmann & Dowker, 2009). Even though, in spite of their abstract nature, it is quite remarkable to see how easily people perceive, use, and manipulate these numerosities and numbers (Rousselle & Noël, 2008). From early childhood, children develop in this environment rich in quantitative information and numerical experiences (Rousselle & Noël, 2007). First, they observe for example one, two, or maybe even five objects flying above them when they lay in their pen; or they play with three balls. Next, they hear adults using numbers to count, to measure, to use money, or to tell the time and give the date; or they see Arabic symbols in shops, in streets, and in games as well as on cars, on book pages, or on television. From two years on, children become able to count themselves and learn to grasp the quantitative relationship between collections. They acquire the smallest number words (up till three) at first, which leads them consecutively to grasp the larger numbers (from three on) at a later stage (Mix, 2009). As such, not surprising, most children come to kindergarten with some sense of number (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). Besides this well-known fact that children learn to count in toddlerhood, it is even more remarkable that infants are found to be sensitive to quantity already in the first year of life (Wynn, 1992). Indeed, infants nonverbally discriminate sets of numerosities (Brannon, Abbott, & Lutz, 2004; Cordes & Brannon, 2008, 2009a; Starr, Libertus, & Brannon, 2013a; Van Loosbroek & Smitsman, 1990; Xu, 2003; Xu & Spelke 2000; Xu, Spelke, & Goddard, 2005).

Small number discrimination

A review of the literature shows that from infancy on children rely on two systems to process quantities (Feigenson, Dehaene, & Spelke, 2004; Xu, 2003). It is mainly assumed that these two systems are differentiated one from another based on the number of items that could be discriminated by means of each system separately.

The first system, the *object-file* system, allows for a discrete and exact representation of a limited number of items (Kahneman & Treisman, 1984; Leslie, Xu, Tremoulet, & Scholl, 1998). This system originates from visual attention literature (Kahneman, Treisman, & Gibbs, 1992; Trick & Pylyshyn, 1994) and proposes that for sets of three and fewer an exact one-to-one correspondence representation of items is used. This allows for a precise discrimination between numerosities in the so called "small" number range, enabling *small number discrimination* (e.g., Feigenson & Carey, 2003). The object-file system differentiates itself from the *analog magnitude* system that supports a less precise, approximate representation of a larger set of items (Feigenson et al., 2004). Number discrimination through this second system is ratio dependent. This means that the relation between two given numerosities is its key feature or, in short, that numerosities with a larger mutual ratio are easier to be discriminated. For example, 6-month-olds discriminate differences at a 1:2 ratio (8 vs. 16), but not at a 2:3 ratio (8 vs. 12) as shown in the study of Xu and Spelke (2000). With age, however, this ability to discriminate larger sets becomes more precise, meaning that two numerosities with a smaller ratio can successfully be discriminated as well (Xu & Arriaga, 2007). Nonetheless, based on infant studies investigating number discrimination, it is generally assumed that the analog magnitude system operates on numerosities larger than three, enabling *large number discrimination* (e.g., Xu et al., 2005).

Although this "divide" between the two systems has long prevailed number discrimination literature (see Cantrell & Smith, 2013 for a review), the connection between set sizes (i.e., small vs. large) and these systems (i.e., object-file vs. analog magnitude) might be less strict. The claim that small numerosities are only processed by object-files, for example, is tentative. Indeed, from the successful discrimination of small from large numerosities (e.g., 1 vs. 4, Cordes & Brannon, 2009a), it could be suggested that the analog magnitude system may also be used to process small next to large numerosities (Cordes & Brannon, 2009a). This is further supported by ratio dependency of small number discrimination (i.e., success for 1 vs. 3 and 1 vs. 2, but failure with 2 vs. 3) in Starr et al. (2013a). Just as for large number discrimination, success thus depended on the ratio of two offered numerosities in line with the ratio dependency characteristic. Nevertheless, this does not have to preclude the existence of the object-file system.

Subitizing

The object-file system is involved not only in abilities in infancy but also within the context of processing small numbers later on in childhood. From the age of 4 years, children are able to enumerate numerosities up till three in a rapid and accurate way, which is referred to as the ability to *verbally subitize* (Lefevre, Fast, Skwarchuck, Smith-Chant, & Bisanz, 2010). The term subitizing was originally used by Kaufman, Lord, Reese, and Volkmann (1949) to define the rapid (40-100 ms/item), automatic, and accurate assessment of small quantities of up to three (or four) items (Clements, 1999; Kaufman et al., 1949; Koontz & Berch, 1996; Nan, Knösche, & Luo, 2006; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008; Trick & Pylyshyn, 1993). As with small number discrimination, the object-file system is assumed to underlie this ability, but the same controversy between the object-file and analog magnitude system also pervades research on the underlying mechanism of verbal subitizing (Dehaene & Changeux, 1993; Feigenson et al., 2004; Revkin et al., 2008).

NUMBER SENSE

Both the ability to nonverbally discriminate small numerosities (i.e., small number discrimination) and the ability to verbally subitize can be subsumed in the concept of *number sense* (e.g., Jordan, 2007; Kaminski, 2002; Wagner & Davis, 2010), which is sometimes also referred to with terms such as *early numeracy* (Aunio, Hautamaki, Sajaniemi, & Van Luit, 2009; Bryant et al., 2011; VanDerHeyden et al., 2001) or *early numerical competencies* (e.g., Powell & Fuchs, 2012).

Nonetheless, regardless of which term or even which definition is given (Berch, 2005), the construct always refers to foundations of building competence in math (Powell & Fuchs, 2012). Therefore, most researchers agree that number sense involves abilities related to counting, magnitude comparisons, number patterns, estimating, and number transformations (Berch, 2005) or, in short, number processing abilities (Dehaene, 2001; Dehaene, Piazza, Pinel, & Cohen, 2003; McCrink & Wynn, 2004).

Gersten, Jordan, and Flojo (2005) concluded that the development of number sense can be enhanced by (in)formal instruction prior to entering school. Moreover, number sense was shown to lay the foundation for learning formal math concepts and skills (Gersten & Chard, 1999; Jordan, Kaplan, Locuniak, & Ramineni, 2007).

In general, two perspectives on number sense can be found in the literature (Berch, 2005). The first low-order perspective defines number sense as a biologically based or innate perceptual sense of quantity that appears to develop without or with little verbal input or instruction early in life (Butterworth, 1999; Dehaene, 1997) and that is restricted to nonsymbolic stimuli (Jordan & Levine, 2009). This view limits the features of number sense to elementary intuitions about quantity (Dehaene, 1997, 2001; Geary, 1995), constituting a primary, preverbal, innate, and nonsymbolic number sense (Butterworth, 1999; Dehaene, 1997; Jordan & Levine, 2009). This in turn, forms the basis for the development of a secondary, verbal kind of number sense (Feigenson et al., 2004). In contrast with the first perspective on number sense, this second perspective considers number sense as highly dependent on the input a child receives (Clements & Sarama, 2007). From this perspective, number sense is considered as a higher-order and acquired conceptual sense-making of mathematics (Berch, 2005) that is much more prone to experience and schooling, and involves symbolic stimuli (Jordan & Levine, 2009). Although the foundational lower order perspective is incorporated in the higher order perspective as well, number sense now comprises also an understanding of mathematical principles and relationships, fluency and flexibility with operations, procedures and numerical expressions involving a solid symbolic number knowledge.

To summarize, a primary, lower order, preverbal, innate, and nonsymbolic number sense needs to be distinguished from a secondary, higher order, verbal, acquired, and symbolic number sense. These two closely connected perspectives on number sense are not in contrast with each other. The preverbal number knowledge provides the foundation for the symbolic number knowledge (Jordan & Levine, 2009). Moreover, it is agreed on that these early symbolic (and verbal) number competencies are necessary for extending knowledge with small numbers to knowledge with larger numbers, and learning school-based mathematics (Jordan, Glutting, & Ramineni, 2010).

A sense of number: subitizing and small number discrimination

Whether subitizing can be covered by either the preverbal or verbal perspective on number sense, depends on how subitizing as an ability is operationalized or described. The traditional use of the word subitizing implies a perceptual-verbal ability that uses number words to express numerosity (Benoit, Lehalle, & Jouen, 2004). This is in line with the initial consideration of subitizing as an explicit quantification operator (Klahr & Wallace, 1976 following Kaufman et al., 1949). Using this operationalization, subitizing can be perceived as an aspect of verbal number sense (i.e., following the second perspective). According to Benoit et al. (2004), the subitizing ability could also be operationalized, though, as a perceptual-preverbal ability. Verbal naming is not required in this format, but the focus is on numerosity whatever the perceptual configuration of number (Benoit et al., 2004). This description of subitizing is covered by the primary or preverbal perspective on number sense, which limits the features of number sense to quantitative intuitions that also include the rapid and accurate perception of small numerosities (Dehaene, 1997, 2001; Geary, 1995). According to this view, the development of number sense starts with a precise representation of small numbers (Feigenson & Carey, 2003; Watson, Maylor, Allen, & Bruce, 2007), which is – in its turn – involved in this perceptual-preverbal operationalization of subitizing. Such an early developmental trajectory of number sense is supported by evidence on infants' and toddlers' successful small number discrimination (e.g., Cordes & Brannon, 2009b; Feigenson & Carey, 2003, 2005), which is considered as a low-order (Berch, 2005), basic, and/or innate (Xu & Arriaga, 2007) number sense.

A sense of number: numerical competencies in kindergarten

Both kinds of subitizing (i.e., perceptual-verbal subitizing and perceptual-preverbal subitizing, from now on referred to as verbal subitizing and small number discrimination respectively) can be understood as part of a developmental pathway that ends with true awareness of the cardinal meaning (e.g., knowing that the word two refers to sets with two items) of small number words (Benoit et al., 2004).

Benoit et al. (2004) confirmed that children are better able to name small sets up till three when stimuli are represented simultaneously instead of sequentially. Therefore, verbal subitizing (and not counting) appears to be the developmental pathway for acquiring the meaning of the first few number words (Benoit et al., 2004). Children thus first map number words onto small sets of three or less items through subitizing (e.g., Le Corre, Van de Walle, Brannon, & Carey, 2006). For larger sets, counting is needed. As children learn to count and understand cardinality, they further learn to represent larger numbers and see that each number has an unique successor (Le Corre & Carey, 2007; Sarnecka & Carey, 2008). Furthermore, verbal subitizing may not only be considered as a foundation or precursor of counting, it may also be used in addition and subtraction (Powell & Fuchs, 2012). Counting and arithmetic operations are closely related too, because counting is also often involved in solving addition and subtraction number combinations (Baroody, Bajwa, & Eiland, 2009). Both addition and subtraction (arithmetic operations) and counting can be considered as higher order number sense (Jordan & Levine, 2009). Following Powell and Fuchs (2012), the term *numerical competencies in kindergarten* is used here to delineate these abilities.

Counting. Counting has been described as the key ability forming the bridge between innate number sense and more advanced culturally expected arithmetic abilities (Butterworth, 2005b). Although closely related, two separately mastered aspects of counting need to be distinguished (Dowker, 2005). Procedural counting is defined as children’s ability to perform a mathematical task, for example, when a child can successfully determine that there are five objects in an array (LeFevre et al., 2006). Conceptual counting constitutes the understanding of the five principles formulated by Gelman and Gallistel (1978), namely, three essential (i.e., word-object correspondence, stable order, and cardinality) and two unessential principles (i.e., abstraction and order irrelevance) that do not result in incorrect counting. Conceptual counting knowledge reflects therefore children’s understanding of why a procedure works or whether a procedure is legitimate (Bisanz & LeFevre, 1992; Hiebert & Lefevre, 1986; LeFevre et al., 2006). Previous studies indicated that children master the essential principles at the age of 4 to 5 years (Le Corre & Carey, 2007; Stock, Desoete, & Roeyers, 2009).

Arithmetic operations. Arithmetic operations find themselves, on their turn, on the border between early numerical competencies (such as counting) and more advanced mathematical knowledge acquired through formal schooling (Purpura & Lonigan, 2013). These exercises prompt the understanding of the composition and decomposition of groups by differentiating sets and subsets (Purpura & Lonigan, 2013). The ability to solve purely nonverbal calculation exercises emerges at around 4 years of age (Levine, Jordan, & Huttenlocher, 1992), whereas many children have an understanding of numerical transformations (i.e., addition and subtraction) in verbal story problems or number fact problems at 5 or 6 years of age (Huttenlocher, Jordan, & Levine, 1994; Levine et al., 1992). The ability to solve visually supported story problems emerges somewhere in between.

THE IMPORTANCE OF NUMBER SENSE

The sense of number sense for mathematical abilities

Most children enter kindergarten demonstrating some degree of number sense (Powell & Fuchs, 2012). Individual differences, however, exist as shown by a diversity in mathematical knowledge (Klibanoff, et al., 2006; Zulauf, Schweiter, & von Aster, 2003). Whereas some children have an impressive array of math skills, others evince fewer skills: some children already appreciate quantities, know their number names, and match sets based on their cardinality or order sets in terms of numerosity, and are able to solve simple additions and subtractions, whereas others still struggle to identify numbers and barely count from 1 to 10 (Klibanoff et al., 2006; Lembke & Foegen, 2009; Zulauf et al., 2003). These individual differences motivated researchers to study number sense in relation to and as a predictor of later math achievement (e.g., DiPema, Lei, & Reid, 2007; Dowker, 2008; Mazocco & Thompson, 2005; Jordan et al., 2007; Stock, et al., 2009). As such, numerical competencies in kindergarten were found to predict later achievement on primary school arithmetic (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Desoete & Grégoire, 2006; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Stock et al., 2009). Indications for the importance of numerical competencies can be found in these studies (Powell & Fuchs, 2012). This can be illustrated for some number sense aspects (i.e., verbal subitizing, counting, and arithmetic operations) in kindergarten that are pivotal for this dissertation.

First, various studies demonstrated that verbal subitizing is an important factor in mathematical development (Landerl, Bevan, & Butterworth, 2004; Penner-Wilger et al., 2007; Träff, 2013), and longitudinal research showed that verbal subitizing is a domain-specific predictor for later mathematical performance over and above domain-general abilities (Krajewski & Schneider, 2009; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; LeFevre et al., 2010; Reigosa-Crespo et al., 2012).

Second, whereas procedural counting knowledge is predictive for numerical facility, conceptual counting knowledge predicts untimed mathematical achievement (Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009). Counting as a whole, in its turn, influences the development of adequate mathematical abilities and early mathematical strategies (Aunola et al., 2004; Fuson, 1988; Le Corre et al., 2006; Wynn, 1990).

Third, several studies demonstrated a relationship between arithmetic operations and math achievement (Jordan et al., 2009, 2010). Furthermore, arithmetic operations, as part of a larger early numerical competencies battery, have been proven predictive for later mathematical abilities and, in particular, for applied problem solving (Jordan et al., 2010).

The sense of number sense for mathematical disorders

Empirical evidence suggests that the earlier at-risk children are detected, the better this is to prevent the impact of learning disorders later on (Coleman, Buysse, & Neitzel, 2006). It is therefore imperative to reveal early predictors of *mathematical learning disorders* (MLD). In the Diagnostic and Statistical Manual of Mental Disorders (5th ed.; DSM-5; APA, 2013), the term MLD refers to the specific learning disorder with a significant degree of impairment in mathematics, manifesting itself in difficulties with number sense, mastering number facts, mathematical reasoning, or calculation skills.

Predictors and core deficits might be addressed as key targets in remediation (Gersten et al., 2005). Within this scope, there has recently been much interest in such factors in kindergarten (e.g., Krajewski & Schneider, 2008, 2009; Mazzocco & Thompson, 2005; Stock, Desoete, & Roeyers, 2010). Meanwhile, several predictors have frequently been studied and have increasingly been found to be an impairment in MLD (Butterworth, 2005; Fisher, Gebhart, & Hartnegg, 2008; Landerl et al., 2004; Mussolin, Mejias, & Noël, 2010; Piazza et al., 2010; von Aster, & Shalev, 2007; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). This again can be illustrated for the pivotal kindergarten concepts of the current dissertation, namely, verbal subitizing, counting, and arithmetic operations, as will be discussed in detail in the section below.

Verbal subitizing is considered as a core deficit in atypical numerical processing by some authors (e.g., Fischer et al., 2008; Schleifer & Landerl, 2011). This is supported by studies in which children with MLD serially count items within the subitizing range, whereas typically achieving (TA) children subitize the same amount of items (e.g., Bruandet, Molko, Cohen, & Dehaene, 2004; Butterworth, 1999; Moeller, Neuburger, Kaufmann, Landerl, & Nuerk, 2009). Although it is demonstrated that children with MLD are in general slower in subitizing tasks compared to TA children (e.g., Koontz & Berch, 1996; Landerl et al., 2004; Schleifer & Landerl, 2011), some studies additionally revealed that this verbal subitizing deficit does not necessarily applies for all children with MLD. Desoete and Grégoire (2006), for example, found a subitizing deficit in 33% of the children of 8.5 years old with a clinical diagnosis of MLD. In addition, Fischer et al. (2008) found that between 43% and 79% of the subjects in the age range of 7 to 17 years with MLD performed below the 16th percentile of the peer control groups on subitizing tasks. As for number sense in terms of counting and arithmetic operations, deficient counting abilities (e.g., Dowker, 2005; Geary, Bowthomas, & Yao, 1992; LeFevre et al., 2006) and lower performance on mathematical story problems have been found in children with MLD compared to TA peers (Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000).

PARENT-CHILD INTERACTION

In line with Berch (2005), number sense can be considered as an innate preverbal basic sense of number that evolves through experience to a more complex acquired number sense including the understanding of mathematical principles and procedures. Individual differences in children's number sense might therefore not solely be due to children's (innate) characteristics or abilities, but also vary depending on received input. This input can be formal schooling as soon children enter kindergarten (e.g., Berch, 2005; Gersten et al., 2005; Kroesbergen, Van Luit, & Aunio, 2012), which cannot account, though, for differences earlier in life when schooling is not yet applicable.

In this context, children's home experiences including *numerical parent-child interaction* step into the limelight. For primary school aged children, but also younger children from kindergarten age, the frequency of such home activities between parent and child relates positively to the children's numerical abilities (e.g., Benigno & Ellis, 2004; Huntsinger, Jose, Larson, Balsink Krieg, & Shaligram, 2000; LeFevre, Clarke, & Stringer, 2002; Kleemans, Peeters, Segers, & Verhoeven, 2012; LeFevre et al., 2009; Pan, Gauvain, Liu, & Cheng, 2006; Tudge & Doucet, 2004). Overall, children advantaged in terms of home number experiences have better mathematical knowledge or number sense than those who are disadvantaged. The latter ones are less or even not encouraged at all by their home environment to engage in mathematical or numerical activities (e.g., Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009).

Early numerical parent-child interaction

A considerable variability can be found in the frequency and type of numerical activities in which kindergartners are regularly engaged at home (e.g., Blevins-Knabe & Musun-Miller, 1996; Saxe, Guberman, & Gearhart, 1987). As Powell and Fuchs (2012) pointed out, exposure to early numerical activities plays an important role in the establishment of early numerical competencies for kindergartners (Baroody & Benson, 2001; Jung, 2011; Skwarchuk, 2009): the more exposure through games, stories, or play, the more they understand the building blocks of mathematics (Ramani & Siegler, 2008).

Although research on this topic in children who did not yet enter kindergarten is scarce, encouraging parents to interact already with their toddlers on numeracy might also positively impact upon children's mathematical attainment, as illustrated by Levine, Suriyakham, Rowe, Huttenlocher, and Gunderson (2011). This research group (Levine et al., 2011) demonstrated that in 14- to 30-month-olds the frequency of parental talk about numbers predicted the children's cardinal knowledge at 46 months of age. Consequently, not only the importance of concurrent numerical parent-child interaction for mathematical abilities in kindergarten (e.g., Blevins-Knabe & Musun-Miller, 1996) is supported, but also the predictive value of these kinds of interactions or activities at younger age. As such, findings on early numeracy experiences might resemble the well-established effect of early literacy experiences on the development of literacy later on from kindergarten on but also from toddlerhood and even from infancy (e.g., Dieterich, Assel, Swank, Smith, & Landry, 2006; Hood, Conlon, & Andrews, 2008; Karrass & Braungart-Rieker, 2005; Pancsofar, Vernon-Feagans, & Family Life Project, 2010; Roberts, Jurgens, & Burchinal, 2005).

OUTLINE OF THIS DISSERTATION

Empirical evidence suggests that it is imperative to reveal early predictors within the scope of risk detection to prevent the impact of MLD later on (Coleman et al., 2006), because these factors might be addressed as key targets in remediation programs (Gersten et al., 2005). In MLD, there has recently been much interest in aspects of number sense (Butterworth, 2005; Fischer et al., 2008; Landerl et al., 2004; Mussolin et al., 2010; Piazza et al., 2010; von Aster, & Shalev, 2007; Wilson et al., 2006) in kindergarten (e.g., Krajewski & Schneider, 2008, 2009; Mazzocco & Thompson, 2005; Stock et al., 2010). Since individual differences in number sense differentiate between children's later outcome, early detection of kindergartners at risk for MLD became possible (e.g., Dowker 2008).

Nevertheless, number sense is assumed to be present even before this kindergarten age in the more basic form of number discrimination (Xu & Arriaga, 2007). As a consequence, individual differences in number sense may already occur from infancy on, which has already been confirmed by Libertus and Brannon (2010) – as far as known pioneers in this topic. Up till now, however, research on number sense focused mainly on kindergartners but not on younger infants or toddlers. In addition, studies on number discrimination in these younger children mainly described group results, thus ignoring individual differences between children.

From the preverbal perspective on number sense, research on number sense may however focus on younger children, considering (small) number discrimination as a suitable candidate to predict (later) typical and atypical mathematical development. In typical development this question could already be answered confirmative for infants' large number discrimination, because this was found to predict mathematical abilities later on (Starr, Libertus, & Brannon, 2013b). Information on whether small number discrimination holds the same promise is not yet available. Therefore, individual differences in small number discrimination need to be studied at first, to determine whether this allows for differentiation between young children's mathematical outcome. This in turn, could enable very early risk detection, if below-average number discrimination performance would relate to atypical and, more specifically, impaired mathematical development. The other way around, better or above-average number discrimination performance at very young age might additionally reveal strengths in mathematical development later on.

In typical mathematical development, next to (the more evident) child competencies to predict later outcome, attention is also given to (primary) environmental aspects, such as numerical parent-child interaction. Exposure to this specific kind of interaction plays an important role in the establishment of early numerical competencies for kindergartners (Baroody & Benson, 2001; Jung, 2011; Skwarchuk, 2009). Although research on this topic in children who did not yet enter kindergarten is scarce, encouraging parents to engage in numerical activities with their toddler (i.e., at younger age than kindergarten age) might be rewarding for children's mathematical attainment as illustrated by Levine et al. (2011).

Research objectives

As a contribution to the described line of research, this doctoral thesis aimed at expanding on the knowledge about the value of small number discrimination and numerical parent-child interaction (i.e., mothers) for either children's concurrent or later numerical competencies in kindergarten. To achieve this main goal, three more specific research objectives were addressed in four separate empirical studies. Each of these studies are reported in the following chapters respectively. The first two studies had a cross-sectional design. In the last two studies, children were followed up longitudinally. Before introducing the chapters in particular, the main goals of this dissertation are outlined below. For more information on how each of the pivotal concepts were assessed, see Appendix (attached to this chapter) for a more detailed description.

First, the two mentioned components modeling the developmental pathway of subitizing (i.e., the traditional verbal format of subitizing and number discrimination as its preverbal variant) were explored in atypically and typically developing adolescents (*Chapter 2*) and typically developing infants (*Chapter 3*) respectively.

Second, from this preverbal perspective on number sense, it was further explored whether small number discrimination could indeed be considered as a precursor of later mathematical abilities or numerical competencies in kindergarten (*Chapter 3* and *4).* Besides aiming at testing a candidate number set (i.e., 1 vs. 3) to further investigate small number discrimination, it was questioned whether discrimination of these numerosities in infancy and toddlerhood could differentiate between children's number sense in kindergarten. Moreover, it was tested how both number discrimination performances (i.e., in infancy and toddlerhood) mutually relate.

Third, this doctoral research intended to incorporate both a perspective on early child characteristics such as number discrimination performance (*Chapter 4*), as well as a parental perspective focusing on early *numerical mother-child interaction* (*Chapter 5*). So, at last, it was studied whether number sense in toddlerhood (i.e., small number discrimination) and kindergarten (i.e., numerical competencies in kindergarten) could be explained from a perspective focusing on a specific kind of mother-child interaction.

Chapter overview

All the chapters correspond to individual manuscripts that are published (*Chapter 2* and *Chapter 3*), under review (*Chapter 4*), or have been submitted (*Chapter 5*). Chapters may therefore partially overlap as each manuscript is self-containing.

Chapter 2 explores the verbal subitizing abilities of adolescents with and without MLD. The aim of this study was to detect impaired subitizing in participants with MLD in contrast to their typically developing peers. This would confirm previous research that considers impaired subitizing abilities as one of the possible core deficits of MLD.

Chapter 3 describes a study on whether infants can discriminate the small numerosities one versus three, as this number set was expected to yield positive results based on literature review. The main goal about unraveling the value of number discrimination for concurrent and later numerical competencies was kept in mind here. A positive group result would indicate that probably most infants are capable of discriminating between these small numerosities, although it would still be possible to identify who was (not) successful, which leads to the goal of the next chapter.

Chapter 4 builds on *Chapter 3* as it reports on whether individual differences in small number discrimination performance in infancy differentiate between children's number discrimination in toddlerhood or their numerical competencies in kindergarten later on. The aim was to explore the predictive value of number sense both in infancy and toddlerhood as operationalized by number discrimination.

Chapter 5 aimed at confirming the relationship between numerical mother-child interaction and numerical competencies in kindergarten. In addition, this chapter wanted to highlight the value of toddler's numerical mother-child interaction for their concurrent small number discrimination performance or later numerical competencies in kindergarten. Doing this, this chapter complements *Chapter 4* by adding a parental perspective, next to a focus on child characteristics, to this research on early predictors.

Chapter 6, finally, provides an overview and a general discussion summarizing the most important findings of this doctoral research. To conclude, limitations and implications are given in support of the empirical relevance of this dissertation.

REFERENCES

American Psychiatric Association [APA] (2013). *Diagnostic and statistical manual of mental disorders* (5th ed.). Arlington, VA: American Psychiatric Publishing.

Aunio, P., Hautamaki, J., Sajaniemi, N., & Van Luit J. E. H. (2009). Early numeracy in low-performing young children. *British Educational Research Journal, 35*, 25–46. http://dx.doi.org/10.1080/01411920802041822

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology, 96*, 699-713. http://dx.doi.org/10.1037/0022-0663.96.4.699

Baroody, A. J., Bajwa, N. P., & Eiland, M. (2009). Why can't Johnny remember the basic facts? *Developmental Disabilities Research Reviews, 15*, 69–79. http://dx.doi.org/10.1002/ddrr.45. [PubMed: 19213010]

Baroody, A.J., & Benson, A. (2001). Early number instruction. *Teaching Children Mathematics, 8,* 154-158.

Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development, 19*, 291–307. http://dx.doi.org/10.1016/j.cogdev.2004.03.005

Benigno, J. P., & Ellis, S. (2004). Two is greater than three: Effects of older siblings on parental support of preschoolers' counting in middle-income families. *Early Childhood Research Quarterly, 19*, 4-20. http://dx.doi.org/10.1016/j.ecresq.2004.01.006

Berch, D.B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*, 333-339. http://dx.doi.org/10.1177/00222194050380040901

Bisanz, J., & LeFevre, J. (1992). Understanding elementary mathematics. In J.I.D. Campbell (Ed.), *The Nature and Origins of Mathematical Skills*, (pp. 113-136). Amsterdam: North-Holland Elsevier Science Publishers.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early*

Development & Parenting, 5, 35-45. http://dx.doi.org/10.1002/(sici)1099-0917(199603)5:1<35::aid-edp113>3.0.co;2-0

Brannon, E. M., Abbott, S., & Lutz, D. J. (2004). Number bias for the discrimination of large visual sets in infancy. *Cognition, 93,* B59-568. http://dx.doi.org/ 10.1016/j.cognition.2004.01.004

Bruandet, M., Molko, N., Cohen, L., & Dehaene, S. (2004). A cognitive characterization of dyscalculia in Turner syndrome. *Neuropsychologia*, *42*, 288–298. http://dx.doi.org/10.1016/j.neuropsychologia.2003.08.007

Bryant, D. P., Bryant, B. R., Roberts, C., Vaughn, S., Pfannenstiel, K. H., Porterfield, J., & Gersten, R. (2011). Early numeracy intervention program for first-grade students with mathematics difficulties. *Exceptional Children, 78*, 7–23.

Butterworth, B. (1999). *The mathematical brain.* London: Macmillan.

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, 46,* 3-18. http://dx.doi.org/10.1111/j.1469-7610.2004.00374.x

Cantrell, L., & Smith, L. B. (2013). Open questions and a proposal: A critical review of the evidence on infant numerical abilities. *Cognition, 128,* 331-352. http://dx.doi.org/10.1016/j.cognition.2013.04.008

Clements, D. H. (1999). Subitizing: What Is It? Why Teach It? *Teaching Children Mathematics, 5*, 400-405. http://dx.doi.org/2200/20140225183550216T

Clements, D. H., & Sarama, J. (2007). Early Childhood mathematics learning. In J. Lester (Ed.), *Second handbook of research on mathematics teaching and learning.* (pp. 461-555). New York: Information Age Publishing.

Coleman, M. R., Buysse, V., & Neitzel, J. (2006). Recognition and response. An early intervening system for young children at risk for learning disabilities. *Research synthesis and recommendations.* UNIC FPG Child Development Institute.

Cordes, S., & Brannon, E. M. (2008). The difficulties of representing continuous extent in infancy: Using number is just easier. *Child Development, 79*, 476-489. http://dx.doi.org/10.1111/j.1467-8624.2007.01137.x

Cordes, S., & Brannon, E. M. (2009a). Crossing the divide: infants discriminate small from large numerosities. *Developmental Psychology, 45*, 1583-1594.

http://dx.doi.org/10.1037/a0015666

Cordes, S., & Brannon, E. M. (2009b). The relative salience of discrete and continuous quantity in young infants. *Developmental Science, 12*, 453-463. http://dx.doi.org/10.1111/j.1467-7687.2008.00781.x

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Dehaene, S. (2001). Precis of the number sense. *Mind & Language, 16*, 16-36. http://dx.doi.org/10.1111/1468-0017.00154

Dehaene, S., & Changeux, J. P. (1993). Development of elementary numerical abilities – A neuronal model. *Journal of Cognitive Neuroscience, 5*, 390-407. http://dx.doi.org/10.1162/jocn.1993.5.4.390.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20*, 487-506. http://dx.doi.org/ 10.1080/02643290244000239

Desoete, A., & Grégoire, J. (2006). Numerical competence in young children and in children with mathematics learning disabilities. *Learning and Individual Differences, 16*, 351-367. http://dx.doi.org/10.1016/j.lindif.2006.12.006

Desoete, A., Stock, P., Schepens, A., Baeyens, D., & Roeyers, H. (2009). Classification, Seriation, and Counting in Grades 1, 2, and 3 as Two-Year Longitudinal Predictors for Low Achieving in Numerical Facility and Arithmetical Achievement? *Journal of Psychoeducational Assessment, 27,* 252-264. http://dx.doi.org/10.1177/0734282908330588

Dieterich, S. E., Assel, M. A., Swank, P., Smith, K. E., & Landry, S. H. (2006). The impact of early maternal verbal scaffolding and child language abilities on later decoding and reading comprehension skills. *Journal of School Psychology, 43*, 481-494. http://dx.doi.org/10.1016/j.jsp.2005.10.003

DiPema, J. C., Lei, P. W., & Reid, E. E. (2007). Kindergarten predictors of mathematical growth in the primary grades: an investigation using the early childhood longitudinal study - kindergarten cohort. *Journal of Educational Psychology, 99,* 369-379. http://dx.doi.org/10.1037/0022-0663.99.2.369

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience, and education.* New York: Psychology Press.

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. [Article]. *Developmental Science, 11*, 650-654. http://dx.doi.org/10.1111/j.1467-687.2008.00713.x

Feigenson, L. & Carey, S. (2003). Tracking individuals via object-files: Evidence from infants' manual search. *Developmental Science, 6*, 568-584. http://dx.doi.org/10.1111/1467-7687.00313

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object *Cognition,* 97, 295-313, http://dx.doi.org/10.1016/j.cognition.2004.09.010

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Fischer, B., Gebhardt, C., & Hartnegg, K. (2008). Subitizing and visual counting in children with problems acquiring basic arithmetic skills. *Optometry and Vision Development*, *39*, 24-29. Retrieved from http://www.covd.org/Portals/0/24-29SubitizingVisualCounting.pdf

Fuson, C. K. (1988). *Children's counting and concepts of number*. New york: Springer-Verlag.

Geary, D. C. (1995). Reflections of evolution and culture in children's cognition - implications for mathematical development and instruction. *American Psychologist, 50*, 24-37. http://dx.doi.org/10.1037//0003-066x.50.1.24

Geary, D. C., Bowthomas, C. C., & Yao, Y. H. (1992). Counting knowledge and skill in cognitive addition: A comparison of normal and mathematically disabled children. *Journal of Experimental Child Psychology, 54*, 372-391. http://dx.doi.org/10.1016/0022-0965(92)90026-3

Gelman, R., & Galistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education, 33,* 18-28. http://dx.doi.org/10.1177/002246699903300102

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*, 293-304. http://dx.doi.org/10.1177/00222194050380040301

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93*, 615-626. http://dx.doi.org/10.1037//0022-0663.93.3.615

Hierbert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hierbert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hood, M., Conlon, E., & Andrews, G. (2008). Preschool home literacy practices and children's literacy development: A longitudinal analysis. *Journal of Educational Psychology, 100*, 252-271. http://dx.doi.org/10.1037/0022-0663.100.2.252

Huntsinger, C. S., Jose, P. E., Larson, S. L., Balsink Krieg, D., & Shaligram, C. (2000). Mathematics, vocabulary, and reading development in Chinese American and European American Children over the primary school years. *Journal of Educational Psychology, 92*, 745-760, http://dx.doi.org/10.1037//0022-0663.92.4.745

Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General, 123,* 284-296. http://dx.doi.org/10.1037/0096-3445.123.3.284

Jordan, N. C. (2007). The need for number sense. *Educational Leadership*, *65*, 63-66. http://dx.doi.org/10.1037/0096-3445.123.3.284

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences, 20*, 82-88. http://dx.doi.org/10.1016/j.lindif.2009.07.004

Jordan, N. C., & Hanich, L. B. (2000). Mathematical thinking in second grade children with different forms of LD. *Journal of Learning Disabilities, 33*, 567-578. http://dx.doi.org/10.1177/002221940003300605

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice (Blackwell Publishing Limited), 22*, 36-46. http://dx.doi.org/10.1111/j.1540-5826.2007.00229.x

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*, 850-867. http://dx.doi.org/10.1037/a0014939.

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*, 60-68. http://dx.doi.org/10.1002/ddrr.46

Jung, M. (2001). Number relationships in preschool. *Teaching Children Mathematics, 17*, 550-557.

Kahneman, D., & Treisman, A. (1984). Changing views of attention and automaticity. In R. Parasuraman & D. R. Davies (Eds.), *Varieties of attention* (pp. 29-61). Orlando, FL: Academic Press.

Kahneman, D., Treisman, A., & Gibbs, B. J. (1992). The reviewing of object files: Object specific integration of information. *Cognitive Psychology, 24*, 174-219. http://dx.doi.org/10.1016/0010-0285(92)90007-O

Kaminski, E. (2002). Promoting mathematical understanding: Number sense in action. *Mathematics Education Research Journal*, *14*, 133-49. http://dx.doi.org/ 10.1007/BF03217358

Karrass, J., & Braungart-Rieker, J. M. (2005). Effects of shared parent infant book reading on early language acquisition. *Journal of Applied Developmental Psychology, 26*, 133-148. http://dx.doi.org/10.1016/j.appdev.2004.12.003

Kaufmann, L., & Dowker, A. (2009). Typical Development of Numerical Cognition: Behavioral and neurofunctional issues Introduction. *Journal of Experimental Child Psychology, 103*, 395-399. http://dx.doi.org/10.1016/j.jecp.2009.05.003

Kaufman, E.L., Lord, M.W., Reese, T.W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology, 62*, 498-525. http://dx.doi.org/10.2307/1418556

Klahr, D., & Wallace, J. G. (1976). *Cognitive development: An information-processing view*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Kleemans, T., Peeters, M., Segers, E., & Verhoeven, L. (2012). Child and home predictors of early numeracy skills in kindergarten. *Early Childhood Research Quarterly, 27*, 471-477. http://dx.doi.org/10.1016/j.ecresq.2011.12.004

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*, 59-69. http://dx.doi.org/10.1037/0012-1649.42.1.59

Koontz, K. L., & Berch, D. B. (1996). Identifying simple numerical stimuli: Processing inefficiencies exhibited arithmetic learning disabled children. *Mathematical Cognition, 2*, 1-23. http://dx.doi.org/10.1080/135467996387525

Krajewski, K., & Schneider, W. (2008). Early development of quantity to number-word linkage as a precursor of mathematical school achievement and mathematical difficulties: Findings from a four-year longitudinal study. *Learning and Instruction*, *19*, 513-526. http://dx.doi.org/10.1016/j.learninstruc.2008.10.002

Krajewski, K., & Schneider, W. (2009). Exploring the impact of phonological awareness, visual-spatial working memory, and preschool quantity-number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology*, *103*, 516-531. http://dx.doi.org/10.1016/j.jecp.2009.03.009

Kroesbergen, E. H., Van Luit, J. E. H., & Aunio, P. (2012). Mathematical and cognitive predictors of the development of mathematics. *British Journal of Educational Psychology, 82*, 24-27. http://dx.doi.org/10.1111/j.2044-8279.2012.02065.x

Kroesbergen, E. H., Van Luit, J. E. H., Van Lieshout, E., Van Loosbroek, E., & Van de Rijt, B. A.M. (2009). Individual differences in early numeracy: The role of executive functions and subitizing. *Journal of Psychoeducational Assessment, 27*, 226-236. http://dx.doi.org/10.1177/0734282908330586

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8-9 year old students. *Cognition, 93,* 99-125. http://dx.doi.org/10.1016/j.cognition.2003.11.004

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*, 395–438. http://dx.doi.org/10.1016/j.cognition.2006.10.005

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Revisiting the ompetence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52*, 130–169. http://dx.doi.org/10.1016/j.cogpsych.2005.07.002

LeFevre, J., Clarke, T., & Stringer, A. P. (2002). Influences of language and parental involvement on the development of counting skills: Comparisons of French- and English-speaking Canadian children. *Early Child Development and Care, 172*, 283–300. http://dx.doi.org/10.1080/03004430212127

LeFevre, J. A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., & Bisanz, J. (2010). Pathways to Mathematics: Longitudinal Predictors of Performance. *Child Development, 81*, 1753-1767. http://dx.doi.org/0009-3920/2010/8106-0011

LeFevre, J. A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home Numeracy Experiences and Children's Math Performance in the Early School Years. *Canadian Journal of Behavioural Science, 41*, 55-66. http://dx.doi.org/10.1037/a0014532

LeFevre, J. A., Smith-Chant, B. L., Fast, L., Skwarchuk, S. L., Sargla, E., Arnup, J. S., Kamawar, D. (2006). What counts as knowing? The development of conceptual and procedural knowledge of counting from kindergarten through Grade 2. *Journal of Experimental Child Psychology, 93*, 285-303. http://dx.doi.org/10.1016/j.jecp.2005.11.002

Lembke, E., & Foegen, A. (2009). Identifying early numeracy indicators for kindergarten and first-grade students. *Learning Disabilities Research & Practice, 24*, 12–20. http://dx.doi.org/10.1111/j.1540-5826.2008.01273.x

Leslie, A. M., Xu, F., Tremoulet, P., & Scholl, B. J. (1998). Indexing and the object concept: Developing “what” and “where” systems. *Trends in Cognitive Sciences, 2*, 10-18. http://dx.doi.org/10.1016/S1364-6613(97)01113-3

Levine, S. C., Jordan, N. C., & Huttenlocher, J. (1992). Development of calculation abilities in young children. *Journal of Experimental Child Psychology, 53*, 72-103. http://dx.doi.org/10.1016/s0022-0965(05)80005-0

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2011). What counts in the development of young children's number knowledge? *Developmental Psychology, 47*, 302-302. http://dx.doi.org/10.1037/a0022101

Libertus, M. E., & Brannon, E. M. (2010). Stable individual differences in number discrimination in infancy. *Developmental Science, 13*, 900-906. http://dx.doi.org/ 10.1111/j.1467-7687.2009.00948.x

Mazzocco, M. M. M., & Thompson, R. E. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities: Research and Practice, 20*, 142-155. http://dx.doi.org/10.1111/j.1540-5826.2005.00129.x

McCrink, K., & Wynn, K. (2004). Large-number addition and subtraction by 9-month-old infants. *Psychological Science, 15*, 776-781. http://dx.doi.org/10.1111/j.0956-7976.2004.00755.x

Mix, K. S. (2009). How Spencer made number: First uses of the number words. *Journal of Experimental Child Psychology, 102*, 427-444. http://dx.doi.org/10.1016/j.jecp.2008.11.003

Moeller, K., Neuburger, S., Kaufmann, L., Landerl, K., & Nuerk, H. C. (2009). Basic number processing deficits in developmental dyscalculia: Evidence form eye tracking. *Cognitive development, 24*, 371-386. doi: 10.1016/j.cogdev.2009.09.007

Mussolin, C., Mejias, S., & Noël, M. P. (2010). Symbolic and nonsymbolic number comparison in children with and without dyscalculia. *Cognition, 115*, 10-25. http://dx.doi.org/10.1016/j.cognition.2009.10.006

Nan, Y., Knösche, T. R., & Luo, Y.-J. (2006). Counting in everyday life: Discrimination and enumeration. *Neuropsychologia*, *14*, 1103-1113. http://dx.doi.org/10.1016/j.neuropsychologia.2005.10.020

Pan, Y., Gauvain, M., Liu. Z., & Cheng, L. (2006). American and Chinese parental involvement in young children's mathematics learning. *Cognitive Development, 21*, 17-5. http://dx.doi.org/10.1016/j.cogdev.2005.08.001

Pancsofar, N., Vernon-Feagans, L., & Family Life Project, I. (2010). Fathers' early contributions to children's language development in families from low-income rural communities. *Early Childhood Research Quarterly, 25*, 450-463. http://dx.doi.org/10.1016/j.ecresq.2010.02.001

Penner-Wilger, M., Fast, L., LeFevre, J., Smith-Chant, B. L., Skwarchuk, S., Kamawar, D., & Bisanz, J. (2007). The foundations of numeracy: Subitizing, finger gnosia, and fine-motor ability. In D.S. McNamara & J.G. Trafton (Eds.), *Proceedings of the 29th Annual Cognitive Science Society* (pp. 1385-1390). Austin, TX: Cognitive Science Society.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., . . . Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*, 33-41. http://dx.doi.org/10.1016/j.cognition.2010.03.012.

Powell, S. R., & Fuchs, L. S. (2012). Early Numerical Competencies and Students with Mathematics Difficulty. *Focus on Exceptional Children, 44*, 1-16. http://dx.doi.org/10.1111/j.1540-5826.2011.00345.x

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal, 50,* 178-209. http://dx.doi.org/10.3102/00028312124 65332

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children’s numerical knowledge through playing number board games. *Child Development, 79*, 375-394. http://dx.doi.org/10.1111/j.1467-8624.2007.01131.x

Reigosa-Crespo, V., Valdes-Sosa, M., Butterworth, B., Estävez, N., Rodräguez, M., Santos, E., . . . Lage, A. (2012). Basic numerical capacities and prevalence of developmental dyscalculia: the Havana survey. *Developmental Psychology, 48*, 123e135. http://dx.doi.org/10.1037/a0025356.

Revkin, S. K., Piazza, M., Izard, V., Cohen, L., & Dehaene, S. (2008). Does subitizing reflect numerical estimation? *Psychological Science, 19*, 607-614. http://dx.doi.org/ 10.1111/j.1467-9280.2008.02130.x

Roberts, J. E., Jurgens, J., & Burchinal, M. (2005). The role of home literacy practices in preschool children's language and emergent literacy skills. *Journal of Speech Language and Hearing Research, 48*, 345-359. http://dx.doi.org/10.1044/1092-4388(2005/024)

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*, 361-395. http://dx.doi.org/10.1016/j.cognition.2006.01.005

Rousselle, L., & Noël, M. P. (2008). The development of automatic numerosity processing in preschoolers: Evidence for numerosity-perceptual interference. *Developmental Psychology, 44*, 544-560. http://dx.doi.org/10.1037/0012-1649.44.2.544

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108,* 662-674. http://dx.doi.org/10.1016/j.cognition.2008.05.007

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52*, piii-viii, 3-162.

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental science, 14,* 280-291. http://dx.doi.org/10.1111/j.1467-7687.2010.00976.x

Skwarchuk, S. L. (2009). How do parents support preschoolers' numeracy learning experiences at home? *Early Childhood Educational Journal, 37*, 189-197. http://dx.doi.org/10.1007/s10643-009-0340-1

Starr, A., Libertus, M. E., & Brannon, E. M. (2013a). Infants show ratio-dependent number discrimination regardless of set size. *Infancy*, *18*, 927–941. http://dx.doi.org/10.1111/infa.12008

Starr, A., Libertus, M. E., & Brannon, E. M. (2013b). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences of the United States of America (PNAS), 110,* 18116-18120. http://dx.doi.org/10.1073/pnas.1302751110

Stock, P., Desoete, A., & Roeyers, H. (2009). Mastery of the counting principles in toddlers: A crucial step in the development of budding arithmetic abilities? *Learning and Individual Differences, 19*, 419-422. http://dx.doi.org/ 10.1016/j.lindif.2009.03.002

Stock, P., Desoete, A., & Roeyers, H. (2010). Detecting children with arithmetic disabilities from kindergarten: Evidence from a 3-year longitudinal study on the role of reparatory arithmetic abilities. *Journal of Learning Disabilities, 43*, 250-268. doi: 10.1177/0022219409345011

Träff, U. (2013) The contribution of general cognitive abilities and number abilities to different aspects of mathematics in children. *Journal of Experimental Child Psychology*, *116*, 139-156. http://dx.doi.org/10.1016/j.jecp.2013.04.007

Trick, L. M. & Pylyshyn, Z. W. (1993). What enumeration studies can show us about spatial attention – Evidence for limited capacity preattentive processing. *Journal of Experimental Psychology-Human Perception and Performance, 19*, 331-351. http://dx.doi.org/10.1037/0096-1523.19.2.331

Trick, L., & Pylyshyn, Z. (1994). Why are small and large numbers enumerated differently? A limited capacity preattentive stage in vision. *Psychological Review, 101*, 80-102. http://dx.doi.org/10.1037//0033-295X.101.1.80

Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19*, 21-39. http://dx.doi.org/10.1016/j.ecresq.2004.01.007

VanDerHeyden, A. M., Broussard, C., Snyder, P., George, J., Lafleur S. M., & Williams, C. (2011). Measurement of kindergartners' understanding of early mathematical concepts. *School Psychology Review*, *40*, 296-306.

Van Loosbroek, E., & Smitsman, A. W. (1990). Visual perception of numerosity in infancy. *Developmental Psychology, 26*, 916-922. http://dx.doi.org/10.1037/0012-1649.26.6.911.b

von Aster, M. G., & Shalev, R. S. (2007). Number development and developmental dyscalculia. *Developmental Medicine and Child Neurology, 49*, 868-873. http://dx.doi.org/10.1111/j.1469-8749.2007.00868.x

Wagner, D., & Davis, B. (2010). Feeling number: Grounding number sense in a sense of quantity. *Educational Studies in Mathematics, 74,* 39-51. http://dx.doi.org/10.1007/s10649-009-9226–9.

Watson, D. G., Maylor, E. A., Allen, G., & Bruce, L. A. M. (2007). Early visual tagging: Effects of target-distractor similarity and old age on search, subitization, and counting. *Journal of Experimental Psychology: Human Perception and Performance, 33*, 549-569. http://dx.doi.org/10.1037/0096-1523.33.3.549

Wilson, A. J., Revkin, S. K., Cohen, D., Cohen, L., & Dehaene, S. (2006). An open trial assessment of “The Number Race”, an adaptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions, 2:*20. http://dx.doi.org/10.1186/1744-9081-2-20. http://www.behavioralandbrainfunctions.com/content/2/1/20

Wynn, K. (1990). Children's understanding of coutning. *Cognition, 36*, 155-193. http://dx.doi.org/10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Childrens' acquisition of the number words and the counting sytem. *Cognitive Psychology, 24*, 220-251. http://dx.doi.org/10.1016/0010-0285(92)90008-p

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25. http://dx.doi.org/10.1016/S0010-0277(03)00050-7

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology, 25*, 103-108. http://dx.doi.org/10.1348/026151005x90704

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, B1-B11. http://dx.doi.org/10.1016/S0010-0277(99)00066-9

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8*, 88-101. http://dx.doi.org/10.1111/j.1467-7687.2005.00395.x

Zulauf, M. W., Schweiter, M., & von Aster, M. (2003). The preschool year: A sensitive period for the development of numerical abilities. *Kindheit Und Entwicklung, 12*, 222-230.

APPENDIX

In what follows, a detailed description will be given of the tasks used in this doctoral research. Figure 1 illustrates the key concepts of this dissertation (*small number discrimination*, *verbal subitizing*, and *numerical competencies in kindergarten* as aspects of *number sense*, investigated next to *numerical mother-child interaction*), along with the specific measures that were chosen to operationalize the aspects at different time points. The different time points are indicated with the ages of the subjects concerned as well as with the terms used in this dissertation to delineate the specific age periods.

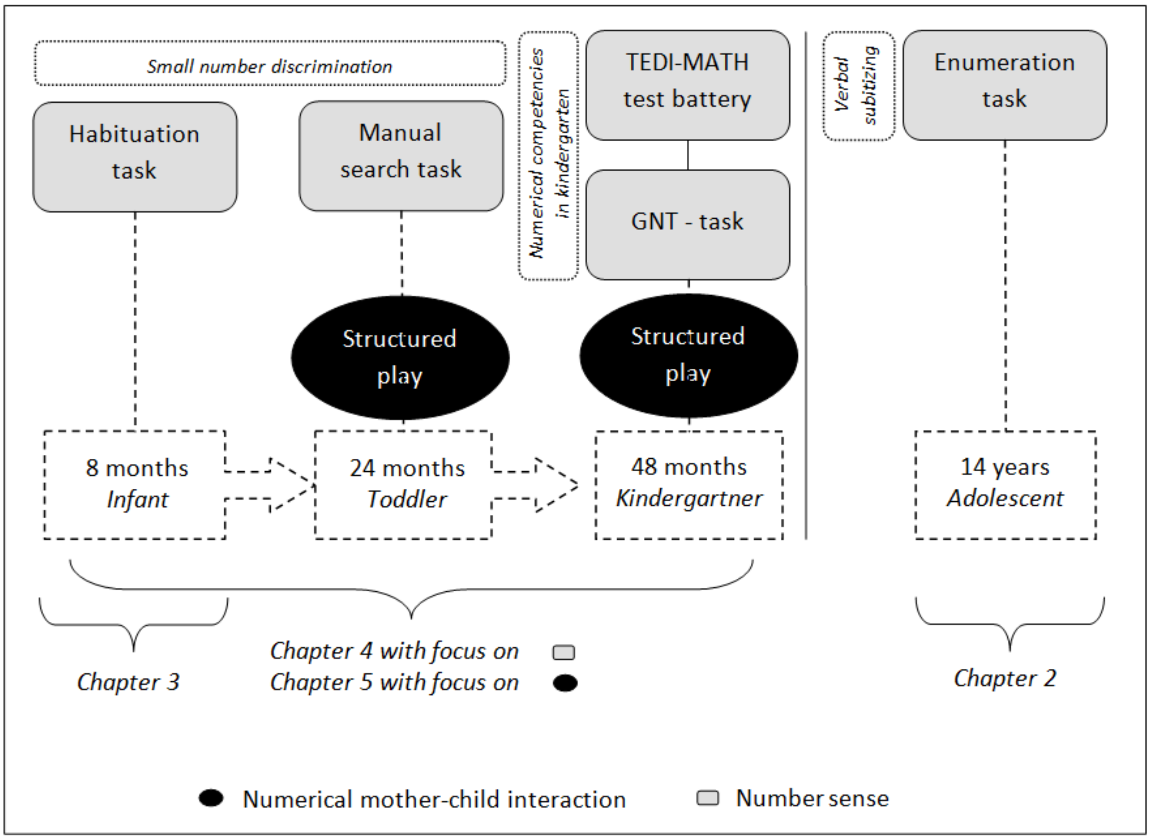

Figure 1. Summary of (measures of) key concepts of this dissertation.

The left part of the figure represents the follow-up study in which the same children were studied longitudinally at the ages of 8 months, 24 months, and 48 months. The right part of the figure shows the cross-sectional study with the older children. In addition, Figure 1 shows for the different parts in which chapter they are handled on.

All children from the longitudinal study participated at sessions designed within the scope of a large-scale study for the Belgian government realised by the Ghent University and Catholic University of Leuven as a partnership within the support center Welfare, Public Health, and Family (http://www.steunpuntwvg.be/jong, see Grietens, Hoppenbrouwers, Desoete, Wiersema, & Van Leeuwen, 2010) with focus on issues related to youth, development and parenting, and health and behavior. The tasks described below were therefore always part of a broader research protocol also including tasks and measurements targeting other research goals and concepts than the ones described in this dissertation – except for the kindergarten session – in which the order of the tasks was defined randomly per child (i.e., infants, toddlers, and adolescents). The total duration of the session at the different ages was about 100, 120 and, 150 minutes respectively. All participants received a reward after completion of one full session (i.e., an age-appropriate toy for infants and kindergartners, a storybook for toddlers, and a movie ticket for adolescents).

Small number discrimination

Habituation task. A number discrimination task based on the habituation paradigm was used to assess infants' small number discrimination ability, following the lead of Xu et al. (e.g., Xu, 2003, Xu & Arriaga, 2007). Children sat on the parent's lap in front of the Tobii T60 eye tracker. This eye tracker device was integrated in a 17" monitor with a refresh rate of 60 Hz (Tobii Technology, 2007). Children were seated at a distance of approximately 60 cm from the screen of the eye tracker. Parents were instructed to remain neutral and not to elicit the child's attention during task administration. Duration of the task was estimated at about 5 to 10 minutes.

At the beginning of the habituation task, an attention grabber accompanied with sound appeared successively in the corners and the middle of the screen to indicate the infants' window of looking. Only after a successful five-point calibration, the experiment began whereafter a well-known cartoon-figure was shown and each following trial was introduced by a sound (to sustain infants' attention).

The main task itself consisted of two phases: a habituation and a test phase. Infants were randomly assigned to one of two habituation conditions. Half of the children was habituated to one-item-arrays, whereas the other half was habituated to three-item-arrays. During this habituation phase, six different displays were presented repeatedly in a randomly defined order until the habituation criterion was met or when 14 habituation trials were completed. The habituation criterion implied a 50% reduction in looking time over three consecutive trials, relative to the first three trials. After the habituation phase, infants were presented with six test displays containing the habituated (old) or new number of items in alternation (counterbalanced for order across participants) in the test phase. An illustration of the two phases of the number discrimination task according to the habituation paradigm is shown in Figure 2 below.

The stimuli that were used for this task were one- and three- element-arrays of red dots on a white square background, displayed in the center of the monitor. In order to maximally attract and sustain the attention of the infants, the dots were colored red (Franklin et al., 2008; Maier, Barchfeld, Elliot, & Pekrun, 2009; Zemach & Teller, 2007).

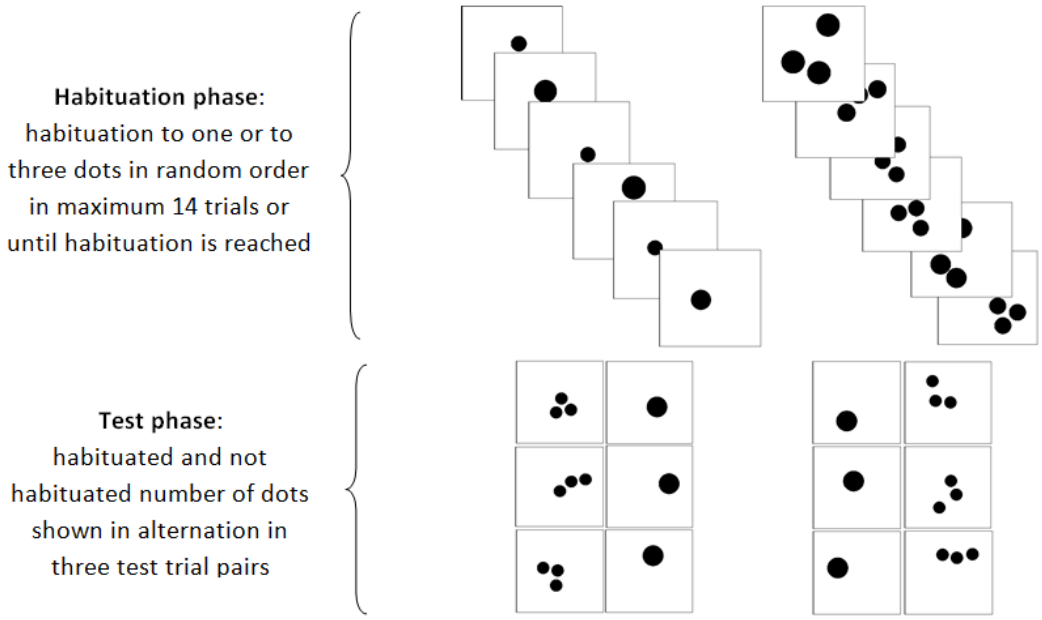

Figure 2. Example of habituation and test phase (for each test pair).

Furthermore, stimuli were controlled for continuous variables (*item size* and *inter-item distance* at item level and the related set-parameters *total item size* and *total occupied set area*) according to the procedure of Dehaene, Izard, and Piazza (2005). The stimuli were designed so that, besides the change in number, all parameter values presented in the test phase were also presented during habituation – thus being equally non-novel. This could be established by randomly selecting one parameter (item or set level) from a fixed distribution regardless of number while the related parameter varied with number in habituation. For the test stimuli, this procedure was reversed. In this experiment, *total item size* and *total occupied area* were fixed during habituation while the correlated set-level-parameters varied with number (Dehaene et al., 2005).

Habit X 1.0, a software program developed for performing the habituation paradigm (Cohen, Atkinson, & Chaput, 2004), was used during task administration. One experimenter saw the infants' looking behavior via a *live viewer* application on a portable computer (connected with the eye tracker) running the software *Tobii Studio* (Tobii Technology, 2007). This live viewer showed infants' eye fixations on stimuli during the task. Looking behavior was recorded in *Habit X 1.0* by holding down a button as long as infants were looking at the stimuli and releasing it when they looked away.

Experimenters were blind to the experimental condition to which children were assigned during task administration and during coding of the looking times afterwards. Real looking times were coded in Tobii Studio from the eye tracking data by two researchers who created areas of interest (with margins of 2 cm) around each dot per array. As such, *total fixation duration* at all dots in one display could be identified. These coded looking times were used for further analyses.

Manual search task. A manual search paradigm was used to assess toddlers' small number discrimination, following the lead of Feigenson and Carey (2003). Children sat on their parent's lap at an empty table in front of the experimenter. Parents were told that some balls would be hidden into a box during a search task to explore how children reacted and that there was no wrong reaction. Parents could redirect the child's attention to the box when necessary, but were not allowed to help. When the child wanted to communicate, they could nod or say "I see". Task duration was about 5 to 10 minutes.

After introduction of the wooden box (25 cm x 12,5 cm x 31.5 cm) to the child, he or she was made familiar with the task: The experimenter placed a colorful marble in the box through the slit at the front, which was oriented to the toddlers. Next, the child was stimulated to search for the marble through the slit. When the child had taken back the marble, he or she could place it in the “Pound a ball”-toy to raise his or her motivation. After familiarization, the main task started as illustrated in Figure 3.

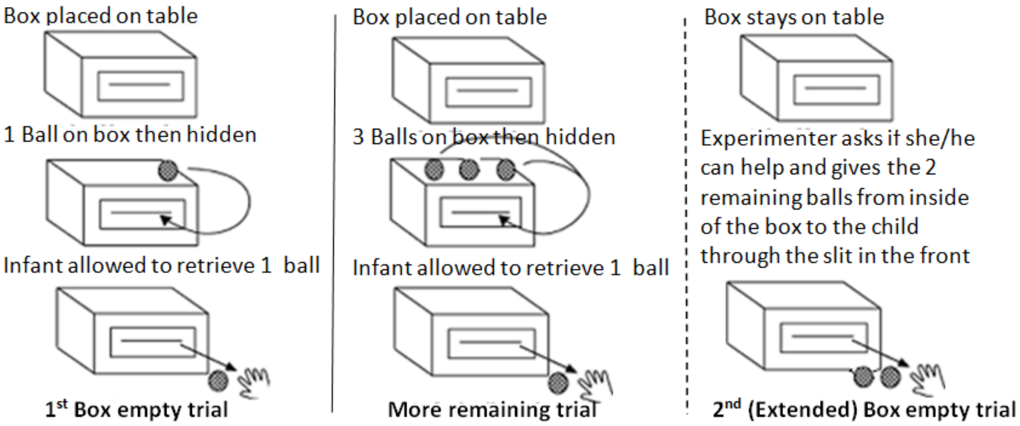

Figure 3. Demonstration of manual search task (for each trial type). Adapted from “On the limits of infants’ quantification of small object arrays,” by L. Feigenson and S. Carey, 2005, *Cognition*, *97*, p. 301. Copyright 2004 by Elsevier B.V.

Stimuli that were used in the task were green-colored balls made of expanded polystyrene with a diameter of 2.5 cm. The main task consisted of three kinds of trials. During the *box empty* trial type, the researcher placed one ball in the box, whereafter the child was allowed to retrieve it. It was expected that children with developed numeracy would only attempt once to retrieve the ball, as they are presumed to expect that the box is empty after retrieving the ball. In a second, *more remaining* trial type, the researcher supposedly placed three balls in the box, whereafter the child was allowed to retrieve the balls. Critical for this trial type, however, is the fact that the experimenter took two balls out of the (open) backside of the box, out of sight of the child. It was expected that children with developed numeracy would try to retrieve all three balls anyhow. In a third “*extended*” *box empty* trial type, which always occurred after a more remaining trial, the experimenter proposed to help and gave back the two “supposedly hidden” balls to the child, which were previously taken away by the experimenter in the more remaining trial.

Of importance is that children with developed numeracy would understand that there are no balls left in the box after this last trial. Therefore they would undertake no further attempts to retrieve a ball.

Numerical competencies in kindergarten

Numerical competencies in kindergarten entailed *counting, arithmetic operation*s, and *cardinality*. To assess these competencies, the *Test for the Diagnosis of Mathematical Competencies* (TEDI-MATH; Grégoire, Noël, & Van Nieuwenhoven, 2004) and a variant of the *Give-a-Number* task (GNT, Sarnecka & Carey, 2008) were used.

TEDI-MATH: counting. Counting abilities were assessed using two subtests of the TEDI-MATH (Grégoire et al., 2004), a Belgian individual assessment battery constructed to detect mathematical problems from the second year of preschool until the third grade in elementary school. The psychometric value of the battery was tested on a sample of 550 Dutch-speaking Belgian children (Grégoire, 2005). The TEDI-MATH has proven to be a well-validated and reliable instrument (Desoete, 2006, 2007; Desoete, Roeyers, Schittekatte, & Grégoire, 2006) of which the predictive value has been demonstrated in several studies (Desoete & Grégoire, 2006; Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009). Table 1 provides some examples of the used excercises. Duration of task administration of the involved TEDI-MATH subtests was about 30 minutes long.

Table 1. *Sample exercises of the counting tasks*

Procedural counting	*Conceptual counting*
'Count up to 6'	'Count all objects' - 'How many objects are there in total' - 'How many objects are there if you start counting with the leftmost object'
'Count from 3'	'Put as many objects on this board as there are on this one'
'Count from 5 up to 9'	'Here you can see some snowmen wearing a hat' - After taking away all the hats and putting them underneath a box, the experimenter asks: 'How many hats are there covered under this box'

Procedural counting (subtest 1) was assessed using accuracy in counting row (up till 31) and counting forward to an upper bound (up till 9 at maximum) and/or from a lower bound (from 3 at minimum). The task consisted of eight items and had a maximum raw score of 8. Conceptual counting (subtest 2) was assessed by judging the validity of counting procedures based on the five basic counting principles formulated by Gelman and Gallistel (1978). In order to investigate these principles, children had to judge the counting of linear and nonlinear patterns of objects (range 5-12) and were asked questions about the counted amount of objects. Furthermore, they had to construct two numerically equivalent amounts of objects (range 5-7) while using counting as a problem-solving strategy in a riddle. The maximum total raw score was 13.

TEDI-MATH: arithmetic operations. Arithmetic operations were assessed using subtest 5.1 of the TEDI-MATH (Grégoire et al., 2004). A series of six visually supported additions and subtractions were presented to all children. Figure 4 provides an example. Additions and subtractions involved single-digit numbers varying from 2 up till 8.

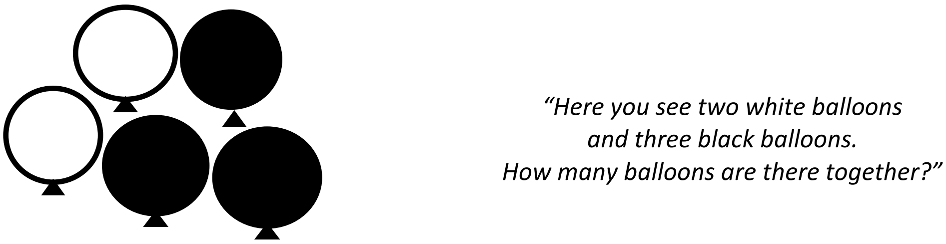

Figure 4. Arithmetic operations - Example of simple addition exercise.

GNT: cardinality. All children were tested with the variant of the GNT as described by Sarnecka and Carey (2008) to determine whether they knew the exact meaning of numbers ranging from one to six. Duration of the task was about 5 minutes.

Children were asked to give a number ("N") of objects to a puppet. Each request of "N" items, was followed by the question whether they gave "N" items. This procedure was restated, until children answered positively (i.e., the child said "yes" as a response to the question). Following the standard protocol, first, one object was asked ("N = 1"), followed by the request for three objects ("N = 3"). This is, two objects ("N = 2") were always skipped in case a child knew the cardinal meaning of the number word "one").

The task was then proceeded as follows: When a child responded correctly to a request "N", the next request was "N + 1" and otherwise "N – 1". Requests continued until at least two successes at a given "N" and at least two failures at "N + 1" were obtained. A credit was given if the child had at least twice as many successes as failures for that numeral. Failure included giving the wrong number of items. Each child's *knower-level* corresponded to the highest number he or she reliably generated. An example of the administration of the different trials of the GNT is given in Figure 5: The number "five" is the highest number reliably generated by the child in this example.

Also in line with Sarnecka and Carey (2008), children who had at least twice as many successes as failures for trials of "five" and "six" were called *cardinal-principle-knowers* (in short, *cardinality-knowers*), whereas all others were called *subset-knowers* (e.g., Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006).

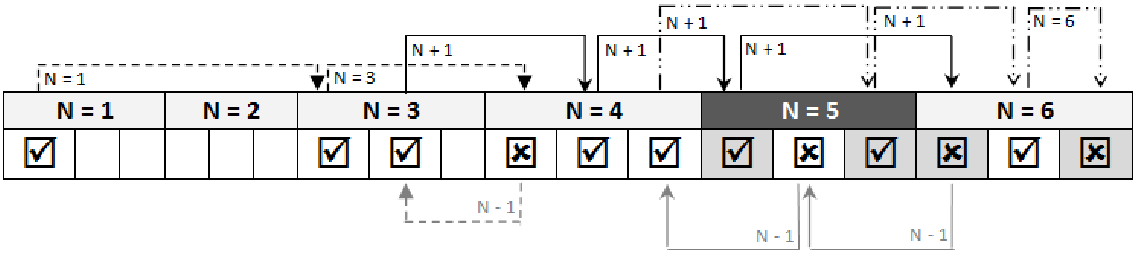

Figure 5. Cardinality - Example of cardinality-knower with "N = 5".

The child in the example, as shown in Figure 5, is a cardinality-knower with two successful trials for "N = 5" and two failures on trials for "N + 1" (i.e., the number "six").

Verbal subitizing: enumeration task

The adolescents' verbal subitizing abilities were tested by means of a computerized enumeration task similar to the one described by Fischer, Gebhardt, and Hartnegg (2008). Participants were instructed to say aloud the number of squares that appeared on a 17" monitor as quickly and accurately as possible. Duration of the task was estimated at about 5 minutes.

Each of the trials began with a central fixation point presented for 500 ms. A display containing one to nine square boxes was then centrally presented at fixation until a vocal response was detected. Stimuli were retrieved from Maloney, Risko, Ansari, and Fugelsang (2010) and were black squares on a white background. The *individual area*, *total area*, and *density* of the squares were varied to insure that participants could not use non-numerical cues to make a correct decision (see Dehaene et al., 2005; Maloney et al., 2010). Verbal responses were collected using a voice key and were manually put in by the researcher.

Two practice phases and one test phase were contained in the enumeration task. In the first practice phase, the adolescent was presented with five displays of randomly chosen numerosities (varying between one and nine) with a presentation and response time of 5,000 ms, so the stimulus remained visible during response time. The second practice phase consisted of 10 displays of randomly chosen numerosities (varying between one and nine) with a presentation time of 120 ms – in line with the study of Hannula, Räsänen, and Lehtinen (2007) and Fischer et al. (2008) – and a mask of 100 ms. Participants had a total response time of 5,000 ms from presentation of the stimulus onwards. The test phase included 72 trials (each numerosity of one to nine was presented eight times) with a presentation time of 120 ms, a mask of 100 ms, and an overall total response time of 5,000 ms. The short presentation time of 120 ms prevented adolescents from counting to enumerate the items (see Fischer et al., 2008). Both accuracy and mean reaction times (on correct trials) were used for analysis. Figure 6 provides an illustration of a test trial of the enumeration task.

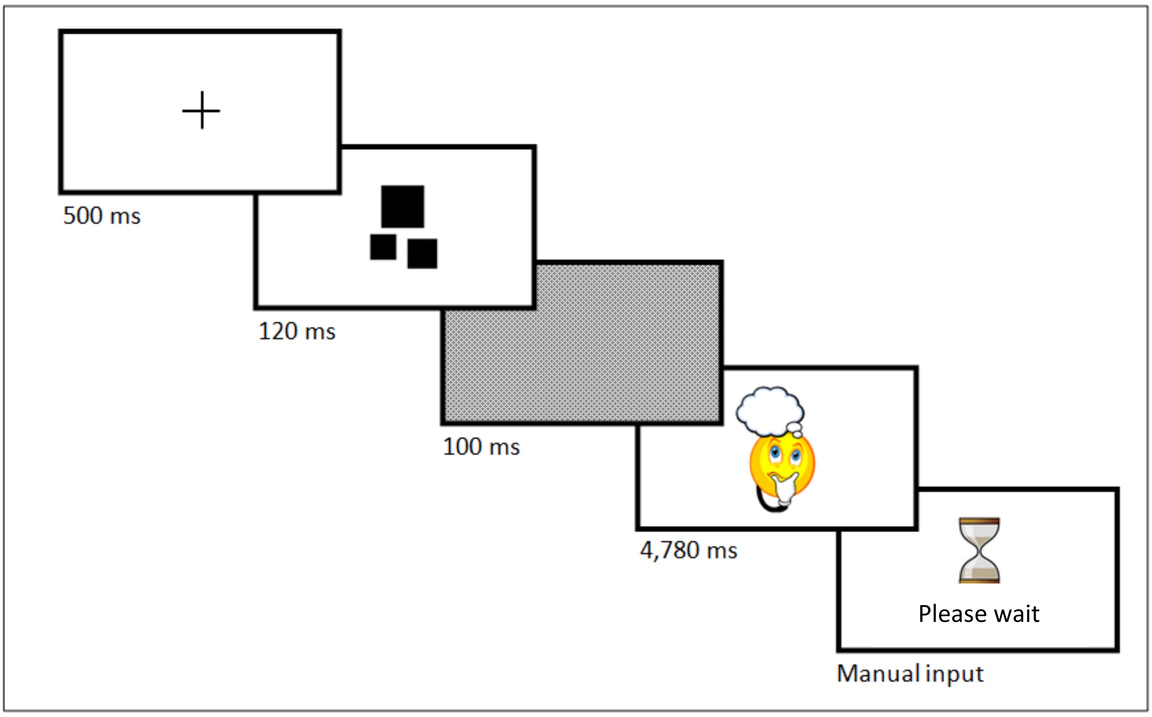

Figure 6. Verbal subitizing – example of a test trial of the enumeration task.

Numerical mother-child interaction: structured play

The structured play aimed to measure the frequency of spontaneous numerical mother-child interaction during a representative situation for a numerical home activity, reproduced in a research setting at 24 months and 48 months of age. Parent and child sat on a pink carpet fabricated of soft plastic and were instructed to build a house with a set of "Lego-Duplo"- blocks according to a model as shown in Figure 7.

24 months of age

48 months of age

Figure 7. Model for structured play at 24 months and 48 months of age respectively.

The model they needed to rebuild contained ordinary blocks and two blocks with numerical information. The set of blocks for mother and child was almost identical as the set of blocks of which the model was built. A notable difference were the two (24 months) or three (48 months) additional blocks with different numerical information. At 24 months of age this implied one block with a picture of three rabbits on it and one block with number “2” printed on it. At 48 months of age this were two blocks with a picture of four apples and a picture of six bees respectively and one block with number “7” printed on it. The purpose of this differential numerical information (i.e., numerical cues) was to give all the dyads the opportunity to focus on these numerical cues and inconsistency. Both mother and child were blind for the true intention of the play. They were asked to play in a similar way as at home. After the full instruction was given, parent and child were left alone in the room and after 5 minutes the play was impeded.

The structured plays were recorded on tape and numerical mother-child interaction was coded manually afterwards. A coding scheme (Table 2) for both time points was developed based on the most often observed actions that occurred during a sample of structured play situations (Adriaensens & Desmedt, 2011) and further inquired consulting available literature on numerical experiences (Benigno & Ellis, 2004; Blevins-Knabe & Musun Miller, 1996; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlochter, & Gunderson, 2010; Tudge & Doucet, 2004).

Actions of mothers that numerically stimulated the child and responses of children on mother’s request related to numeracy-related topics and concepts were coded as *numerical mother-child interaction*. As opposed to these reactions of children on prompts of the mother, spontaneous actions of children related to numeracy were not coded in this category. Coding items entailed could be divided into the following categories: singularity versus plurality (e.g., saying there are one/more apples printed on a block); specific number use (e.g., reciting a counting sequence, using number words or ordinal numbers); practicing numerical/mathematical concepts (e.g., equality, ordinality, quantity [comparison], shape, spatial relations); actions that elicit certain numerical routines in children or the children’s response to these prompts (e.g., asking “How many”, asking to group or to count objects, asking to name numbers, or trigger the use of matching). The frequency of the coded (re)actions was coded and summed up to one final score.

Table 2. *Coding scheme for numerical mother-child interaction*

Category	*Items*
Plurality	Language (game-related and plural when applicable)
	Quantity (e.g., apples, bees, …) ~ numerical blocks
Singularity	Language (game-related and singular when applicable)
	Quantity (e.g., apple, bee, …) ~ numerical blocks
Specific number use	Counting (all possible combinations of at least 2 numbers: showing, with no/the intention to be repeated by child)
	Saying small numbers: 1, 2, 3
	Saying large numbers: up to 10
	Saying ordinal numbers (e.g., first, second, ...)
Numerical/mathematical concepts	about (in)equality: (not) the same as, like (this), different, equal, also, such as, another, ...
	about ordinality: more, less, smaller, bigger, another one, as much as, too much, ...
	about (in)equality + ordinality: (not) the same number
	about quantity/size: a lot, few, nothing, small, big, enough, everything, all, ...
	about location in space: on (top of), over, next to, in front of, in, beneath, in the middle of, in between, aside, on the other side, here, ...
	about the shape of the blocks: square, flat, triangular, ...
	about the color of the blocks: yellow, orange, red, green, ...
Numerical routines	e.g., "What is on the blocks?" (about numbers 3, 7, recognition of numbers)
	e.g., "What is on the blocks?" (about apples, bees; recognition of pictures)
	Asking about/using the words "how much"
	Asking about/using the word "number"/"amount"
	Using the word "counting", for example while encouraging the child to count
	Counting objects (in function of showing, with the intention to be repeated by child)
	Counting the amount of objects (in function of cardinality)
	Counting objects together with child (in function of reciting the counting sequence)
	Counting the amount of objects together with child (in function of cardinality)
	Counting (of objects) instead of child
	Grouping objects (with the intention to be repeated by child)
	(Encouraging of) use of matching (1 for you, 1 for me ...)

REFERENCES

Adriaensens, L., & De Smedt, L. (2011). *Toddlers (24 months) and their first steps into the the land of numeracy holding the hand of the mother: What is the mother's contribution?* (Master’s thesis). Ghent University, Department of Psychology and Educational Sciences.

Benigno, J. P., & Ellis, S. (2004). Two is greater than three: Effects of older siblings on parental support of preschoolers' counting in middle-income families. *Early Childhood Research Quarterly, 19*, 4-20. http://dx.doi.org/10.1016/j.ecresq.2004.01.006

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development & Parenting, 5*, 35-45. http://dx.doi.org/10.1002/(sici)1099-0917(199603)5:1<35::aid-edp113>3.0.co;2-0

Cohen, L. B., Atkinson, D. J., & Chaput, H. H. (2004). A new program for obtaining and organizing data in infant perception and cognition studies (Version 1.0). Austin: University of Texas.

Dehaene, S., Izard, V., & Piazza, M. (2005). Control over non-numerical parameters in numerosity experiments. *Unpublished manuscript.* Available at http://www.unicog.org/.../DocumentationDotsGeneration.doc.

Desoete, A. (2006). Validiteitsonderzoek met de TEDI-MATH [Validity research on the TEDI-MATH]. *Diagnostiek-wijzer, 9*, 140-157.

Desoete, A. (2007). De TEDI-MATH. De plaats van de Tedi-Math in de diagnostiek van dyscalculie in Vlaanderen [The TEDI-MATH. The place of the Tedi-Math in the assessment of dyscalculia in Flanders]. *Caleidoscoop, 19*, 6-19.

Desoete, A., & Grégoire, J. (2006). Numerical competence in young children and in children with mathematics learning disabilities. *Learning and Individual Differences, 16*, 351-367. http://dx.doi.org/10.1016/j.lindif.2006.12.006

Desoete, A., Roeyers, H., Schittekatte, M., & Grégoire, J. (2006). Dyscalculie-gevoelige kennis en vaardigheden in het basisonderwijs in Vlaanderen, Wallonië en Frankrijk. [Dyscalculia-sensitive knowledge and skills in elementary education in the Flemish and French speaking part of Belgium and in France]. *Pedagogische Studiën, 83*, 105-121.

Desoete, A., Stock, P., Schepens, A., Baeyens, D., & Roeyers, H. (2009). Classification, seriation, and counting in grades 1, 2, and 3 as two-year longitudinal predictors for low achieving in numerical facility and arithmetical achievement? *Journal of Psychoeducational Assessment, 27*, 252-264. http://dx.doi.org/10.1177/0734282908330588

Feigenson, L. & Carey, S. (2003). Tracking individuals via object-files: Evidence from infants' manual search. *Developmental Science, 6*, 568-584. http://dx.doi.org/10.1111/1467-7687.00313

Fischer, B., Gebhardt, C., & Hartnegg, K. (2008). Subitizing and visual counting in children with problems acquiring basic arithmetic skills. *Optometry and Vision Development, 39*, 24-29.

Franklin, A., Pitchford, N., Hart, L., Davies, I. R. L., Clausse, S., & Jennings, S. (2008). Salience of primary and secondary colours in infancy. *British Journal of Developmental Psychology, 26*, 471-483. http://dx.doi.org/10.1348/026151007X256672

Gelman, R., & Galistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Grégoire, J. (2005). Développement logique et compétences arithmétiques. Le modèle piagétien est-il toujours actuel ? [Development of logical competences in arithmetics. Is the Piagetian model still up to date?]. In M. Crahay, L. Verschaffel, E. De Corte & J. Grégoire (Eds.), *Enseignement et apprentissage des mathématiques* (pp. 57-77). Bruxelles: De Boeck.

Grégoire, J., Noël, M., & Van Nieuwenhoven, C. (2004). *TEDI-MATH*. Antwerpen: Harcourt.

Grietens, H., Hoppenbrouwers, K., Desoete, A., Wiersema, R., & Van Leeuwen, K. (2010). *JOnG! Theoretische achtergronden, onderzoeksopzet en verloop van het eerste meetmoment [JOnG! Theoretical backgrounds, study design and course of the first assessment].* Leuven: Steunpunt, Welzijn, Volksgezondheid en Gezin (SWVG).

Hannula, M. M., Räsänen, P., & Lehtinen, E. (2007). Development of counting skills: Role on numerosity and subitizing-based enumeration. *Mathematical Thinking, 9*, 51-57. http://dx.doi.org/10.1080/10986060709336605

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*, 59-69. http://dx.doi.org/10.1037/0012-1649.42.1.59

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*, 395–438. http://dx.doi.org/10.1016/j.cognition.2006.10.005

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Revisiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52*, 130–169. http://dx.doi.org/10.1016/j.cogpsych.2005.07.002

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2011). What counts in the development of young children's number knowledge? *Developmental Psychology, 47*, 302-302. http://dx.doi.org/10.1037/a0022101

Maier, M. A., Barchfield, P., Elliot, A. J., & Pekrun, R. (2009). Context specificity of implicit preferences: The case of human preference for red. *Amercan Psychological Association, 9*, 734-738. http://dx.doi.org/10.1037/a0016818

Maloney, E. A., Risko, E. F., Ansari, D., & Fugelsang, J. (2010). Mathematics anxiety affects counting but not subitizing during visual enumeration. *Cognition, 114*, 293-297. http://dx.doi.org/10.1016/j.cognition.2009.09.013

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108,* 662-674. http://dx.doi.org/10.1016/j.cognition.2008.05.007

Tobii Technology Inc., Falls Church, VA, USA, Product description: Tobii T/X Series eye trackers, May 2007.

Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19*, 21-39. http://dx.doi.org/10.1016/j.ecresq.2004.01.007

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25. http://dx.doi.org/10.1016/S0010-0277(03)00050-7

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology, 25*, 103-108. http://dx.doi.org/10.1348/026151005x90704

Zemach, I.K., & Teller, D.Y. (2007). Infant color vision: Infants’ spontaneous color preferences are well behaved. *Vision Research, 47*, 1362-1367. http://dx.doi.org/10.1016/j.visres.2007.02.002

CHAPTER 2

ENUMERATION OF SMALL AND LARGE NUMEROSITIES IN ADOLESCENTS WITH MATHEMATICAL LEARNING DISORDERS[1]

ABSTRACT

The accuracy and speed in an enumeration task were investigated in adolescents with typical and atypically poor development of arithmetic skills. The number naming performances on small and large nonsymbolic numerosities of 18 adolescents with *mathematical learning disorders* (MLDs) and 28 typically achieving age-matched adolescents were compared. A mixed logistic regression model showed that adolescents with MLD were not significantly less accurate on numerosities within the *subitizing* range than control peers. Moreover, no significant differences in reaction times were found between both groups. Nevertheless, we found that within the control group adolescents with higher ability tended to respond faster when taking into account the whole range (1-9) of numerosities. This correlation was much weaker in the MLD group. When looking more closely at the data, however, it became clear that the correlation between accuracy and speed within the control group differed in direction depending on the range (subitizing or counting) of the numerosities. As such, our findings did not support an impaired ability of subitzing in MLD. However, the data stressed a different correlation between speed and accuracy for both groups of adolescents and a different behavioral pattern depending on the numerosity range as well. Implications for the understanding and approach of MLD are considered.

---

[1] Based on Ceulemans, A., Titeca, D., Loeys, T., Hoppenbrouwers, K., Rousseau, S., & Desoete, A. (2014). Enumeration of small and large numerosities in adolescents with mathematical learning disorders. *Research in Developmental Disabilities, 35,* 27-35. http://dx.doi.org/10.1016/j.ridd.2013.10.018

Mathematical literacy is important in our society (e.g., Vanmeirhaeghe, 2012). Numbers and mathematics are inherently present in everyday life; each day we are confronted with it while paying in the shop, baking a cake, travelling by train, ... However, it is a fact that in some children, determining numerosity gives stress (e.g., Vanmeirhaeghe, 2012). Although specific *mathematical learning disorders* (MLDs) have serious educational consequences, this area has received less attention than it deserves, contrary to specific reading disorders (Dowker, 2005; Tymms, 1999). The estimated prevalence of MLD lies between 3% and 14% of the population depending on the country of study and the used criteria (American Psychiatric Association [APA], 2013; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Dowker, 2005; Shalev, Manor, & Gross-Tsur, 2005).

In the *Diagnostic and Statistical Manual of Mental Disorders* (5th ed.; DSM-5; APA, 2013), the term MLD refers to the specific learning disorder with a significant degree of impairment in mathematics, manifesting itself in difficulties with mastering *number sense,* number facts, mathematical reasoning, or calculation skills. In accordance with the definition in the DSM-5 (APA, 2013) as described below, three criteria are used to determine whether a child has a clinical diagnosis of MLD, namely, the severeness criterion, the resistance criterion, and the exclusion criterion (Fuchs et al., 2007). The mathematics abilities of individuals with MLD situate themselves substantially and quantifiably below those expected for the individual’s chronological age, causing interference with academic performance (APA, 2013). This is known as the severeness criterion (Fuchs et al., 2007). In addition, the symptoms persist for at least 6 months despite the provision of interventions that target the specific difficulties (APA, 2013). This is referred to as the resistance criterion or a lack of responsiveness to intervention (RTI; Fuchs et al., 2007). Finally, the MLD-related problems cannot be better accounted for by intellectual disabilities or external factors (such as inadequate educational instruction) that could provide sufficient evidence for scholastic failure (APA, 2013), also known as the exclusion criterion (Fuchs et al., 2007).

There are several models trying to describe or explain the mechanisms underlying quantity processing deficits in children with MLD. Some models focus on immature counting and calculation strategies, deficits in working memory or deficits in retrieving facts from semantic long-term memory, problems with visual-spatial elaboration, and executive deficits (e.g., Geary, 2011; Passolunghi & Siegel, 2004). However, other researchers consider the aforementioned deficits as higher order problems of children with MLD resulting from a low-level deficient or imprecise number representation (e.g., Butterworth, 2005a,b; Butterworth, Varma, & Laurillard, 2011). From this perspective, MLD is the result of a specific disability in basic numerical processing, rather than the consequence of a deficit in other cognitive abilities such as outlined above (Landerl, Bevan, & Butterworth, 2004; Noël & Rousselle, 2011).

Within the field of MLD, *subitizing* or the rapid (40-100 ms/item), automatic, and accurate assessment of small quantities of up to three (or four) items (Kaufman, Lord, Reese, & Volkmann, 1949; Koontz & Berch, 1996; Trick & Pylyshyn, 1993) is investigated as a core deficit in this basic numerical processing (e.g., Fischer, Gebhardt, & Hartnegg, 2008; Schleifer & Landerl, 2011). According to some studies, children with MLD serially count items within the subitizing range, whereas typically achieving (TA) children subitize the same amount of items (Bruandet, Molko, Cohen, & Dehaene, 2004; Butterworth, 1999; Moeller, Neuburger, Kaufmann, Landerl, & Nuerk, 2009). Although it is demonstrated that children with MLD are slower in subitizing tasks compared to TA children (Koontz & Berch, 1996; Landerl, Bevan, & Butterworth, 2004; Schleifer & Landerl, 2011), there is no consensus on this subitizing problem, since some studies do not support children with MLD being slower on small numbers (De Smedt & Gilmore, 2011; Rouselle & Noël, 2007). In addition, some studies revealed that, indeed, some of the children with MLD (but not all of them) have subitizing problems. Desoete and Grégoire (2006), for example, found a subitizing deficit in 33% of the children of aged 8.5 years old with a clinical diagnosis of MLD. Fischer et al. (2008) found that between 43% and 79% of the participants in the age range of 7 to 17 years with MLD performed below the 16th percentile of the peer control groups on subitizing tasks.

In the studies mentioned above different tasks were used, making studies difficult to compare. In some studies, stimuli were presented during a short time span, disabling counting and urging individuals to use subitizing (e.g., Fischer et al., 2008). In other studies, individuals were allowed to count because stimuli were shown until a response was given (e.g., Moeller et al., 2009). Although the former method is the best way to assess rapid enumeration of a small set of items without counting, the latter is used more often.

This study aimed to enlarge the knowledge about subitizing in MLD using an enumeration task presenting numerosities (up till nine items) only for a short time to MLD and TA adolescents. The main question was whether these groups differ in accuracy and reaction time for either small (up till four items) or larger numbers (from five to nine items). In line with Fischer et al. (2008), who used a similar task to investigate enumeration in subjects with and without MLD (age 7-17 years), it was expected that the MLD group would perform both slower and less accurate than the TA group, especially regarding the small numerosities within the subitizing range.

METHOD

Participants and procedure

Participants were 18 adolescents with MLD and 28 TA adolescents between 13 and 16 years old. Age, IQ, and gender of the participants are described in Table 1. As shown, no significant differences in age (*p* = .482) or gender (*p* = .953) were found between the groups. However, a significant difference in IQ was found between the groups (*p* = .002). All individuals were living in Flanders, the Dutch-speaking part of Belgium. About half of the 46 participants were a subsample of a larger cohort study of JOnG! (from which this study is only one part). This large-scale study was carried out by the universities of Ghent and Leuven at the request of the Belgian government (http://www.steunpuntwvg.be/jong, see Grietens, Hoppenbrouwers, Desoete, Wiersema, and Van Leeuwen, 2010). Additional adolescents (*n* = 22) for the current study were recruited from mainstream and special education schools and an informed consent was obtained for each participant. The present study – as part of the larger study – was approved by the ethical committees of the Ghent University and the Catholic University of Leuven. The research took place in rooms provided by local *Pupil Guidance Centres* [Centra voor Leerlingenbegeleiding, CLB] in Flanders. Master students in educational sciences were trained to administer the tests of the testprotocol used in this study. Table 1 also shows the scores on the standardized math and readings tests.

Table 1. *Descriptive characteristics of the participants*

	TA Group (*n* = 28)		MLD Group (*n* = 18)		
Measure	*M*	*(SD)*	*M*	*(SD)*	*t(1, 44)*
Age	14.43	*(0.57)*	14.33	*(0.77)*	0.48
IQ	103.29	*(9.90)*	93.17	*(10.58)*	3.29**
EMT	15.29	*(16.11)*	10.28	*(2.35)*	1.31
KLEPEL	16.39	*(16.50)*	10.78	*(2.49)*	1.43
TTR	78.25	*(18.94)*	43.28	*(21.37)*	5.81***
KTR-R	63.82	*(25.22)*	10.56	*(8.95)*	8.59***
Gender	Boys	Girls	Boys	Girls	$\chi^2(1)$
	8	20	5	13	0.003

Note. TA = typically achieving. MLD = mathematical learning disorder. IQ = Intelligence Quotient. EMT = *Eén-Minuut test* [One-minute-test]. TTR = *Tempotest Rekenen* [Arithmetic Number Fact Test]. KRT-R = *Kortrijkse Rekentest - Revisie* [Kortrijk Arithmetic Test Revision].
** *p* ≤.010. *** *p* ≤ 0.001

In order to be included in the sample of this study as a participant with MLD, adolescents needed to have a clinical diagnosis of impairing learning difficulties as indicated by parent-report within the scope of the larg-scale study (JOnG!). Whether it concerned the specific diagnosis of MLD was questioned by telephone afterwards.

In Flanders, a standardized test on the memorization of arithmetic facts and a test on accurate and fluent calculation are commonly used to explore the severeness criterion of impaired mathematical abilities. The second criterion implies the persistence of number fact or calculation difficulties despite the provision of targeted interventions (nonresponsiveness to remediation; Desoete et al., 2010). Finally, to meet the exclusion criterion, mathematical problems may not be due to a lack in education, a sensory deficit, or another behavioral or developmental disorder.

In the current study, this formal diagnosis of MLD (fulfilling the three criteria as described above) was confirmed as follows. The participants had to be at least of average intelligence and had to score below the 25th percentile (in line with Geary, 2004) on fluent calculation or memorization of arithmetic facts when compared to a norm group. This would demonstrate the affected academic skills and significant interference with academic performance as suggested in DSM-5 (APA, 2013). Furthermore, reading scores achieved by MLD adolescents had to exceed the 25th percentile in order to exclude a comorbid diagnosis of a specific learning disorder with impairment in reading. In the TA group, adolescents had to be at least of average intelligence and needed to have scores above the 25th percentile on mathematics and reading. In TA adolescents, there was no parental concern on academic or other developmental problems as indicated by parent-report within the scope of the larger cohort study of JOnG!.

Measures

Intelligence. An estimated IQ was calculated, using an abbreviated version of the Dutch *Wechsler Intelligence Scale for Children - Third Edition* (WISC-III; Wechsler, 2005). This shortened version was recommended by Grégoire (2000), has a high correlation (r = .93) with full scale IQ (Kaufman, Kaufman, Balgopal, & McLean, 1996), and consists of four subtests: Vocabulary, Similarities, Picture Arrangement, and Block Design. Total duration of administration of this shortened version was estimated at about 30 minutes.

Mathematics. All adolescents were tested with the *Arithmetic Number Fact Test* (Tempo Test Rekenen [TTR]; De Vos, 1992) and the *Kortrijk Arithmetic Test Revision* (Kortrijkse Rekentest-Revisie [KRT-R]; Baudonck et al., 2006).

The TTR is a test on memorization of arithmetic facts, consisting of five subtests concerning arithmetic number fact problems: addition, subtraction, multiplication, division, and mixed exercises. The participants had to solve as many items as possible in five minutes and they could work 1 minute on every subtest. The TTR is a frequently used test in Flemish education and scientific research as a measure of the memorization of arithmetic facts (e.g., Bachot, Gevers, Fias, & Roeyers, 2005; Callens, Tops, & Brysbaert, 2012; Stock, Desoete, & Roeyers, 2010; Tops, Callens, Lammertyn, Van Hees, & Brysbaert, 2012; Zhao, Valcke, Desoete, Burny, & Imbo, 2013). Moreover, the test has been standardized in Flanders on a sample of 10,059 children in total (Ghesquière & Ruijssenaars, 1994).

The KRT-R is a standardized test of mathematical achievement which requires that individuals solve mental arithmetic and number knowledge tasks. The KRT-R is frequently used in Flemish education as a measure of accurate calculation skills (e.g., Stock et al., 2010). The psychometric value of the KRT-R has been demonstrated on a sample of 3,246 children in total. A validity coefficient (correlation with teacher ratings) and reliability coefficient (Cronbach's α) of respectively .65 and .83 were found. In addition, the test-retest value was .78 (Baudonck et al., 2006). Participants could solve the tasks of the KRT-R in a period of maximum 45 minutes.

Reading. All adolescents were tested with standardized Dutch reading measures. Total duration was about 5 minutes. Word reading accuracy and fluency were assessed by the *One Minute reading Test* (EMT; Brus & Voeten, 2010) and pseudo-word reading by the KLEPEL (Van den Bos, Spelberg, Scheepstra, & de Vries, 2010). Both tests consist of lists of 116 unrelated words. Adolescents were instructed to read as many words as possible in 1 (EMT) or 2 minutes (KLEPEL) without making errors. The raw scores consisted of the number of words read correctly. Both tests were validated in Flanders on 10,059 children (Ghesquière & Ruijssenaars, 1994), with a reliability of .76 for the EMT and .91 for the KLEPEL (Van den Bos, Spelberg, Scheepstra, & de Vries, 1994).

Numerosity. All adolescents were tested with an enumeration test. In this task, stimuli were displayed on a 17” monitor. Verbal responses were collected using a microphone headset. Each trial began with a fixation point presented for 500 ms. A display containing one to nine square boxes was then centrally presented at fixation until a vocal response was detected. Participants were instructed to say aloud the number of squares on the screen. All squares were black on a white background. The *individual area*, *total area*, and *density* of the squares were varied to ensure that participants could not use non-numerical cues to make a correct decision (see Dehaene, Izard, & Piazza, 2005; Maloney, Risko, Ansari, & Fugelsang, 2010). There were two practice phases and one test phase. In the first practice phase, adolescents were presented with five different screens with squares. The presentation time of the stimuli was limited to 5,000 ms but the reaction time was unlimited. The second practice phase consisted of 10 screenshots of different numbers of squares situated at random on the screen. The presentation time was 120 ms – in line with the study of Hannula, Räsänen, and Lehtinen (2007) and Fischer et al. (2008) – and the participants had to react within 5 seconds after beginning of the presentation. The test session consisted of 72 samples maintaining the same format of the second practice phase. Both accuracy and reaction time (only correct trials were included) were measured. Total duration of administration of the enumeration task was estimated at about 5 minutes.

Analyses

In order to define the subitizing and counting range for both groups, multiphase models (Cudeck & Klebe, 2002) were fitted to the response times for each individual separately as a function of the numerosity. More specifically, a linear-quadratic model with varying change point at τ was assumed with a continuity between both segments. The difference between the medians of the estimated individual change points in the MLD group and TA group was assessed using a Wilcoxon-rank test.

To assess the difference between response times in both groups, a linear mixed model with crossed random effects (a random effect for each participant and a random effect for each stimulus) and fixed effects for numerosity (as a factor), group, and their interaction was fitted. The variance of the random effects for the participants reflected the variability in speed between participants, whereas the variance of the random effects of the stimuli reflected the variability in the intensity of the stimuli (Loeys, Rosseel, & Baten, 2011). Similarly, a mixed logistic regression model with crossed random effects with the same fixed effects was used to assess the difference in accuracy (i.e., the probability of giving a correct response). Here, the variance of the random effects for the participants reflected the variability in capacity between participants, whereas the variance of the random effects of the stimuli reflected the variability in the difficulty level of the stimuli. Using a joint modeling framework for the response time and accuracy (Loeys et al., 2011), one can use the correlation of the random effects for the participants from both models to explore the *speed-accuracy trade-off* (SAT; e.g., Shouten & Bekker, 1967; Wickelgren, 1977).

Analyses were performed in R 2.15.3 (R Core team, 2013) for the linear mixed model and in SAS® 9.3 (SAS Institute Inc[2], Cary NC, 2011) for the linear-quadratic model change point analysis.

RESULTS

The median change point in the TA group was 3.9 (Interquartile Range from 3.5 to 4.2), whereas in the MLD group the median change point was 3.6 (Interquartile Range from 2.8 to 4.6). The difference in medians between both groups was not statistically significant (Wilcoxon $Z = 1.13$, $p = .256$). The 1-4 range was, therefore, defined as the subitizing range and the 5-9 range as the counting range in all subsequent analyses. The mean of the accuracy scores, calculated as the percentage of correct responses at each numerosity, are presented by group in Figure 1.

---

[2] SAS Institute Inc. SAS and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc., Cary, NC, USA.

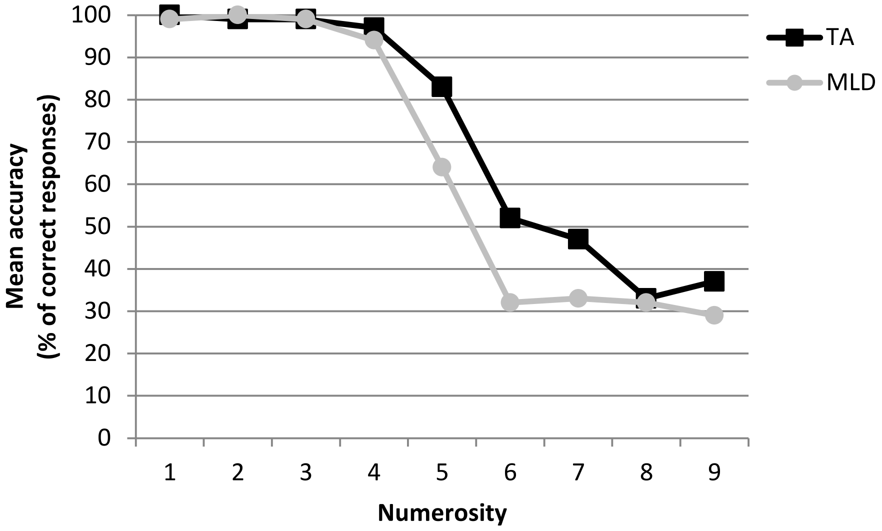

Figure 1. Mean accuracy (in percentage of correct responses) for the presented numerosities during the enumeration task.

The overall mean accuracy on the enumeration task was 72.07% (*SE* = 1.00) for adolescents without MLD and 64.81 % (*SE* = 1.33) for adolescents with MLD. Based on mixed logistic regression neither an overall significant difference nor a difference at any specific numerosity was found between groups (all *p* >.300). When grouping the numerosities into categories according to the defined subitizing range and counting range, a significant difference (*p* = .001) was observed in accuracy between the TA group ($M_{counting}$ = 50.54 %, *SE* = *1.49;* $M_{subitizing}$ = 99.00 %, *SE* = 0.33) and the MLD group ($M_{counting}$ = 38.19 %, *SE* = 1.81*;* $M_{subitizing}$ = 98.09 %, *SE* = 0.57) in the counting range (with the TA group being more accurate than the MLD group), but not in the subitizing range (*p* = .479).

Next, the mean reaction times on the enumeration task at each of the numerosities are presented by group in Figure 2.

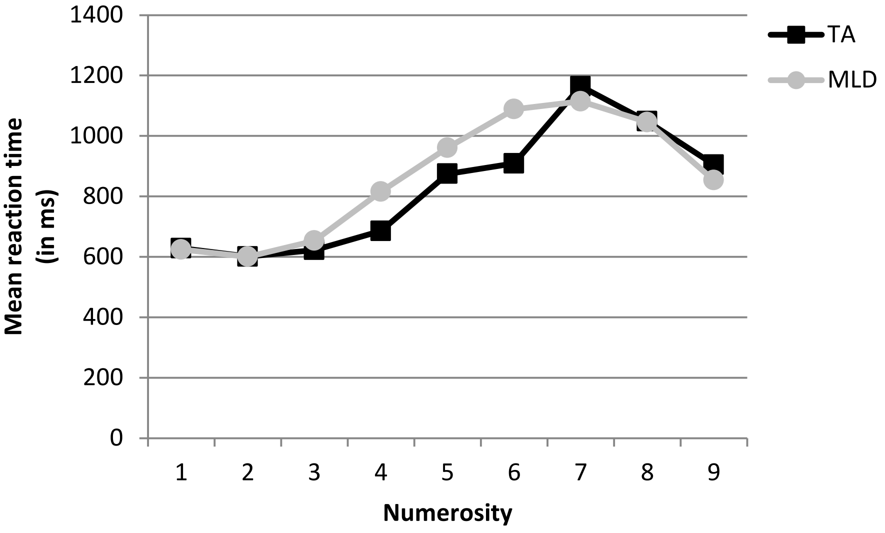

Figure 2. Mean reaction time (in milliseconds) for the presented numerosities during the enumeration task.

The overall mean reaction time on the enumeration task was 844.15 ms (*SE* = 48.20) for adolescents without MLD and 878.73 ms (*SE* = 57.13) for adolescents with MLD. Based on the mixed model for the response time, no overall significant difference was found between both groups (*p* = .300) at the 5% significance level. When looking at each numerosity level separately, the largest differences were found at numerosities 4 and 6 (*p* = .019 and *p* = .013 respectively), but after applying a conservative Bonferroni correction, these differences were no longer considered significant. When grouping the numerosities into categories according to the defined subitizing range and counting range, a marginally significant difference (*p* = .051) was observed in response times between the TA group ($M_{counting}$ = 1,005.37, *SE* = 69.73; $M_{subitizing}$ = 636.44, *SE* = 28.22) and the MLD group ($M_{counting}$ = 1,087.01, *SE* = 86.96; $M_{subitizing}$ = 651.04, *SE* = 34.77) in the counting range (with the TA group responding faster than the MLD group), but not in the subitizing range (*p* = .800).

Finally, based on the joint modeling approach, the SAT was explored. In the TA group, increasing speed was significantly associated with higher accuracy; the correlation equaled .34 (95 % CI from .05 to .63). In contrast, this association was much weaker in the MLD group; the correlation only equaled .09 (95 % CI from -.34 to .46).

In other words, it was observed that whereas in the control group adolescents with a higher ability (accuracy) tended to respond faster, this correlation was much weaker in the MLD group. Using Fisher r-to-z transformations, however, no significant difference between the correlation coefficients was found ($z = 0.81$, $p = .418$). Adjusted for IQ, the same results could be found (correlation equaled .34 with 95 % CI from .03 to .64 in the TA group, and .08 with 95 % CI from -.32 to .64 in the MLD group).

The associations between speed and accuracy, though, were mostly driven by the results in the counting range. Indeed, in the subitizing range, the correlations were even numerically negative and equaled -.10 (95 % CI from -.51 to .42) and -.05 (95 % CI from -.51 to .55) in the TA group and MLD group respectively. However, it should be noted that in this range (i.e., subitizing), the error rate was rather low for all participants and, hence, the variability between participants was low. In the counting range on the other hand, the correlations between speed and accuracy equaled .34 (95 % CI from .01 to .64) and .04 (95 % CI from -.37 to .45) in the TA group and MLD group respectively.

DISCUSSION

Preparatory analyses revealed that the subitizing range could be defined from numerosities one to four, whereas the counting range encompassed those from five to nine for both groups of adolescents. From the graphs on the reaction time and accuracy data, it is clear that all participants switched to a slower process of enumeration in larger numerosities (from five onwards). They also tended to answer less accurate within this range of numerosities. Considering possible differences between the groups, no overall significant differences were found in accuracy or reaction time when taking into account either the whole range of numerosities or the numerosities within the subitizing range (1-4). For the counting range (5-9), however, analyses revealed a significant difference in accuracy and a marginally significant difference in reaction time between the TA and the MLD group, with the former group being more accurate and responding faster during the enumeration task within this range.

To conclude, no evidence was found in the MLD group neither for a less accurate performance nor for a slower processing of numerosities within the subitizing range. This is in line with De Smedt and Gilmore (2011) and Rousselle and Noël (2007), but in contrast with some other studies (e.g., Fischer et al., 2008; Moeller et al., 2009; Schleifer & Landerl, 2011). This lack of evidence of impaired subitizing skills in the current study might be due to specific task instructions or specific sample characteristics. First, compared with the study of Moeller and colleagues (2009), our task urged participants to rely on subitizing instead of counting skills, whereas no time constraints were set in the other study. Second, our sample included both more and older participants as compared with this study (Moeller et al., 2009). However, even when comparing our study with one using a similar task in (partially) the same age group, such as the one by Fischer et al. (2008), differences in reaction time or accuracy in subitizing performance between MLD and TA individuals could not be replicated. Because MLD might not be as homogeneous as assumed (some studies revealed subtypes, e.g., Geary, 2004; Pieters, Roeyers, Rosseel, Van Waelvelde, & Desoete, 2013; Robinson, Menchetti, & Torgesen, 2002; Temple, 1991), this might suggest that not all MLD individuals have a subitizing deficit.

The data indicated, furthermore, that problems in MLD are not located at the level of rapid or accurate encoding of small quantities. Rather, the combination of speed and accuracy in enumerating quantities up till nine seems to have a different outcome in adolescents with MLD compared to their TA peers. Although no significant difference was found between the groups on the speed-accuracy correlation, it was demonstrated that whereas TA adolescents with higher enumeration accuracy also responded faster, their peers with MLD did hardly show this tendency. Correction for IQ did not reveal any difference in the results. Moreover, IQ was no significant predictor of reaction time or accuracy. The finding in the TA group points at an automatization process, which enables them to respond both fast and accurate during an enumeration task. The task in the current study implicitly required adolescentes to make an association between the nonsymbolic (e.g., group of items with different quantities) internal representation of magnitude and a symbolic modality of the same magnitude. This is, participants needed to verbally name the number word of the represented quantity on the screen.

Because individuals with MLD experience difficulties with the automatic association of symbols to the internal representation of magnitude (e.g., Rubinsten & Henik, 2005), it is likely that they would use more time to overthink the correct response to improve their performance on an enumeration task. This is in line with standard theories on SAT (e.g., Shouten & Bekker, 1967; Wickelgren, 1977), according to which one might expect that slower responding is beneficial for accuracy because individuals have more time to consider the correct response (e.g., Falkenstein, Hoormann, Christ, & Hohnsbein, 2000). The fact that our results, however, did not reflect the use of a SAT strategy in MLD individuals could be once more due to our specific task instructions. Because the stimuli were only presented during a short time span, participants were not advantaged when taking more time to overthink their responses. As such, trading speed for accuracy was not beneficial in this context. In contrast, enumeration tasks in which the design enables participants to count the presented numerosities could trigger this trade-off in MLD individuals.

Nevertheless, the lack of a substantial speed-accuracy correlation suggests that the ability to enumerate quantities (small or large) is not (yet) automated in adolescents with MLD. Moreover, when taking into account the subitizing and counting range, a different behavioral pattern was observed between the TA and MLD group. For the MLD group, the speed-accuracy correlation was negligible considering both ranges respectively. For the TA group, in contrast, a less negligible – though, still low – negative correlation could be observed for the subitizing range and a positive correlation for the counting range. The latter result (i.e., the positive speed-accuracy correlation in the TA group) is consistent with the aforementioned overall speed-accuracy correlation for the whole range. The former (i.e., the negative speed-accuracy correlation in the TA group), however, shows a relation in the opposite direction, meaning that the more time subjects took to overthink their answer, the better they performed. Although this finding was somehow against expectations – especially regarding small numerosities for which one is mainly expected to use subitizing skills (rapidly naming the accurate number) – this result indicated a strategy switch consistent with the range to which a numerosity belongs.

It seemed that whereas in TA adolescents the range defined the answer strategy, this was not the case in their MLD peers. The marginally significant difference in reaction time and the significant difference in accuracy between the TA and the MLD group in the counting range (with the TA group reacting faster and more accurate) as well as the speed-accuracy correlation specifically for the counting but not the subitizing range, pointed in this direction. It might be that TA adolescents senses that taking less time during an enumeration task within the counting range was more efficient. This would suggest that, in this case, the TA individuals relied on a faster estimation process compared to the more time-consuming strategy of counting. Moreover, a counting strategy was not feasible anyway given the restricted presentation time. The higher accuracy for TA adolescents for numerosities within this counting range also mirrored the beneficial use of their strategy.

This theoretical consideration could result in some practical implications regarding assessment, intervention, and support of individuals with MLD. First, the assessment of number naming should at least take into account both speed and accuracy as well as the combination of both aspects. Second, based on the lack of observed speed-accuracy correlation, adolescents with MLD seem to have problems with adjusting their behavior according to the specific demands of a learning situation. This implies, consequently, that they have more troubles to orient and choose the most efficient mathematical strategy. Therefore, it might be useful to focus on declarative, conditional, and procedural knowledge and to help them understand what, when, and how strategies work and why these strategies – such as taking more time – are useful to solve specific math-related problems. Not knowing which strategy to choose may underlie the RTI of children and adolescents with MLD. Moreover, assessment should aim at detecting strong and weak skills in adolescents with MLD in order to develop reasonable adjustments or STICORDI advices. STICORDI stands for stimulation, compensation, remediation, and dispensation advices based on specific needs of the individual child (Henneman, 1989). This could imply that if the aforementioned intervention would not suffice in improving basic numerical skills, compensatory mechanisms (such as a calculator) should be offered to improve performance in individuals with MLD when they are put under time pressure.

The results of this study should be interpreted with care because the number enumeration skills were only assessed in a small group of adolescents with and without MLD. Obviously, sample size is not a problem to detect significant differences. However, when analyses have insufficient power and are not significant, a risk of type 2- or β-mistakes cannot be excluded (Field, 2009, pp. 55-56). Additional research with a larger group of participants is indicated anyway. Furthermore, accuracy within the subitizing range (1-4) was very high, resulting in hardly any variation in this measure, making the estimation of the relation between speed and accuracy more difficult. In addition, number enumeration is only one possible paradigm to assess the ability to process numerosities. Although an attempt was made to explain the results related to enumeration of numerosities within the counting range (5-9), the task that was used in the current study aimed to especially investigate subitizing more in depth. Other paradigms such as traditional number comparison tasks (assessing the abilities to discriminate especially two – especially larger [Halberda & Feigenson, 2008] – quantities in order to point out the largest of both [Gersten et al., 2012]), might therefore be more suitable to draw conclusions on performances in the counting range. Future research should thus consider other paradigms or combine paradigms within and outside the subitizing range (Defever, Sasanguie, Gebuis, & Reynvoet, 2011; Kadosh, Muggleton, Silvanto, & Walsh, 2010; van Opstal & Verguts, 2011).

To summarize, adolescents with MLD show a different profile regarding speed-accuracy performance on an enumeration task compared to TA peers. Within this scope, future research should address this topic more in detail to unravel the role of subitizing as a possible core deficit of MLD.

REFERENCES

American Psychiatric Association [APA]. (2013). *Diagnostic and statistical manual of mental disorders* (5th ed.). Arlington, VA: American Psychiatric Publishing.

Bachot, J., Gevers, W., Fias, W., & Roeyers, H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. *Psychology Science, 47*, 172-183. Retrieved from http://www.psychologie-aktuell.com

Barbaresi, W. J., Katusic, S. K., Colligan, R. C., Weaver, A. L., & Jacobsen, S. J. (2005). Learning disorder: Incidence in a population-based birth cohort, 1976-82, Rochester, Minn. *Ambulatory Pediatrics, 5*, 281-289. http://dx.doi.org/10.1367/A04-209R.1

Baudonck, M., Debusschere, A., Dewulf, B., Samyn, F., Vercaemst, V., & Desoete, A. (2010). *Kortrijkse Rekentest Revisie (KRT-R) [Kortrijk Arithmetic Test Revision]*. Kortrijk, Belgium: Revalidatiecentrum Overleie.

Bruandet, M., Molko, N., Cohen, L., & Dehaene, S. (2004). A cognitive characterization of dyscalculia in Turner syndrome. *Neuropsychologia*, *42*, 288–298. http://dx.doi.org/10.1016/j.neuropsychologia.2003.08.007

Brus, B. T., & Voeten, M. J. M. (2010). *Eén-Minuut-Test (EMT): vorm A en B [One-Minute-Test: form A and B]*. Nijmegen, The Netherlands: Berkhout.

Butterworth, B. (1999). *The mathematical brain.* London: Macmillan.

Butterworth, B. (2005a). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, 46,* 3-18. http://dx.doi.org/10.1111/j.1469-7610.2004.00374.x

Butterworth, B. (2005b). Developmental dyscalculia. In J.I.D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 455 – 467). Hove: Psychology Press.

Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: From brain to education. *Science, 332*, 1049-1053. http://dx.doi.org/10.1126/science.1201536

Callens, M., Tops, W., & Brysbaert, M. (2012). Cognitive profile of students who enter higher education with an indication of dyslexia. *Plos One, 7*, e38081. http://dx.doi.org/10.1371/journal.pone.0038081

Cudeck, R., & Klebe, K. J. (2002). Multiphase mixed-effects models for repeated measures data. *Psychological Methods, 7,* 41-63. http://dx.doi.org/10.1037/1082—989X.7.1.41

Defever, E., Sasanguie, D., Gebuis, T., & Reynvoet, B. (2011). Children's representation of symbolic and nonsymbolic magnitude examined with the priming paradigm. *Journal of Experimental Child Psychology, 109*, 174-186. http://dx.doi.org/ 10.1016/j.jecp.2011.01.002

Dehaene, S., Izard, V., & Piazza, M. (2005). Control over non-numerical parameters in numerosity experiments. *Unpublished manuscript.* Available at http://www.unicog.org/…/DocumentationDotsGeneration.doc.

De Smedt, B., & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of Experimental Child Psychology, 108,* 278-292. http://dx.doi.org/10.1016/j.jecp.2010.09.003

Desoete, A., Ghesquière, P., De Smedt, B., Andries, C., Van den Broeck, W., & Ruijssenaars, W. (2010). Dyscalculie: Standpunt van onderzoekers in Vlaanderen en Nederland *[Dyscalculia: Point of view of researchers in Flanders and the Netherlands]*. *Logopedie, 23*, 4-9.

Desoete, A., & Grégoire, J. (2006). Numerical competence in young children and in children with mathematics learning disabilities. *Learning and Individual Differences*, *16*, 351-367. http://dx.doi.org/10.1016/j.lindif.2006.12.006

De Vos, T. (1992). *Tempo Test Rekenen (TTR) [Arithmetic Number Fact Test]*. Nijmegen, The Netherlands: Berkhout.

Dowker, A. (2005). *Individual differences in arithmetic. Implications for psychology, neuroscience and education*. Hove, United Kingdom: Psychology Press.

Falkenstein, M., Hoormann, J., Christ, S., & Hohnsbein, J. (2000). ERP components on reaction errors and their functional significance: a tutorial. *Biological Psychology, 51*, 87-107. http://dx.doi.org/10.1016/S0301-0511(99)00031-9

Field, A. (2009). *Discovering Statistics Using SPSS (third edition).* London: Sage.

Fischer, B., Gebhardt, C., & Hartnegg, K. (2008). Subitizing and visual counting in children with problems acquiring basic arithmetic skills. *Optometry and Vision Development*, *39*, 24-29.

Fuchs, L. S., Fuchs, D., Compton, D. L., Bryant, J. D., Hamlett, C. L., & Seethaler, P. M. (2007). Mathematics screening and progress monitoring at first grade: Implications for responsiveness to intervention. *Exceptional Children, 73*, 311-330. http://dx.doi.org/10.1177/1053451211414188

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37,* 4-15. http://dx.doi.org/10.1177/00222194040370010201

Geary, D. C. (2011). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of Developmental and Behavioral Pediatrics, 32*, 250-263. http://dx.doi.org/ 10.1097/DBP.0b013e318209edef

Gersten, R., Clarke, B., Jordan, N. C., Newman-Gonchar, R., Haymond, K., & Wilkins, C. (2012). Universal screening in mathematics for the primary grades: Beginnings of a research base. *Exceptional Children*, *78*, 423-445.

Ghesquière, P., & Ruijssenaars, A. (1994). *Vlaamse normen voor studietoetsen rekenen en technisch lezen lager onderwijs [Flemish standards for study evaluation of mathematics and technical reading in primary schools].* Leuven, Belgium: Catholic University of Leuven, Center of Educational and Professional Guidance.

Grégoire, J. (2000). *L'évaluation clinique de l'intelligence de l'enfant*. Liège, Belgium: Mardaga.

Grietens, H., Hoppenbrouwers, K., Desoete, A., Wiersema J. R., & Van Leeuwen, K. (2010). *JOnG! Theoretische achtergronden, onderzoeksopzet en verloop van het eerste meetmoment. [JOnG! Theoretical backgrounds, study design and course of the first assessment].* Leuven: Steunpunt Welzijn Volksgezondheid en Gezin (SWVG).

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the ”number sense”: The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*, 1457-1465. http://dx.doi.org/10.1037/a0012682

Hannula, M. M., Räsänen, P., & Lehtinen, E. (2007). Development of counting skills: Role on numerosity and subitizing-based enumeration. *Mathematical Thinking, 9*, 51-57. http://dx.doi.org/10.1080/10986060709336605

Henneman, K. (1989). *Methoden voor behandeling van ernstige spellingsproblemen bij oudere leerlingen. Symposium Dyslexie en Dysorthografie*. *[Treatment of severe spelling problems in older students].* Gent: Omega Editions.

Kadosh, R. C., Muggleton, N., Silvanto, J., & Walsh, V. (2010). Double dissociation of format-dependent and number-specific neurons in human parietal cortex. *Cerebral Cortex, 20*, 2166-2171. http://dx.doi.org/10.1093/cercor/bhp273

Kaufman, A. S., Kaufman, J. C., Balgopal, R., & McLean, J. E. (1996). Comparison of three WISC-III short forms: Weighing psychometric, clinical, and practical factors. *Journal of Clinical Child Psychology, 25*, 97-105. http://dx.doi.org/10.1207/s15374424jccp2501_11

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology, 62*, 498-525. http://dx.doi.org/10.2307/1418556

Koontz, K. L., & Berch, D. B. (1996). Identifying simple numerical stimuli: Processing inefficiencies exhibited arithmetic learning disabled children. *Mathematical Cognition, 2*, 1-23. http://dx.doi.org/10.1080/135467996387525

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8-9 year old students. *Cognition, 93,* 99-125. http://dx.doi.org/10.1016/j.cognition.2003.11.004

Loeys, T., Rosseel, Y., & Baten, K. (2011). A joint modeling approach for reaction time and accuracy in psycholinguistic experiments. *Psychometrika, 76,* 487-503. http://dx.doi.org/10.1007/s11336-011-9211-Y

Maloney, E. A., Risko, E. F., Ansari, D., & Fugelsang, J. (2010). Mathematics anxiety affects counting but not subitizing during visual enumeration. *Cognition, 114*, 293-297. http://dx.doi.org/10.1016/j.cognition.2009.09.013.

Moeller, K., Neuburger, S., Kaufmann, L., Landerl, K., & Nuerk, H. C. (2009). Basic number processing deficits in developmental dyscalculia: Evidence form eye tracking. *Cognitive development, 24*, 371-386. http://dx.doi.org/10.1016/j.cogdev.2009.09.007

Noël, M. P., & Rousselle, L. (2011). Developmental changes in the profiles of dyscalculia: an explanation based on a double exact-and-approximate number representation model. *Frontiers in Human Neuroscience, 5*. http://dx.doi.org/10.3389/fnhum.2011.00165

Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology, 88*, 348-367. http://dx.doi.org/10.1016/j.jecp.2004.04.002

Pieters, S., Roeyers, H., Rosseel, Y., Van Waelvelde, H., & Desoete, A. (2013). Identifying subtypes among children with developmental coordination disorder and mathematical learning disabilities, using model-based clustering. *Journal of Learning Disabilities*. http://dx.doi.org/10.1177/0022219413491288

R Core Team (2013). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org/

Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research & Practice, 17,* 81-89. http://dx.doi.org/10.1111/1540-5826.00035

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*, 361-395. http://dx.doi.org/ 10.1016/j.cognition.2006.01.005

Rubinsten, O., & Henik, A. (2005). Automatic activation of internal magnitudes: a study of developmental dyscalculia. *Neuropsychology, 19*, 641-648. http://dx.doi.org/10.1037/0894-4105.19.5.641

SAS Institute Inc. (2011). SAS® 9.3. Cary, North Carolina: SAS Institute Inc.

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental science, 14,* 280-291. http://dx.doi.org/10.1111/j.1467-7687.2010.00976.x

Shalev, R. S., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: a prospective six-year follow-up. *Developmental Medicine & Child Neurology, 47*, 121-125. http://dx.doi.org/10.1017/s0012162205000216

Shouten, J. F. & Bekker, J. A. M. (1967). Reaction time and accuracy. *Acta Psychologica, 27*, 143-153. http://dx.doi.org/10.1016/0001-6918(67)90054-6

Stock, P., Desoete, A., & Roeyers, H. (2010). Detecting children with arithmetic disabilities from kindergarten: Evidence from a 3-year longitudinal study on the role of reparatory arithmetic abilities. *Journal of Learning Disabilities, 43*, 250-268. http://dx.doi.org/10.1177/0022219409345011

Temple, C. M. (1991). Procedural dyscalculia and number fact dyscalculia – double dissociation in developmental dyscalculia. *Cognitive Neuropsychology, 8,* 155-176. http://dx.doi.org/10.1080/02643299108253370

Tops, W., Callens, M., Lammertyn, J., Van Hees, V., & Brysbaert, M. (2012). Identifying students with dyslexia in higher education. *Annals of Dyslexia, 62*, 186-203. http://dx.doi.org/10.1007/s11881-012-0072-6

Trick, L. M. & Pylyshyn, Z. W. (1993). What enumeration studies can show us about spatial attention – Evidence for limited capacity preattentive processing. *Journal of Experimental Psychology-Human Perception and Performance, 19*, 331-351. http://dx.doi.org/10.1037/0096-1523.19.2.331

Tymms, P. (1999). Baseline assessment, value-added and the prediction of reading. *Journal of Research in Reading, 22*, 27-36. http://dx.doi.org/10.1111/1467-9817.00066

Van den Bos, K. P., Spelberg, L. H. C., Scheepstra, A. J. M., & de Vries, J. R. (1994). *De Klepel: vorm A en B [Klepel: form A and B]*. Nijmegen, The Netherlands: Berkhout.

Van den Bos, K. P., Spelberg, L. H. C., Scheepstra, A. J. M., & de Vries, J. R. (2010). *De Klepel: vorm A en B [Klepel: form A and B]*. Nijmegen, The Netherlands: Berkhout.

Vanmeirhaeghe, B. (2012). *Divided by numbers. Studying with dyscalculia.* Documentary: http://www.studerenmetdyscalculie.be/synopsis. Artevelde University College: Gent, Belgium.

van Opstal, F., & Verguts, T. (2011). The origins of the numerical distance effect: The same-different task. *Journal of Cognitive Psychology, 23*, 112-120. doi: 10.1080/20445911.2011.466796

Wechsler, D. (2005). *Wechsler Intelligence Scale for Children, Derde Editie NL (WISC-III NL). Handleiding en verantwoording door Kort, W., Schittekatte, M., Dekker, P. H.,Verhaeghe, P., Compaan, E. L., Bosmans, M., & Vermeir, G.* Amsterdam, The Netherlands: Hartcourt Test Publishers.

Wickelgren, W. A. (1977). Speed-accuracy tradeoff and information processing dynamics. *Acta Psychologica, 41*, 67-85. doi: 10.1016/0001-6918(77)90012-9

Zhao, N., Valcke, M., Desoete, A., Burny, E., & Imbo, I. (2013). Differences between Flemish and Chinese primary students' mastery of basic arithmetic operations. *Educational Psychology: An International Journal of Experimental Educational Psychology.* Online publication. doi 10.1080/01443410.2013.832150

CHAPTER 3

SMALL NUMBER DISCRIMINATION IN EARLY HUMAN DEVELOPMENT: THE CASE OF ONE VERSUS THREE[1]

ABSTRACT

Infancy research assumes the existence of two systems in early *number discrimination*. Several studies using the habituation paradigm supported a *large number discrimination* system when controlling for continuous stimulus properties. According to this paradigm, successful discrimination is assumed when children look longer at a new number of visual presented objects after habituation to another number when this new and habituated number are shown in alternation after a habituation phase. In contrast to many studies on the large number system, few studies addressed the *small number discrimination* system and only one[2] reported a positive result when controlling for continuous variables. As yet, the range of small numbers has not been explored entirely.

The current study aims to fill this gap by investigating in infants the number set 1 vs. 3 which has not been studied before within the field of early number discrimination. Participants were 16 full term 8-month-olds of whom their number discrimination ability was assessed with a computerized habituation task in combination with an eye tracking device. Eye tracking served as an accurate measure for looking time in infants. Stimuli were controlled for continuous variables and attention was given to different approaches to analyze data retrieved from the habituation paradigm. The main results showed that 8-month-olds discriminated one from three dots by looking longer at one number after habituation to the other number (p = .009). This supports small number discrimination in infancy. The results retrieved through other approaches are discussed.

---

[1] Based on Ceulemans, A., Loeys, T., Warreyn, P., Hoppenbrouwers, K., Rousseau, S., & Desoete, A. (2012). Small number discrimination in early human development: The case of one versus three. *Education Research International, Article ID 964052, 5 pages*. http://dx.doi.org/10.11155/2012/964052

[2] In 2013, a new publication appeared on small numbers (i.e., Starr, Libertus & Brannon, 2013)

In the course of the last two decades, studies have revealed that preverbal children rely on an *object-file* system to process small (< 4) numbers[3] and an *analog magnitude* system to process large (> 3) ones (Feigenson, Dehaene, & Spelke, 2004; Xu, 2003). The first system enables a discrete, exact representation of a limited number of items (Kahneman & Treisman, 1984; Leslie, Xu, Tremoulet, & Scholl, 1998). This concept of an object-file system originates from visual attention literature (Kahneman, Treisman, & Gibbs, 1992; Trick & Pylyshyn, 1994) and proposes that for sets of three (or four) or less items, infants have an exact one-to-one correspondence representation of these items. This allows young children to make a precise discrimination between a number of objects within this small number range. The second system enables a less precise, approximate representation of a larger number of items (Feigenson et al., 2004). *Number discrimination* is then ratio dependent according to *Weber's law*: discrimination becomes less precise with increasing numerosity and the ratio between numbers determines its ease (Barth, Kanwisher, & Spelke, 2003; Brannon & Terrace, 2000; Izard & Dehaene, 2008; Whalen, Gallistel, & Gelman, 1999). A crucial question for both systems, however, remains upon which variables the discrimination is based (Feigenson, Carey, & Spelke, 2002). Some studies suggest that children use the *discrete* variable *number* (Cordes & Brannon, 2008; Xu, Spelke, & Goddard, 2005), whereas others believe that they rely on *continuous* variables such as the *total occupied area* of all the items together (Clearfield & Mix, 1999; Rousselle, Palmers, & Noël, 2004). Controlling for these continuous variables in stimuli is therefore essential to rule out that infants use these variables instead of the discrete variable number to discriminate number sets.

Tasks based on the habituation paradigm are frequently used to study number discrimination in very young children (e.g., Xu, 2003; Xu & Arriaga, 2007). In these tasks, children see a specific number of stimuli (e.g., dots) until they are habituated to it (or until they have received a maximum number of habituation trials, mostly 14 trials). Afterwards, they see, in alternating order, the same number and a new number. Longer looking time at the novel number or dishabituation (Berk, 2007) is considered to be an indication of discriminating between the given numbers (Xu, 2003; Xu & Spelke, 2000).

---

[3] The term "number" – which is used throughout this manuscript – refers to a set of nonsymbolic stimuli or numerosities rather than the Arabic numeral or number word

Various studies have investigated *large number discrimination* with this paradigm and have evidenced that 6-month-olds differentiate large numbers when the ratio is 1:4 as in 4 vs.16 (Wood & Spelke, 2005), 1:3 as in 7 vs. 21 (Cordes & Brannon, 2008) or 1:2 as in 4 vs. 8, 8 vs. 16, or 16 vs. 32 (Lipton & Spelke, 2003; Wood & Spelke, 2005; Xu, 2003; Xu & Spelke, 2000; Xu et al., 2005). Findings on small numbers diverge between studies with young infants (Antell & Keating, 1983; Clearfield & Mix, 1999; Cordes & Brannon, 2009b). Early studies using habituation found that newborns and infants, ranging from 4 to 7 months old, discriminate between small numbers (Antell & Keating, 1983; Starkey & Cooper, 1980). However, these studies did not control for continuous variables that covary with number. Later studies that did control for these variables found that infants could neither discriminate one from two items at 6 months of age (Xu et al., 2005) nor two from three elements at 6-8 months of age (Clearfield & Mix, 1999). However, a recent replication of the Clearfield and Mix study (Clearfield & Mix, 1999) reported a positive result for these latter numbers (i.e., 2 vs. 3) in 7-month-olds (Cordes & Brannon, 2009b). This study supported *small number discrimination* using habituation while controlling for continuous variables. The difference between this finding and the finding in the original paper by Clearfield and Mix (1999) might additionally result from changes in data collection and analysis. Cordes and Brannon (2009b) used computer-generated images presented on computer monitors (instead of computer-generated drawings mounted on white foam board). Furthermore, they not only included looking times of all test trial pairs in their analysis (instead of only the looking times of the first test pair), but also took into account the three last habituation trials (instead of only the last habituation trial as in the original study [Clearfield & Mix, 1999]).

As reported above, there is plenty of evidence for large number discrimination in infants using the habituation paradigm. For small numbers, however, this is not the case. To date, one number set has been proven unsuccessful (1 vs. 2; Xu et al., 2005) whereas another comparison yields contrasting results (2 vs. 3; Clearfield & Mix, 1999; Cordes & Brannon, 2009b). As a consequence, a full understanding of small number discrimination has not yet been reached. In the small number range, however, all number combinations can easily be investigated.

The current study aimed to extend the previous ones by investigating the number set 1 vs. 3. This specific small number set is the only one which has not been investigated before with the habituation paradigm. Moreover, an eye tracking system was used to measure looking time more accurately than previously done in studies on number discrimination. More specifically, instead of using the reflection of the computer screen in the infant's eye (e.g., Xu & Arriaga, 2007) or considering the direction of the child's face (e.g., Cordes & Brannon, 2009a), looking time was registered on the basis of the infant's gazes to the presented stimuli using eye tracking during task administration.

METHOD

Participants

Participants were part of a birth cohort of 3,000 babies born between May 2008 and April 2009, living in different Flemish districts in Belgium. They were recruited within the scope of a longitudinal large-scale study for the Belgian government realised by the Ghent University and Catholic University of Leuven as a partnership within the support center Welfare, Public Health, and Family (http://www.steunpuntwvg.be/jong, see Grietens, Hoppenbrouwers, Desoete, Wiersema, & Van Leeuwen, 2010). *Child & Family*, a governmental agency with responsibility for young children and families in Flanders (http://www.kindengezin.be), invited parents to this study. From the parents who had sent back a signed informed consent, 10% were randomly invited with a letter to participate in an additional multidisciplinary study, approved by the related academic ethical committees, of which this study is one part. Parents could fill in a new informed consent and if they consented, they were contacted by telephone. Research took place at Child & Family facilities. The study reported here included 16 full term infants of which there were nine boys and seven girls. The age of the infants varied from 31 weeks (7.75 months) to 34 weeks (8.5 months), with a mean age of 32.5 weeks (8.13 months, *SD* = 1.10 weeks).

Stimuli

The method of this study was based on the methodology of Xu et al. (e.g., Xu, 2003; Xu & Arriaga, 2007). A task based on the habituation paradigm was used. Stimuli were one- and three-element-arrays of dots on a white square background, displayed in the center of the eye tracker monitor. In order to maximally attract and sustain the attention of the infants, the dots were colored red (Franklin et al., 2008; Maier, Barchfeld, Elliot, & Pekrun, 2009; Zemach & Teller, 2007). Furthermore, stimuli were controlled for continuous variables (*item size* and *inter-item distance* at item level and the related set-parameters *total item size* and *total occupied set area*) according to the procedure of Dehaene, Izard, and Piazza (2005). The stimuli were designed so that, besides the change in number, all parameter values presented in the test phase were also presented during habituation, thus being equally non-novel. This could be established by randomly selecting one parameter (item or set level) from a fixed distribution regardless of the number while the related parameter varied with number in habituation stimuli. For the test stimuli, this procedure was reversed. In this experiment, *total item size* and *total occupied area* were fixed during habituation while the correlated set-level-parameters varied with number (Dehaene et al., 2005). See Figure 1 for an example of stimuli from the habituation task.

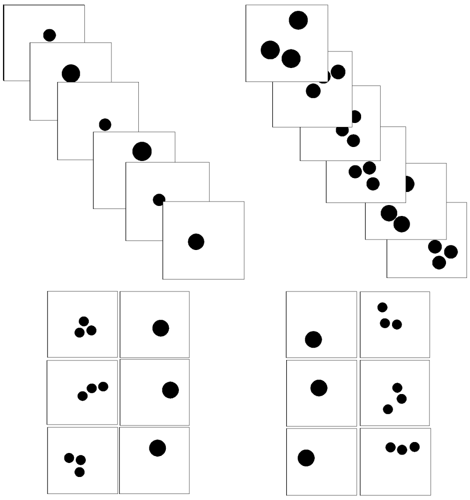

Figure 1. Example of stimuli used in the habituation and test phase (for each test trial pair).

Apparatus

Children sat on their parent’s lap in front of a *Tobii T60 eye tracker* (Tobii Technology, May, 2007) at a viewing distance of approximately 60 cm. This eye tracking device is integrated in a 17” TFT monitor with a refresh rate of 60 Hz and an accuracy of 0.5 degrees allowing freedom of head movement (44x22x30 cm). Parents were instructed to remain neutral and not to elicit the child’s attention during task administration. *Habit X 1.0*, a software program developed for performing the habituation paradigm (Cohen, Atkinson, & Chaput, 2004), was used for the task. One experimenter saw the infants’ looking behavior via the *live viewer* application, on a portable computer (connected with the eye tracker) running the software *Tobii Studio* (Tobii Technology, May, 2007). This live viewer showed the eye fixations of infants on the presented stimuli during the habituation task. Looking behavior was recorded in Habit X 1.0 by holding down a button when an infant was looking at the stimuli and releasing it when he or she looked away from the stimuli. Experimenters were blind to the experimental condition (see procedure for more information on this issue) to which children were assigned. Real looking times were coded afterwards in Tobii Studio from the eye tracking data by two researchers who created areas of interest (with margins of 2 cm) around each dot per array. As such, total fixation duration at all dots in one display could be identified. These coded looking times were then used for the final analysis. Inter-rater reliability was calculated using Pearson’s *r*. This was .97, indicating a good reliability between the two researchers (who both coded seven infants in a pilot study).

Procedure

At the beginning of the task, an attention grabber accompanied with sound appeared successively in the four corners and the middle of the eye tracker screen to indicate the infants’ window of looking. Only after a successful five-point calibration, the experiment began. Then, a well-known cartoon-figure was shown and each following trial was introduced by a sound (to sustain the infants’ attention during the task).

Looking time was valid (and consequently recorded in Habit X 1.0) from the moment infants looked at least for 0.5 seconds at a stimuli until they looked away for 2 seconds continuously from a stimuli (or when they kept looking for a maximum of 120 seconds in total). The task consisted of a habituation and a test phase. Infants were randomly assigned to one of two habituation conditions: Half of the infants were habituated to one-dot-arrays, the others to three-dot-arrays. Six different displays were presented in (repeating) random order until the infant met the habituation criterion (a 50% reduction in looking time over three consecutive trials, relative to the first three trials) or until 14 trials were completed. All infants reached the habituation criterion. Afterwards, infants were presented with six test displays containing the habituated (old) or the new number of dots in alternation (counterbalanced for order across participants).

Statistical analysis

First, a paired samples t-test comparing looking time on the first three and last three habituation trials was conducted to confirm whether infants did habituate. The main analysis focused on looking patterns exhibited during the test phase, which is a common practice (Xu, 2003; Xu & Arriaga, 2007; Xu & Spelke, 2000). Following Xu et al. (2005), outlying scores (> 2 *SD* from the mean for each condition) were excluded from analysis. In accordance with Cordes and Brannon (2009a), these looking times were replaced with the next longest looking time observed for all infants in each condition. Furthermore, to avoid loss of observations with one or more missing outcomes (due to technical failure) with a repeated measures ANOVA, a linear mixed model analysis was conducted on the looking times. This analysis tested whether looking time at the new number of dots was longer than looking time at the old number of dots in accordance with Xu et al. (Xu, 2003; Xu & Arriaga, 2007; Xu et al., 2005; Xu & Spelke, 2000). The consistency of the effect across trial pair, habituation condition, and sex was also assessed. In addition, the method of analysis used by Cordes and Brannon (2009b) was compared to that of the study by Clearfield and Mix (1999).

RESULTS

Figure 2 shows the mean looking time (LT) for the first three and last three habituation trials. A paired samples t-test revealed a significant reduction in LT from the first three habituation trials (*M* = 3.10, *SD* = 2.13) to the last three habituation trials (*M* = 1.81, *SD* = 1.49), *t (*15*)* = 4.15, *p* = .001, Cohen's *d* = 0.70. Figure 2 also displays the mean LT at the old and new number (i.e., test trial type) across all three test trial pairs. The linear mixed model analysis revealed a significant larger LT to the new (*M* = 2.11, *SD* = 1.25) compared to the old number (*M* = 1.78, *SD* =1.16), *F*(1,12.59) = 9.45, *p* = .009, Cohen's *d* = 0.21. No other effects of test trial pair, habituation condition, or sex were found with *F*(2,14.95) = 0.54, *F*(1,13.06) = 0.35, *F*(1,13.05) = 2.45 respectively and all *p* > .050. The paired samples t-test comparing mean LT on the last three habituation trials (*M* = 1.81, *SD* = 1.49) with mean LT on the new number across all test trials pairs (*M* = 2.11, *SD* = 1.25), revealed no significant difference between the two measures, *t (*15*)* = -0.87, *p* = .397. The paired samples t-test comparing LT on the last habituation trial (*M* = 1.50, *SD* = 1.94) with LT on the first of each type of test trial, showed a trend for an increasing looking time on the trial with a change in number (*M* = 2.58, *SD* = 1.48), *t (*15*)* = -1.76, *p* = .099, Cohen's *d* = 0.63.

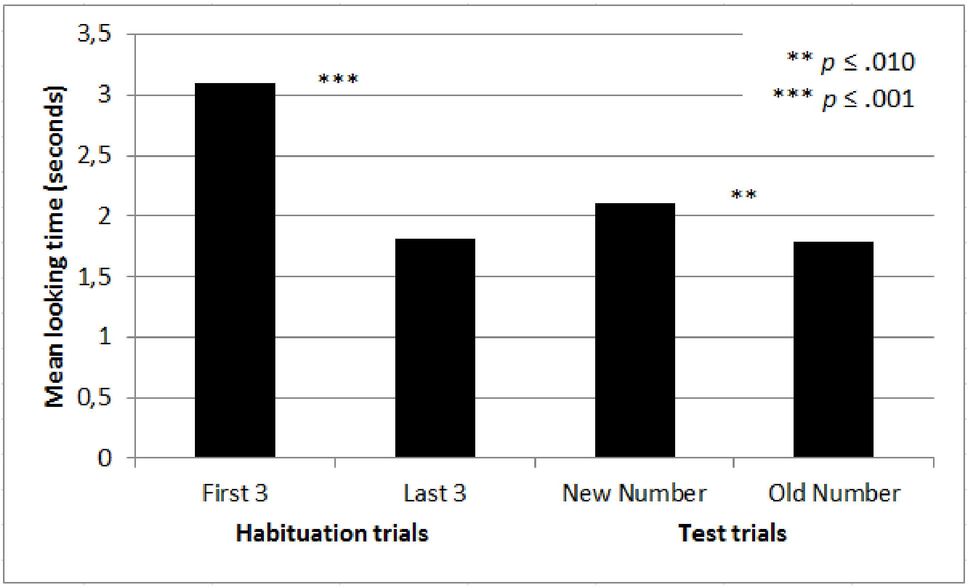

Figure 2. Mean looking times for the first three and last three habituation trials and for the new and the old number trials across all test pairs.

DISCUSSION

Previous research has demonstrated that infants rely on two different systems to process numbers: the object-file system is held responsible for small numbers (Kahneman & Treisman, 1984; Leslie et al., 1998) and the analog magnitude system for large numbers (Feigenson et al., 2004). Up till now, evidence for the discrimination of small numbers has not been found for all small numbers using habituation (Clearfield & Mix, 1999; Cordes & Brannon, 2009b; Xu et al., 2005). To extend previous work, the current experiment investigated number discrimination of the small number set 1 vs. 3. Special attention was given to the use of accurate looking time measures using eye tracking during administration of a habituation task on number discrimination. Data, furthermore, were analyzed following several approaches from habituation studies on number discrimination (Clearfield & Mix, 1999; Cordes & Brannon, 2009b; Xu, 2003; Xu & Arriaga, 2007; Xu et al., 2005; Xu & Spelke, 2000).

Results, retrieved according to the analysis approach of Xu et al. (Xu, 2003; Xu & Arriaga, 2007; Xu et al., 2005; Xu & Spelke, 2000), revealed infants' ability to discriminate these small numbers during a habituation task. Indeed, the linear mixed model analysis showed a significant (small) effect of number on the looking times. This successful discrimination extends previous research on small numbers. However, questions about the nature of number discrimination within this range remain. Why was number discrimination unsuccessful for 1 vs. 2 (Xu et al., 2005) in contrast with the success of the 1 vs. 3 set in this study? Why do findings on the 2 vs. 3 set vary (Clearfield & Mix, 1999; Cordes & Brannon, 2009b)? Because of the feature of precision, which is ascribed to the object-file system, it would be logical that all sets (1 vs. 2, 2 vs. 3, and 1 vs. 3) in the small number range could be equally well discriminated. Therefore, it might be possible that infants would be successful in discriminating 1 vs. 2 in a replication of the experiment of Xu et al. (2005). Reinvestigating the number set 2 vs. 3 had also this result (Cordes & Brannon, 2009b) compared to the original study (Clearfield & Mix, 1999).

Though, referring to the introduction of this paper, the difference in this recent finding and that of Clearfield and Mix (1999) is assumed, inter alia, to result from changes in data collection and analysis. Related to this, data from the current study confirmed the hypothesis of revealing different results retrieved from each approach (Clearfield & Mix, 1999; Cordes & Brannon, 2009b). To summarize, results from all approaches together showed the following: Against expectations, looking time at the new number did not significantly differ from looking time on the last three habituation trials according to the analysis approach of Cordes and Brannon (2009b). As expected, for successful number discrimination, looking time at the new number did differ significantly from looking time at the old number during the test phase following the analysis approach of Xu et al. (Xu, 2003; Xu & Arriaga, 2007; Xu et al., 2005; Xu & Spelke, 2000). The former finding, based on a comparison between looking time on the last three habituation trials and looking time at the new number across all test trial pairs, suggests the absence of dishabituation to the new presented number. Nevertheless, analysis according to Clearfield and Mix (1999), only taking into account the last habituation trial and the first test trial pair, does actually suggest a trend of dishabituation. Since different methods of processing data influence outcome, one may therefore question the expectation of the habituation paradigm, namely, that looking time at the new number in the test trials exceeds (i.e., dishabituation) the looking time at the last three habituation trials or at the habituated number in test trials. This matter, however, is subject to further research and will not be discussed in detail. When retrieving data following the main approach by Xu et al. (Xu, 2003; Xu & Arriaga, 2007; Xu et al., 2005; Xu & Spelke, 2000), it can however be concluded that the approach of Clearfield and Mix (1999) supports the results somehow (i.e., revealing a trend).

To further discuss 1 vs. 3, its success might be explained by the large ratio of the number set, considering the failure of the 1 vs. 2 set (Xu et al., 2005). Although ratio dependency is not known as a feature of the object-file system, (Kahneman & Treisman, 1984; Leslie et al., 1998) raising the ratio might facilitate number discrimination[4] as with large numbers (Feigenson et al., 2004; Izard & Dehaene, 2008).

---

[4] This idea was supported later on by Starr et al. (2013)

This explanation, however, is in contrast with the positive evidence for the discrimination of 2 vs. 3 (Cordes & Brannon, 2009b) which has a smaller ratio than 1 vs. 2. Another explanation therefore is the age of the children. The 8-month-olds were two months older than infants in the study of Xu et al. (2005). It is possible that infants are able to discriminate small numbers only from a specific age on. The negative result (Clearfield & Mix, 1999) may be due to the inclusion of 6-month-olds, whereas the result of the replication (Cordes & Brannon, 2009b) may be explained by the older infants (7 months of age). Despite the fact that infants discriminate large numbers from 6 months on (Xu, 2003), this might not count for small numbers[5]. As far as known, the effect of age on small number discrimination has not been investigated yet.

Nonetheless, it should be mentioned that the method in this study did differ from previous research, which may have led to the different finding. An eye tracking system was used to enhance accuracy of looking time measures. The online-registration of where the child was looking at enabled recording of looking behavior on grounds of the infant's gazes instead of either the reflection of a computer screen in the infant's eye (e.g., Xu & Arriaga, 2007) or just the direction of the child's face as in previous research (e.g., Cordes & Brannon, 2009a). One can question whether an infant is looking at a random area on the computer screen or is really attentive to the presented stimuli by looking straight at it. Without devaluating the previous mentioned habituation methods, eye tracking obviously takes away any doubt in this matter. As described under "apparatus" in the method section, analysis techniques of looking data such as areas of interest within an area defined by the computer screen's boundaries helps to unravel the child's looking behavior. As such, the use of eye tracking in combination with the known habituation paradigm from previous studies is more precise than the use of the paradigm only. This method reduces noise and therefore increases the possibility of revealing significant differences. This difference in methodology may also be an explanation for the smaller mean looking times registered in this study (see Figure 2) than those reported in previous studies on number discrimination (e.g., Xu & Arriaga, 2007).

---

[5] Small number discrimination was studied by Starr et al. (2013) in 6-month-olds, however, by using a new developed paradigm.

Regardless of the small effect of the main result, small number discrimination of 1 vs. 3 in infants is supported. However, not all previous and novel issues about the nature of small number discrimination can be solved, because only one number set was used. A study examining all three small number sets in the same infants with the same precise method seems indicated to establish a better understanding of small number discrimination[6]. This might help to reach a better insight in the ability of discriminating small numbers in infants. Within the scope of prevention, early detection of individual differences may be established through this better knowledge. Consequently, it might be possible to distinguish children at risk from typically developing children.

---

[6] All small number sets were studied by Starr et al. (2013) using however a new developed paradigm.

REFERENCES

Antell, S. E., & Keating, D. P. (1983). Perception of Numerical Invariance in Neonates. *Child Development, 54*, 695-701. http://dx.doi.org/10.1111/j.1467-8624.1983.tb00495.x

Barth, H., Kanwisher, N., & Spelke, E. (2003). The construction of large number representations in adults. *Cognition, 86*, 201-221. http://dx.doi.org/10.1016/S0010-0277(02)00178-6

Berk, L. E. (2007). *Development through the lifespan.* (4th ed. - International Edition) Boston: Allyn & Bacon, Inc.

Brannon, E. M., & Terrace, H. S. (2000). Representation of the numerosities 1-9 by rhesus macaques (Macaca mulatta). *Journal of Experimental Psychology-Animal Behavior Processes, 26*, 31-49. http://dx.doi.org/10.1037/0097-7403.26.1.31

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10*, 408-411. http://dx.doi.org/10.1111/1467-9280.00177

Cohen, L. B., Atkinson, D. J., Chaput, H. H. (2004). A new program for obtaining and organizing data in infant perception and cognition studies (Version 1.0). Austin: University of Texas.

Cordes, S., & Brannon, E. M. (2008). The difficulties of representing continuous extent in infancy: Using number is just easier. *Child Development, 79*, 476-489. http://dx.doi.org/10.1111/j.1467-8624.2007.01137.x

Cordes, S., & Brannon, E. M. (2009a). Crossing the Divide: Infants Discriminate Small From Large Numerosities. *Developmental Psychology, 45*, 1583-1594. http://dx.doi.org/10.1037/a0015666

Cordes, S., & Brannon, E. M. (2009b). The relative salience of discrete and continuous quantity in young infants. *Developmental Science, 12*, 453-463. http://dx.doi.org/10.1111/j.1467-7687.2008.00781.x

Dehaene, S., Izard, V., & Piazza, M. (2005). Control over non-numerical parameters in numerosity experiments. *Unpublished manuscript.* Available at http://www.unicog.org/.../DocumentationDotsGeneration.doc

Feigenson, L., Carey, S., & Spelke, E. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology, 44*, 33-66. http://dx.doi.org/10.1006/cogp.2001.0760

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Franklin, A., Pitchford, N., Hart, L., Davies, I. R. L., Clausse, S., & Jennings, S. (2008). Salience of primary and secondary colours in infancy. *British Journal of Developmental Psychology, 26*, 471-483. http://dx.doi.org/10.1348/026151007x256672

Grietens, H., Hoppenbrouwers, K., Desoete, A., Wiersema, R., & Van Leeuwen, K. (2010). *JOnG! Theoretische achtergronden, onderzoeksopzet en verloop van het eerste meetmoment [JOnG! Theoretical backgrounds, study design and course of the first assessment].* Leuven: Steunpunt, Welzijn, Volksgezondheid en Gezin (SWVG).

Izard, V., & Dehaene, S. (2008). Calibrating the mental number line. *Cognition, 106*, 1221-1247. http://dx.doi.org/10.1016/j.cognition.2007.06.004

Kahneman, D., Treisman, A. (1984). Changing views of attention of automaticity In R. Parasuraman, Davies, D.R. (Ed.), *Varieties of attention* (pp. 29-61). Orlando: Adacemic Press.

Kahneman, D., Treisman, A., & Gibbs, B. J. (1992). The reviewing of object files: Object specific integration of information. *Cognitive Psychology, 24*, 175-219. http://dx.doi.org/10.1016/0010-0285(92)90007-O

Leslie, A. M., Xu, F., Tremoulet, P. D., & Scholl, B. J. (1998). Indexing and the object concept: developing 'what' and 'where' systems. *Trends in Cognitive Sciences, 2*, 10-18. http://dx.doi.org/10.1016/S1364-6613(97)01113-3

Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense: Large-number discrimination in human infants. *Psychological Science, 14*, 396-401. http://dx.doi.org/10.1111/1467-9280.01453

Maier, M. A., Barchfeld, P., Elliot, A. J., & Pekrun, R. (2009). Context specificity of implicit preferences: The case of human preference for red. *Emotion, 9*, 734-738. http://dx.doi.org/10.1037/a0016818

Rousselle, L., Palmers, E., & Noël, M. P. (2004). Magnitude comparison in preschoolers: What counts? Influence of perceptual variables. *Journal of Experimental Child Psychology, 87*, 57-84. http://dx.doi.org/10.1016/j.jecp.2003.10.005

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science, 210*, 1033-1035. http://dx.doi.org/10.1126/science.7434014

Starr, A., Libertus, M. E., & Brannon, E.M. (2013). Infants show ratio-dependent number discrimination regardless of set size. *Infancy, 18,* 927-941.

Tobii Technology Inc., Falls Church, VA, USA, Product description: Tobii T/X Series eye trackers, May 2007.

Trick, L. M., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? A limited capacity preattentive stage in vision. *Psychological Review, 101*, 80-102. http://dx.doi.org/10.1037/0033-295x.101.1.80

Whalen, J., Gallistel, C. R., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. *Psychological Science, 10*, 130-137. http://dx.doi.org/10.1111/1467-9280.00120

Wood, J. N., & Spelke, E. S. (2005). Chronometric studies of numerical cognition in five-month-old infants. *Cognition, 97*, 23-39. http://dx.doi.org/10.1016/j.cognition.2004.06.007

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25. http://dx.doi.org/10.1016/s0010-0277(03)00050-7

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology, 25*, 103-108. http://dx.doi.org/10.1348/026151005x90704

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, B1-B11. http://dx.doi.org/10.1016/S0010-0277(99)00066-9

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8*, 88-101. http://dx.doi.org/10.1111/j.1467-7687.2005.00395.x

Zemach, I. K., & Teller, D. Y. (2007). Infant color vision: Infants' spontaneous color preferences are well behaved. *Vision Research, 47*, 1362-1367. http://dx.doi.org/10.1016/j.visres.2007.02.002

CHAPTER 4

THE SENSE OF NUMBER SENSE: THE PREDICTIVE VALUE OF NUMBER DISCRIMINATION IN INFANCY AND TODDLERHOOD FOR NUMERICAL COMPETENCIES IN KINDERGARTEN[1]

ABSTRACT

Number sense as a predictor to later math outcome is mostly studied in kindergartners, but is already known in infancy as *number discrimination.* Extending previous research on the predictive value of *large number discrimination*, this study explored the role of infants' and toddlers' *small number discrimination* for *numerical competencies in kindergarten*. Only toddlers' small number discrimination related to numerical competencies in kindergarten, raising thoughts about the task, age, set size, stability and development of number discrimination, or other influencing factors. When approaching successful number discrimination more strictly, the relationship could not be confirmed anymore, highlighting the importance of how to define success. Future research should study all small set sizes (not only 1 vs. 3) and a broader range of numerical competencies in kindergarten in a larger sample. Nevertheless, while infants' small number discrimination might be too early to predict numerical competencies in kindergarten, performance in toddlerhood might be addressed in the future to establish a measure to detect at-risk mathematical development.

---

[1] Based on Ceulemans, A., Titeca, D., Loeys, T., Hoppenbrouwers, K., Rousseau, S., & Desoete, A. (revision submitted). The sense of number sense: The predictive value of number discrimination in infancy and toddlerhood for numerical competencies in kindergarten. *Learning and Individual Differences.*

Infants are found to nonverbally discriminate between sets with a different number of items. This so called *number discrimination* (e.g., Xu & Arriaga, 2007) can be subsumed in the concept of *number sense* (e.g., Jordan, 2007; Kaminski, 2002; Wagner & Davis, 2010) as an innate sense of quantity that develops without or with little verbal input early in life (Butterworth, 1999; Dehaene, 1997; Jordan & Levine, 2009). Most children thus enter kindergarten demonstrating some sense of number (Powell & Fuchs, 2012). Individual differences, however, exist as shown by a diversity in mathematical knowledge (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Zulauf, Schweiter, & von Aster, 2003) and motivated researchers to study number sense as a predictor of later mathematical achievement (e.g., DiPema, Lei, & Reid, 2007; Dowker, 2008; Mazocco & Thompson, 2005; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Stock, Desoete, & Roeyers, 2009). Although number discrimination is considered as a basic number sense present from infancy on (Xu & Arriaga, 2007), glosses could be raised.

As yet, studies on number discrimination were mainly restricted to concurrent group results (e.g., Xu, Spelke, & Goddard, 2005). Besides, studies that investigated number sense as a predictor mainly focused on kindergartners (e.g., Jordan, Kaplan, Ramineni, & Locuniak, 2009). Recently, it is however shown that individual differences in number sense do already occur from infancy (Libertus & Brannon, 2010) and do relate to later mathematical outcome (Starr, Libertus, & Brannon, 2013b). It should be noted that this concerns *large* – as opposed to *small* – *number discrimination*, with the latter being another format of this ability and moreover the focus of the current study. Overall, small number discrimination has been connected with *object-files* and large number discrimination with *analog magnitudes* as underlying systems (Feigenson, Dehaene, & Spelke, 2004; Xu, 2003, and see Cantrell & Smith, 2013 for a review).

While the object-file system allows for an exact representation of a limited number (up to three) of items (Kahneman & Treisman, 1984; Kahneman, Treisman, & Gibbs, 1992; Leslie, Xu, Tremoulet, & Scholl, 1998; Trick & Pylyshyn, 1994), the analog magnitude system allows for an approximate representation of a larger set of items (Feigenson et al., 2004). In the latter case, discrimination is ratio dependent: Sets with a larger ratio are easier to discriminate. For example, Xu and Spelke (2000) demonstrated that 6-month-olds discriminate at a 1:2 ratio (8 vs. 16), but not at a 2:3 ratio (8 vs. 12).

Nonetheless, the claim that small numbers are only processed by object-files is tentative, as the successful discrimination of small from large numerosities (Cordes & Brannon, 2009a) and the finding that number discrimination is ratio dependent regardless of set size (Starr, Libertus, & Brannon, 2013a) might be incompatible with this assumption. Moreover, children probably have access to both systems, but on which one they actually rely might be paradigm-related (Feigenson & Carey, 2003).

Reviewing literature on small number discrimination, three paradigms step into the limelight: the habituation (Clearfield & Mix, 1999; Cordes & Brannon, 2009b; Xu et al., 2005), the manual search (Feigenson & Carey, 2003; 2005), and (only) recently the numerical change detection (Libertus & Brannon, 2010) paradigm. Habituation can be described as "learning" that reflects a changing responsiveness toward reiterated information, leading children to less heed stimuli which are repeatedly shown (Bornstein, Pêcheux, & Lécuyer, 1988). The paradigm relies on a preference for novelty (e.g., Colombo & Mitchell, 2009) which is in this case a new number of items. Like the name suggests, the manual search task relies on how children search for a certain amount of objects that are being hidden after presentation (Feigenson & Carey, 2003). Reaching/searching for objects is an action aimed at retrieving individual objects. Therefore, children are less prone to draw attention on the perceptual features (i.e., size, color, and shape) and give attention to the number of objects (Feigenson & Carey, 2005). Recently, the numerical change detection paradigm was developed by Libertus and Brannon (2010) and based on a paradigm invented by Ross-Sheehy, Oakes, and Luck (2003) to test infants' visual short-term memory. By means of two peripheral offered streams of rapidly changing images – relying on infants' preference for numerical change above constant numerosity – it was modified to test infants' ability to detect numerical changes. Regarding small number discrimination, the numerical change paradigm is assumed to activate the analog magnitude system (Starr et al., 2013a), whereas the manual search task would prompt the use of the object-file system (e.g., Barner, Thalwitz, Wood, Yang, & Carey, 2007; Feigenson & Carey, 2003, 2005). Divergent findings on small number discrimination – with failure for 1 vs. 2 (Xu et al., 2005), success for 2 vs. 3 (Cordes & Brannon, 2009b) and 1 vs. 3 (Ceulemans et al., 2012) – leaves however the question on which system is triggered (by this paradigm) unresolved.

The current study tried to further disentangle the role of number sense (as operationalized by number discrimination) in addition to earlier topic-related studies (Libertus & Brannon, 2010; Starr et al., 2013b). For this purpose, number discrimination was assessed in children at the age of 8 months (infants, T1) and 24 months (toddlers, T2) using an age-appropriate task (i.e., habituation and manual search paradigm respectively) at both time points. To date, number discrimination studies mostly used habituation tasks in younger infants (mostly aged 6 up till 10 months; e.g., Cordes & Brannon, 2009b; Xu, 2003, Xu & Arriaga, 2007; Xu et al., 2005), whereas the manual search task has more often been used in (older) toddlers (aged 1 to 2 years; e.g., Barner et al., 2007; Feigenson & Carey, 2003, 2005). Furthermore, in line with Starr et al. (2013b), the following numerical competencies in kindergarten, in addition to general intelligence, were tested in these children at the age of 48 months (kindergartners, T3): counting, arithmetic operations, and cardinality knowledge.

Three research objectives were formulated. First, it was investigated whether performance on the habituation task (T1) related to numerical competencies in kindergarten (T3). Second, this was examined for performance on the manual search task (T2). In other words, were infants' and toddlers' number discrimination performances predictive to later numerical competencies in kindergarten? When a specific relationship between a number discrimination measure and a numerical outcome measure was significant, it was further explored whether number discrimination still had an additional value when taking into account intelligence. Finally, in the third research objective, it was studied whether number discrimination performance at 8 months and 24 months of age were significantly related demonstrating stability throughout development.

Number discrimination in this study focused on small numerosities. From the age of 2 years on, children learn to count by acquiring consecutively the meaning of the first number words (Mix, 2009) in a first stage. This leads them to learn larger number words in a later stage. As such, investigating the predictive value of small number discrimination to later mathematical outcome – even from infancy on, but certainly at the critical age of 24 months – seemed to be a meaningful addition to previous research on the predictive value of large number discrimination (Starr et al., 2013b).

Based on the findings of Starr et al. (2013b), it was expected that infants' number discrimination (T1) would relate significantly to numerical competencies in kindergarten (T3). Consequently, number discrimination in toddlerhood (T2) was also expected to relate significantly to these competencies (T3), because the assessed number discrimination tasks at both ages – although different in design – are assumed to tap the same number sense ability.

Small set sizes were previously investigated with the habituation and manual search task (e.g., Cordes & Brannon, 2009b; Feigenson & Carey, 2003; 2005; Xu, 2003). In order to make a prediction possible from number discrimination to later outcome, at least some children needed to successfully discriminate the numerosities. Accordingly, the small set size with the largest ratio (1 vs. 3) was chosen, because this set warranted success with both tasks (Ceulemans et al., 2012; Feigenson & Carey, 2003, 2005).

In addition to previous studies, only providing binary information in terms of success or failure based on one overall task performance (Starr et al., 2013b), this study took into account successes and failures on different test trials of the tasks instead. As such, the study aimed at taking the binary information to a higher level and making it sensitive to individual differences. Moreover, the particular *cut-off* (i.e., a positive difference score larger than zero) mainly used to define success in number discrimination studies (e.g., Feigenson & Carey, 2003, 2005; Starr et al., 2013b; Xu, 2003, Xu & Arriaga, 2007) was questioned by taking into account the reliability of the measures.

METHOD

Participants

Participants came from a large-scale birth cohort living in different Flemish districts in Belgium, recruited within the scope of a longitudinal (governmental) study (http://www.steunpuntwvg.be/jong, see Grietens, Hoppenbrouwers, Desoete, Wiersema, & Van Leeuwen, 2010) of which the current reported study is only one part.

Children were randomly selected to participate in some cross-sectional studies. However, due to practical limitations (e.g., project expiration, availability of complete data sets), it was only possible to follow up some of them until the age of 48 months. Eventually, parents of 31 (out of 39) children consented to participate with their child at the ages of 8 months (T1), 24 months (T2), and 48 months (T3). See Table 1 for further details on the sample characteristics about age, IQ, sex, parents' educational level and family income.

Table 1. *Descriptive sample characteristics*

	M		*(SD)*
Age (*in months*)			
8 months (T1)	8.10		(1.16)
24 months (T2)	23.55		(1.18)
48 months (T3)	48.42		(0.92)
IQ[a]			
T3	101.33		(12.53)
	Boys (*n*)		Girls (*n*)
Sex			
T1,T2,T3	15		16
	Mothers (*n*)		Fathers (*n*)
Educational level (T1)[b]			
Primary education	1		0
Higher secondary education	7		15
Higher education	23		13
	Low (*n*)[d]	Medium (*n*)[e]	High (*n*)[f]
Family income (T1)[c]	2	13	13

Note. T1 = time point (1) at 8 months of age. T2 = time point (2) at 24 months of age. T3 = time point (2) at 48 months of age. IQ = Intelligence Quotient.

[a] IQ retrieved from the Wechsler Preschool and Primary Scale of Intelligence – Third edition (WPPSI-III-NL; Wechsler, 2002). [b] Information unknown for 3 of 31 fathers. [c] Three families did not disclose information on income. [d] Income < €1500. [e] €1501. < income < €3000. [f] Income > €3000.

Procedure and measures

At T1 and T2, number discrimination was assessed. At T3, children's counting, arithmetic operations, and cardinality knowledge were tested. All tasks were part of a broader protocol (see Appendix attached to *Chapter 1* for more information). Research (T1, T2) was conducted in a distraction-free room at *Child & Family* facilities (with governmental responsibility for guidance and support of young children and families in Flanders, http://www.kindengezin.be) and at the children's home (T3). The number discrimination tasks were assessed while children sat on a parent's lap. Parents were instructed to remain neutral and not to elicit attention or communication. Tests on numerical competencies in kindergarten were assessed individually, in absence of any parents, in the same order for all children. Parents signed an informed consent and the study was approved by the ethical commissions of the involved faculties. All test leaders (graduate students) received training in the assessment and interpretation of the tests.

Number discrimination performance: Habituation task. Children received a number discrimination task (T1) following habituation (e.g., Xu, 2003; Xu & Spelke, 2000, Xu et al., 2005) using one- and three-element-arrays of red dots on a white background. Stimuli were controlled for continuous variables (i.e., *item size*, *inter-item distance*, *total item size*, and *occupied set area*) according to the procedure of Dehaene, Izard, and Piazza (2005). The task consisted of a phase aimed at habituating children randomly to one of these arrays, using six different displays shown in repeating random order. In a test phase in which six displays contained the habituated and new dot arrays in alternation (counterbalanced for order across participants), longer looking at the novel arrays was considered as successful discrimination (Xu, 2003; Xu & Arriaga, 2007). The method of this study was based on the methodology of Xu et al. (e.g., Xu, 2003; Xu & Arriaga, 2007). Expanding on previous studies using this paradigm (Cordes & Brannon, 2009a; Xu et al., 2005), habituation software (i.e., *Habit X 1.0*; Cohen, Atkinson, & Chaput, 2004) was combined with eye tracking (Tobii T60; Tobii Technology, 2007). Looking times were coded afterwards from eye tracking data in *Tobii Studio* software (Tobii Technology, 2007) by identifying total eye fixation duration per dot array. Experimenters and coders were blind to the conditions to which children were assigned.

Inter-rater reliability between the two coders (r = .97) was good. See also Table 2 for more information on the different measures retrieved from this habituation task.

Analyses focused on the difference between looking time at the habituated and the new number of dots per test trial pair, which is a common practice (e.g., Xu, 2003; Xu & Arriaga, 2007; Xu & Spelke, 2000). This resulted in three difference scores. According to a *lenient* approach of success, one credit was given for each score larger than zero, in line with the mainly used definition of successful discrimination (e.g., Feigenson & Carey, 2003, 2005; Starr et al., 2013b; Xu, 2003, Xu & Arriaga, 2007). According to a *restricted* approach, one credit was only given for each score larger than a *reliable change index* (RCI). These indexes – calculated following Morley (2013) for the difference scores from all trial pairs – helped to find out whether differences between looking times were reliable. This method is generally used to define a meaningful change (e.g., Jacobson & Truax, 1999) or to evaluate clinical data for which no control group is available to compare the sample group with (e.g., Fenton & Morley, 2013).

Number discrimination performance: Manual search task. A manual search task presenting a 1 vs. 3 comparison as described by Feigenson and Carey (2005) was administered at T2. A wooden box (25 cm x 12.5 cm x 31.5 cm) had a slit at the front oriented to the toddlers and an opening at the backside of the box which was oriented to the experimenter who faced, in his/her turn, the child at an empty (besides the box) table. Parents were told that some balls would be hidden by the experimenter (through the slit in front of the box) to explore how children reacted and that no wrong reaction existed. The task consisted of three kinds of trials as illustrated more in detail in Figure 1: a first *box empty* trial, a *more remaining* trial and a second variant of the box empty trial (*“extended” box empty),* which always followed after a more remaining trial. See also Table 2 for more information on the measures retrieved from this manual search task.

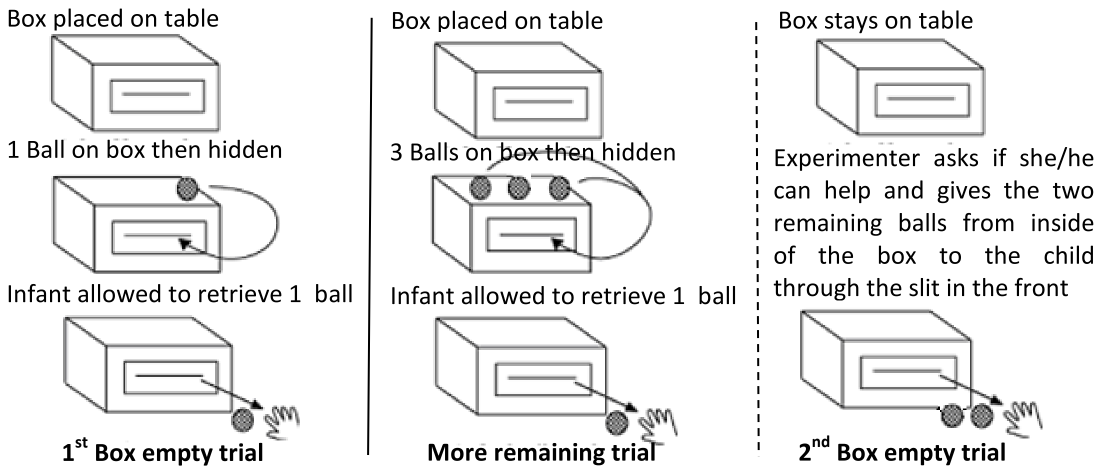

Figure 1. Different trial types of the manual search task. Adapted from "On the limits of infants' quantification of small object arrays," by L. Feigenson and S. Carey, 2005, *Cognition, 97*, p. 301. Copyright 2004 by Elsevier B.V.

Each of the trial types was presented twice and the order of the trials was counterbalanced. Children could search through the slit for 10 seconds after each type of trial. It was expected that children would search longer after the more remaining trials than after the box empty trials. This would indicate successful discrimination. Cumulative searching time, was coded manually afterwards using *The Observer XT* software (http://www.noldus.com).

Searching was defined as the period during which knuckles of one or both child's hands passed through the slit. Grasping of the slit did not count (Feigenson & Carey, 2003, 2005). Because administration of the search task revealed that children also looked into the box to search for the (supposedly) hidden balls, "looking through the slit" was additionally considered as searching.

For the manual search task (T2) outcome measures equivalent to the habituation task (T1) were constructed. Subtracting searching time after box empty trials from searching time after more remaining trials resulted in four difference scores. A positive difference score was considered as indicative for success and credited with one point if it was larger than zero for the lenient measure. For the restricted measure, one credit was only given if the difference score was larger than the specified RCI (Morley, 2013).

Respectively, four indexes were calculated for the difference scores resulting from the four possible subtractions between searching times on the different trial pairs.

Table 2. *Description of number discrimination tasks and related measures*

Tasks	*Description*	*Maximum*[a]	*Reliability*[b]
Habituation (T1):			
Lenient habituation success (LHS)	Credit difference score > 0	3/3	.21
Restrictive habituation success (RHS)	Credit difference score > RCI	1/3	
Manual Search (T2):			
Lenient manual search success (LMSS)	Credit difference score > 0	4/4	.79
Restrictive manual search succes (RMSS)	Credit difference score > RCI	4/4	

Note. T1 = time point (1) at 8 months of age. T2 = time point (2) at 24 months of age.
[a] All minimum scores equaled 0.
[b] Reliability of difference scores as measured with Cronbach's α.

Numerical competencies in kindergarten. NCK (T3) were assessed using counting and arithmetic operations subtests of the *Test for the Diagnosis of Mathematical Competencies* (TEDI-MATH; Grégoire, Noël, & Van Nieuwenhoven, 2004). The psychometric value of this assessment battery was tested on 550 Dutch-speaking Belgian children (Grégoire, 2005) and has proven to be conceptually accurate and clinically relevant. Its predictive value has been demonstrated in several studies (Desoete & Grégoire, 2006; Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009; Stock, Desoete, & Roeyers, 2007). A *Give-a-Number* task (GNT), designed by Wynn (1990, 1992) and adjusted by Sarnecka and Carey (2008), was used to additionally tap cardinality.

Counting subtests. To assess counting (T3), two subtests of the TEDI-MATH (Grégoire et al., 2004) were used. Procedural knowledge included accuracy in reproducing a counting sequence starting from one (up till 31), counting up to an upper bound (e.g., “count to 9”) and/or from a lower bound (e.g., “count from 3”). Conceptual knowledge implied the validity of counting procedures based on the counting principles of Gelman and Gallistel (1978). Children had to judge the counting of linear and nonlinear patterns of objects and were asked questions about the counted amount of objects (e.g., “How many objects are there?”). Furthermore, they had to construct two numerically equivalent amounts of objects for which they needed to use counting as a problem-solving strategy in a riddle.

Arithmetic operations subtest. At last, arithmetic operations were assessed by presenting a series of visually supported additions and subtractions to the children. See also Table 3 for more information on the measures retrieved from the different subtests of the TEDI-MATH.

Give-a-number task. In addition, all children were tested with the GNT-variant (Sarnecka & Carey, 2008) to determine whether they knew the exact meaning of numbers (cardinality) from one to six. Children were asked to give "N" objects to a puppet, followed by the question whether they gave "N" items. This question was restated until children answered positively. First, one object then three objects were asked. After a correct answer, the next request was "N + 1", otherwise "N – 1". Requests continued until at least two successes at "N" and at least two failures at "N + 1" were obtained. A credit was given if the child had at least twice as many successes as failures for that numeral. Failure included giving the wrong number of items. Each child's *knower-level* corresponded to the highest number he or she reliably generated. In line with Sarnecka and Carey (2008), children who had at least twice as many successes as failures for trials of "five" and "six" were called *cardinality-knowers*, whereas all others were called *subset-knowers* (Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006). See also Table 3 for more information on the GNT-task and the groups of cardinality- and subset-knowers defined by the (highest) score on this task.

Table 3. *Description of tasks on numerical competencies and related measures*

Tasks (T3)	*Description*	*Maximum*[a]	*Reliability*[b]
TEDI-MATH:			
Procedural counting	Items on counting sequence	5/8	.62
Conceptual counting	Items on counting principles	11/13	.76
Arithmetic operations	Simple additions & subtractions	6/6	.73
GNT:			
Cardinality-knowers (*n* = 13)	Cardinality from number "5" on	6/6	.82
Subset-knowers (*n* = 18)	Cardinality below number "5"	4/6	

Note. T3 = time point (3) at 48 months of age. TEDI-MATH = *Test for the Diagnosis of Mathematical Competencies (*Grégoire et al., 2004*)*. GNT = (score on) *Give-a-Number* task (Sarnecka & Carey, 2008).

[a] All minimum scores equaled, except for the GNT (i.e., 1 for subset-knowers and 5 for cardinality-knowers).

[b] Reliability of subscale as measured with Cronbach's α.

Analysis

Since graphical inspection revealed no strong evidence against non-normality, parametric tests were used. More specifically, because of graphically supported linear trends linear regressions were performed (SPSS Version 21.0, IBM Corp., 2012) to explore the relationship between number discrimination and TEDI-MATH measures. In case of a significant result, a hierarchical multiple linear regression was conducted to determine the additional effect of number discrimination on top of intelligence. Furthermore, independent samples t-tests were used to reveal whether cardinality- and subset-knowers (GNT; Sarnecky & Carey, 2008) differed on number discrimination.

RESULTS

See Table 4 for an overview of descriptives and intercorrelations of the variables.

Table 4. *Summary of intercorrelations, means and standard deviations for number discrimination, numerical competencies, and intelligence measures*

Measure	1	2	3	4	5	6	7	8	9	*n*	*M*	*(SD)*
1. LHS	--									31	1.45	*(0.93)*
2. RHS	-.10	--								31	0.23	*(0.43)*
3. LMSS	.05	-.21	--							31	2.42	*(1.48)*
4. RMSS	-.01	.10	-.17	--						31	1.45	*(1.43)*
5. Pc	-.23	.14	.20	-.03	--					31	1.45	*(1.61)*
6. Cc	-.13	-.14	.38*	-.03	.38*	--				31	4.39	*(2.70)*
7. Ao	-.19	.05	.31^{t}	-.07	.41*	.57***	--			31	1.97	*(1.80)*
8. GNT	-.21	.01	.16	.19	.58***	.49**	.35^{t}	--		31	3.84	*(1.81)*
Cardinality-knowers										13	5.69	*(0.48)*
Subset-knowers										18	2.50	*(1.04)*
9. IQ^{a}	-.24	.14	.11	.36^{t}	.55**	.53**	.34^{t}	.59***	--	30	101.33	*(12.53)*

Note. LHS = Lenient habituation success. RHS = Restrictive habituation success. LMSS = Lenient manual search success. RMSS = Restrictive manual search success. Pc = Procedural counting. Cc = Conceptual counting. Ao = Arithmetic operations. GNT = *Give-a-Number* task (Sarnecka & Carey, 2008). IQ = Intelligence Quotient.

^{a} IQ retrieved from the Wechsler Preschool and Primary Scale of Intelligence - Third edition (WPPSI-III-NL; Wechsler, 2002) for all children except for one child of whom full scale IQ could not be calculated (n = 30).

$^{t}p \leq .100$. * $p \leq .050$. ** $p \leq .010$. *** $p \leq .001$.

Using the LHS, linear regression revealed no significant relationship between infants' number discrimination (, T1) and numerical competencies in kindergarten (T3). Furthermore, cardinality- and subset-knowers did not differ significantly (GNT, T3) on their LHS (T1) as indicated by an independent samples t-test. Using the RHS (T1) provided the same results for numerical competencies and the difference (T1) between cardinality- and subset-knowers (GNT, T3). See Table 5 for the statistical values.

Table 5. *Summary of linear regressions for numerical competencies*

	Procedural counting				
Variable	*B*	*SE(B)*	*β*	*t(30)*	*p*
LHS	-.40	.31	-.23	-1.28	.211
RHS	0.52	.70	.14	.75	.458
LMSS	.22	.20	.20	1.09	.286
RMSS	-.04	.21	-.03	-.18	.858

	Conceptual counting				
Variable	*B*	*SE(B)*	*β*	*t(30)*	*p*
LHS	-.37	.54	-.13	-.68	.501
RHS	.87	1.17	-.14	-.74	.464
LMSS	.70	.31	.38	2.24	.033*[a]
RMSS	-.06	.35	-.03	-.16	.875

	Arithmetic operations				
Variable	*B*	*SE(B)*	*β*	*t(30)*	*p*
LHS	-.37	.35	-.19	-1.05	.302
RHS	.23	.78	.05	.29	.775
LMSS	.37	.22	.31	1.73	.094[t b]
RMSS	-.09	.23	-.07	-.39	.701

	Cardinality					
Variable	Cardinality-knowers (*n* = 13)		Subset-knowers (*n* = 18)		*t(29)*	*p*
	M	*(SD)*	*M*	*(SD)*		
LHS	1.23	*(0.60)*	1.61	*(1.09)*	1.14	.266
RHS	0.23	*(0.44)*	0.22	*(0.43)*	-.05	.957
LMSS	2.69	*(1.60)*	2.22	*(1.40)*	-.87	.391
RMSS	1.77	*(1.59)*	1.22	*(1.31)*	1.05	.302

Note. LHS = Lenient habituation success. RHS = Restrictive habituation success. LMSS = Lenient manual search success. RMSS = Restrictive manual search success.

[a] $R^2 = .15$. [b] $R^2 = .10$

[t] $p \leq .100$. * $p \leq .050$.

Linear regression analysis with the LMSS (T2) as a predictor revealed no significant relationship with procedural counting (T3). However, for conceptual counting and arithmetic operations, a significant (r = .38) and marginally significant (r = .31) relationship was found respectively. Even on top of IQ, a marginally significant effect was found on conceptual counting, B = .57, $SE (B)$ = .29, β = .30, t(27) = 1.97, p = .060. For arithmetic operations the relationship disappeared B = .37, $SE (B)$ = .22, β = .29, t(27) = .17, p = .107. Furthermore, no significant difference was found between cardinality- and subset-knowers (GNT, T3) on this LMSS (T2) as shown by an independent samples t-test. Conducting the same analysis with the RMSS (T2) revealed the same findings regarding the relationship with procedural counting. For conceptual counting and arithmetic operations, however, no relationship was found anymore. Furthermore, no significant difference was found between cardinality- and subset-knowers (GNT, T3) on this RMSS (T2) as shown by an independent samples t-test. Finally, the infants' (T1) and toddlers' (T2) number discrimination measures were not significantly related, neither using the lenient measures, B = .08, $SE (B)$ = .30, β = .05, t(29) = .28, p = .782, nor using the restricted measures, B = .34, $SE (B)$ = .62, β = .10, t(29) = .54, p = .590.

DISCUSSION

General findings

This study aimed to shed light on infants' (T1) and toddlers' (T2) number discrimination in relation to numerical competencies in kindergarten (T3). Results showed that no significant relationship could be found between infants' number discrimination (8 months, T1) and later numerical outcome in kindergarten (48 months, T3). This contrasts with the previous demonstrated relationship between number discrimination at 6 months of age and later mathematical abilities (Starr et al., 2013b). Important to note, however, is that Starr et al. (2013b) used another paradigm, a younger cohort, and also probed large number instead of small number discrimination.

These differences could be responsible for other findings between this study and the current study. A more detailed outline on each of these differences is presented below.

First, Starr et al. (2013a) highlighted that a numerical change detection paradigm would be more likely to invoke analog magnitudes than the habituation paradigm. Task-dependency of the recruited numerical system may therefore be a plausible explanation for different findings between the current study and the study of Starr et al. (2013b). Ratio dependency is a well-known characteristic of numerical representation using analog magnitudes (e.g., Xu & Spelke, 2000, see Cantrell & Smith, 2013 for a review). Because infants successfully discriminated the set sizes 1 vs. 3 and 1 vs. 2, but not 2 vs. 3 – having the most difficult ratio and hereby revealing ratio dependency – it was concluded that the numerical change paradigm elicits the analog magnitude system to represent small numerosities. On the contrary, to date, not all small set sizes are investigated simultaneously in the same group of infants using habituation. Moreover, divergent findings (Ceulemans et al., 2012; Cordes & Brannon, 2009b; Xu et al., 2005) with success for the set sizes 1 vs. 3 (Ceulemans et al., 2012) and 2 vs. 3 (Cordes & Brannon, 2009b) – suggesting the recruitment of object-files –, and failure of the set size 1 vs. 2 (Xu et al., 2005) – even undermining the use of both object-files and analog magnitudes – makes it difficult to draw any conclusion on which system is triggered to discriminate small numbers with the habituation paradigm. Another explanation might be the age of the infants. Starr et al. (2013b) stated that at 6 months of age the relationship between numerical representation using analog magnitudes and burgeoning math might be at its strongest. Therefore, it is possible that this relationship could not be replicated in older infants (8 months of age), in case this also holds for small number discrimination. Though, because small (1 vs. 3) instead of large number discrimination (6 vs. 24, 5 vs. 15, 6 vs. 18, 8 vs. 16, or 10 vs. 20; Starr et al., 2013b) was investigated, it may well be that the representation of small and large numbers simply contributes differently to later numerical competencies in kindergarten. This could be, finally, a third possible explanation for the different findings of the current study and the one by Starr et al. (2013b).

Although no significant relationship could be found between number discrimination in infancy (T1) and numerical competencies in kindergarten (T3), the predictive value of toddlers' number discrimination (T2) for these competencies could be demonstrated for at least some numerical aspects (i.e., arithmetic operations and even on top of IQ, conceptual counting) when using the lenient approach of successful number discrimination. Aside from the studied age, this finding is in line with the study of Starr et al. (2013b). From different points of view, some explanations could be provided for the current pattern of results across all time points.

First, small number discrimination at the age of 8 months is possibly not yet stable enough to reliably predict numerical functioning over a period longer than three months (i.e., from 8 months of age to 24 or 48 months of age) in contrast to stable (large) number discrimination abilities between 6 and 9 months of age (Libertus & Brannon, 2010). Second, the development of number discrimination may bloom in the first half year of life, stabilize, and again take a leap at 24 months of age, because children start to count and manipulate small numbers to further elaborate their counting skills with large numbers (Mix, 2009). Therefore, it might be more likely to find a significant relationship between number discrimination at 24 months of age and later numerical competencies than with the 8-month-measure. Third, also factors, such as numerical mother-child interactions or educational systems that vary across countries, may influence lower or higher number discrimination ability in infancy or toddlerhood.

Important to keep in mind, however, is that for these results a merely positive difference score was interpreted as success (in line with Starr et al., 2013b). These results vanished when using a restricted approach of success taking into account a particular cut-off (i.e., RCI; Morley, 2013). RCI-analysis takes into account the reliability of tasks to determine an index, which is then available to decide whether a difference in participants' behavior across trials is real and not just due to, for example, measurement error (Morley, 2013). Defining success using RCI-analysis therefore seems to lead to more reliable conclusions and demands further exploration, without detracting from success merely defined by a positive difference score (e.g., Feigenson & Carey, 2003, 2005; Starr et al., 2013b; Xu, 2003, Xu & Arriaga, 2007). Perhaps with a larger sample, positive significant results can be found with this advanced approach as well.

That no significant (mutual) relationship was found between infants' (T1) and toddlers' (T2) number discrimination is suggestive for the presumption that both tasks trigger a different underlying numerical representation system or simply appeal to different abilities. Given that habituation triggers object-files in small number discrimination (although not exclusively) and the informed knowledge that the manual search paradigm does this for sure – based on success for all small set sizes (Feigenson & Carey, 2003, 2005) – the first presumption seems less likely. It can alternatively be stated that both tasks appeal to different abilities. Related to this, Cantrell and Smith (2013) questioned indeed the suitability of the manual search tasks in order to study number discrimination because infants' performances may require more than mere discrimination of quantities in these search tasks. Infants need to remember amounts and their locations, and are required to base behavior on this knowledge. Manual search tasks are assumed therefore to be more demanding than other discrimination tasks, because they also dependent upon visual working memory, object representation, and knowledge of "more". This, in turn, aligns with the notion of the use of the manual search task in infants who are relatively older than those children who participate in general in conducted habituation studies (Cantrell & Smith, 2013).

Limitations and implications

The current study tried to disentangle the role of small number discrimination for later mathematical outcome, but some limitations remain to inspire future research. First, the small sample size might explain marginally significant results. Although trends can indicate relevant findings, future research needs to incorporate a larger sample. Second, only some competencies were studied despite the wide range of math-related abilities. A nonsymbolic task, for example, was not integrated at T3 in contrast with Starr et al. (2013b). One may expect to find a positive correlation between a nonsymbolic performance and number discrimination performance because the latter situates itself also on a nonsymbolic level by relying on internal mental number representations (Feigenson et al., 2004) to nonverbally discriminate two different sets of numerosities.

Third, only the set 1 vs. 3 was investigated, implying a clear interpretation based on one ratio. Including all small set sizes could however provide more insight in small number discrimination. Moreover, although standards were followed for the administration of the habituation task (e.g., Xu, 2003; Xu & Spelke, 2000, Xu et al., 2005), its reliability was low. Including more trials would be indicated as long as infants' attention-span – which is rather short, though – is taken into account as well. Fourth, from the few frequently used paradigms to investigate small number discrimination, the tasks in this study were age-appropriate resulting in different paradigms at T1 and T2 making comparison of number discrimination abilities more difficult (yet, not impossible). It seems therefore crucial to assess the same different paradigms both at one and across various time points in the same children.

Despite these limitations, some implications could be drawn from the current study results. First, small number discrimination at the age of 8 months (T1) might be too early to predict later outcome at 24 months (T2) and 48 months (T3) of age. From 24 months (T2) on, it seems possible to predict some numerical competencies in kindergarten (T3) taking into account how children repeatedly succeed on a series of trials within a task and not just an overall performance. Following this line of thought about the value of toddlers' number discrimination, it might be valuable to follow up (clinically) low performers on number discrimination to establish sensitive measures to detect at-risk development for later mathematical problems. Especially for those at higher risk for these problems (i.e., for example, siblings of children with a mathematical learning disorder; Shalev et al., 2001) this could be worthwhile. If, moreover, problems might be reduced by providing at-risk children opportunities to improve their skills (Clements & Sarama, 2011; DiPema et al., 2007; Fuchs, 2011), additional numerical stimulation in toddlerhood might be beneficial. Within this context, agencies, or initiatives in support of parenting (such as local parent support consultations, parenting shops, or parenting support centers), might play a sensitizing role toward parents. Nevertheless, implications are tentative, since different approaches of success unfold other patterns of results.

Conclusion

The current study questioned the predictive value of infants' and toddlers' number discrimination for later numerical competencies in kindergarten. Only when using a *lenient* approach of toddlers' number discrimination, a relationship between number discrimination with later conceptual counting and arithmetic operations in kindergarten could be observed. When taking into account intelligence, this relationship only held for conceptual counting.

Several hypotheses may underpin the importance of toddlerhood above infancy: stability, development, the format (small instead of large number discrimination), and other plausible influencing factors of number discrimination. Nevertheless, the demonstrated relationship may inspire future research to follow up children with low number discrimination in order to find out whether at-risk detection in toddlers for mathematical problems is possible. A restricted approach of number discrimination (which could not confirm the relationship with any aspect of the measured numerical competencies in kindergarten) additionally suggested the importance of how to define success to reveal significant results.

REFERENCES

Barner, D., Thalwitz, D., Wood, J., Yang, S.-J., & Carey, S. (2007). On the relation between the acquisition of singular-plural morpho-syntax and the conceptual distinction between one and more than one. *Developmental Science, 10*, 365-373. http://dx.doi.org/10.1111/j.1467-7687.2007.00591.x

Bornstein, M. H., Pêcheux, M. G., & Lécuyer, R. (1988). Visual habituation in human infants: Development and rearing circumstances. *Psychological Research, 50*, 130-133. http://dx.doi.org/10.1007/BF00309213

Butterworth, B. (1999). *The mathematical brain.* London: Macmillan.

Cantrell, L., & Smith, L. B. (2013). Open questions and a proposal: A critical review of the evidence on infant numerical abilities. *Cognition, 128,* 331-352. http://dx.doi.org/10.1016/j.cognition.2013.04.008

Ceulemans, A., Loeys, T., Warreyn, P., Hoppenbrouwers, K., Rousseau, S. & Desoete, A. (2012). Small number discrimination in early human development: The case of one versus three. *Education Research International*, *2012*, Article ID 964052, 5 pages.

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10*, 408-411. http://dx.doi.org/10.1207/s15327647jcd0203_1

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics intervention. *Science, 333,* 968-970. http://dx.doi.org/10.1126/science.1204537

Cohen, L. B., Atkinson, D. J., & Chaput, H. H. (2004). A new program for obtaining and organizing data in infant perception and cognition studies, (Version 1.0). Austin: University of Texas.

Colombo, J., & Mitchel, D. W. (2009). Infant visual habituation. *Neurobiology of Learning and Memory, 92*, 225-234. http://dx.doi.org/10.1016/j.nlm.2008.06.002

Cordes, S., & Brannon, E. M. (2009a). Crossing the divide: infants discriminate small from large numerosities. *Developmental Psychology, 45*, 1583-1594.

http://dx.doi.org/10.1037/a0015666

Cordes, S., & Brannon, E. M. (2009b). The relative salience of discrete and continuous quantity in young infants. *Developmental Science, 12*, 453-463. http://dx.doi.org/10.1111/j.1467-7687.2008.00781.x

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Dehaene, S., Izard, V., & Piazza, M. (2005). Control over non-numerical parameters in numerosity experiments. *Unpublished manuscript.* Available at http://www.unicog.org/.../DocumentationDotsGeneration.doc

Desoete, A., & Grégoire, J. (2006). Numerical competence in young children and in children with mathematics learning disabilities. *Learning and Individual Differences, 16*, 351-367. http://dx.doi.org/10.1016/j.lindif.2006.12.006

Desoete, A., Stock, P., Schepens, A., Baeyens, D., & Roeyers, H. (2009). Classification, Seriation, and Counting in Grades 1, 2, and 3 as Two-Year Longitudinal Predictors for Low Achieving in Numerical Facility and Arithmetical Achievement? *Journal of Psychoeducational Assessment, 27,* 252-264. http://dx.doi.org/10.1177/0734282908330588

DiPema, J. C., Lei, P. W., & Reid, E. E. (2007). Kindergarten predictors of mathematical growth in the primary grades: an investigation using the early childhood longitudinal study - kindergarten cohort. *Journal of Educational Psychology, 99,* 369-379. http://dx.doi.org/10.1037/0022-0663.99.2.369

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. [Article]. *Developmental Science, 11*, 650-654. http://dx.doi.org/10.1111/j.1467-687.2008.00713.x

Feigenson, L. & Carey S. (2003). Tracking individuals via object-files: Evidence from infants' manual search. *Developmental Science, 6*, 568-584. http://dx.doi.org/10.1111/1467-7687.00313

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object *Cognition, 97*, 295-313, http://dx.doi.org/10.1016/j.cognition.2004.09.010

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Fenton, G., & Morley, S. (2013). A tale of two RCTs: Using randomized controlled Trials to benchmark routine clinical (psychological) treatments for chronic pain. *Pain, 154,* 2108-2119. http://dx.doi.org/ dx.doi.org/10.1016/j.pain.2013.06.033

Fuchs, L. S. (2011). Mathematics intervention at the secondary prevention level of a multitier prevention system: Six key principles. Retrieved from http://www.rtinetwork.org/essential/tieredinsstruction/tier2/mathintervention

Gelman, R., & Galistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Grietens, H., Hoppenbrouwers, K., Desoete, A., Wiersema, R., & Van Leeuwen, K. (2010). *JOnG! Theoretische achtergronden, onderzoeksopzet en verloop van het eerste meetmoment [JOnG! Theoretical backgrounds, study design and course of the first assessment].* Leuven: Steunpunt, Welzijn, Volksgezondheid en Gezin (SWVG).

Grégoire, J. (2005). Développement logique et compétences arithmétiques. Le modèle piagétien est-il toujours actuel ? [Development of logical competences in arithmetics. Is the Piagetian model still up to date?]. In M. Crahay, L. Verschaffel, E. De Corte & J. Grégoire (Eds.), *Enseignement et apprentissage des mathématiques* (pp. 57-77). Bruxelles: De Boeck.

Grégoire, J., Noël, M., & Van Nieuwenhoven, C. (2004). *TEDI-MATH*. Antwerpen: Harcourt.

IBM Corp. (2012). *IBM SPSS Statistics for Windows*, Version 21.0. Armonk, NY: IBM Corp.

Jacobson, N. S, & Truax P. (1991). Clinical significance: A statistical approach to defining meaningful change in psychotherapy research. *Journal of Consulting and Clinical Psychology, 59*, 12-19. http://dx.doi.org/10.1037/0022-006X.59.1.12

Jordan, N. C. (2007). The need for number sense. *Educational Leadership*, *65*, 63-66. http://dx.doi.org/10.1037/0096-3445.123.3.284

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice (Blackwell Publishing Limited), 22*, 36-46. http://dx.doi.org/10.1111/j.1540-5826.2007.00229.x

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*, 850-867. http://dx.doi.org/10.1037/a0014939

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*, 60-68. http://dx.doi.org/10.1002/ddrr.46

Kahneman, D., & Treisman, A. (1984). Changing views of attention and automaticity. In R. Parasuraman & D. R. Davies (Eds.), *Varieties of attention* (pp. 29-61). Orlando, FL: Academic Press.

Kahneman, D., Treisman, A., & Gibbs, B. J. (1992). The reviewing of object files: Object specific integration of information. *Cognitive Psychology, 24*, 174–219. http://dx.doi.org/10.1016/0010-0285(92)90007-O

Kaminski, E. (2002). Promoting mathematical understanding: Number sense in action. *Mathematics Education Research Journal*, *14*, 133-49. http://dx.doi.org/10.1007/BF03217358

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L.V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*, 59-69. http://dx.doi.org/10.1037/0012-1649.42.1.59

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*, 395–438. http://dx.doi.org/10.1016/j.cognition.2006.10.005

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Revisiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52*, 130–169. http://dx.doi.org/10.1016/j.cogpsych.2005.07.002

Leslie, A. M., Xu, F., Tremoulet, P., & Scholl, B. J. (1998). Indexing and the object concept: Developing “what” and “where” systems. *Trends in Cognitive Sciences, 2*, 10-18. http://dx.doi.org/10.1016/S1364-6613(97)01113-3

Libertus, M. E., & Brannon, E. M. (2010). Stable individual differences in number discrimination in infancy. *Developmental Science, 13*, 900–906. http://dx.doi.org/

10.1111/j.1467-7687.2009.00948.x

Mazzocco, M. M. M., & Thompson, R. E. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities: Research and Practice, 20*, 142-155. http://dx.doi.org/10.1111/j.1540-5826.2005.00129.x

Mix, K. S. (2009). How Spencer made number: First uses of the number words. *Journal of Experimental Child Psychology, 102*, 427-444. http://dx.doi.org/10.1016/j.jecp.2008.11.003

Morley, S. (2013). A rough guide to evaluating your Pain Management Programme: The analysis of individual patient data and benchmarking. *Unpublished manuscript*, University of Leeds.

Powell, S. R., & Fuchs, L. S. (2012). Early Numerical Competencies and Students with Mathematics Difficulty. *Focus on Exceptional Children, 44*, 1-16. http://dx.doi.org/10.1111/j.1540-5826.2011.00345.x

Ross-Sheeby, S., Oakes, L. M., & Luck, S. J. (2003). The development of visual short-term memory capacity in infants. *Child Development, 74*, 1807-1822. http://dx.doi.org/ 10.1046/j.1467-8624.2003.00639.x

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108,* 662-674. http://dx.doi.org/10.1016/j.cognition.2008.05.007

Shalev, R. S., Manor, O., Kerem, B., Ayali, M., Badichi, N., Friedlander, Y., & Gross-Tsur, V. (2001). Developmental dyscalculia is a familial learning disability. *Journal of Learning Disabilities, 34*, 59-65. doi: 10.1177/002221940103400105

Starr, A., Libertus, M. E., & Brannon, E. M. (2013a). Infants show ratio-dependent number discrimination regardless of set size. *Infancy*, *18*, 927–941. http://dx.doi.org/10.1111/infa.12008

Starr, A., Libertus, M. E., & Brannon, E. M. (2013b). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences of the United States of America (PNAS), 110,* 18116-18120. http://dx.doi.org/10.1073/pnas.1302751110

Stock, P., Desoete, A., & Roeyers, H. (2007). Early markers for arithmetic difficulties. *Educational and Child Psychology, 24*, 28-39.

Stock, P., Desoete, A., & Roeyers, H. (2009). Mastery of the counting principles in toddlers: A crucial step in the development of budding arithmetic abilities? *Learning and Individual Differences, 19*, 419-422. http://dx.doi.org/ 10.1016/j.lindif.2009.03.002

Tobii Technology Inc., Falls Church, VA, USA, Product description: Tobii T/X Series eye trackers, May 2007.

Trick, L., & Pylyshyn, Z. (1994). Why are small and large numbers enumerated differently? A limited capacity preattentive stage in vision. *Psychological Review, 101*, 80–102. http://dx.doi.org/10.1037//0033-295X.101.1.80

Wagner, D., & Davis, B. (2010). Feeling number: Grounding number sense in a sense of quantity. *Educational Studies in Mathematics, 74,* 39-51. http://dx.doi.org/10.1007/s10649-009-9226–9.

Wechsler, D. (2002). *Wechsler preschool and primary scale of intelligence - third edition*. *Dutch translation by Hendriksen,J., & Hurks, P.* Amsterdam, Nederland: Pearson.

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*, 155-193. http://dx.doi.org/10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Childrens' acquisition of the number words and the counting sytem. *Cognitive Psychology, 24*, 220-251. http://dx.doi.org/10.1016/0010-0285(92)90008-p

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25. http://dx.doi.org/10.1016/S0010-0277(03)00050-7

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology, 25*, 103-108. http://dx.doi.org/10.1348/026151005x90704

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, B1-B11. http://dx.doi.org/10.1016/S0010-0277(99)00066-9

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8*, 88-101. http://dx.doi.org/10.1111/j.1467-7687.2005.00395.x

Zulauf, M. W., Schweiter, M., & von Aster, M. (2003). The preschool year: A sensitive period for the development of numerical abilities. *Kindheit Und Entwicklung, 12*, 222-230.

CHAPTER 5

A GUIDED WALK INTO NUMERICAL COMPETENCIES IN KINDERGARTEN: THE PREDICTIVE VALUE OF NUMERICAL MOTHER-CHILD INTERACTIONS IN TODDLERS[1]

ABSTRACT

This study explored the relationship between *numerical mother-child interaction* and numerical competencies in 31 children during a follow-up from toddlerhood till kindergarten. All children were tested on their ability to discriminate small numbers (*small number discrimination)* in toddlerhood and on their *numerical competencies in kindergarten*. Through a structured play the frequency of the mother-child interaction at both time points was observed. Maternal involvement at home and sensitivity during the structured play situation were taken into account as additional control measures.

The study confirmed the concurrent positive linear relationship between those interactions and some numerical competencies in kindergarten above maternal sensitivity. Such relationship with small number discrimination could not be confirmed in toddlerhood. However, the results showed a predictive contribution of toddlers' numerical mother-child interactions to some numerical competencies in kindergarten. The results, furthermore, underline that numerical mother-child interaction may show differential relationships with numerical competencies in kindergarten depending on the age at which these interactions are assessed. Implications are discussed in terms of fostering numeracy. At last, limitations and suggestions for future research are outlined.

---

[1] Based on Ceulemans, A., Titeca, D., Loeys, T., Hoppenbrouwers, K., Rousseau, S., & Desoete, A. (submitted). A guided walk into numerical competencies in kindergarten: The predictive value of numerical mother-child interaction in toddlers. *Early Childhood Research Quarterly.*

Parent-child interaction and child development

Parent-child interactions play a key role in various domains of children's development such as adaptation to school (e.g., Tan & Goldberg, 2009), musical development (e.g., McPherson, 2009), social and communicative competences, and cognitive development (e.g., Alvarenga & Piccinini, 2009; Landry, Smith, & Swank, 2006; Landry, Smith, Swank, Assel, & Vellet, 2001). As a measure of the quality of the interaction between parent and child, *sensitivity* (Bigelow et al., 2010) refers to the parents' ability to adjust their behavior and structure the interaction in such a way it fits their children's level of functioning (Bigelow et al., 2010; Page, Wilhelm, Gamble, & Card, 2010). *Involvement* refers to the establishment of a qualitative home learning environment (e.g., Melhuish et al., 2008) through one-on-one interactions between parent and child targeting the development of academic skills as a proximal form of parent involvement (Sy, Gottfried, & Gottfried, 2013). With regard to this academic development, the preschool and kindergarten years represent a critical juncture because achievement at early age is highly related to later achievement (e.g., Krajewski & Schneider, 2009). Two central domains in this context are reading (more in general, literacy) and mathematics (more in general, numeracy) as pointed out by Purpura, Hume, Sims, and Lonigan (2011) because the skills in these specific fields are found to be strong predictors for academic performance (Aunio & Niemivirta, 2010; Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Claessens, Duncan, & Engel, 2009; Duncan et al., 2007; LeFevre et al., 2010; Melhuish et al., 2008; National Institute of Child Health & Development, 2005; Ortiz, Stowe, & Arnold, 2001). It is not surprising therefore – as stated in Dougherty (2003) – measures to improve numeracy and literacy are often given priority in policies intended to help those with lowest educational attainment.

To illustrate, reading acquisition is a central challenge in children's developmental trajectories and a key determinant to overall educational success during elementary school (Cunningham & Stanovich, 1997; Duncan et al. 2007). General consensus exists that literacy begins to emerge during infancy (Scarborough, 2002; Whitehurst & Lonigan, 2002). Evidence suggests continuity from early literacy in preschool to literacy achievement in school (Aram, 2005; Aram & Levin, 2004; Levin, Ravid, & Rapaport, 2001; Shatil, Share, & Levin, 2000) and even to higher education (Cunningham & Stanovich, 1997). A good start is likely to help children become active in literacy (Aram & Biron, 2004) supported by the believe that early success may set positive life-course trajectory, leading to good academic outcomes whereas hampered literacy skills may lead to less desirable outcomes (e.g., Butler, Marsh, Sheppard, & Sheppard, 1985; Sénéchal & LeFevre, 2002; Stainthorp & Hughes, 2004; Wagner et al., 1997). Some children do for example fail to acquire literacy skills despite adequate intelligence and opportunity (Heath et al., 2014). These children manifesting early literacy difficulties (Costa et al., 2013) represent a vulnerable group at risk for underachievement trajectories throughout childhood and beyond, with long lasting consequences (Campbell, Pungello, Miller-Johnson, Burchinal, & Ramey, 2001; Heckman, 2006; Maughan et al., 2009). Moreover, they rarely catch up with their peers (Juel, 1988; Prior, Sanson, Smart, & Oberklaid, 2000). The identification of such children before they begin to struggle at school is a matter of significant concern to educators and policymakers as well as to parents (Heath et al., 2014). Screening at-risk children accurately requires the understanding of which factors contribute to children’s early literacy skill development (Lonigan, Burgess, & Anthony, 2000). Several important early predictors can be found within specific contexts in which individual patterns of literacy take place, determined at both individual and social level (Heath et al., 2014; Kern & Friedman, 2008). At an individual level, child characteristics may differentiate in early academic outcomes (Kern & Friedman, 2008): Within the child, there is a cluster of skills known to be fundamental to literacy development (e.g., Alloway & Gathercole, 2005; Schatschneider, Fletcher, Francis, Carlson, & Foorman, 2004). At the social level, attributes of the home environment, for example, may create a context for learning (Fitzgerald, Spiegel, & Cunningham, 1991): Literacy achievement in the early school years appears to be rooted in early childhood experiences with activities such as engaging in literacy activities with family members (Lonigan et al., 2000; Sénéchal & LeFevre, 2002). Early home factors, with for example specific literacy experiences, are therefore potentially influential in early educational milestones and subsequent academic outcomes (Kern & Friedman, 2008) that can be targeted in interventions for at-risk children (e.g., Lam, Chow-Yeung, Wong, Lau, & Tse, 2012).

In support of this assumption, parent-child interactions in general were found to affect the development of language and literacy (e.g., Dieterich, Assel, Swank, Smith, & Landry, 2006; Hood, Conlon, & Andrews, 2008). For example, the overall responsiveness (i.e., sensitivity) of mothers to their child predicts later language and literacy development (Bornstein & TamisLeMonda, 1989; Dodici, Draper, & Peterson, 2003). In this respect, not only general parental influence (i.e., parental involvement or parental sensitivity) seems valuable, but also more domain-related home experiences. For example, the verbal responsiveness of mothers stimulates the child's language (e.g., Baumwell, TamisLeMonda, & Bornstein, 1997) and, in turn, later reading (Dieterich et al., 2006). Studies on the role of these specific literacy experiences as a predictor have focused upon a few months old infants (e.g., Karrass & Braungart-Rieker, 2005; Pancsofar, Vernon-Feagans, & Family Life Project, 2010), toddlers of only a few years old (e.g., Dodici et al., 2003), and children from kindergarten (e.g., Dieterich et al., 2006; Hood et al., 2008; Roberts, Jurgens, & Burchinal, 2005) or primary school (Sénéchal & LeFevre, 2002).

As with literacy (Hooper, Roberts, Sideris, Burchinal, & Zeisel, 2010), children who start school with poor knowledge and skills in numeracy (Jordan, Kaplan, Locuniak, & Ramineni, 2007) are also unlikely to catch up with their peers. Previous research revealed substantial variability in levels of math understanding as early as the preschool years (e.g., Ginsburg, Klein, & Starkey, 1998; Young-Loveridge, 1991). Longitudinal studies have shown that relative achievement levels in this domain are fairly stable throughout primary and secondary school years (Fogelman, 1983; Young-Loveridge, 1991). Nevertheless, disparities in math achievement evident at school entry increase as students advance through the school system: The gap between the most and the least competent students becomes larger over time (e.g., Fogelman, 1983). This suggests that strengthening numeracy in the early school years could be of great benefit to student's mathematical learning in the long-term (Young-Loveridge, 2004). Research shows that intervention programs (and thus knowing which predictors need to be targeted) can be effective in reducing these disparities in math achievement (Gervasoni, 2001; 2002). Again the need for research on predictors (both at child and social level) steps into the limelight. As such, similar as in the domain of literacy, one context to find predictors for numeracy is the (general) home environment. First, in line with the established role of overall sensitivity (Dodici et al., 2003) and parental involvement on literacy skills (Sy et al., 2013), the effect of maternal sensitivity (Morrison, Rimm-Kauffman, & Pianta, 2003) and the home environment offered by parents on (informal) mathematical knowledge or numeracy skills of children has also been demonstrated (Anders et al., 2012; Blevins-Knabe, Whiteside-Mansell, & Selig, 2007). To continue, in line with the domain of literacy, one might also focus on domain-related aspects of this home environment with regard to numeracy. It is remarkable however that fewer studies exist on (home) numeracy experiences than on (home) literacy experiences (e.g., Dieterich et al., 2006; Hood et al., 2008; Sénéchal & LeFevre, 2002). One reason might be that parents have a bias toward literacy activities and therefore especially focus on it in young children (e.g., LeFevre et al., 2009; Skwarchuk, 2009) initially leading fewer researchers to study experiences that foster mathematical knowledge outside the school (Blevins-Knabe & Musun-Miller, 1996; Benigno & Ellis, 2004; Tudge & Doucet, 2004). Nevertheless, this does not detract from the fact that parents do engage with their children in numerical activities at home, which is kind of acknowledged by a growing interest in numerical experiences more recently (e.g., Kleemans et al., 2012; LeFevre et al., 2009; Skwarchuk et al., 2014).

Unfortunately, as opposed to literature on literacy experiences covering a wide age range even tracing back this source of input to infancy, a review of the literature learns that the majority of studies on numerical experiences cover at best kindergarten age (or older) with hardly any – as far as known – studies in (also) younger children (Blevins-Knabe & Musun-Miller, 1996; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2011). In line with the focus on very early literacy experiences (e.g., Dodici et al., 2003), one might also give attention to very early numeracy experiences because numbers pervade numerous everyday activities (Kaufmann & Dowker, 2009; Rousselle & Noël, 2007, 2008). As such, children grow up – from infancy on – in this environment rich in quantitative experiences (Rousselle & Noel, 2007) and are even found to be sensitive to quantity in these first years of life (Wynn, 1992). This ability to nonverbally discriminate (two) numerosities is considered by some authors as a very basic sense of number (i.e., *number discrimination*; Xu & Arriaga, 2007).

Additionally, from 2 years onwards, children learn to count by acquiring consecutively the first number words (Mix, 2009). Investigating this toddler age in particular – a milestone in children's numeracy development – within the context of early numerical experiences through parental input seems therefore especially meaningful. *Numerical parent-child interaction* might be as valuable already in toddlerhood as it is mostly demonstrated from kindergarten on for both concurrent and prospective mathematical achievement of children (e.g., Kleemans et al., 2012). Moreover, within the framework of targeting known predictors in intervention, starting intervention as early as possible is likely to be beneficial, and prevention is preferable to remediation because of the difficulty of remediating failure later (Brandt, 1993; Slavin, Karweit, & Wasik, 1993). This encourages the detection of very early predictors (in this case, numerical parent-child interaction for numeracy). The section below provides a brief insight in the literature on kindergartners.

Early numerical experiences through parent-child interaction in kindergarten

Available studies on early numerical experiences are mostly based on parent interviews (e.g., Blevins-Knabe & Musun-Miller, 1996) or questionnaires about a range of mathematical activities (e.g., Kleemans et al., 2012; Lefevre et al., 2009; Saxe, Guberman, & Gearhart, 1987; Skwarchuk, Sowinski, & LeFevre, 2014). Parents report that child and parent are regularly engaged in home activities involving numbers with a considerable variability in frequency (which correlated with the children's actual mathematical performance) and type (Blevins-Knabe & Musun-Miller, 1996; Saxe et al., 1987). As for literacy (e.g., Hood et al. 2008; LeFevre et al., 2009), parent-child interactions concern activities that could be related both directly and indirectly to numeracy. Recently, the distinction between those activities is made using terms as *formal* and *informal* (e.g., Skwarchuk et al., 2014). Formal or direct activities explicitly relate to the acquisition of skills such as counting, recognizing digits or number names (LeFevre et al., 2009), or to the pursuit of academic goals (Tudge & Doucet, 2004). It concerns shared experiences in which parents directly and intentionally teach their children about numbers, quantity, or arithmetic to enhance numeracy knowledge

(Skwarchuk et al., 2014). Informal or indirect experiences have quantitative features but are not geared to teach numeracy (LeFevre et al., 2009) or to reach academic objectives (Tudge & Doucet, 2004). During these interactions, various discussions about numbers, quantity, and concepts may arise (e.g., counting, talking about quantity, naming shapes; Anderson, 1997; Bjorklund, Hubertz, & Reubens, 2004; Skwarchuk, 2009; Vandermaas-Peeler, Boom-garden, Finn, & Pittard, 2012). However, teaching is not the purpose of these activities, but may occur incidentally (Skwarchuk et al., 2014). There is evidence to support the link between those formal and informal numerical experiences and children's numeracy (see Skwarchuk et al., 2014 for a review).

In addition to the aforementioned parent report studies, some scholars prefer observational methods, because observation may reveal informal math in everyday life (Tudge, 2009). Parents might be unaware of the support they provide during joint activities and therefore miss some unintentional, indirect numerical elements in daily life (Tudge & Doucet, 2004; Tudge, Li, & Stanley, 2008). Play, for example, during parent-child interactions does not guarantee mathematical development, but offers opportunities to draw the child's attention to numeracy even when parents are unaware of it (Benigno & Ellis, 2004; Sarama & Clements, 2008; Starkey, Klein, & Wakeley, 2004). These observational studies on numerical experiences point in the direction of opportunities provided by daily activities to promote aspects of numeracy development (Benigno & Ellis, 2004; Tudge & Doucet, 2004). Benigno and Ellis (2004), for example, observed that mothers took advantage of opportunities provided by playing a board game to promote counting in their 4-year-old children. Although parents thus (possibly) unconsciously engage with their children in numerical experiences, the provision of numerical experiences seems to be perceived as rather uncommon (Tudge & Doucet, 2004), especially compared to literacy experiences to which a higher value of importance is attributed by parents (Blevins-Knabe, 2008). Despite this concern, similar results were demonstrated using an approach combining observational data and parent reports (Huntsinger et al., 2000; Skwarchuk, 2009): Even though, some of these studies were not conclusive (e.g., Blevins-Knabe, Berghout Austin, Musun, Eddy, & Jones, 2000) the overall pattern of results provided by these studies indicates that kindergartners who receive more numerical home experiences have better mathematical skills compared to those with fewer experiences (e.g., Blevins-Knabe & Musun-Miller, 1996; Kleemans et al., 2012; LeFevre et al., 2009). Research on this topic in children who did not yet enter kindergarten is scarce, but encouraging parents to talk about numbers already with their toddlers might also positively impact upon children's mathematical attainment (Levine et al., 2011). Levine et al. (2011) demonstrated that in 14- to 30-month-olds, the frequency of parental number talk predicted the children's cardinal knowledge (e.g., knowing that the word "four" refers to sets with "4" items) at 46 months of age. Consequently, not only the importance of concurrent numerical parent-child interaction for math in kindergarten (e.g., Blevins-Knabe & Musun-Miller, 1996) is supported, but also the predictive value at younger age.

The current study

The brief review above shows that language experiences are commonly acknowledged as important from infancy on (e.g., Karrass & Braungart-Rieker, 2005). Little is known, however, about early numerical experiences in children who did not yet enter kindergarten. Nevertheless, even very young children encounter magnitudes and learn, in addition, to count from the age of 2 years onwards. As such, also toddlers' numerical home experiences (Levine et al., 2011) can be highlighted with respect to later mathematical abilities, but the importance of parental numerical input in these younger children – especially on the crucial age of 2 years in toddlerhood – for concurrent numerical performance remains undisclosed.

The current study questioned in line with Levine and colleagues (2011) whether numerical parental input in children younger than kindergarten age (i.e., toddlers) could be valued (equally) as in children of kindergarten age for children's *numerical competencies* later on (*in kindergarten*). In addition, the current study also focused on toddler's concurrent numerical performance. To this end, an explorative study was set up with the general aim to examine the frequency of numerical parent-child interaction in relation to both children's concurrent number discrimination ability in toddlerhood and children's concurrent and prospective numerical competencies in kindergarten.

Three specific research questions were formulated within this broader study aim: First, the current study aimed at confirming the positive relationship between the amount of kindergartners' numerical parent-child interaction and concurrent numerical competencies. Second, the value of (the frequency of) this kind of interaction in toddlerhood for toddlers' concurrent number discrimination performance was questioned. At last, the third research objective dealt with whether the frequency of numerical parent-child interactions in toddlerhood already predicted numerical competencies later on (i.e., at kindergarten age). In line with the findings of the aforementioned kindergarten studies, it was expected to find positive concurrent (e.g., Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009) and prospective (e.g., Kleemans et al., 2012) relationships for the younger children investigated in this study. Furthermore, the findings of this study were intended to guide supplementary studies: If the predictive value of numerical parent-child interaction in toddlerhood could be demonstrated, future research should focus on this predictor within the framework of prevention, digging in to concrete aspects and types of interaction that are important.

A manual search task (e.g., Feigenson & Carey, 2003) and a standardized test battery on mathematical competencies (Grégoire, Noël, & Van Nieuwenhoven, 2004), along with a cardinality knowledge test (Sarnecka & Carey, 2008), were used respectively to assess toddlers' number discrimination and numerical competencies at kindergarten age in the same children. Number discrimination focused on small numerosities (i.e., not more than four items; Feigenson, Dehaene, & Spelke, 2004), as 2-year-olds first acquire the smaller number words (up till three) leading them to learn the larger numbers later on (Mix, 2009). A nonverbal task was used to assess a sense of number because not all 2-year-olds were expected to already actively (and verbally) recite for example a counting sequence (i.e., at this age children only "start" to learn counting: Primary understanding of amounts emerges, but only very basic discrimination is possible [e.g., Fuson, 1988; Van de Rijt, 1996]). Investigating the concurrent and predictive value of small number discrimination seemed thus meaningful. Because small set sizes (i.e., with small numerosities) are often investigated with the manual search task (e.g., Cordes & Brannon, 2009; Feigenson & Carey, 2003, 2005; Xu, 2003), this study relied on this paradigm to measure toddlers' *small number discrimination* ability.

To map the parent-child interactions, observational data were used guided by the concern that numerical experiences might be unreliably reported on by parents (Fluck, Holgate, & Linell, 2005; Tudge, 2009) in even younger children. Observation comprised a short structured play situation between the parent and the child. Furthermore, numerical specificity of the relationship (if found) between parental input and numerical outcome was explored by taking into account (general) parental sensitivity and parental involvement: The additional value of (domain-specific) numerical parent-child interaction for early number discrimination performance or numerical competencies was questioned on top of these (domain-general) control measures.

METHOD

Participants

Participants were part of a larger birth cohort of babies, living in different Flemish districts in Belgium, recruited within the scope of a large-scale longitudinal study (http://www.steunpuntwvg.be/jong, see Grietens, Hoppenbrouwers, Desoete, Wiersema, & Van Leeuwen, 2010), of which the reported study is one part. From this larger cohort, children were randomly selected to participate in an in-depth study on early number sense and later numerical competencies. Eventually, parents (all mothers) of 31 children consented to participate with their child at the age of 24 (T1) and 48 months (T2).

For parents who cannot be with their children full-time, many *child-minding* options are currently available in Flanders for children between the ages of 0-3 years, both formal (e.g. day nurseries or day care, child-minding families, …) and informal (e.g. grandparents or other family members, friends, neighbours, …). In the current sample child-minding options were divided as follows: no provision (n = 2), informal provision (n = 6), formal provision (n = 14; with for n = 5: day care and for n = 9: child-minding families), and a combination of informal and formal provision (n = 8; with for n = 5: combined with day care, with for n = 2: combined with child-minding families, and for n = 1: combined with day care and child-minding families). For one child this information was unknown.

In the Flemish part of Belgium, children typically attend preschool (usually termed *kindergarten*, for which reason a distinction was made in this dissertation between infants, toddlers, and kindergartners) when they are 2.5 years old, and enter elementary school (i.e., first grade) at 6 years of age. Although preschool education is not compulsory, the vast majority of children do attend school usually for three years. Compulsory education, according to a defined curriculum, starts in first grade. At T2, all children had received one year of preschool education and were assumed to have received similar preschool experiences concerning preparatory math. See Table 1 for sample descriptives.

Table 1. *Descriptive sample characteristics*

		M	*(SD)*	
Age (*in months*)				
	24 months (T1)	23.55	(1.18)	
	48 months (T2)	48.42	(0.92)	
IQ[a]				
T3		101.33	(12.53)	
		Boys (*n*)	Girls (n)	
Sex				
T1,T2,T3		15	16	
		Mothers (*n*)	Fathers (*n*)	
Educational level (T1)[b]				
	Primary education	1	0	
	Higher secondary education	7	15	
	Higher education	23	13	
		Low (*n*)[d]	Medium (*n*)[e]	High (*n*)[f]
Family income (T1)[c]		2	13	13

Note. T1 = time point (1) at 8 months of age. T2 = time point (2) at 24 months of age. T3 = time point (2) at 48 months of age; IQ = Intelligence Quotient.

[a] IQ retrieved from the Wechsler Preschool and Primary Scale of Intelligence – Third edition (WPPSI-III-NL; Wechsler, 2002) for all children except for one child of whom full scale IQ could not be calculated (*n* = 30).

[b] Information unknown for 3 of 31 fathers. [c] Three families did not disclose information on income. [d] income < €1500. [e] €1501 < income < €3000. [f] income > €3000.

Parents signed an informed consent before participation with their child and the study was approved by the ethical commissions of the Faculties that were involved in the large-scale study. All test leaders (graduate students) received training in the assessment and interpretation of the tests.

Procedure

Children were tested at T1 with a number discrimination task and at T2, counting skills, arithmetic operations, and cardinality knowledge were tested. Intellectual abilities were additionally assessed at T2. In addition, at both time points, numerical mother-child interaction was assessed through a structured play observation. All tasks were part of a broader protocol for each of the time points (See Appendix attached to *Chapter 1*).

Research at T1 took place at facilities of *Child & Family*, which is a governmental service with responsibility for the guidance and support of young children (i.e., from birth until 3 years of age) and their families (http://www.kindengezin.be). The tests at T2 were assessed at the children's home, because children do not attend these services anymore at the age of 48 months and in order to facilitate conditions to maximize the parental motivation to participate in this additional (time point of the) study.

In all settings research was conducted in a distraction-free room. The number discrimination task at T1 was assessed while children sat on their mother's lap. Parents were instructed to remain neutral and not to elicit the child's attention during task administration. At T1, children participated furthermore in a structured play observation with the mother. The tests on numerical competencies in kindergarten at T2 were assessed individually in absence of any parents, in the same order for all participants (i.e., counting, arithmetic operations, and cardinality) after administration of an intelligence test (WPPSI-III-NL; Wechsler, 2002). Both at T1 and T2, the mothers were asked to fill out a questionnaire about their own child's home experiences (i.e., questions on the frequency of general parental involvement in the home environment).

Number discrimination performance in toddlerhood. A manual search task presenting a 1 vs. 3 set as described by Feigenson and Carey (2005) was used to assess children's number discrimination at T1. Children sat on their parent's lap at an empty table in front of the experimenter. A wooden box (25 cm x 12.5 cm x 31.5 cm) had a slit at the front oriented to the toddlers and an opening at the backside of the box which was oriented to the experimenter. Parents were told that some balls would be hidden into this box to explore how children reacted and that no wrong reaction existed.

Parents could only redirect their child's attention when (really) necessary, but were furthermore not allowed to help and were asked to further minimize communication.

In this task, three kinds of trials existed. First, there was a *box empty* trial after which children were allowed to retrieve a hidden ball. Second, a *more remaining* trial followed, wherein the researcher hid three balls, but surreptitiously took away two, allowing the child to only retrieve one ball. Third, there was a second ("*extended*") *box empty* trial – always following the more remaining trial – in which the experimenter inserted again the balls that he took away through the backside of the box and offered the child to help, resulting in the child retrieving all (once) hidden balls. Each of the trial types were presented twice and the order of the trials was counterbalanced.

Children were allowed to search through the slit for ten seconds after each kind of trial. It was expected that children would search longer after the more remaining trials than after the box empty trials. Infants' cumulative searching time was coded manually afterwards, using *The Observer XT* (http://www.noldus.com) software for analysis of observational data. Following Feigenson and Carey (2003, 2005), searching was defined as the period during which the knuckles of one or both child's hands passed through the slit and grasping of the slit itself did not count. Because administration of the manual search task revealed that children also looked into the box through the slit to search for the (supposedly) hidden balls, periods of looking through the slit were considered additionally as searching behavior. Two experimenters – who coded the observational data of the manual search task – achieved an averaged inter-rater reliability of .79 percentage of agreement.

Searching times after more remaining trials and box empty trials were compared, assuming that longer searching after the first kinds of trials indicated success (Feigenson & Carey, 2003; 2005). Subtracting searching time after box empty trials from searching time after more remaining trials resulted in four difference scores. Consequently, infants could achieve a maximum score of four for this task. Reliability of the difference scores (and the range) can be found in Table 2. Expanding on the mainly used definition of successful discrimination (e.g., Feigenson & Carey, 2003, 2005), according to which a positive difference score (greater than zero) is indicative for success, one credit was only given for each positive difference score larger than a defined reliable change index (RCI).

This RCI was computed following the procedure of Morley (2013) and helped to find out whether the difference between the searching times at the different trial types was “real” or reliable. This method is generally used for defining a meaningful change (e.g., Jacobson & Truax, 1991) and/or evaluating clinical data for which no control group is available against which the sample group can be compared (e.g., Fenton & Morley, 2013).

Numerical competencies in kindergarten. Numerical competencies in kindergarten (T2) were assessed using on the one hand, counting and arithmetic operation subtests of the *Test for the Diagnosis of Mathematical Competencies* (TEDI-MATH; Grégoire, Noël, & Van Nieuwenhoven, 2004) and on the other hand, a variant of the *Give-a-Number* task (GNT) designed by Wynn (1990, 1992) and adjusted by Sarnecka and Carey (2008), to tap cardinality knowledge.

Counting subtests. As earlier mentioned, to assess counting abilities, two subtests of the TEDI-MATH (Grégoire et al., 2004) were used. The psychometric value of this Belgian individual assessment battery was tested on a sample of 550 Dutch-speaking Belgian children (Grégoire, 2005) and has proven to be conceptually accurate and clinically relevant. Its predictive value has been demonstrated in several studies (Desoete & Grégoire, 2006; Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009; Stock, Desoete, & Roeyers, 2007). Counting items embraced both procedural and conceptual counting knowledge.

Procedural knowledge included accuracy in counting row and counting forward to an upper bound and/or from a lower bound. The task consisted of eight items. Conceptual knowledge implied the validity of counting procedures based on the five basic counting principles formulated by Gelman and Gallistel (1978). Children had to judge the counting of linear and nonlinear patterns of objects and were asked questions about the counted amount of objects (e.g., “How many objects are there in total?”). Furthermore, they had to construct two numerically equivalent amounts of objects and had to use counting as a problem-solving strategy in a riddle.

Arithmetic operations subtest. Arithmetic operations were also assessed using a subtest of the TEDI-MATH (Grégoire et al., 2004). A series of six visually supported additions and subtractions were presented to the children.

Give-a-number task. The GNT (Sarnecka & Carey, 2008) was used to determine whether children knew the exact meaning of the numbers ranging from one to six. Children were asked to give a number of objects to a puppet. Each request of "N" items was followed by the question whether they gave "N" items. This procedure was restated until children answered positively. First, one object was asked and then three objects. When a child responded correctly, the next request was "N + 1" and otherwise "N – 1". Requests continued until at least two successes at a given "N" and at least two failures at "N + 1" were obtained.

A credit was given if the child had at least twice as many successes as failures for that numeral. Failures included giving the wrong number of items. Each child's *knower-level* corresponded to the highest number he or she reliably generated. In line with Sarnecka and Carey (2008), children with at least twice as many successes as failures for "five" and "six" were called *cardinality-knowers*, all others were called *subset-knowers* (e.g., Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006).

Table 2. *Description of tasks on number discrimination and numerical competencies*

Tasks	*Description*	*Maximum*[a]	*Reliability*[b]
Manual search task (T1)	Nonverbal quantity discrimination	4/4	.79
TEDI-MATH (T2):			
Procedural counting	Items on counting sequence	5/8	.62
Conceptual counting	Items on counting principles	11/13	.76
Arithmetic operations	Simple additions & subtractions	6/6	.73
GNT (T2):			
Cardinality-knowers (*n* = 13)	Cardinality from number "5" on	6/6	.82
Subset-knowers (*n* = 18)	Cardinality below number "5"	4/6	

Note. T1 = time point (1) at 24 months of age. T2 = time point (2) at 48 months of age. TEDI-MATH = *Test for the Diagnosis of Mathematical Competencies* (Grégoire et al., 2004). GNT = *Give-a-Number* task (Sarnecka & Carey, 2008).

[a] All minimum scores equaled, except for the GNT (i.e., 1 for subset-knowers and 5 for cardinality-knowers).

[b] Reliability of subscale as measured with Cronbach's α.

Table 2 provides a short overview of descriptives of the measures on number discrimination (T1) and the measures on numerical competencies in kindergarten (T2). Scores for numerical competencies in kindergarten (T2) were the number of correct exercises on the subtests. Furthermore, the definition of cardinality-knowers (Sarnecka & Carey, 2008) was used to group children into cardinality- and subset-knowers (e.g., Le Corre & Carey, 2007; Le Corre et al., 2006) based on the highest numeral of which they knew the exact meaning during assessment of the GNT.

Numerical mother-child interaction and maternal sensitivity. A structured play situation aimed to measure the frequency of spontaneous numerical behaviors of mothers as well as equivalent mother-child interactions. In addition, the same observation was used to grasp a sense of the mothers' overall sensitivity. See Table 3 for more information on the measures. Due to time and practical constraints, no data were available for two out of the 31 children at T1. At T2 no missing data were encountered.

Mother and child sat on a carpet fabricated of soft plastic and were instructed to build a house with a set of "Lego-Duplo" blocks according to a specific pre-built model. The model contained ordinary blocks and additional blocks with numerical information. The set of blocks that mother and child could use during the play was almost identical to the modeling set of blocks. The only notable difference were the two (T1) or three (T2) additional blocks, with different numerical information. The purpose was to give all the participants the opportunity to focus on (inconsistent) numerical cues during the play. Both mother and child were blind for the real intention of the observation and were asked to play as they were used to do at home. After all of the instructions were given, mother and child were left alone in the room and after 5 minutes the play was impeded.

Numerical mother-child interaction. The structured play was recorded on video and all numerical actions of mothers and/or children were coded manually afterwards. A coding scheme was developed based on the most often observed actions that occurred during a sample of structured play situations (Adriaensens & De Smedt, 2011) and was further inquired consulting available literature on numerical home experiences (Benigno & Ellis, 2004; Blevins-Knabe & Musun-Miller, 1996; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Levine et al., 2011; Tudge & Doucet, 2004).

All actions of mothers that stimulated the children numerically and responses of the children on mothers' request with regard to numeracy-related topics were coded as "numeracy interaction between mother and child". Spontaneous actions of children related to numeracy were not coded within this category, and were moreover not further taken into account for analysis because hardly any appeared during the time of observation. All actions were given a score according to their frequency during the structured play situation. The sum of all these frequency scores on numeracy interaction items resulted, eventually, in a "total numeracy interaction score". Reliability of this scale for the current study as well as the range of scores on this scale (T1 and T2) can be found in Table 3. At T1 two experimenters achieved an average inter-rater reliability of .88 percentage of agreement. At T2, two experimenters achieved an average inter-rater reliability of .84 percentage of agreement.

Maternal sensitivity. In line with other research (e.g., Feldman & Masalha, 2010), the Coding Interactive Behavior (CIB) system (Feldman, 1998) was used to assess parental (i.e., maternal) sensitivity during the structured play. The CIB is a global rating system of parent-child interaction that includes 42 codes rated on a scale of 1 (low) to 5 (high) that leads to eight theoretically derived parent, child, and dyadic composites on diverse aspects of mother-child interaction that show high levels of internal consistency (e.g., Feldman, 2000; Feldman, Eidelman, & Rotenberg, 2004; Feldman & Klein, 2003). For each code, the observer assigns a single score after viewing the entire interaction. Several viewings are required to complete the coding. At T1, the coder achieved an average percentage of agreement of .84 with an officially trained coder by the lab of Feldman (i.e., the first author of this paper). At T2, the inter-rater reliability between two observers averaged .91 with the same trained coder.

In this study, the composite of parental sensitivity was used, which included the following codes: parent acknowledgment of child signals, maintenance of visual contact, expression of positive affect, appropriate vocal quality, resourcefulness in handling child's distress or expanding the interaction, consistency of style, and display of an affective range that matches child's readiness to interact. Reliability for this composite as well as the range of scores on this composite (T1 and T2) can be found in Table 3.

Maternal involvement. A parent questionnaire was used to map some experiences of children in the daily home context. Items on maternal involvement (*n* = 10 at T1 and *n* = 15 at T2) were retrieved from the respective scale *Parental Involvement in Developmental Advance* (PIDA) from a Dutch translation of the StimQ-Toddler (12-36 months) and StimQ-Preschool (36-72 months) interview respectively (Dreyer, Mendelsohn, & Tamis-LeMonda, 1996). Each item described a possible action or activity with the child on initiative of the parent in the home environment on which parents could indicate “Yes” or “No” with one credit given for each positive answer (e.g., “Do you play make-believe games with your child in which you sit at the table or on the floor?” at T1 or “Do you often have the opportunity to point to things in the street or around the house and name them for your child?” at T2). The score on the PIDA scale was calculated in line with the (manual) instructions as the total number of each “Yes”. All StimQ forms have high internal consistency, as shown by a Cronbach’s α ranging from .88 to .93 (consult http://pediatrics.med.nyu.edu/developmental/research/the-belle-project/stimq-cognitive-home-environment for more information on the StimQ forms). Reliability of the PIDA-scale as well as the range of the scores on this scale (T1 and T2) for this study can be found in Table 3. At T1, 24 out of 31 mothers filled out all items of the scale, at T2, no missing data were encountered.

Table 3. *Description of measures on numerical mother-child interaction, maternal sensitivity, and maternal involvement*

Measures	*Description*	*Range*	*Reliability*[a]
Structured play observation:			
Numerical mother-child interaction (T1)	Frequency numerical activities	0 - 57	.64
Numerical mother-child interaction (T2)	Frequency numerical activities	12 - 109	.70
PIDA-scale:			
Mater involvement (T1)	score on PIDA-scale	9 - 18	.71
Maternal involvement (T2)	score on PIDA-scale	3 - 14	.63
Sensitivity-scale CIB[c]-coding system:			
Maternal sensitivity (T1)	Mothers’ level of sensitivity	2.40 - 4.55	.87
Maternal sensitivity (T2)	Mothers’ level of sensitivity	2.60 - 4.70	.80

Note. T1 = time point (1) at 24 months of age. T2 = time point (2) at 48 months of age. PIDA = *Parent Development in Developmental Advance* (Dreyer et al., 1996). CIB = *Coding Interactive Behavior* (Feldman, 1998).

[a] Reliability of subscale as measured with Cronbach’s α.

Analysis

Linear regression analysis (executed in SPSS version 21.0; IBM Corp., 2012) was performed to explore the earlier described research questions. Graphical inspection of the data revealed that error terms were normally distributed. For all investigated relationships, not only linear relationships, but also quadratic relationships were explored in any case presenting the data graphically gave sufficient cause for doing so. Only when a significant quadratic relationship could be demonstrated, this was mentioned along with the results of the linear relationship between certain variables. Significant relationships between numerical mother-child interaction and number discrimination performance or numerical competencies in kindergarten were further tested on their numerical specificity by taking into account the mentioned control variables: maternal involvement and maternal sensitivity. However, each control factor was only considered when it correlated significantly with the relevant outcomes.

RESULTS

As an introduction, Table 4 provides the correlations between all measures. Bearing in mind the objectives of the current study, correlations between the predictor(s) numerical mother-child interaction and numerical competencies as outcome measure are especially of interest, together with simultaneously correlating control variables (i.e., involvement and sensitivity) with those same outcome measures.

This means that the significant correlations between numerical mother-child interaction at T1 and T2 (p = .016; with a significant higher amount of interactions at T2 than at T1: t (28) = - 8.65, p < .001); and between numerical mother-child interaction at T2 and the score on arithmetic operations at T2 (p = .025) are important. In addition, the marginally significant correlations between numerical mother-child interaction at T1 and the score on conceptual counting at T2 (p = .068); and between maternal involvement at T2 and the score on arithmetic operations at T2 (p = .066) are also informative within the context of the three formulated research questions of this study.

Table 4. *Intercorrelations between numerical mother-child interactions, maternal involvement and maternal sensitivity, number discrimination, numerical competencies, and cardinality*

Measure		1		2		3		4	5	6	7	8	*n*	*M*	*(SD)*
		T1	T2	T1	T2	T1	T2	T1	T2	T2	T2	T2			
1. Numerical mother-child interaction	T1	--											29	23.52	*(13.72)*
	T2	.44*	--										31	59.90	*(25.84)*
2. Maternal involvement	T1	.22	.32	--									24	14.00	*(2.75)*
	T2	.03	.37*	.27	--								31	8.00	*(2.61)*
3. Maternal sensitivity	T1	.37*	.00	-.08	-.17	--							29	3.98	*(0.54)*
	T2	.08	.20	.46*	.22	.17	--						31	3.79	*(0.63)*
4. Number discrimination	T1	.02	-.14	-.16	-.05	-.10	-.11	--					31	1.45	*(1.43)*
5. Procedural counting	T2	.15	.29	$.39^t$	.29	.08	.15	-.03	--				31	1.45	*(1.61)*
6. Conceptual counting	T2	$.34^t$	-.00	-.05	.12	.23	.07	-.03	.38*	--			31	4.39	*(2.70)*
7. Arithmetic operations	T2	.28	.40*	.29	$.33^t$	.24	.20	-.07	.41*	.57**	--		31	1.97	*(1.80)*
8. Give-a-Number	T2	.08	-.11	.06	.08	-.04	-.13	.19	.58**	.49**	$.35^t$	--	31	3.84	*(1.81)*

Note. T1 = time-point 1 (24 months). T2 = time-point 2 (48 months). GNT = (score on the) *Give-a-Number* task (Sarnecka & Carey, 2008).
$^t p \leq .100$. * $p \leq .050$. ** $p \leq .010$.

As could be expected, Table 4 displays that all numerical competencies were (marginally) significantly related in this sample. Finally, a significant correlation shows that the two measures of numerical mother-child interaction were mutually related.

Numerical mother-child interaction and numerical competencies in kindergarten

Linear regression analysis with numerical mother-child interaction in kindergarten (T2) revealed no significant linear relationship with procedural counting knowledge (T2), $F(1,29) = 2.60$, $p = .118$, and conceptual counting knowledge, $F(1,29) = 0.00$, $p = .986$. For arithmetic operations, however, a significant positive linear relationship, $F(1,29) = 5.56$, $p = .025$, $R^2 = .161$ with an effect size of $r = .40$, was found which remained marginally significant on top of concurrent maternal involvement, $F_{change}\ (1,28) = 3.11$, $p = .089$, $R^2_{\text{change}} = .09$. Mediation analysis revealed, in addition, that the known relationship between maternal involvement and later numerical competencies (in this case, arithmetic operations) was (marginally significantly) mediated by numerical mother-child interaction in kindergarten, $\beta = .32$, $p = .089$. Furthermore, there was no significant relationship between numerical mother-child interaction in kindergarten (T2) and group membership of cardinality-knowers or subset-knowers (T2) as demonstrated by a linear regression analysis, $F(1,29) = 0.34$, $p = .565$.

Numerical mother-child interaction and number discrimination in toddlerhood

Linear regression analysis with numerical mother-child interaction in toddlerhood (T1) demonstrated no significant linear relationship with the concurrent number discrimination performance as measured with the manual search task (T1), $F(1,27) = 0.01$, $p = .932$.

Numerical mother-child interaction in toddlerhood and numerical competencies in kindergarten

Linear regression analysis with numerical mother-child interaction in toddlerhood (T1) revealed no significant linear relationship with procedural counting knowledge (T2), *F*(1,27) = .62, *p* = .439, and a marginally significant relationship with conceptual counting knowledge (T2), *F*(1,27) = 3.60, *p* = .068, R^2 = .118 with an effect size of *r* = .34. Although, initially, no significant linear relationship was found between numerical mother-child interaction in toddlerhood (T1) and arithmetic operations in kindergarten (T2), *F*(1,27) = 2.29, *p* = .142, a significant quadratic (positive) relationship could be found instead, *F*(2,26) = 3.68, *p* = .039, R^2 = .22 with an effect size of *r* = .47. Figure 1 shows the significant relationships between arithmetic operations (T2) and numerical mother-child interaction in toddlerhood (T1) and kindergarten (T2) respectively.

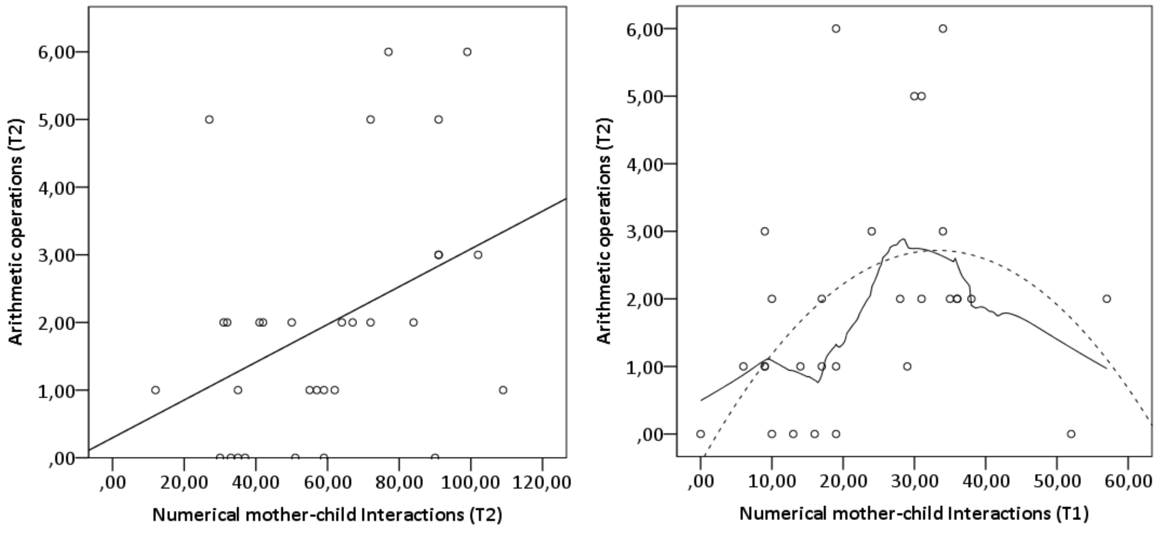

Figure 1. The linear and quadratic relationship between numerical mother-child interactions and arithmetic operations. T1 = time-point 1 (24 months); T2 = time-point 2 (48 months).

Furthermore, numerical mother-child interaction in toddlerhood (T1) had no significant relationship with the group membership of cardinality-knowers and subset-knowers (T2) as demonstrated by a linear regression analysis, *F*(1,27) = 0.18, *p* = .672.

DISCUSSION

A bulk of research exists on early literacy experiences through parent-child interaction from infancy on (e.g., Karrass & Braungart-Rieker, 2005). Considering the study field on early numeracy experiences, the current study questioned whether numerical mother-child interaction in children younger than kindergarten age could have a similar influence on (concurrent or later) numerical abilities.

Through three specific research objectives, the value of numerical mother-child interaction was explored for the concurrent number discrimination ability or number sense in toddlerhood as well as for the concurrent and prospective numerical competencies in kindergarten. The first aim of the study was to confirm the positive relationship between kindergartners' (aged 48 months) numerical mother-child interaction and concurrent numerical competencies in kindergarten, which has been reported extensively in previous research (e.g., Blevins-Knabe & Musun-Miller, 1996; LeFevre et al., 2009). The second and third aim were to extend this research and to investigate numerical mother-child interaction at younger age (the age of 24 months). More specifically, the concurrent value of the frequency of numerical mother-child interaction in toddlerhood for number discrimination performance was explored (second aim). Moreover, it was questioned whether these toddlerhood interactions already affected the same competencies later on in kindergarten (third aim).

Numerical mother-child interaction

As a preamble to the results that gave an answer to the specified research questions, a first exploration of numerical mother-child interaction will be discussed. Both in toddlerhood and in kindergarten, the structured play session was 5 minutes long. Derived from the descriptives of both measures given in Table 3, an average rate of one interaction per 13 seconds could be defined in toddlerhood. In kindergarten, an average rate of one interaction per 5 seconds could be observed. This implied that, on average, at both time points parent and child engaged in quite a considerable amount of numerical interactions during the structured play situation. Moreover, the constructs at both time points were significantly related to each other, with the kindergarten frequency of numerical mother-child interactions furthermore significantly higher than the frequency that could be observed in toddlerhood. This difference in frequency might be explained by the older age of the children at T2 compared to T1. As children grow older, it is not surprising that they become more able to engage in more (numerical) interactions. Another explanation for this higher amount of observed numerical mother-child interactions, however, might also be due to the difference in settings at both time points: At T1, research took place in Child & Family facilities but at T2, all tests were assessed at the children's home. It is possible that both child and parent needed some time to adjust before they felt at ease in these facilities at the first moment of testing in order to act in such a way as they would do in their own home environment. The order of the different tasks that were administered, though, was always randomly determined. Therefore, some *dyads* had already a considerable time to adjust to this unfamiliar setting which makes this explanation possibly less likely to be generalized to the whole sample of participants.

Numerical mother-child interaction and numerical competencies in kindergarten

No significant relationships were found between kindergartners' numerical mother-child interaction and concurrent numerical competencies in kindergarten related to neither procedural nor conceptual counting or cardinality knowledge. For the score on arithmetic operations, however, a significant linear relationship was (initially) found between numerical mother-child interaction and concurrent numerical competencies in kindergarten: How kindergartners performed on visually supported simple addition and subtraction exercises related to more numerical mother-child interaction. As such, this specific numerical kind of interaction might be perceived as a factor that stimulates the child's development of numerical abilities positively. However, it might also be well that parents who talk more about numbers or act more on numeracy do so because their children themselves are (more) interested in numeracy or because their children have better numerical abilities (Levine et al., 2011). Children might accordingly provoke numerical interaction with their mothers themselves.

Therefore, it cannot be ruled out that mothers who know this are sensitive to these specific needs, although this might not be clearly demonstrated in the current observation (i.e., structured play situation) because spontaneous actions of children themselves were not coded. Though, children may exhibit these behaviors in the home environment, and mothers may take this into account and generalize it to other situations or contexts leading them to engage more in numerical activities/interactions with their child anyway. The finding by Saxe et al. (1987) supports this idea of a bidirectional effect between parent and child (Bell, 1968, 1979; Sameroff, 1975) since mothers reported that they often adjusted their activities in line with their children's early numerical abilities and that their children, in their turn, adjusted their goals to the efforts that were undertaken by the mothers. As such, children’s academic performance may shape parental practices, which again would predict children’s subsequent achievement (Sy et al., 2013). For the current data, this could mean that either more numerical mother-child interaction resulted in better performance in arithmetic operations or that children who were more successful in arithmetic operations elicited more numerical interaction. No causal relationship could be drawn, however, because of the cross-sectional nature of the analysis of these concurrent measured constructs.

Nonetheless, important to note is that the value of this specific kind of interaction could still be demonstrated in some way (i.e., marginally significant) even when taking into account maternal involvement as a plausible explaining control factor. In line with Blevins-Knabe et al. (2007), it was reasonable to expect an effect of maternal involvement on numerical competencies: A significant relationship was indeed found between concurrent maternal involvement and the score on arithmetic operations. However, numerical mother-child interaction in kindergarten not only had a marginally significant additional value on top of this maternal involvement, but also mediated marginally significantly the expected relationship between maternal involvement and the specific numerical competency of solving arithmetic operations. Therefore, it was suggested that the influence of this specific kind of interaction on children’s numerical competencies in kindergarten could not entirely be due to an overall higher parental (in this case, maternal) involvement. Findings should however be interpreted with care because the sample of the current study entailed only a small number of children.

Sample size is not a problem for retrieving significant differences, but when analyses have insufficient power and are not significant (at .050 level), a risk of type 2- or β-mistakes cannot be excluded (Field, 2009, pp. 55-56). The marginal significant relationships that were found might turn significant in larger samples. Additional research with a larger group of participants is therefore definitely warranted. The extent to which mothers were sensitive to their children's needs was not significantly related to the ability to perform arithmetic operations. Consequently, sensitivity was not taken into account as a control measure for the relationship between numerical mother-child interaction and children's numerical competencies in kindergarten.

Numerical mother-child interaction and number discrimination performance in toddlerhood

Another aim of this study was to investigate the relationship of these early numerical experiences (T1) in toddlerhood with the concurrent early numeracy performance of children demonstrated by a number discrimination performance on a manual search task (T1). No significant relationship was present in the current dataset.

At first glance, this suggests that numerical mother-child interaction in toddlerhood does not yet influence concurrent early numeracy, at least not the kind of numeracy measured with the manual search task. Given that in the current study numerical mother-child interaction in toddlerhood predicted also the score on simple additions and subtractions later on in kindergarten, it is not inconceivable that numerical interaction in the current sample would have related to another kind of performance that more resembled the (complexity of the) one on the arithmetic operation subtest. Indeed, it can be assumed that abilities to perform arithmetic operations entail (apart from the obviously less developed capacities of children in toddlerhood than in kindergarten in general) skills that can be situated on another (i.e., more complex) level than the skills that are required to nonverbally discriminate numerosities (as measured with the manual search task). One candidate performance is for example the one on a task based on the visual expectation paradigm (e.g., Berger, Tzur, & Posner, 2006; Kobayashi, Hiraki, Mugitani, & Hasegawa, 2004; McCrink & Wynn, 2004; Wynn, 1992).

During visual expectation tasks, objects are presented to the infant on a stage. The scene is then hidden by a screen and an object is visually removed or added behind the screen. When the screen is dropped afterwards, either a possible or an impossible outcome is shown. Looking longer at the impossible (instead of the possible) outcome is considered as an indication of successful discrimination. Moreover, it is regarded as an indication of the ability to nonverbally solve a subtraction or an addition. This paradigm has already been used in infants from age 5 months and older (e.g., Wynn, 1992) and thus could grasp a sense of a very basic form of performing arithmetic operations in these children.

However, it might also be that the manual search task was simply not sensitive enough to detect individual differences in number discrimination and therefore not able to reveal a significant relationship with numerical mother-child interaction. The kind of task itself, the low number of trials per child, or the small sample size may account for this nonsignificant result.

Despite of this nonsignificant result, some remarks can be made. First, it is still conceivable that a concurrent relationship does exist between numerical mother-child interaction and other basic skills at this age. This question could not be answered based on the current dataset because only one aspect of numeracy was highlighted in toddlerhood. Future research should overcome this limitation by examining a broader range of numerical abilities to elucidate this hypothesis. Second, that numerical mother-child interaction in toddlerhood did not show a concurrent relationship with number discrimination does not detract from the fact that numerical mother-child interaction in toddlerhood could still have a predictive value for any later numerical outcome. This was explored in the next research question of which the results are discussed below.

Numerical mother-child interaction in toddlerhood and numerical competencies in kindergarten

Regarding the predictive value of numerical mother-child interaction in toddlerhood (T1) for later numerical competencies in kindergarten (T2), results were partially in line with the findings that resulted from the first objective of this study.

Similar to the relationship demonstrated between numerical mother-child interaction in kindergarten (T2) and concurrent numerical competencies (T2), also at 24 months of age the frequency of this specific kind of interaction marginally significantly predicted the score on arithmetic operations in kindergarten. This finding, together with the nonsignificant relationship between these interactions (both at T1 and T2) and procedural counting or cardinality knowledge, may prudently point toward the role of numerical parent-child interaction acting on a more complex level than (mere) counting. Especially when taking into account the complexity level of the assessed measures in the current study, one might acknowledge that arithmetic operations were supposed to be more difficult than the items on procedural counting or cardinality knowledge. Indeed, arithmetic operations find themselves on the border between early numerical skills and more advanced math knowledge acquired through formal teaching (Purpura & Lonigan, 2013), whereas counting and cardinality knowledge are often used in or involved in carrying out these operations (Powell & Fuchs, 2012). Furthermore, toddlers' numerical mother-child interaction showed a (marginally significant) linear relationship with later conceptual counting. In addition, it also related (marginally significant) quadratically to later arithmetic operations. In kindergarten, this kind of interaction did not relate anymore to conceptual knowledge whereas the relationship with arithmetic operations remained (marginally) significant but became linear. This train of thought might support the idea of arithmetic operations being more complex than conceptual counting.

To recall the resemblance between both measurements of numerical mother-child interaction with regard to the performance on (later or concurrent) arithmetic operations, an important note should be made: Whereas the concurrent relationship between numerical mother-child interaction and arithmetic operations at kindergarten age was linear, the prospective relationship of numerical mother-child interaction at toddler age with arithmetic skills at kindergarten age was quadratic in nature (see Figure 1). In kindergarten, this implied that more numerical mother-child interaction correlated with higher scores on arithmetic operations (and vice versa). In toddlerhood, however, it seemed that more numerical mother-child interaction only predicted higher performance on arithmetic operations in kindergarten to some extent. At higher rates of these numerical interactions the performance on arithmetic operations declined again.

This finding may align with the higher complexity level of arithmetic operations (as suggested earlier). Moreover, it suggests that early numerical stimulation through mother-child interaction might be worthwhile for later arithmetic, although its positive effect is not unlimited. Within a child's *zone of proximal development*, parents can be seen as intuitive tutors who may facilitate development (Wells, 1999). This "zone" refers to the distance between the actual and potential developmental level, with the latter achieved under adult guidance or in collaboration with more capable peers (Vygotsky, 1978). Because of its reciprocal and bidirectional nature (Van Geert 1994; Valsiner 1994), an explanation may be found in the receptivity of the zone of proximal development which is in the hands of children (Chak, 2001). A child's readiness to move forward does not only depend on the appropriateness of the cognitive demand, provided in this case through numerical mother-child interaction, but also on the child's motivation to engage in the activity. Children can be out of the zone of proximal development not only because of their lack of ability, but also simply because they are not in tune with a task (Veresov, 2000).

Nevertheless, the finding from the previous research objective that no concurrent relationship could be found between numerical mother-child interaction and number sense was indeed not detrimental to the value of numerical mother-child interaction: Its predictive value was demonstrated by a prospective relationship with some numerical aspects in kindergarten. Higher scores on conceptual counting knowledge could be traced back to more observed numerical mother-child interactions in toddlerhood. For arithmetic operations, these interactions seemed worthwhile without exaggerating in its provision. The current findings highlighted long-term effects (i.e., prospective relationships) of numerical mother-child interaction rather than short-term effects (i.e., concurrent relationships), which in this case were not even perceived.

Numerical specificity of mother-child interaction

Finally, an important issue to this study related to the domain-specific nature of the influence of parental numerical interaction on children's early numerical abilities.

The findings seemed to point in the direction of numeracy-specific relationships between mother-child interaction and aspects of numeracy, because overall no significant relationships were found between general involvement or sensitivity and neither the manual search task performance in toddlerhood nor any other of the numerical competencies in kindergarten. Only once, a (marginally) significant relationship existed between maternal involvement in kindergarten and the concurrent performance on arithmetic operations as well as the observed numerical mother-child interaction in kindergarten. Nonetheless, controlling for the effect of this general parental factor, the relationship between the frequency of these interactions and the numerical competency measure remained marginally significant.

Therefore, it can be concluded (although with some caution because of the only marginally significant results) that the influence of numerical interaction on children’s early numeracy performance is not (only) due to an overall involvement of parents toward their children. In line with Hong, Yoo, You, and Wu (2010), it could be expected that parental involvement and sensitivity with a domain-specific focus (i.e., numerical cues) may yield different results than a general approach of both factors. The current data might have underpinned this assumption because general parental involvement and sensitivity could not fully explain the results in this study. However, more in-depth analysis is needed with focus on the difference between domain-general involvement and sensitivity, and domain-specific involvement and sensitivity during coding of the interactions. For now, coding was restricted to the frequency of numerical mother-child interactions without investigating its nature (i.e., numerical involvement or sensitivity).

Implications

To explore the impact of numerical parent-child interaction at the age of 2 years on children’s budding numeracy, longitudinal research needed to clarify the predictive value of those interactions for children’s later mathematical outcome. This was the aim of the current study. It was expected that early numerical experiences through mother-child interaction would influence children’s later outcome in line with Levine et al. (2011).

The confirmative findings of the current study imply that an additional focus on numeracy (next to literacy) by agencies in support of parenting in preschoolers – including infants and toddlers – could be worthwhile. As Skwarchuk (2009) already concluded, society is aware of the benefits of early literacy exposure, but it may also be beneficial to improve the image of numeracy learning by informing parents, practitioners, and policymakers about the merit of numerical mother-child interaction, even in toddlers. Therefore, more in-depth research is needed to identify which specific aspects and types of numerical parent-child activities promote numeracy knowledge (the most) and how they can be incorporated properly (Skwarchuk, 2009). Whereas toddlers need to have opportunities to explore and get to know early numerical concepts on their own, parents may function as facilitators within the zone of proximal development. Fostering number sense within relative boundaries now can have long-term gains later on.

At clinical level, this research lays the foundation for a follow-up of those children who received less numerical mother-child interaction during the observation at the age of 2 years. If less parental input on numeracy would be predictive of later mathematical problems, additional numerical stimulation of children at risk could be worthwhile. At-risk children are those who perform less due to specific child or sociocultural factors, such as siblings of children with a mathematical disorder who have a higher risk on having this disorder too (e.g., Shalev et al., 2001) or children from families with a low income (e.g., Jordan et al., 2007). After all, positive parenting practices can protect children from, for example, the disadvantages of financial strain (Gershoff, Aber, Raver, & Lennon, 2007; Yeung, Linver, & Brooks-Gunn, 2002).

Limitations and suggestions for future research

It follows that there are some limitations with regard to this study. First, all participating parents were mothers. Therefore, no data on father-child interaction could be included, although fathers are more and more involved in studies to gain insight in children’s development (Pancsofar et al., 2010). It remains a challenge to include fathers or even siblings in studying childrens’ development (e.g., Dai & Heckman, 2013).

Moreover, one-on-one interactions between parent and child may not reflect the context in which children live in a representative way. Children also attend for example day care (Benigno & Ellis, 2004) where they meet peers and other caregivers. Other interaction-partners, therefore, but also *triads* (see Benigno & Ellis, 2004 for example) instead of dyads could provide more insight on numerical experiences of children. More and longer situations to observe such numerical interactions, in addition to different contexts simultaneously, might be indicated to get a full picture of children's daily numerical experiences.

More observational data, but also more specific coding is indispensable to unveil various aspects and different types of interaction that may differ in their value for later numerical outcome. Related to this topic it should be remarked that the context of the situation that was used to tap the frequency of the numerical mother-child interaction was informal; referring to for example *mathematical play* in which mathematics are embedded (Ginsburg, 2006) play continues spontaneously as it does sometimes with block play. In the current study, numerical cues were incorporated in the play with the intention to create opportunities to model and support children's numeracy. As such, (some) movements in this activity could be related with mathematics. However, because mothers (and children) were not aware of the true intention of the observation (as to code the frequency of numerical mother-child interaction), the situation can be compared with informal numerical experiences taking place at home where there might be no intentions on the part of caregivers to enhance numeracy. This may have elicited especially indirect experiences in the current study. Nevertheless, probably as a result of the offered toys (i.e., blocks of which some specifically contained clear numerical cues) as well as the structured and time-restricted character of the play situation, prospection of coded items seemingly revealed mainly direct activities. This intertwining of the play context and the (un)intention of the observed interaction made it difficult to apply the "direct-indirect" distinction straightforward to the current data. Otherwise, it would have been self-evident to disentangle which type of parent-child interactions are of most importance with regard to current or prospective numerical functioning in toddlerhood and kindergarten respectively (see Skwarchuk, Sowinski, & LeFevre, 2014).

This would be informative in providing parents, practitioners, and policymakers with practical guidelines to address the importance of numerical mother-child interaction.

Moreover, different aspects or types of interactions may also predict different aspects of numerical competencies (see Skwarchuk et al., 2014). Additional information could be harvested in this respect by studying a broader range of numerical abilities or competencies, both in toddlerhood and kindergarten. Supplementary factors, such as specific family and parent characteristics can also be taken into further consideration. This study dealt with families recruited from a subpopulation that already participated in the broader project of which this study was only one part. Therefore, the parents might have had an unusual high interest in engaging in educational activities with their children (e.g., Benigno & Ellis, 2004). Another limitation is that only families with a middle or high family income were included, disabling the exploration of the influence of socioeconomic status (SES). From previous studies, it is known that middle-SES mothers engage their children in more complex number activities than low-SES mothers, leading to better developed skills (e.g., Starkey et al., 2004). It would therefore be interesting for future research to also take into account low-income families to accurately investigate the influence of SES on both numerical interaction and performance. Not only SES, but also more domain-specific factors such as parents' attitudes toward mathematics, their own mathematical abilities or knowledge, and their academic expectations are potential predictors of home practices (e.g., LeFevre, 2009; Skwarchuk, 2009; Skwarchuk et al., 2014).

Conclusion

Despite the aforementioned limitations, the strength and unique contribution of the current study to previously conducted research on early numeracy lies in the predictive value of the frequency of numerical mother-child interactions even in toddlerhood. This holds for at least some aspects of children's numerical competencies as measured in kindergarten. This is, mother-child interaction was found to be related to later conceptual counting (shown by a marginally significant linear relationship) and to later arithmetic operations (shown by a marginally significant quadratic relationship).

Moreover, this study informs about the different kind of relationship of numerical mother-child interaction with arithmetic operations when comparing results from toddlerhood with results from kindergarten. As this relationship was confirmed to be linear in kindergarten, one could – so to speak – simply promote engagement in this kind of interaction. However, because the positive effect on numerical competencies in kindergarten of the same interactions in toddlerhood seemed not unlimited, fostering numerical development within relative boundaries is indicated at this younger age. Emphasis is therefore mainly on empowerment of opportunities to foster numerical development within children's zone of proximal development (even) from toddlerhood on. Caution is, however, warranted because of the small sample size and marginal significance of the main findings of the current study. It is vital for future research to replicate the findings in a larger group of children and to clarify long-term effects and specific aspects of these numerical mother-child experiences on typical and atypical development of numeracy to further explore its clinical relevance.

REFERENCES

Adriaensens, L., & De Smedt, L. (2011). *Toddlers (24 months) and their first steps into the the land of numeracy holding the hand of the mother: What is the mother's contribution?* (Master's thesis). Ghent University, Department of Psychology and Educational Sciences.

Alloway, T. P., & Gathercole, S. E. (2005). The role of sentence recall in reading and language skills of children with learning difficulties. *Learning and Individual Differences, 13*, 271-282. http://dx.doi.org/10.10168j.lind.2005.05.001

Alvarenga, P., & Piccinini, C. A. (2009). Child-rearing practices and social development indicators in three-year-old children. *Psicologia-Reflexao E Critica, 22*, 191-199. http://dx.doi.org/10.1590/S0102-79722009000200004

Anders, Y., Rossbach, H. G., Weinert, S., Ebert, S., Kuger, S., Lehrl, S., & von Maurice, J. (2012). Home and preschool learning environments and their relations to the development of early numeracy skills. *Early Chilhood Research Quarterly, 27*, 231-244. http://dx.doi.org/10.1016/j.ecresq.2011.08.003

Anderson, A. (1997). Families and mathematics: A study of parent-child interactions. *Journal for Research in Mathematics Education, 28*, 484-511.

Aram, D. (2005). The continuity in children's literacy achievements: A longitudinal perspective from kindergarten to school. *First Language,* 25, 259-289. 10.1177/0142723705050339

Aram, D., & Biron, S. (2004). Joint storybook reading and joint writing interventions among low SES preschoolers: differential contributions to early literacy. *Early Childhood Research Quarterly, 19,* 588-610. http://dx.doi.org/ 10.1016/j.ecresq.2004.10.003

Aram, D., & Levin, I. (2004). The role of maternal mediation of writing to kindergartners in promoting literacy achievements in school: A longitudinal prespective. *Reading & Writing: An Interdisciplinary Journal, 17*, 47-61. http://dx.doi.org/10.1023/B:READ.0000032665.14437.e0

Aunio, P., & Niemivirta, M. (2010). Predicting children's mathematical performance in grade one by early numeracy. *Learning and Individual Differences, 20*, 427-435. http://dx.doi.org/10.1016/j.lindif.2010.06.003

Aunola, K., Leskinen, E., Lerkkanen, M.-K., & Nurmi, J.-E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology, 96*, 699-713. http://dx.doi.org/10.1037/0022-0663.96.4.699

Baumwell, L., Tamis-LeMonda, C. S., & Bornstein, M. H. (1997). Maternal verbal sensitivity and child language comprehension. *Infant Behavior & Development, 20*, 247-258. http://dx.doi.org/10.1016/s0163-6383(97)90026-6

Bell, R. Q. (1968). A reinterpretation of the direction of effects in studies of socialization. *Psychological Review, 75*, 81-95. http://dx.doi.org/10.1037/h0025583

Bell, R. Q. (1979). Parent, child and reciprocal influences. *American Psychologist, 34*, 821-826. http://dx.doi.org/10.1037/0003-066x.34.10.821

Benigno, J. P., & Ellis, S. (2004). Two is greater than three: Effects of older siblings on parental support of preschoolers' counting in middle-income families. *Early Childhood Research Quarterly, 19*, 4-20. http://dx.doi.org/10.1016/j.ecresq.2004.01.006

Berger, A., Tzur, G., & Posner, M. I. (2006). Infant brains detect arithmetic errors. *Proceedings of the National Academy of Science of the United States of America, 103*, 12649-12653. http://dx.doi.org/10.1073:pnas.0605350103

Bigelow, A. E. Maclean, K., Proctor, J., Myatt, T., Gillis, R., & Power, M. (2010). Maternal sensitivity throughout infancy: Continuity and relation to attachment security. *Infant Behaviour & Development, 33*, 50-60. http://dx.doi.org/10.1016/j.infbeh.2009.10.009

Bjorklund, D., Hubertz, M., & Reubens, A. (2004). Young children's arithmetic strategies in social context: How parents contribute to children's strategy development while playing games. *International Journal of Behavioral Developments, 28*, 347−357. http://dx.doi.org/10.1080/01650250444000027

Blevins-Knabe, B. (2008). Fostering early numeracy at home. Encyclopedia of Language and Literacy Development. Londonn ON: Canadian Language and Literacy

Research Netwerk, Retrieved from http://www.literacyencyclopedia.ca/pdfs/topic.php.topId=245

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development & Parenting, 5*, 35-45. http://dx.doi.org/10.1002/(sici)1099-0917(199603)5:1<35::aid-edp113>3.0.co;2-0

Blevins-Knabe, B., Berghout Austin, A., Musun, L. Eddy, A., & Jones, R. M. (2000). Familiy home care providers' and parents' beliefs and practices concerning mathematics with young children. *Early Child Development and Care, 165*, 41-58. http://dx.doi.org/10.1080/0300443001650104

Blevins-Knabe, B., Whiteside-Mansell, L., & Selig, J. (2007). Parenting and mathematical development. *Academic Exchange Quarterly, 11*, 76-80.

Bornstein, M. H., & Tamis-LeMonda, C. S. (1989). Maternal responsiveness and cognitive development in children. In M. H. Bornstein (Ed.), *Maternal responsiveness: Characteristics and consequences* (Vol. 43, pp. 49-61). San Francisco: Jossey-Bass.

Brandt, R. (1993). On misuse of testing: A conversation with George Madaus. *Educational Leadership, 46*, 26-29.

Butler, S. R., Marsh, H. W., Sheppard, M. J., Sheppard, J. L. (1985). Seven-year longitudinal study of the early prediction of reading achievement. *Journal of Educational Psychology*, *77*, 349-361. http://dx.doi.org/10.1037/0022-0663.77.3.349

Campbell, F. A., Pungello, E. P., Miller-Johnson, S., Burchinal, M., & Ramey, C. T. (2001). The development of cognitive and academic abilities: Growth curves from an early childhood educational experiment. *Developmental Psychology, 37,* 231-242. http://dx.doi.org/10.1037/0012-1649.37.2.231

Chak, A. (2001). Adult sensitivity to children’s learning in the zone of proximal development. *Journal for the Theory of Social Behaviour, 31*, 383-395. http://dx.doi.org/10.1111/1468-5914.00166

Claessens, A., Duncan, G., & Engel, M. (2009). Kindergarten skills and fifth-grade achievement: Evidence from the ECLS-K. *Economics of Education Review, 28*, 415-427. http://dx.doi.org/10.1016/j.econedurev.2008.09.003

Cordes, S., & Brannon, E. M. (2009). The relative salience of discrete and continuous quantity in young infants. *Developmental Science, 12* (3), 453-463. http://dx.doi.org/10.1111/j.1467-7687.2008.00781.x

Costa, H. C., Perdry, H., Soria, C., Pulgar, S., Cusin, F., & Dellatolas, G. (2013). Emergent literacy skills, behavior problems and familial antecedents of reading difficulties: A follow-up study of reading achievement from kindergarten to fifth grade. *Research in Developmental Disabilities, 34*, 1018-1035. http://dx.doi.org/10.1016/j.ridd.2012.11.029

Cunningham , A. E., & Stanovich, K. E. (1997). Early reading acquisition and its relation to reading experience and ability 10 years later. *Developmental Psychology, 5*, 111-123. http://dx.doi.org/10.1037/0012-1649.33.6.934

Dai, X. & Heckman, J. J. (2013). Older siblings' contributions to young child's cognitive skills. *Economic Modelling, 35*, 235-248. http://dx.doi.org/10.1016.j.econmod.2013.07.003

Desoete, A., & Grégoire, J. (2006). Numerical competence in young children and in children with mathematics learning disabilities. *Learning and Individual Differences, 16*, 351-367. http://dx.doi.org/10.1016/j.lindif.2006.12.006

Desoete, A., Stock, P., Schepens, A., Baeyens, D., & Roeyers, H. (2009). Classification, seriation, and counting in grades 1, 2, and 3 as two-year longitudinal predictors for low achieving in numerical facility and arithmetical achievement? *Journal of Psychoeducational Assessment, 27,* 252-264. http://dx.doi.org/10.1177/0734282908330588

Dieterich, S. E., Assel, M. A., Swank, P., Smith, K. E., & Landry, S. H. (2006). The impact of early maternal verbal scaffolding and child language abilities on later decoding and reading comprehension skills. *Journal of School Psychology, 43*, 481-494. http://dx.doi.org/10.1016/j.jsp.2005.10.003

Dodici, B. J., Draper, D. C., & Peterson, C. A. (2003). Early parent-child interactions and early literacy development. *Topics in Early Childhood Special Education, 23*, 124-136. http://dx.doi.org/10.1177/02711214030230030301

Dougherty, C. (2003). Numeracy, literacy, and earnings: Evidence from the National Longitudinal Survey of Youth. *Economics of Education Review, 22,* 551-521. http://dx.doi.org/10.1016/S0272-7757(03)00040-2

Dreyer, B. P., Medelsohn, A. L., & Tamis-LeMonda, C. S. (1996). Assessing the child's cognitive home environment through parental report: Reliability and validity. *Early Development & Parenting*, *5*, 271-287. http://dx.doi.org/10.1002/(SICI)1099-0917(199612)5:4<271::AID-EDP138>3.0.CO;2-D

Duncan, G. J., Claessens, A., Magnuson, K., Klebanov, P., Pagani, L. S., Feinstein, L., … Japel, C. (2007). Schoolreadiness and later achievement. *Developmental Psychology, 43*, 1428-1446. http://dx.doi.org/10.1037/0012-1649.43.6.1428

Feigenson, L. & Carey S. (2003). Tracking individuals via object-files: Evidence from infants' manual search. *Developmental Science, 6*, 568-584. http://dx.doi.org/10.1111/1467-7687.00313

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object *Cognition,* 97, 295-313, http://dx.doi.org/10.1016/j.cognition.2004.09.010.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Feldman, R. (1998). Coding interactive behavior manual. *Unpublished manual*, Bar-Ilan University, Israel.

Feldman, R. (2000). Parents' convergence on sharing and marital satisfaction, father involvement, and parent–child relationship at the transition to parenthood. *Infant Mental Health Journal, 21,* 176–191. http://dx.doi.org/10.1002/1097-0355(200007)21:3<176::AID-IMHJ3>3.0.CO;2-4

Feldman, R., Eidelman, A. I., & Rotenberg, N. (2004). Parenting stress, infant emotion regulation, maternal sensitivity, and the cognitive development of triplets: A model for parent and child influences in a unique ecology. *Child Development, 75*, 1774–1791.

Feldman, R., & Klein, P. (2003). Toddlers' self-regulated compliance with mother, caregiver, and father: Implications for theories of socialization. *Developmental Psychology, 39*, 630–692. http://dx.doi.org/10.1037/0012-1649.39.4.680

Feldman, R., & Masalha, S. (2010). Parent–child and triadic antecedents of children's social competence: Cultural specificity, shared process. *Developmental Psychology, 46*, 455–467. http://dx.doi.org/10.1037/a0017415

Fenton, G., & Morley, S. (2013). A tale of two RCTs: Using randomized controlled Trials to benchmark routine clinical (psychological) treatments for chronic pain. *Pain, 154,* 2108-2119. http://dx.doi.org/10.1016/j.pain.2013.06.033

Field, A. (2009). *Discovering Statistics Using SPSS (third edition).* London: Sage.

Fitzgerald, J. Spiegel, S. L., & Cunningham, J. W. (1991). The relationship between parental literacy level and perceptions of emergent literacy. *Journal of Reading Behavior, 23*, 191-213. http://dx.doi.org/10.1080/10862969109547736

Fluck, M., Holgate, M., & Linnell, M. (2005). Does counting count for 3- to 4-year-olds? Parental assumptions about preschool children's understanding of counting and cardinality. *Social Development, 14*, 498-513. http://dx.doi.org/10.1111/j.1467-9507.2005.00313.x

Fogelman, K. (1983). *Growing up in Great Britain: Papers from the national child developmental study.* London: Macmillan.

Fuson, C. K. (1988). *Children's counting and concepts of number*. New york: Springer-Verlag.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Gershoff, E. T., Aber, J. L., Raver, C. C., & Lennon, M. C. (2007). Income is not enough: Incorporating material hardship into models of income associations with parenting and child development. *Child Development*, *78*, 70–95. http://dx.doi.org/10.1111/j.1467-8624.2007.00986.x

Gervasoni, A. (2001). Specialised programs for students who are low attaining in mathematics. Do they help? In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond (Proceedings of the 24th annual conference of the Mathematics Research Group of Australasia*, pp. 249-256). Sydney, NSW, 30 June - 4 July.

Gervasoni, A. (2002). Intervention in mathematics: Is assistance more effective in Grade 1 or Grade 2? In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Mathematics education in the South Pacific (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia,* Auckland, pp. 301-308). Syndney: MERGA.

Ginsburg, H. P. (2006). Mathematical play and playful mathematic: A guide for early education. In G. D. Singer, R. M. Golinkoff, & K. Hirsh-Pasek. *Play = learning, How many* (pp. 145-165). New York: Oxford University Press.

Ginsburg, H. P., Klein, A., & Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In I. E. Sigel & K. A. Renninger (Eds.), Handbook of child psychology: Child psychology in practice (Vol. 4, 5th ed., pp. 401–476). New York: John Wiley & Sons.

Grégoire, J. (2005). Développement logique et compétences arithmétiques. Le modèle piagétien est-il toujours actuel ? [Development of logical competences in arithmetics. Is the Piagetian model still up to date?]. In M. Crahay, L. Verschaffel, E. De Corte & J. Grégoire (Eds.), *Enseignement et apprentissage des mathématiques* (pp. 57-77). Bruxelles: De Boeck.

Grégoire, J., Noël, M., & Van Nieuwenhoven, C. (2004). *TEDI-MATH*. Antwerpen: Harcourt.

Grietens, H., Hoppenbrouwers, K., Desoete, A., Wiersema, R., & Van Leeuwen, K. (2010). *JOnG! Theoretische achtergronden, onderzoeksopzet en verloop van het eerste meetmoment [JOnG! Theoretical backgrounds, study design and course of the first assessment].* Leuven: Steunpunt, Welzijn, Volksgezondheid en Gezin (SWVG).

Heath, S. M., Bishop, D. V. M., Bloor, K. E., Boyle, G. L., Fletcher, J., Hogben, J. H., … Yeong, S. H. M. (2014). A spotlight on preschool: The influence of family factors on children's early literacy skills. *Plos One, 9*, e95255. http://dx.doi.org/10.1371/journal.pone.0095255

Heckman, J. J. (2006). Skill formation and the economics of investing in disadvantaged children. *Science, 312*, 1900-1902. http://dx.doi.org/10.1126/science.1128898

Hong, S., Yoo, S. K., You, S., & Wu, C. C. (2010) The reciprocal relationship between parental involvement and mathematics achievement: autoregressive crosslagged modeling. *The Journal of Experimental Education, 78*, 419-439. http://dx.doi.org/10.1080/00220970903292926

Hood, M., Conlon, E., & Andrews, G. (2008). Preschool home literacy practices and children's literacy development: A longitudinal analysis. *Journal of Educational Psychology, 100*, 252-271. http://dx.doi.org/10.1037/0022-0663.100.2.252

Hooper, S. R., Roberts, J., Sideris, J., Burchinal, M. & Zeisel, S. (2010). Longitudinal predictors of reading and math trajectories through middle school African-American versus Causian students across two samples. *Development Psychology, 46*, 1018-1029. http://dx.doi.org/10.1037/a0018877

Huntsinger, C. S., Jose, P. E., Larson, S. L., Balsink Krieg, D., & Shaligram, C. (2000). Mathematics, vocabulary, and reading development in Chinese American and European American Children over the primary school years. *Journal of Educational Psychology, 92*, 745-760, http://dx.doi.org/10.1037//0022-0663.92.4.745

IBM Corp. (2012). *IBM SPSS Statistics for Windows*, Version 21.0. Armonk, NY: IBM Corp.

Jacobson, N. S, & Truax P. (1991). Clinical significance: A statistical approach to defining meaningful change in psychotherapy research. *Journal of Consulting and Clinical Psychology, 59*, 12-19. http://dx.doi.org/10.1037/0022-006X.59.1.12

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice (Blackwell Publishing Limited), 22*, 36-46. http://dx.doi.org/10.1111/j.1540-5826.2007.00229.x

Juel, C. (1988). Learning to read and to write: A longitudinal study of 54 children from first through fourth grades. *Journal of Educational Psychology, 80,* 437-447. http://dx.doi.org/10.1037/0022-0663.80.4.437

Karrass, J., & Braungart-Rieker, J. M. (2005). Effects of shared parent infant book reading on early language acquisition. *Journal of Applied Developmental Psychology, 26*, 133-148. http://dx.doi.org/10.1016/j.appdev.2004.12.003

Kaufmann, L., & Dowker, A. (2009). Typical Development of Numerical Cognition: Behavioral and neurofunctional issues Introduction. *Journal of Experimental Child Psychology, 103*, 395-399. http://dx.doi.org/10.1016/j.jecp.2009.05.003

Kern, M. L., & Friedman, H. S. (2008). Early educational milestones as predictors of lifelong academic achievement, midlife adjustment, and longevity. *Journal of Applied Developmental Psychology, 30*, 419-430. http://dx.doi.org/10.1016/j.appdev.2008.12.025

Kleemans, T., Peeters, M., Segers, E., & Verhoeven, L. (2012). Child and home predictors of early numeracy skills in kindergarten. *Early Childhood Research Quarterly, 27*, 471 - 477. http://dx.doi.org/10.1016/j.ecresq.2011.12.004

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*, 59-69. http://dx.doi.org/10.1037/0012-1649.42.1.59

Kobayashi, T., Hiraki, K., Mugitani, R., & Hasegawa, T. (2004). Baby arithmetic: one object plus one tone. *Cognition, 91*, 23-34. http://dx.doi.org/10.1016/j.cognition.2003.09.004

Krajewski, K., & Schneider, W. (2009). Exploring the impact of phonological awareness, visual-spatial working memory, and preschool quantity-number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology*, *103*, 516-531. http://dx.doi.org/10.1016/j.jecp.2009.03.009

Lam, S., Chow-Yeung, K., Wong, B. P. H., Lau, K. K., & Tse, S. I. (2012). Involving parents in paired reading with preschoolers: Results from a randomized controlled trial. *Contemporary Educational Psychology, 38,* 126-135. http://dx.doi.org/ 10.1016/j.cedpsych.2012.12.003

Landry, S. H., Smith, K. E., & Swank, P. R. (2006). Responsive parenting: Establishing early foundations for social, communication, and independent problem-solving skills. *Developmental Psychology, 42*, 627-642. http://dx.doi.org/10.1037/0012-1649.42.4.627

Landry, S. H., Smith, K. E., Swank, P. R., Assel, M. A., & Vellet, S. (2001). Does early responsive parenting have a special importance for children's development or is consistency across early childhood necessary? *Developmental Psychology, 37*, 387-403. http://dx.doi.org/10.1037//0012-1649.37.3.387

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*, 395–438. http://dx.doi.org/10.1016/j.cognition.2006.10.005

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Revisiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology, 52*, 130–169. http://dx.doi.org/10.1016/j.cogpsych.2005.07.002

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: longitudinal predictors of performance. *Child Development, 81*, 1753-1767. http://dx.doi.org/10.1111/j.1467-8624.2010.01508.x

LeFevre, J.-A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home Numeracy Experiences and Children's Math Performance in the Early School Years. *Canadian Journal of Behavioural Science, 41*, 55-66. http://dx.doi.org/10.1037/a0014532

Levin, I., Ravid, D., & Rapaport, S. (2001). Morphology and spelling among Hebrew-speaking children: From kindergarten to first grade. *Journal of Child Language, 28*, 741-772. http://dx.doi.org/10.1017/S0305000901004834

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2011). What counts in the development of young children's number knowledge? *Developmental Psychology, 47*, 302-302. http://dx.doi.org/10.1037/a0022101

Lonigan C. J., Burgess S. R., & Anthony, J. L. (2000). Development of emergent literacy and early reading skills in preschool children: Evidence from a latent-variable longitudinal study. *Developmental Psychology, 36,* 596-613. http://dx.doi.org/10.1037//OOI2-1649.36.5.596

Maughan, B. B., Messer, J. J., Collishaw, S. S., Pickles, A. A., Snowling, M. M. Yule, W. W., Rutter, M. (2009). Persistence of literacy problems: Spelling in adolescence and at mid-life. *Journal of Child Psychology and Psychiatry, 50*, 893-901. http://dx.doi.org/10.1111/j.1469-7610.2009.02079.x

McCrink, K., & Wynn, K. (2004). Large-number addition and subtraction by 9-month-old infants. *Psychological Science, 15*, 776-781. http://dx.doi.org/10.1111/j.0956-7976.2004.00755.x

McPherson, G. E. (2009). The role of parents in children's musical development. *Psychology of Music, 37*, 91-110. http://dx.doi.org/10.1177/0305735607086049

Melhuish, E. C., Phan, M. B., Sylva, K., Sammons, P., Siraj-Blatchford, I., & Taggart, B. (2008). Effects of the home learning environment and preschool center experience upon literacy and numeracy development in early primary school. *Journal of Social Issues, 64*, 95-114. http://dx.doi.org/ 10.1111/j.1540-4560.2008.00550.x

Mix, K. S. (2009). How Spencer made number: First uses of the number words. *Journal of Experimental Child Psychology, 102*, 427-444. http://dx.doi.org/10.1016/j.jecp.2008.11.003

Morley, S. (2013). A rough guide to evaluating your Pain Management Programme: The analysis of individual patient data and benchmarking. Unpublished manuscript, University of Leeds.

Morrison, E. F., Rimm-Kauffman, S., & Pianta, R. C. (2003). A longitudinal study of mother-child interactions at school entry and social and academic outcomes in middle school. *Journal of School Psychology, 41*, 185-200. http://dx.doi.org/10.1016/S0022-4405(03)00044-X

National Institute of Child Health & Development (2005). Pathways to Reading: The role of oral language in the transition to reading. *Developmental Psychology, 41*, 428-442. http://dx.doi.org/10.1037/0012-1649.41.2.428

Ortiz, C., Stowe, R. M., & Arnold, D. H. (2001). Parental influence on child interest in shared picture book reading. *Early Childhood Research Quarterly, 16,* 263-281. http://dx.doi.org/10.1016/S0885-2006(01)00101-6

Page, M., Wilhelm, M. S., Gamble, W .C., & Card, N. A. (2010). A comparison of maternal sensitivity and verbal stimulation as unique predictors of infant social-emotional and cognitive development. *Infant Behaviour & Development, 33*, 101-110. http://dx.doi.org/10.1016/j.infbeh.2009.12.001

Pancsofar, N., Vernon-Feagans, L., & Family Life Project, I. (2010). Fathers' early contributions to children's language development in families from low-income rural communities. *Early Childhood Research Quarterly, 25*, 450-463. http://dx.doi.org/10.1016/j.ecresq.2010.02.001

Powell, S. R., & Fuchs, L. S. (2012). Early numerical competencies and students with mathematics difficulty. *Focus on Exceptional Children, 44*, 1-16. http://dx.doi.org/10.1111/j.1540-5826.2011.00345.x

Prior, Sanson, Smart, & Oberklaid (2000). Pathways from infancy to adolescence: Australian temperament project 1983-2000. Melbourne: Austrialian Institute of Family Studies

Purpura, D. J. Hume, L. E., Sims, D. M. & Lonigan, C. J. (2011). Early literacy and early numeracy: The value of including early literacy skills in the prediction of numeracy development. *Journal of Experimental Child Psychology, 110,* 647-658. http://dx.doi.org/10.1016/j.jecp.2011.07.004

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal, 50,* 178–209. doi:10.3102/00028312124 65332

Roberts, J. E., Jurgens, J., & Burchinal, M. (2005). The role of home literacy practices in preschool children's language and emergent literacy skills. *Journal of Speech Language and Hearing Research, 48*, 345-359. http://dx.doi.org/10.1044/1092-4388(2005/024)

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*, 361-395. http://dx.doi.org/10.1016/j.cognition.2006.01.005

Rousselle, L., & Noel, M. P. (2008). The development of automatic numerosity processing in preschoolers: Evidence for numerosity-perceptual interference. *Developmental Psychology, 44*, 544-560. http://dx.doi.org/10.1037/0012-1649.44.2.544

Sameroff., A. J. (1975). Transactional models in early social relations. *Human Development, 18*, 65-79. http://dx.doi.org/10.1159/000271476

Sarama, J., & Clements, D. H. (2008). Mathematics in early childhood. In O. N. Saracho & B. Spodek (Series Eds.) & O. N. Saracho & B. Spodek (Vol. Eds.) *Contemporary perspectives in early childhood education: Mathematics in early childhood education* (pp. 67-94). Greenwich, CT: Information Age Publishing.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition, 108,* 662-674. http://dx.doi.org/10.1016/j.cognition.2008.05.007

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52*, piii-viii, 3-162.

Scarborough, H. (2002). Connecting early language and literacy to later reading (dis)abilities. In S. B. Neuman & D. K. Dickinson (Eds.), *Handbook of early literacy development* (pp. 97-110). New York: Guilford.

Schatschneider, C., Fletcher, J. M., Francis, D. J., Carlson, C. D., & Foorman, B. R. (2004). Kindergarten prediction of reading skills: A longitudinal comparative analysis. *Journal of Educational Psychology, 96*, 265-282. http://dx.doi.oirg. 10.1037/0022-00663.96.2.265

Sénéchal, M., & LeFevre, J. A. (2002). Parental involvement in the development of children's reading skill: A five-year longitudinal study. *Child Development, 73*, 445-460. http://dx.doi.org/10.1111/1467-8624.00417

Shalev, R. S., Manor, O., Kerem, B., Ayali, M., Badichi, N., Friedlander, Y., & Gross-Tsur, V. (2001). Developmental dyscalculia is a familial learning disability. *Journal of Learning Disabilities*, *34*, 59-65. http://dx.doi.org/10.1177/002221940103400105

Shatil, E., Share, D.C., & Levin, I. (2000). On the contribution of kindergarten writing to grade one literacy: A longitudinal study in Hebrew. *Applied Psycholinguistics, 21*, 1-21. http://dx.doi.org/10.1017/S0142716400001016

Skwarchuk, S. L. (2009). How do parents support preschoolers' numeracy learning experiences at home? *Early Childhood Educational Journal, 37*, 189-197. http://dx.doi.org/10.1007/s10643-009-0340-1

Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology, 121*, 63-84. http://dx.doi.org/10.1016/j.jecp.2013.11.006

Slavin, R. E., Karweit, N. L., & Wasik, B. A. (1993). Preventing early school failure: What works? *Educational Leadership, 50*, 10-18.

Stainthorp, R., & Hughes, D. (2004). What happens to precocious readers' performance by the age of eleven? *Journal of Research in Reading, 27*, 357-372. http://dx.doi.org/10.1111/j.1467-9817.2004.00239.x

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*, 99-120. http://dx.doi.org/10.1016/j.ecresq.2004.01.002

Stock, P., Desoete, A., & Roeyers, H. (2007). Early markers for arithmetic difficulties. *Educational and Child Psychology, 24*, 28-39.

Sy, S. R., Gottfried, A. W., & Gottfried, A. E. (2013). A transactional model of parental involvement and children's achievement from early childhood through adolescence. *Parenting science and practice, 13*, 133-152. http://dx.doi.org/10.1080/15295192.2012.709155

Tan, E. T., & Goldberg, W. A. (2009). Parental school involvement in relation to children's grades and adaptation to school. *Journal of Applied Developmental Psychology, 30*, 442-453. http://dx.doi.org/10.1016/j.appdev.2008.12.023

Tudge, J. (2009). Methods of assessment of Young children's informal mathematical experiences. *Encyclopedia of Language and Literacy Development* (pp. 1-7). London, ON: Canadian Language and Literacy Research Network.

Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19*, 21-39. http://dx.doi.org/10.1016/j.ecresq.2004.01.007

Tudge, J. R. H., Li, L., & Stanley, K. (2008). The impact of method on assessing young children's everyday mathematics experiences. In O.N. Saracho & B. Spodek (Series Eds.) & O.N. Saracho & B. Spodek (Vol. Eds.), *Contemporary perspectives* *in early childhood education: Mathematics in early childhood education* (pp. 187-214). Greenwich, CT: Information Age Publishing.

Valsiner, J. (1994). Vygotskian dynamics of development: Commentary. *Human Development*, *37*, 366–369. http://dx.doi.org/10.1159/000278281

Vandermaas-Peeler, M., Boomgarden, E., Finn, L., & Pittard, C. (2012). Parental support of numeracy during a cooking activity with four-year-olds. *International Journal of Early Years Education, 20*, 78-93. http://dx.doi.org/10.1080/09669760.2012.66323

Van de Rijt, B. A. M. (1996). *Voorbereidende rekenvaardigheiden bij kleuters* [Early mathematical competence in young children]. Doctoral dissertation. Utrecht University. Doetinchem, The Netherlands: Graviant.

Van Geert, P. (1994). Vygotskian dynamics of development. *Human Development*, *37*, 346–365. http://dx.doi.org/10.1159/000278280

Veresov, N. (2000). *Some considerations on play and learning in the zone of proximal development.* Paper presented at IVS Conference, Zvenigorod, Russia.

Vygotsky, L.S. (1978). *Mind in Society*. Cambridge, MA: Harvard University Press.

Wagner, R. K., Torgesen, J. K., Rashotte, C. A., Hecht, S. A., Barker, T. A., Burgess S. R., … Garon, T. (1997). Changing relations between phonological processing abilities and word-level reading as children develop from beginning to skilled readers: A 5-year longitudinal study. *Developmental Psychology, 33*, 468-479. http://dx.doi.org/10.1037/0012-1649.33.3.468

Wechsler, D. (2002). *Wechsler preschool and primary scale of intelligence - third edition*. *Dutch translation by Hendriksen, J., & Hurks, P.* Amsterdam, Nederland: Pearson.

Wells, G. (1999). *Dialogic inquiry: Toward a sociocultural practice and theory of education.* Cambridge, United Kingdom: Cambridge University Press.

Whitehurst, G. J., & Lonigan, C. J. (2002). Emergent literacy: Development from prereaders to readers. In S. B. Neumann & D. K. Dickinson (Eds.), *Handbook of early literacy development* (pp. 11-29). New York: Guilford.

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*, 155-193. http://dx.doi.org/10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Childrens' acquisition of the number words and the counting sytem. *Cognitive Psychology, 24*, 220-251. http://dx.doi.org/10.1016/0010-0285(92)90008-p

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25. http://dx.doi.org/10.1016/S0010-0277(03)00050-7

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology, 25*, 103-108. http://dx.doi.org/10.1348/026151005x90704

Yeung, W. J., Linver, M. R., & Brooks-Gunn, J. (2002). How money matters for young children's development: Parental investment and family processes. *Child Development, 73,* 1861-1879. http://dx.doi.org/10.1111/1467-8624.t01-1-00511

Young-Loveridge, J. M. (1991). *The development of children's number concepts from ages five to nine. Early mathematics learning project: Phase II. Vol. I: Report of findings.* Hamilton, NZ: University of Waikato.

Young-Loveridge, J. M. (2004). Effects on early numeracy of a program using number books and games. *Early Childhood Research Quaterly, 19*, 82-98. http://dx.doi.org/10.1016/j.ecresq.2004.01.001

CHAPTER 6

GENERAL DISCUSSION

This doctoral thesis aimed at expanding on the knowledge about the value of *small number discrimination* and *numerical mother-child interaction* for either concurrent or prospective *numerical competencies in kindergarten*. To this end, more specific research objectives were addressed in four separate empirical studies. A summary and discussion of the main findings is enclosed in this final chapter. Along with limitations of the current studies, suggestions for future research are outlined in addition. To conclude, practical implications and recommendations are discussed.

RECAPITULATION OF RESEARCH GOALS

In the last decades, evidence has accumulated that early detection of children at risk for *mathematical learning disorders* (MLDs) might be possible as early as in kindergarten, because individual differences in *number sense* differentiate between children's later mathematical outcome (e.g., DiPema, Lei, & Reid, 2007; Dowker, 2008; Mazzocco & Thompson, 2005; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Stock, Desoete, & Roeyers, 2009). Moreover, deviations in the development of number sense characterize atypical or impaired mathematical development or abilities (e.g., Dowker, 2005; Geary, Bowthomas, & Yao, 1992; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000; LeFevre et al., 2006). This early detection of atypical development is important to set up early intervention in order to prevent vulnerable children from falling further behind or to reduce the impact of mathematical difficulties (Coleman, Buysse, & Neitzel, 2006; Fuchs et al., 2007; Gersten, Jordan, & Flojo, 2005; Pasnak, Cooke, & Hendricks, 2006; Passolunghi & Lanfranchi, 2012).

Although number sense is assumed to be present even before kindergarten age, enabling perhaps an even earlier screening of children at risk (Xu & Arriaga, 2007), research on individual differences in infants' and toddlers' number sense and their predictive value to later outcome is still in its infancy. Individual differences in number sense seem to occur though already in infancy as demonstrated for *(large) number discrimination* (Libertus & Brannon, 2010). As children also learn from input they receive, the value of numerical mother-child interaction in toddlers (for either concurrent or numerical functioning later on) was studied, along with the predictive value of number discrimination for later numerical outcome at young age.

This theoretical framework resulted in three specific research objectives, which were addressed in four empirical studies as described in the previous chapters of this doctoral thesis. As a guideline through this discussion the goals are recapituled below.

First, it was explored whether subitizing in its traditional verbal format as one aspect of number sense (e.g., Jordan & Levine, 2009) could be indicated as a characteristic of atypical (and specifically problematic) mathematical development (e.g., Fischer, Gebhardt, & Hartnegg, 2008; Schleifer & Landerl, 2011). Subitizing abilities of atypically developing adolescents with MLD and typically achieving peers without MLD were therefore compared by means of an enumeration paradigm in *Chapter 2*. In this enumeration task, *verbal subitizing* – and not counting – was triggered in line with for example Fischer et al. (2008). It was expected to detect below-average subitizing skills in adolescents with MLD compared to their typically achieving peers without MLD.

Second, from the preverbal perspective on number sense (e.g., Berch, 2005), the ability to quickly and accurately enumerate small numbers (verbal subitizing) studied in *Chapter 2* was traced back to a rudimentary perceptual-preverbal ability in *Chapter 3*. As such, the ability to discriminate small numbers already in infancy, as a developmental precursor of the traditional format of subitizing (Benoit, Lehalle, & Jouen, 2004), was explored in a group of typically developing 8-month-old infants. To this end, a habituation task was used in line with, for example, Xu (2003) and Xu and Arriaga (2007). In this task, children were presented with a specific number of stimuli (e.g., dots) until they were habituated to it (or had a maximum number of habituation trials). Afterwards, they saw, in alternating order, the same and a new number of these stimuli. Looking longer at the novel number (i.e., dishabituation, Berk, 2007) was considered to be an indication of discriminating between the numerosities (e.g., Xu, 2003: Xu & Spelke, 2000). Based on a review of the number discrimination literature (e.g., Clearfield & Mix, 1999; Cordes & Brannon, 2009a, 2009b; Xu, Spelke, & Goddard, 2005), a specific number set (i.e., 1 vs. 3) was chosen to function as a candidate precursor of later mathematical outcome in *Chapter 4*. By searching for a candidate precursor, unraveling the value of *small number discrimination* for numerical competencies was kept in mind as one of the main goals of this dissertation. Finding a positive group result would indicate that probably most infants are capable of discriminating between the given numerosities one and three. However, analyzing results at individual level would still make it possible to identify children who were not successful, from children who were successful.

Third, building on *Chapter 3*, it was examined in *Chapter 4* whether individual differences in this small number discrimination performance in infancy differentiated between children’s later number sense. Outcome variables were children’s performance on (small) number discrimination in toddlerhood and children’s early *numerical competencies in kindergarten*. The relationship between children’s number discrimination performance in infancy and toddlerhood was also further explored. Children from the studied samples were 8 months, 24 months, and 48 months of age in infancy, toddlerhood, and kindergarten respectively. As already mentioned, a habituation task was used to measure number discrimination in infancy. In toddlerhood, another age-appropriate number discrimination paradigm was selected as a measure, namely, the manual search task. In this task children’s search time should reflect their discrimination ability (Barner, Thalwitz, Wood, Yang, & Carey, 2007; Feigenson & Carey, 2003, 2005). Children were allowed to search for objects (e.g., balls) into an opaque box, after a number of objects were presented. In some trials, though, the experimenter surreptitiously took away some of the objects. It was therefore expected that children would search longer after these kinds of trials, then after those allowing them to take all objects back from the box. Numerical competencies in kindergarten were measured using subtests of the *Test for the Diagnosis of Mathematical Competencies* (TEDI-MATH; Grégoire, Noël, & Van Nieuwenhoven, 2004). With this standardized test battery children’s mastery of the verbal counting sequence and its flexibility was tested, as well as how well they understood and used basic counting principles. Finally, children were presented with simple additions and subtractions in a pictorially supported format. It was hypothesized that number discrimination in infancy and toddlerhood would predict numerical competencies in kindergarten and that infants’ number discrimination would predict a variant of this number discrimination performance in toddlerhood.

This dissertation wanted to include studies on predictors from both a child and a parent perspective. The child perspective was already presented in *Chapter 4*, which handled number discrimination as an early innate child characteristic of number sense. In the fifth study described in *Chapter 5* – guided through the perspective of number sense as acquired through experience and as opposed to the innate perspective (e.g., Berch, 2005) – mother-child interaction as a form of parental input was included.

More specifically, in *Chapter 5*, it was studied whether number sense in toddlerhood (i.e., small number discrimination) and kindergarten (i.e., numerical competencies in kindergarten) could be explained by *numerical mother-child interaction*. In line with previous findings of a positive relationship between kindergartners' numerical mother-child interaction and their concurrent numerical competencies (e.g., Blevins-Knabe & Musun-Miller, 1996; Kleemans, Peeters, Segers, & Verhoeven, 2012), it was expected to demonstrate confirmative results at kindergarten age. In addition, it was hypothesized that the interactions at younger age (i.e., toddlerhood) would also predict these kindergartners' competencies. Finally, it was questioned whether the toddlers' mother-child experiences would already influence the children's concurrent number discrimination performance at toddler age. The experiences were supposed to do so, given that toddlers already encounter magnitudes and learn to count and that the importance of parental numerical input for concurrent numerical functioning has already been evidenced in kindergarten. Number sense measures were the same as in the previous reported studies of the current doctoral thesis: a manual search task for toddlers' number discrimination and a standardized test battery for kindergartners' numerical competencies. The frequency of toddlers' and kindergartners' numerical mother-child interactions was observed during a structured play situation at both time points.

COVERING CONCLUSIONS AND DISCUSSION

About small numbers: the importance for mathematical development

In MLDs, the ability to rapidly and accurately assess small quantities up to three (or four) items (verbal subitizing; Kaufman, Lord, Reese, & Volkmann, 1949; Koontz & Berch, 1996; Trick & Pylyshyn, 1993) has been investigated as a possible impairment (e.g., Fischer et al., 2008; Schleifer & Landerl, 2011). It has been demonstrated that children with MLD either serially count items within the subitizing range whereas typically achieving peers subitize (e.g., Bruandet, Molko, Cohen, & Dehaene, 2004;

Butterworth, 1999; Moeller, Neuburger, Kaufmann, Landerl, & Nuerk, 2009) or are slower in subitizing tasks (e.g., Koontz & Berch, 1996; Landerl, Bevan, & Butterworth, 2004; Schleifer & Landerl, 2011) compared to these peers. However, some studies do not support these problems in MLD (e.g., De Smedt & Gilmore, 2011; Rousselle & Noël, 2007). Likewise, *Chapter 2* provided no direct support for less accurate or slower subitizing in MLD when comparing 14-year-olds with MLD with typically achieving peers.

Small number discrimination in infancy and toddlerhood was studied in this dissertation as a potential developmental precursor of this verbal format of subitizing in light of unraveling (very) early predictors of later numerical or mathematical outcome. Now, knowing that verbal subitizing at later age was not found to be indicative for a specific disorder in mathematical abilities, one might question the further relevance of investigating its precursor in very young children. Notwithstanding the aforementioned finding, the study described in *Chapter 2* did show an interesting behavioral pattern when comparing the performance of participants with MLD with their control peers (without MLD) on this specific enumeration task. A combined perspective on speed and accuracy indicated that the individuals with MLD did not change their answer strategy depending on whether they were confronted with a set of stimuli in the *counting* or *subitizing* range. Indeed, regardless of set size (i.e., with a small set of items up to four belonging to the subitizing range and larger sets belonging to the counting range for this specific study) the speed-accuracy correlation was negligible for the MLD group. In the typically achieving group, however, participants might have experienced taking less time as an efficient strategy to enumerate sets within the counting range because they could rely on a faster estimation process than mere counting. This was illustrated by a positive correlation between speed and accuracy within this range for these participants. Although expected, the typically achieving group did not seem to use such a strategy (with subitizing instead of estimating items) within the subitizing range. This suggested a strategy switch consistent with the range to which a numerosity of the set of items belonged. This could not be found in the MLD group. This different behavioral pattern between groups pointed toward different ways to process small and large numerosities. The overall faster and more accurate enumeration performance in the counting range of the typically achieving group compared to the MLD group supports this idea.

Therefore, even though at first glance no evidence was found for (significant) less accurate or slower subitizing skills in MLD, research on basic numerical processing abilities regarding small versus large numerosities seems indicated. Setting small and large number processing in children with MLD compared to typically achieving children alongside each other might be the obvious line to take to elucidate how individuals with MLD process numbers differently from their control peers. Next to an enumeration task that does not allow counting, a magnitude comparison[1] task (e.g., Landerl et aL, 2004; Mazzocco, Feigenson, & Halberda, 2011) might help to get insight in both small versus large number processing abilities. Large number processing is often investigated with this latter kind of task and is assumed to be driven by an approximate number system (Dehaene, 1992; Halberda & Feigenson, 2008) also formerly known as the *analog magnitude* system (e.g., Feigenson, Dehaene, & Spelke, 2004). This system allows for an approximate estimation of quantities. As such, mostly two large sets of nonsymbolic stimuli are used, which need to be compared in the comparison task. It is however still possible that individuals would rely on a subitizing process to compare sets including one or two small numerosities in this magnitude comparison task.

Studying number processing at early age seems indicative to learn about typical and atypical (mathematical) development. With the *object-file* system as the mainly assumed underlying system of both small number discrimination (e.g., Cordes & Brannon, 2009b; Feigenson & Carey, 2003, 2005; Feigenson et al., 2004) and verbal subitizing (Dehaene & Changeux, 1993; Feigenson et al., 2004; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008), the former can be valued as a developmental precursor of the latter. Extending this line of reasoning, knowing that the analog magnitude system is assumed to support both large number discrimination (e.g., Feigenson et al., 2004; Xu, 2003) and magnitude comparison skills (Dehaene, 1992; Halberda & Feigenson, 2008), the ability to discriminate large numerosities can be seen as a precursor of the latter. While for large number discrimination evidence for the existence of number processing early in childhood – from infancy on – is obvious (e.g., Xu, 2003), this is rather less clear for small number discrimination. Doubts could be raised from divergent findings (e.g.,

---

[1] The ability to distinguish numerosities by indicating the larger of two (Gersten et al., 2012)

Cordes & Brannon, 2009b; Xu et al., 2005), although additional information on this topic appeared since the start of this doctoral research (Starr, Libertus, & Brannon, 2013a). Nonetheless, as an explorative start to comparative research on number discrimination, it seemed meaningful (at the start of this research) to study at first whether infants were able to nonverbally discriminate between small numerosities (*Chapter 3*). *Chapter 3* describes in this respect the successful group result for the number set of one versus three items. Second, the predictive value of small number discrimination for later mathematical achievement was explored (*Chapter 4*).

About number discrimination and numerical mother-child interaction: very early predictors for later mathematical outcome

The main goals of this dissertation included two major topics. First, research was set up to detect predictors at (a yet hardly explored) early age for kindergartner's numerical competencies later on per se. Second, this research wanted to incorporate two aspects, namely, both child and parent predictors. This resulted in two research lines that were elaborated on in *Chapter 4* and *5* of this dissertation. Combining the results of the studies described in these two chapters shed light on the predictive value of number discrimination in toddlerhood (*Chapter 4*) as well as the numerical mother-child interaction within the same age range (24 months).

Both studies extended previous research on these respective topics as they expanded the age of interest from kindergarten (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Jordan, Kaplan, Ramineni, & Locuniak, 2009) to an age range starting already from toddlerhood. This is an informative subject in light of early screening or detection. Important to note is that not only child characteristics themselves seemed valuable to take into account, but also specific environmental input from, in this case, mother-child interactions. In search for early predictors of (a)typical mathematical development, the importance to further elaborate on studies investigating early number sense and numerical input is hereby highlighted. These findings are compatible with the view that infants have innate abilities guiding later competencies, but also learn from experience as for example through parent-child interaction.

Because of the bidirectional nature of the relationship between parent and child (Bell, 1968, 1979; Sameroff, 1975) and because toddlers' number discrimination (*Chapter 4)* and numerical mother-child interaction (*Chapter 5)* related both with (the same) numerical competencies in kindergarten (i.e., conceptual counting knowledge), one might have expected to find a (positive) significant relationship between both predictors. However, this could not be confirmed as outlined more in detail in *Chapter 5*: A bidirectional effect could thus not be supported yet at toddler age in this sample. As expounded in *Chapter 5*, the benefits of numerical mother-child interaction might only be postponed, because children with more early numerical interactions exhibit advantageous numerical competencies in kindergarten. In toddlerhood, children might therefore not immediately show the benefits of these specific kinds of interactions or do otherwise not seem to profit from this parental input by showing an enhanced number discrimination performance at that age.

Next to the common influence on later conceptual counting in kindergarten of both the child (i.e., number discrimination) and environmental characteristic (i.e., numerical mother-child interaction) measured in toddlerhood, toddlers' numerical mother-child interaction had an additional impact on arithmetic operations assessed in kindergarten[2]. Looking more closely at the data, however, pointed in the direction of a rather isolated contribution of the two predictors. Supplementary linear regression analysis indeed disclosed the additional value of one (toddlerhood) predictor above the other with respect to different numerical outcome that were measured at kindergarten age: The number discrimination performance did not contribute significantly to the score on arithmetic operations (as one of the subtests to assess numerical competencies in kindergarten) on top of numerical mother-child interaction, F_{change} (1,25) = 1.03, p = .320, R^2_{change} = .03. The contribution of numerical mother-child interaction in toddlerhood to the score on conceptual counting knowledge (as one of the other subtests to assess numerical competencies) on top of number discrimination was furthermore also nonsignificant, F_{change} (2,25) = 1.46, p = .252, R^2_{change} = .09.

---

[2] This was in line with the hypothesized (marginally significant) relationship that was demonstrated between kindergartners' numerical mother-child interaction and concurrent numerical competencies (in this case also arithmetic operations) on top of maternal involvement. This will be discussed later on.

Without going much further in detail, the suggestion that numerical interaction and number discrimination would have a differential contribution to numerical competencies in kindergarten might be explained by the difference in complexity of each of the predictors and the respective competencies accordingly. Conceptual (counting) knowledge involves interconnected and meaningful knowledge (Baroody, 2003; Hiebert & Lefevre, 1986). From a historical perspective, conceptual mathematical knowledge has been linked to the *incidental learning* theory and the *meaning* theory (Baroody, 2003). According to the incidental learning theory, children should explore the world and actively construct their own understanding (Baroody, 2003). Similarly, the meaning theory advocates the meaningful comprehension of mathematical relations, in relation with the understanding of its mathematical and practical significance (Baroody, 2003; Brownell, 1935). Conceptual counting, as such, can be situated on the level of the child, for which children can use their own basic number sense (or number discrimination ability). Arithmetic operations, in contrast, as well as parent-child interactions find themselves on a more advanced level. Without disacknowledging the possibility that toddlers' numerical mother-child interaction could have related to another concurrent performance that would have been more similar in complexity compared to arithmetic operations, it is therefore not surprising to find no significant relationship between the involved child characteristic (i.e., number discrimination) and parent-child aspect (i.e., numerical interaction).

Besides, only with a "lenient" definition of successful discrimination, a significant relationship could be found between this performance and conceptual counting on top of intelligence. Using by contrast a "restrictive" number discrimination outcome, no significant relationships could be found with any of the later numerical competencies. In that case, one should not even expect a mutual relationship between number discrimination and numerical mother-child interaction. This different pattern of results through different approaches of number discrimination will be elaborated on in the next section.

About defining, analyzing, and measuring outcome: what you choose is what you get

Number discrimination: choosing how to define and to analyze. Keeping in mind the aforementioned main findings of the current dissertation, it is fundamental to linger over the not negligible issue on how to define or to analyze outcome measures, while provisionally not questioning the “how” of measuring concepts yet. Probably, this can be generalized for all studies, but specifically in *Chapter 3* and *4* it was illustrated that the way from raw data to plain results is not always that straightforward. Consulting literature in different domains learns that although same paradigms are used, outcome can be defined differently resulting accordingly in different analyses. It goes without saying that any other definition or analyzing technique has its impact on final results.

In *Chapter 3* and *4*, a habituation and manual search task were used respectively. A review of the available literature gave an overall idea of how to define a successful performance on both tasks (e.g., Clearfield & Mix, 1999; Cordes & Brannon, 2009b; Feigenson & Carey, 2003, 2005; Starr, Libertus, & Brannon, 2013b; Xu, 2003, Xu & Arriaga, 2007). However, other outcomes could be achieved when operationalizing “success” differently. Because studying the predictive value of number discrimination at early age is still in its infancy, it is not surprising there is no golden standard, yet. Moreover, researchers can only be encouraged to explore more in-depth possible ways to approach number discrimination as a predictor to later outcome. Probably, the way of defining outcome depends, after all, on the specific interests of an investigation and might differ across studies. Although combining different approaches to analyze data could be informative, in the end, at least one approach should be used across different studies in order to compare and to generalize research findings.

Number discrimination: choosing how to measure. The previous paragraph did not cover the “how” of measuring concepts. For this doctoral research, specific number discrimination tasks were chosen based on a review of relevant literature, taking into account the different age of the participants in the respective studies. However, there were multiple paradigms available. It is important to mention that other findings could have resulted from using other tasks to measure small number discrimination, because differences might be caused by the number processing system that each task elicits.

Given that infants have access to both systems (i.e., the object-file system and the analog magnitude system) and since it might depend on the kind of task which system is triggered (Feigenson & Carey, 2003), the choice of paradigm might be of crucial importance. Logically, studies about number discrimination initially connected small number discrimination with object-files and large number discrimination with analog magnitudes (see Cantrell & Smith, 2013 for a review), regardless of the task used. However, the claim that small numbers are only processed by object-files is currently tentative, because both the successful discrimination of small from large numerosities in infants (Cordes & Brannon, 2009a) as well as the finding that infants show ratio dependent discrimination regardless of set size also with small numerosities (Starr et al., 2013a) are incompatible with this (straightforward) two-system account. Combining findings from different number discrimination studies or results on various number sets discloses features in paradigms which are characteristic for the underlying systems (e.g., Cordes & Brannon, 2009a, 2009b; Feigenson & Carey, 2003, 2005; Libertus & Brannon, 2010; Starr et al., 2013a; Xu, 2003; Xu et al., 2005). These characteristics give notion of which system is probably used to discriminate numerosities using a particular task. *Ratio dependency*, for example, as the key characteristic of number discrimination by means of the analog magnitude system, is mainly used to decide whether a task induces this specific kind of system (see Cantrell & Smith, 2013 for a review).

Since ratio dependency does not affect small number discrimination measured with the manual search task, it is likely that this paradigm prompts the object-file system (e.g., Feigenson & Carey, 2003, 2005). This means that the connection between small number discrimination and the object-file system thus applies for this paradigm. To our knowledge, large number discrimination has not been investigated yet with the manual search task, so no conclusions can be drawn on this format with this paradigm.

Whereas it is rather clear that the manual search task triggers the object-file system, it is also indisputable that the numerical change paradigm activates the analog magnitude system. This is a rather new kind of paradigm (Libertus & Brannon, 2010) for which is shown that number discrimination is ratio dependent regardless of set size. For both kinds of number discrimination (i.e., small and large), children are assumed to rely on analog magnitudes when performing numerical change tasks.

Regarding habituation tasks, however, the story about the underlying system is more complicated. Initially, it was stated that tasks following this paradigm would (only) activate the analog magnitude system. Indeed, no evidence for success could be found for sets with small numerosities having the same ratio as sets with larger numerosities. An example is the failure of 1 vs. 2 (small numerosities; Xu et al., 2005) compared to the success of 4 vs. 8 (large numerosities; Xu, 2003) in 6-month-old infants. These kinds of findings has led researchers to find support in a two-system account (e.g., Feigenson et al., 2004; Xu 2003): Because small numerosities could not be discriminated using habituation, they must have been processed differently by another system than the large ones (e.g., Xu et al., 2005). This system (i.e., the object-file system) was not thought to be triggered by these habituation tasks. Recently, however, habituation studies also indicated successful small number discrimination (e.g., Cordes & Brannon, 2009b) besides the abundance of positive findings regarding large number discrimination using these kinds of tasks (e.g., Cordes & Brannon, 2008b; Xu, 2003; Xu et al., 2005; Xu & Spelke, 2000). As such, these divergent findings on small number discrimination using habituation, with failure for set 1 vs. 2 (Xu et al., 2005) and success for the sets 2 vs. 3 (Cordes & Brannon, 2009b) and 1 vs. 3 (*Chapter 3*), still raise questions on the triggered system. To elucidate this, however, falls beyond the scope of this dissertation. Though, it illustrates the impact of using different paradigms to measure a concept. Until all questions on this “paradigm-system” issue are answered one should bear this in mind while interpreting results in comparison with other studies.

Numerical mother-child interaction: choosing how to define and to analyze. In *Chapter 5*, it became clear that also numerical mother-child interaction can be approached differently. First, one needs to define numerical mother-child interaction. Mostly, this encompasses not one, but a range of activities (e.g., Benigno & Ellis, 2004; Blevins-Knabe & Musun-Miller, 1996; Klibanoff, Levine, Huttenlocher, Vasileya, & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Tudge & Doucet, 2004). Second, data need to be processed. One way of analyzing is taking into account the overall amount (i.e., frequency) of interactions in line with, for example, Kleemans et al. (2012). This approach was chosen in the study described in *Chapter 5*.

Coding items on parent-child interactions were retrieved from prospection of a sample of mother-child interactions and adapted, as far as possible, in accordance with coded items available in the existing literature on this topic (Benigno & Ellis, 2004; Blevins-Knabe & Musun-Miller, 1996; Klibanoff et al., 2006; Levine et al., 2011; Tudge & Doucet, 2004). A more in-depth way of looking at the data is to differentiate between levels of complexity (or quality; Skwarchuk, 2009) of the observed interaction and/or types of interaction based on purpose (Skwarchuk, Sowinski, & LeFevre, 2014). As with literacy (e.g., Hood, Conlon, & Andrews, 2008; LeFevre, Fast, et al., 2009), parent-child interactions can involve for instance activities that are both directly or indirectly related to numeracy development. Direct experiences relates to the acquisition of numerical skills as an academic goal (LeFevre, Skwarchuk, et al., 2009; Tudge & Doucet, 2004). Indirect experiences are at first glance not geared to teach numeracy as an academic objective (LeFevre, Skwarchuk, et al., 2009; Tudge & Doucet, 2004), but do unintentionally provide learning opportunities. Although researchers try to apply conclusive definitions (e.g., Skwarchuk et al., 2014), practice learns that straightforward interpretations might be complicated by context as illustrated for example in *Chapter 5*. On a next level, as outlined in the same chapter, further analysis of the interactions with focus on numerical parental involvement and sensitivity – apart from general parental involvement and sensitivity – would be informative to unravel the additional value of the specific nature of numerical mother-child interaction above these parental factors in general. Anyway, taking into account these remarks, it can be acknowledged that some aspects need consideration when studying numerical mother-child interaction. Inevitably, a structured or unstructured play situation as well as the time and the number of observations are important factors to reflect on.

Chapter 5 elaborates thus on the findings regarding the frequency of numerical input. Again, although the choice was made to only incorporate the frequency of interactions, it is indicated that future research analyze more in-depth which kinds of numerical interactions have the strongest predictive value to later numerical outcome and how specific these interactions are. Furthermore, as with the definition of successful number discrimination, it is imperative to include comparable items in the concept of numerical interaction across studies in order to generalize findings.

Numerical mother-child interaction: choosing how to measure. Not only number discrimination performance can be measured otherwise, but also numerical mother-child interaction has been investigated differently across studies. In general, numerical mother-child interaction has been studied by means of parent report (e.g., (e.g., Blevins-Knabe & Musun-Miller, 1996; Lefevre, Skwarchuk, et al., 2009; Saxe, Guberman, & Gearhart, 1987) or observation (e.g., Benigno & Ellis, 2004; Tudge & Doucet, 2004).

Fluck, Linnell, and Holgate (2005) questioned the value of parent reports to map children's numerical experiences. They found that parents overestimated the knowledge of their children, possibly due to the unawareness of parents of the support they provide during joint activities (Fluck et al., 2005). Also, Tudge (2009) had doubts about the reliability of such reports, because parents might miss unintentional, indirect numerical elements in daily life (Tudge & Doucet, 2004; Tudge, Li, & Stanley, 2008). Some scholars therefore prefer observation, because this might reveal informal math (Tudge, 2009). Play during parent-child interactions does not guarantee mathematical development, but offers opportunities to draw the child's attention to numeracy even when parents are unaware of it (Benigno & Ellis, 2004; Sarama & Clements, 2008; Starkey, Klein, & Wakeley, 2004). Therefore, observational data were chosen to map the frequency of numerical mother-child interaction, although observation usually provides only a snapshot of daily life. This does not preclude that a combination of a parent report and observation might be the most indicative to fully map numerical home experiences.

LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

Throughout the current and previous chapters, limitations and suggestions for future research were already stipulated. In what follows, the most important considerations will be highlighted along with covering thoughts and recommendations.

About this dissertation: self-containing studies as part of a large-scale project

This dissertation can be situated within the context of a broader large-scale project in which researchers from different universities and disciplines joined (http://www.steunpuntwvg.be/jong; Grietens, Hoppenbrouwers, Desoete, Wiersema, & Van Leeuwen, 2010). This implied obviously lots of advantages but also some drawbacks.

First, because many researchers were involved, it was impossible to fully explore in depth one research topic and all related variables. One protocol was set up to integrate a diversity of interests. Taking into account the attention span of each child at different ages, it is needless to say that the time available for each slot was not infinite. As a consequence, data on the specific topic of this dissertation were limited compared to the wealth of all data handling various aspects of development, parenting, and health. For certain, it would be interesting and innovative to link all aspects in a multifaceted study. In order, however, to not complicate research questions and analysis, it was intentionally chosen to narrow the focus of this project to its current aim, in line with previous studies on number discrimination and/or numerical parent-child interaction. This does not detract, however, from the possibility to analyze more of the available data according to new additional research questions in the future.

Second, this large-scale project provided the possibility to study a large number of children who were selected randomly and lived in different parts of Flanders, Belgium. As such, a quite representative sample could be retrieved, even though, actually most of the parents who consented to participate in the different studies had a middle or high income. Although a lot of children were investigated in separate cross-sectional studies, it was – due to certain practical circumstances such as the expiration term of the project – only possible to follow-up a handful of children longitudinally until the age of 48 months. As the main focus of this dissertation was on the longitudinal aspect, the reason of the small sample size in the included studies could be attributed to these conditions. Partly due to this limitation, the initial aim to also involve large and small vs. large, next to small number discrimination, was reconsidered in between and finally dropped. The cross-sectional data are obviously available for further exploration.

Based on these first comments, related to the larger context of this doctoral project, two major topics can be distinguished on which will be elaborated upon below.

About the sample: how many and who

A general shortcoming of this doctoral research, which is applicable to all studies, are the small sample sizes that were used, especially in *Chapter 4* and *5*. Small samples may have lead to the false conclusion that there are no differences between groups, even though, in reality there are (risk of type 2-mistakes; Field, 2009, pp. 55-56). Albeit the sample sizes of the different studies are sometimes comparable with similar studies in the past (*Chapter* 3; Cordes & Brannon, 2009b; Xu et al., 2005*; Chapter 5*; Skwarchuk, 2009), replication of the results in larger samples is nevertheless needed.

In addition, due to the small sample sizes, a distinction between low-achieving children (scoring below the 25th percentile following Geary, 2004) – or even clinically low-achieving children (scoring below or equal to the 10th percentile following Murphy, Mazzocco, Hanich, and Early, 2007) – and average- or high-achieving children (scoring above the 50th percentile) on numerical competencies in kindergarten was not possible. However, in *Chapter 4* and *5*, it would have been of great interest to explore whether the respective predictors were sensitive enough to differentiate between critical performances, in light of early screening of at-risk children at earlier age. Based on the numerical competencies in kindergarten, it might still be possible that it is easier to detect high-achieving children (who are not at risk) than to detect low-achieving children (who might be at risk).

Furthermore, in line with Stock, Desoete, and Roeyers (2010), different conclusions can be drawn when using restrictive or lenient criteria: It is important to keep in mind that the performance of low-achievers cannot automatically be generalized to that of clinically low-achievers. These children with MLD show qualitatively or quantitatively different profiles than typically or not clinically low-achieving children, depending on different numerical competencies (Mazzocco, Devlin, & McKenney, 2008).

Following the intention about using this line of research for early risk detection of mathematical problems, it would be worthwhile to include groups of children with a presumably higher risk to develop impaired mathematical abilities. Comparing these groups with typically developing children could expose informative between-group differences, which could inform on specific predictors for typical versus atypical (problematic) mathematical development. For example, children born very preterm (gestational age < 32 weeks) face an overall increased risk for poorer academic performance (Johnson, Wolke, Hennessy, & Marlow, 2011; Simms, Cragg, Gilmore, Marlow, & Johnson, 2013). Moreover, although results are mixed (Guarini et al., 2006; Isaacs, Edmonds, Lucas, & Gadian, 2001), studies provided evidence for a deficient low-level numerical processing in preterms (e.g., Simms, Gilmore, et al., 2013), which may be responsible for poor math achievement. In addition, given the importance of genetic factors for mathematics performance (e.g., Hart, Petrill, & Dush, 2010), and more specifically the heritability of MLD (e.g., Alarcon, DeFries, Light, & Pennington, 1997; Shalev et al., 2001), siblings of children with MLD show a greater genetic susceptibility (Shalev et al., 2001). Therefore, siblings in whom – as far as known – the numerical abilities are studied only once (Desoete, Praet, Titeca, & Ceulemans, 2013), constitute another group of interest for these studies on early predictors.

About the scope: studied components and yet to study components

Besides enlarging sample size, also the amount of included factors can be expanded. Of course, the limitation of overlooking different possible powerful factors such as predictors or control variables is inherent in doing research. Choices had to made, but the future is challenged to meet this shortcoming as much as possible. In this respect, some suggestions can be made based on the reflections that were outlined above.

As already suggested implicitly, it would be worthwhile to investigate small and large number discrimination simultaneously, using at least more than one number set per range. Moreover, using different paradigms to measure each performance, per child at each time slot, would be an additional surplus for research on number discrimination.

Only in this way, conclusions can be drawn on, for example, the task dependency of underlying mental number systems. On the sideline, some critical thoughts on the demands of the manual search task were given in *Chapter 4*. Based on these concerns, future research could control number discrimination performance for additional cognitive abilities children might rely on when performing such tasks. The manual search task is likely to involve a combination of cognitive, working memory and motivational processes, and thus might possibly not reveal number discrimination abilities in young children in a precise way. Addressing this concern might help to determine whether this task is still suitable to measure number discrimination in the future.

This train of thought on studying only a tip of the iceberg of what number discrimination is actually about, can be generalized to both numerical competencies in kindergarten and numerical mother-child interaction (as the two remaining key concepts of the current dissertation). It would be, for instance, interesting to directly relate small number discrimination performance to verbal subitizing skills in kindergarten or later. A positive correlation would be in support of the assumption that small number discrimination could be considered as a developmental precursor of verbal subitizing. This was, in fact, investigated in the sideline of this doctoral project. However, because of insufficient data on the subitizing task that was administered, no conclusions could be drawn from the available data.

To continue with listing recommendations for future research, fathers, siblings, and even caregivers and children's peers from day care might be valuable actors, whose role should be explored in future studies on numerical caregiver-child interaction. The past three decades, a steady increase could be observed in studies including both mothers and fathers in understanding children's development (Pancsofar, Vernon-Feagans, & Family Life Project, 2010). This accords with children in two-parent families experiencing one home environment that blends both parents' influences (Martin, Ryan, & Brooks-Gunn, 2007). This greater emphasis on fathers has been rooted in the changing and many roles of fathers in the lives of their children and families, along with an increasing numbers of mothers into the workforce (e.g., Cabrera, Tamis-LeMonda, Bradley, Hofferth, & Lamb, 2000; Gottman, 1998; Lamb & Tamis-LeMonda, 2004).

Next to fathers, increased attention should also be given to the role of siblings in children's development, because of the extent and intensity of interactions between siblings (Dai & Heckmann, 2013; Dunn, 2002; Howe, Ross, & Recchia, 2011). With furthermore the increase in the percentage of women in the workforce and a corresponding rise in the number of children receiving routine care by someone other than their mother (NICDH Early Child Care Research Network, 2001), the majority of young children spend significant amounts of time in nonmaternal care, often with age-mates (NICDH Early Child Care Research Network, 1996; United States Department of Labor, 1994). Two sources of child-care influence are then children's experiences with peers, but also with their caregivers (NICHD Early Child Care Research Network, 2001).

Because all these actors (fathers, siblings, child-care) play an important role in children's development, they might contribute to children's budding numerical competencies. In addition, as part of the general environmental influence on children's development, the socioeconomic status (SES) of the families of participating children can be taken into account. Therefore, it would be meaningful to include more families with a low SES, who – based on income – only constituted a minority of the sample of the large-scale project. This is not surprising as a bulk of evidence suggests that individuals with a higher SES are more likely to participate in scientific research (e.g., Burg, Allred, & Sapp, 1997; Galea & Tracy, 2007; Hille et al., 2005). This probably reflects a greater trust in science and a higher degree of volunteerism in this group (Bak, 2001; Putnam, 1995). Partly due to the small sample size of the studies in this dissertation and this majority of middle- and high-incomes (Ceulemans, Desoete, Van Leeuwen, & Hoppenbrouwers, 2011; Guérin et al., 2013), the impact of SES was not further explored. Moreover, based on literature on either mathematical abilities or numerical interaction (or home numeracy experiences) with an additional focus on SES, one should not expect differences between middle- and high-income families, but rather between these two groups on the one hand and low-income families on the other hand (e.g., Blevins-Knabe & Musun-Miller, 1996; Jordan et al., 2007; Jordan, Kaplan, Olah, & Locuniak, 2006; Starkey et al., 2004). Therefore, recruiting more low-income families in this kind of research is warranted to draw reliable conclusions.

IMPLICATIONS

Considering all findings and directions for future research, it is possible to distinguish implications that are of concern for researchers and implications for practice.

About research and theory

The importance of small numbers and numerical mother-child interaction. Regarding the two concepts that were investigated as possible predictors of later mathematical outcome, some research-related implications could be drawn.

Small versus large. Verbal subitizing was investigated as a preamble to the studies on small number discrimination and was therefore not the main focus. Nevertheless, investigating verbal subitizing in a clinical group (of 14-year-olds with MLD) compared to a group of typically achieving adolescents resulted in more insight in this ability in MLD.

No straightforward impairment could be found on the enumeration of numerosities within the subitizing range in MLD, although this was expected based on previous studies (e.g., Koontz & Berch, 1996; Landerl et al., 2004; Schleifer & Landerl, 2011). The finding that the participants with MLD showed no strategy switch compared to their typically achieving peers when presented with small and large nonsymbolic stimuli is, however, in line with what was already formerly known on strategy use in children with MLD. Whereas adaptive and flexible strategy use is important for math proficiency and adequate mathematical learning (Heinze, Star, & Verschaffel, 2009), it has been demonstrated that children with MLD use different, less efficient or immature, strategies compared to typically achieving children (Geary, 1993). Since strategy use seems to depend on the kinds of presented stimuli, it seems important to take into account both large and small number processing tasks involving stimuli on small and large numerosities simultaneously. It will be important to integrate these components also into assessment, but therefore it seems imperative for research to first continue to disentangle the specific underlying impairments that children with MLD might encounter.

Integrating research on small and large number processing not only seems indicated to get more insight on elemental mechanisms in atypical (impaired) mathematical development, but would also be informative for typical development. In *Chapter 3*, it was discussed that at 6 months of age infants might be too young yet to discriminate between small numerosities. This would explain why in this chapter 8-month-olds were successfully discriminating those numerosities during habituation, whereas in previous studies using the same paradigm success could not be found in younger infants (i.e., 6 months; Xu et al., 2005). However, in *Chapter 4*, it was reasoned, following Starr et al. (2013b), that the relationship between numerical representation (i.e., by means of analog magnitudes) and numerical competencies in kindergarten might be the strongest at 6 months of age. This could clarify why this relationship could not be confirmed anymore in already 8-month-olds in the study described in *Chapter 4.*

These two points warrant some supplementary discussion, as they might intuitively be interpreted as opposite to each other without additional explanation. With this respect, the most important gloss to mention is that in this doctoral research the focus was on small number discrimination instead of large number discrimination. While considering the results of the studies in question, it was suggested in *Chapter 3* that small number discrimination might only be possible from a given age, as an explanation for previous contrasting findings in other studies. In *Chapter 4*, the claim on the strongest relationship between large number discrimination and numerical competencies in kindergarten at 6 months of age was endorsed. No knowledge is however available yet, from the current dissertational studies or previous conducted studies, on whether infants' small number discrimination performance at 6 months of age (measured, for example, with the numerical change paradigm; Starr et al., 2013a) does relate to numerical competencies in kindergarten. This doctoral research only informed on the fact that small number discrimination at the age of 8 months (using a habituation task) did not relate yet with numerical competencies assessed in the same children at kindergarten age. Studying number discrimination at 24 months of age in the same children (using a manual search task), however, revealed a significant relation with (one of) those numerical competencies in kindergarten.

One should have the issue on how to measure number discrimination and define success at the back of his or her mind regarding these findings. Nevertheless, given these considerations, it might be that small number discrimination only shows a strong relationship with numerical competencies in kindergarten rather late in childhood (i.e., toddlerhood), compared to large number discrimination (i.e., infancy). Possibly, large number discrimination is therefore a more suitable candidate for research on predictors of atypical and typical math development, but additional studies focusing on both small and large number discrimination are needed to confirm this assumption. It would be of great interest to investigate how small and large number discrimination, measured with different kinds of paradigms (each task involving both small and large number sets) in the same child, relate to each other. This way, it would be possible to clear out whether either small or large number discrimination in infants or toddlers is more predictive to later numerical outcome.

A model for early predictors. For now, (small) number discrimination, as one aspect of number sense, has proven to be predictive already from toddlerhood on with respect to later outcome (*Chapter 4*). This is still at earlier age than mostly investigated before (e.g., Aunola et al., 2004; Jordan et al., 2009; Stock et al., 2009). This conclusion also holds for numerical mother-child interaction. This construct is demonstrated to be already predictive too from toddlerhood on for later numerical outcome (*Chapter 5*) and is otherwise only scarcely studied at this young age (Levine et al., 2011). This implies that while doing research on early predictors, next to aspects of number sense that lie within the child's own capacities or can be described as a characteristic of the child, attention should also be given to specific environmental input such as parent-child interaction. In addition, it should be taken into account that relationships might not only be linear, but components could also show a quadratic relationship, which might strongly influence interpretations. This was illustrated in *Chapter 5* for numerical mother-child interaction which related linearly with numerical competencies when both constructs were assessed in kindergarten, but displayed a quadratic relationship when measured in toddlerhood and associated with the same numerical competencies in kindergarten.

Multi-approach of definitions, methods, and components. A review of the literature in preparation of this doctoral research, with its self-containing studies, learned that knowledge on early predictors of atypical mathematical development is important within the scope of early detection and intervention (Coleman et al., 2006; Fuchs et al., 2007; Gersten et al., 2005; Pasnak et al., 2006; Passolunghi & Lafranchi, 2012; Powel & Fuchs, 2012). Within this scope, research on early predictors for later typical mathematical achievement is mounting up till now as a valuable source of knowledge (e.g., Aunola et al., 2004; Jordan et al., 2009; Stock et al., 2009). Research on number discrimination is remarkable in this respect, as it shows that even infants have already an innate sense of number (e.g., Cordes & Brannon, 2009; Starr et al., 2013a; Xu, 2003). Within an abundance of studies, they are still trying to unravel the precise nature and underlying processes of this ability. Only recently, number discrimination has been linked for the first time to later numerical competencies (Starr et al., 2013b). The finding that number discrimination performance in infancy is positively related with mathematical abilities in kindergarten (Starr et al., 2013b) should instigate longitudinal research on number discrimination as a possible predictor of atypical mathematical achievement. Along with the understanding that this will be a long-winded work, comes the observation that this innovative research is still in its infancy. Therefore, it was almost inevitable that this doctoral research encountered different approaches and difficulties in operationalizing number discrimination performance with an impact on analysis of data on this subject. Specific standardization of number discrimination tasks seemed convenient, which could be used to identify low- and high-achievers and to reliably link these performances with later outcome and compare results across studies.

In addition, researchers are advised to use a multimethod approach because number discrimination measured with different tasks may trigger different systems (Feigenson & Carey, 2003). Comparing performances on these different tasks in the same children might inform on different formats of number discrimination (i.e., small and large number discrimination). Furthermore, considering the results reported in *Chapter 4* and *5* that illustrate that the studied predictors related to different aspects of numerical competencies in kindergarten, gives the initial impetus to a larger framework.

The broader the range of studied mathematical abilities, the more insight would be obtained in how predictors such as number discrimination performance or numerical mother-child interaction relate to different aspects of numerical competencies later on. As such, it is worthwhile and recommended to adhere to a multicomponential approach in this case as well.

About practice

The sense of small number discrimination and numerical mother-child interaction. This doctoral research informs about the importance of toddlerhood as a predicting age for later mathematical outcome. Both small number discrimination as well as numerical mother-child interaction seem to predict some aspects of numerical competencies in kindergarten. This not only holds implications for future research but also leads to some practical implications that deserve some further elaboration.

As already mentioned in *Chapter 5*, society is aware of the benefits of early literacy exposure through parent-child interaction (outlined in Skwarchuk, 2009). However, the current findings support in line with Skwarchuk (2009) that it may also be beneficial to improve the image of numeracy. Besides toddlers' own number discrimination ability at the age of 24 months, which predicts aspects of numerical competencies in kindergarten (i.e., conceptual counting), toddlers' numerical mother-child interaction could be valued. Since these interactions predict even more complex arithmetic operations, it seems worthwhile to sensitize numerical input through mother-child interaction. Agencies in support of parenting in preschoolers (including infants and toddlers) could for example additionally focus on numeracy, next to literacy.

As parents are already well-informed on the importance of early literacy support for children's later development in this domain, they could analogously be informed about the merit of such an early support in the domain of numeracy too. Following Levine et al. (2011), this could be encouraging for parents, because they might be unsure on how they can stimulate numerical development or they might assume that numerical development is (only) the responsibility of teachers and school, and not of the home environment (e.g., Cannon & Gindsburg, 2008; Evans, Fox, Cremaso, & Mckinnon, 2004).

Focus should however remain on the possibilities rather than the necessity, bearing in mind the quadratic (and not linear) relationship between toddlers' numerical mother-child interaction and later numerical competencies in kindergarten. This kind of relationship implies that "less is more", while at the same time at least some interaction is also more worthwhile than no such specific interaction at all. Although they might not (yet) realize it, parents can add to their children's numeracy, because mathematical development is not only a matter of formal schooling. Already early in childhood children develop step by step to a more advanced sense of number, which is already present in infancy at for example 6 months (large number discrimination; Xu et al., 2003) or 8 months (small number discrimination; *Chapter 3*) of age. This number sense is moreover now shown to relate with numerical competencies in kindergarten from infancy on (large number discrimination; Starr et al., 2013b) and also from toddlerhood on (small number discrimination; *Chapter 4*). In addition, parents should know it is rewarding to stimulate or promote early numerical development within the zone of the child's development. Even if the child does not seem to profit from this input immediately, it might experience advantages later on in math development. This is labeled as the "postponed" benefits of early numerical mother-child interaction in *Chapter 5* of the current dissertation. Although some people may feel that certain topics are too advanced, expectation is everything: Indeed, high academic expectations for children leads to more learning activities than lower expectations (e.g., LeFevre et al., 2009; & Skwarchuk et al., 2014).

Assessment of at-risk infants for MLD and older children with MLD. In order to diminish the impact of mathematical problems later on, early detection of individual differences in numerical development seems indicated (Coleman et al., 2006; Fuchs et al., 2007; Gersten et al., 2005; Pasnak et al., 2006; Passolunghi & Lafranchi, 2012). Within this scope, research on early predictors steps into the limelight.

Concerning small number discrimination in toddlerhood, which is predictive for numerical outcome in kindergarten (*Chapter 4*), it might be important to follow-up those children who performed extremely low (far below average) on number discrimination.

This kind of performance might differentiate children who develop mathematical problems later on from typically developing peers, although future research is needed to confirm this. Especially for specific groups such as siblings of children with MLD or (very) preterms, the establishment of sensitive measures to detect at-risk development for mathematical problems might be worthwhile, because these children are known to entail a higher risk to develop these problems (e.g., Shalev et al;, 2001; Simms, Gilmore, et al., 2013). Likewise, this doctorate lays the foundation to follow up those children who received less or more numerical mother-child interaction at 24 months of age. If less input predicts underachievement and more input predicts higher achievement (even though only to some extent) with regard to later numerical competencies, additional stimulation through numerical mother-child interaction might be considered a protective factor, which might again be interesting for the specific mentioned at-risk groups. If the impact of impaired mathematical abilities might be reduced by providing these at-risk groups opportunities to improve their skills, additional numerical mother-child interaction in toddlerhood could be worthwhile in this respect.

Regarding older children with MLD (i.e., adolescents, *Chapter 2*), this doctoral research informed about the relevance of assessing both small and large number processing with focus on strategy use. Several methods, such as think-aloud protocols (Jacobse & Harskamp, 2012), verbal self-reports (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Peltenburg, van den Heuvel-Panhuizen, & Robitzsch, 2012), experimenter observation (Wu et al., 2008), or nonverbal reaction time measures (Wu et al., 2008; Peters, De Smedt, Torbeyns, Verschaffel, & Ghesquiere, 2014) can be used to shed light on whether a variety and flexibility in strategy use or rather mastery in only one strategy is feasible, suitable, and favorable for children with MLD (e.g., Geary, 1993; Kilpatrick, Swafford, & Findell, 2001; Peters et al., 2014; Verschaffel, Thorbeyns, De Smedt, Luwel, & Van Dooren, 2007).

CONCLUSION

The reported studies of this doctoral research have provided more insight into some early predictors for typical development of later number sense, taking into account both a child and a parent perspective. At the sideline, more information was obtained on processing small and large numbers in MLD by comparing individuals with MLD with typically achieving peers on their performance on an enumeration task. While conducting this research, some issues arised about measuring, defining, and analyzing concepts and data.

The current dissertation aimed at instigating further research on early predictors of atypical and impaired development, as it showed the predictive value of toddlers' small number discrimination and numerical mother-child interaction for later numerical outcome in typical development. In doing so, this doctoral research added to the upcoming literature of early number discrimination as a predictor for later outcome and extends the study field on numerical parent-child interaction in association with numerical competencies. As this kind of research on predictors (very) early in childhood is still in its infancy, there are yet many steps to take, while moving ahead on different roads, and every choice that will be made will eventually define the final destination.

REFERENCES

Alarcon, M., DeFries, J. C., Light, J. G., & Pennington, B. F. (1997). A twin study of mathematics disability. *Journal of Learning Disabilities, 30*, 617-623.

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology, 96*, 699-713. http://dx.doi.org/10.1037/0022-0663.96.4.699

Bak, H. J. (2001). Education and public attitudes toward science: Implications for the "deficit model" of education and support for science and technology. *Social Science Quarterly, 82*, 779-795. http://dx.doi.org/10.1111/0038-4941.00059

Barner, D., Thalwitz, D., Wood, J., Yang, S. J., & Carey, S. (2007). On the relation between the acquisition of singular-plural morpho-syntax and the conceptual distinction between one and more than one. *Developmental Science, 10*, 365-373. http://dx.doi.org/10.1111/j.1467-7687.2007.00591.x

Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise*. Mahwah, NJ: Erlbaum.

Bell, R. Q. (1968). A reinterpretation of the direction of effects in studies of socialization. *Psychological Review, 75*, 81-95. http://dx.doi.org/10.1037/h0025583

Bell, R. Q. (1979). Parent, child and reciprocal influences. *American Psychologist, 34*, 821-826. http://dx.doi.org/10.1037/0003-066x.34.10.821

Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development, 19*, 291–307. http://dx.doi.org/10.1016/j.cogdev.2004.03.005

Benigno, J. P., & Ellis, S. (2004). Two is greater than three: Effects of older siblings on parental support of preschoolers' counting in middle-income families. *Early*

Childhood Research Quarterly, 19, 4-20. http://dx.doi.org/10.1016/j.ecresq.2004.01.006

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*, 333-339. http://dx.doi.org/10.1177/00222194050380040901

Berk, L. E. (2007). *Development through the lifespan.* (4th ed. - International Edition). Boston: Allyn & Bacon, Inc.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development & Parenting, 5*, 35-45. http://dx.doi.org/10.1002/(sici)1099-0917(199603)5:1<35::aid-edp113>3.0.co;2-0

Brownell, W. A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In D. W. Reeve (Ed.), *The teaching of arithmetic (Tenth yearbook, National Council of Teachers of Mathematics*. New York: Bureau of Publications, Teachers College, Columbia University.

Bruandet, M., Molko, N., Cohen, L., & Dehaene, S. (2004). A cognitive characterization of dyscalculia in Turner syndrome. *Neuropsychologia*, *42*, 288–298. http://dx.doi.org/10.1016/j.neuropsychologia.2003.08.007

Burg, J. A. R., Allred, S. L., & Sapp, J. H. (1997). The potential for bias due to attrition in the national exposure registry: An examination of reasons for nonresponse, nonrespondent characteristics, and the response rate. *Toxicology and Industrial Health, 13*, 1-13.

Butterworth, B. (1999). *The mathematical brain.* London: Macmillan.

Cabrera, N. J., Tamis-LeMonda, C. S., Bradley, R. H., Hofferth, S., & Lamb, M. E. (2000). Fatherhood in the Twenty-first Century. *Child Development, 71*, 127-136. http://dx.doi.org/10.1111/1467-8624.00126

Cannon, J., & Gindsburg, H. P. (2008). Doing the math: Maternal beliefs about early mathematics versus language learning. *Early Education and Development, 19*, 238-260. http://dx.doi.org/10.1080/10409280801963913

Cantrell, L., & Smith, L. B. (2013). Open questions and a proposal: A critical review of the evidence on infant numerical abilities. *Cognition, 128,* 331-352. http://dx.doi.org/10.1016/j.cognition.2013.04.008

Ceulemans, A., Desoete, A., Van Leeuwen, K., & Hoppenbrouwers, K. (2011). Peuters en de eerste stapjes in rekenland alleen en aan de hand van mama: Een verkennend rapport over de numerieke ervaringen van tweejarigen in JOnG! *[Toddlers and their first steps into the land of math, alone and with mom: An explorative report on the numerical experiences of 2-year-olds in JOnG!].* Leuven: Steunpunt, Welzijn, Volksgezondheid en Gezin (SWVG).

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10*, 408-411. http://dx.doi.org/10.1207/s15327647jcd0203_1

Coleman, M. R., Buysse, V., & Neitzel, J. (2006). Recognition and response. An early intervening system for young children at-risk for learning disabilities. *Research synthesis and recommendations.* UNIC FPG Child Development Institute.

Cordes, S., & Brannon, E. M. (2008). Quantitative competencies in infancy. *Developmental Science, 11*, 803-808. http://dx.doi.org/10.1111/j.1467-7687.2008.00770.x

Cordes, S., & Brannon, E. M. (2009a). Crossing the divide: infants discriminate small from large numerosities. *Developmental Psychology, 45*, 1583-1594. http://dx.doi.org/10.1037/a0015666

Cordes, S., & Brannon, E. M. (2009b). The relative salience of discrete and continuous quantity in young infants. *Developmental Science, 12*, 453-463. http://dx.doi.org/10.1111/j.1467-7687.2008.00781.x

Dai, X., & Heckman, J. J. (2013). Older siblings' contributions to young child's cognitive skills. *Economic Modelling, 35*, 235-248. http://dx.doi.org/10.1016/j.econmod.2013.07.003

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*, 1-42. doi: 10.1016/0010-0277(92)90049-N

Dehaene, S., & Changeux, J. P. (1993). Development of elementary numerical abilities – A neuronal model. *Journal of Cognitive Neuroscience, 5*, 390-407. http://dx.doi.org/10.1162/jocn.1993.5.4.390

De Smedt, B., & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of Experimental Child Psychology, 108,* 278-292. doi: 10.1016/j.jecp.2010.09.003

Desoete, A., Praet, M., Titeca, D., & Ceulemans, A. (2013). Cognitive phenotype of mathematical learning disabilities: What can we learn from siblings? *Research in Developmental Disabilities, 34*, 404-412. doi: 10.1016/j.ridd.2012.08.022

DiPema, J. C., Lei, P. W., & Reid, E. E. (2007). Kindergarten predictors of mathematical growth in the primary grades: an investigation using the early childhood longitudinal study - kindergarten cohort. *Journal of Educational Psychology, 99,* 369-379. http://dx.doi.org/10.1037/0022-0663.99.2.369

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience, and education.* New York: Psychology Press.

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. [Article]. *Developmental Science, 11*, 650-654. http://dx.doi.org/10.1111/j.1467-687.2008.00713.x

Dunn, J. (2002). *Sibling Relationships.* Oxford, UK: Blackwell Publishing Ltd.

Evans, M. A., Fox, M., Cremaso, L., & McKinnon, L. (2004). Beginning reading: The views of parents and teachers of young children. *Journal of Educational Psychology, 96*, 130-141. http://10.1037/0022-0663.96.1.130

Feigenson, L. & Carey S. (2003). Tracking individuals via object-files: Evidence from infants' manual search. *Developmental Science, 6*, 568-584. http://dx.doi.org/10.1111/1467-7687.00313

Feigenson, L., & Carey, S. (2005). On the limits of infants' quantification of small object *Cognition,* 97, 295-313, http://dx.doi.org/10.1016/j.cognition.2004.09.010

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Field, A. (2009). *Discovering Statistics Using SPSS (third edition).* London: Sage.

Fischer, B., Gebhardt, C., & Hartnegg, K. (2008). Subitizing and visual counting in children with problems acquiring basic arithmetic skills. *Optometry and Vision*

Development, *39*, 24-29. Retrieved from http://www.covd.org/Portals/0/24-29SubitizingVisualCounting.pdf

Fluck, M., Linell, M., & Holgate, M. (2005). Does counting count for 3- to 4-year-olds? Parental assumptions about preschool children's understanding of counting and cardinality. *Social Development, 14*, 496-513. http://dx.doi.org/10.1111/j.1467-9507.2005.00313.x

Fuchs, L. S., Fuchs, D., Compton, D. L., Bryant, J. D., Hamlett, C. L., & Seethaler, P. M. (2007). Mathematics screening and progress monitoring at first grade: Implications for responsiveness to intervention. *Exceptional Children, 73*, 311-330.

Galea, S., & Tracy, M. (2007). Participation rates in epidemiologic studies. *Annals of Epidemiology, 17*, 643-653. http://dx.doi.org/10.1016/j.annepidem.2007.03.013

Geary, D. C. (1993). Mathematical disabilities - cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*, 345-362. http://dx.doi.org/10.1037//0033-2909.114.2.345

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37,* 4-15. http://dx.doi.org/10.1177/00222194040370010201

Geary, D. C., Bowthomas, C. C., & Yao, Y. H. (1992). Counting knowledge and skill in cognitive addition: A comparison of normal and mathematically disabled children. *Journal of Experimental Child Psychology, 54*, 372-391. http://dx.doi.org/10.1016/0022-0965(92)90026-3

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*, 1343-1359. http://dx.doi.org/10.1111/j.1467-8624.2007.01069.x

Gersten, R., Clarke, B., Jordan, N. C., Newman-Gonchar, R., Haymond, K., & Wilkins, C. (2012). Universal screening in mathematics for the primary grades: Beginnings of a research base. *Exceptional Children,* 78, 423-445.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*, 293-304. http://dx.doi.org/10.1177/00222194050380040301

Gottman, J. M. (1998). Toward a process model of men in marriages and families. In: Booth, A.; Crouter, A. C. (eds). *Men in families: When do they get involved? What difference does it make?* (pp. 149-193). Mahwah, NJ: Erlbaum

Grégoire, J., Noël, M., & Van Nieuwenhoven, C. (2004). *TEDI-MATH*. Antwerpen: Harcourt.

Grietens, H., Hoppenbrouwers, K., Desoete, A., Wiersema, R., & Van Leeuwen, K. (2010). *JOnG! Theoretische achtergronden, onderzoeksopzet en verloop van het eerste meetmoment [JOnG! Theoretical backgrounds, study design and course of the first assessment].* Leuven: Steunpunt, Welzijn, Volksgezondheid en Gezin (SWVG).

Guarini, A., Sansavini, A., Giovanelli, G., Alessandroni, R., Faldella, G., Ansari, D., & Karmiloff-Smith, A. (2006). Basic numerical processes in preterms. *World Journal of Pediatrics, 2*, 102-108.

Guérin, C., Roelants, M., Devogelaer, N., Hoppenbrouwers, Van Leeuwen, K., & Desoete A. (2013). *Sociaal profiel, gezondheid en determinanten van gezondheid tijdens het tweede levensjaar van de Vlaamse geboortecohorte JOnG! [Social profile, health and determinants of health during the second year of life in the Flemish birth cohort JOnG!].* Leuven: Steunpunt, Welzijn, Volksgezondheid en Gezin (SWVG).

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology, 44*, 1457-1465. doi:10.1037/a0012682

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology, 93*, 615-626. http://dx.doi.org/10.1037//0022-0663.93.3.615

Hart, S. A., Petrill, S. A., & Dush, C. M. K. (2010). Genetic influences on language, reading, and mathematics skills in a national sample: An analysis using the national longitudinal survey of youth. *Language Speech and Hearing Services in Schools, 41*, 118-128. doi: 10.1044/0161-1461(2009/08-0052)

Heinze, A., Star, J., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM, 41*, 535-540. http://dx.doi.org/10.1007/s11858-009-0214-4

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hille, E. T. M., Elbertse, L., Gravenhorst, J. B., Brand, R., Verloove-Vanhorick, S. P., & Dutch, P.-C. S. (2005). Nonresponse bias in a follow-up study of 19-year-old adolescents born as preterm infants. *Pediatrics, 116*, E662-E666. http://dx.doi.org/10.1542/peds.2005-0682

Hood, M., Conlon, E., & Andrews, G. (2008). Preschool home literacy practices and children's literacy development: A longitudinal analysis. *Journal of Educational Psychology, 100*, 252-271. http://dx.doi.org/10.1037/0022-0663.100.2.252

Howe, Ross, & Recchia (2011). *Sibling Relations in Early and Middle Childhood.* The Wiley-Blackwell Handbook of Childhood Social Development (pp. 356). Oxford, UK: Blackwell Publishing Ltd.

Isaacs, E. B., Edmonds, C. J., Lucas, A., & Gadian, D. G. (2001). Calculation difficulties in children of very low birthweight - A neural correlate. *Brain, 124*, 1701-1707. doi: 10.1093/brain/124.9.1701

Jacobse, A. E., & Harskamp, E. (2012). Towards efficient measurement of metacognition in mathematical problem solving. *Metacognition and Learning, 7*, 133-149. http://dx.doi.org/10.1007/s11409-012-9088-x

Johnson, S., Wolke, D., Hennessy, E., & Marlow, N. (2011). Educational outcomes in extremely preterm children: Neuropsychological correlates and predictors of attainment. *Developmental Neuropsychology, 36*, 74-95. doi: 10.1080/87565641.2011.540541

Jordan, N. C., & Hanich, L. B. (2000). Mathematical thinking in second grade children with different forms of LD. *Journal of Learning Disabilities, 33*, 567-578. http://dx.doi.org/10.1177/002221940003300605

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice (Blackwell Publishing Limited), 22*, 36-46. http://dx.doi.org/10.1111/j.1540-5826.2007.00229.x

Jordan, N. C., Kaplan, D., Olah, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*, 153-175. http://dx.doi.org/10.1111/j.1467-8624.2006.00862.x

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*, 850-867. http://dx.doi.org/10.1037/a0014939.

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*, 60-68. http://dx.doi.org/10.1002/ddrr.46

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology, 62*, 498-525. http://dx.doi.org/10.2307/1418556

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kleemans, T., Peeters, M., Segers, E., & Verhoeven, L. (2012). Child and home predictors of early numeracy skills in kindergarten. *Early Childhood Research Quarterly, 27*, 471-477. http://dx.doi.org/10.1016/j.ecresq.2011.12.004

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*, 59-69. http://dx.doi.org/10.1037/0012-1649.42.1.59

Koontz, K. L., & Berch, D. B. (1996). Identifying simple numerical stimuli: Processing inefficiencies exhibited arithmetic learning disabled children. *Mathematical Cognition, 2*, 1-23. http://dx.doi.org/10.1080/135467996387525

Lamb, M. E., & Tamis-LeMonda, C. S. (2004). The role of the father: An introduction. In: Lamb, M. E. (ed). *The role of the father in child development. 4.* (pp. 1-31). Hoboken, NJ: John Wiley & Sons, Inc.

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8-9 year old students. *Cognition, 93,* 99-125. http://dx.doi.org/10.1016/j.cognition.2003.11.004

LeFevre, J.-A., Fast, L., Sowinski, C., Osana, H., Skwarchuk, S. L., & Manay Quian, N. (2009). Influences of language and parental involvement on the development of coutning skilss: Comparisons of French- and English-speaking Canadian children. *Early Child Development and Care, 172*, 283-300. http://dx.doi.org/ 10.1080/03004430212127

LeFevre, J. A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home Numeracy Experiences and Children's Math Performance in the Early School Years. *Canadian Journal of Behavioural Science, 41*, 55-66. http://dx.doi.org/10.1037/a0014532

LeFevre, J. A., Smith-Chant, B. L., Fast, L., Skwarchuk, S. L., Sargla, E., Arnup, J. S., & Kamawar, D. (2006). What counts as knowing? The development of conceptual and procedural knowledge of counting from kindergarten through Grade 2. *Journal of Experimental Child Psychology, 93*, 285-303. http://dx.doi.org/10.1016/j.jecp.2005.11.002

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2011). What counts in the development of young children's number knowledge? *Developmental Psychology, 47*, 302-302. http://dx.doi.org/10.1037/a0022101

Libertus, M. E., & Brannon, E. M. (2010). Stable individual differences in number discrimination in infancy. *Developmental Science, 13*, 900-906. http://dx.doi.org/ 10.1111/j.1467-7687.2009.00948.x

Martin, A., Ryan, R. M., & Brooks-Gunn, J. (2007). The joint influence of mother and father parenting on child cognitive outcomes at age 5. *Early Childhood Research Quarterly, 22*, 423-439. http://dx.doi.org/10.1016/j.ecresq.2007.07.001

Mazzocco, M. M. M., Devlin, K. T., & McKenney, S. J. (2008). Is it a fact? Timed arithmetic performance of children with mathematical learning disabilities (MLD) varies as a function of how MLD is defined. *Developmental Neuropsychology, 33*, 318-344.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Preschoolers' Precision of the Approximate Number System Predicts Later School Mathematics Performance. *PLoS ONE, 6: e23749*. http://dx.doi.org/10.1371/journal.pone.0023749

Mazzocco, M. M. M., & Thompson, R. E. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities: Research and Practice, 20*, 142-155. http://dx.doi.org/10.1111/j.1540-5826.2005.00129.x

Moeller, K., Neuburger, S., Kaufmann, L., Landerl, K., & Nuerk, H. C. (2009). Basic number processing deficits in developmental dyscalculia: Evidence form eye tracking. *Cognitive development, 24*, 371-386. doi: 10.1016/j.cogdev.2009.09.007

Murphy, M. M., Mazzocco, M. M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities, 40*, 458-478. http://dx.doi.org/10.1177/00222194070400050901

NICDH Early Child Care Research Network. (1996). Characteristics of infant child care: Factors contributing to positive caregiving. *Early Childhood Research Quarterly, 11*, 269-306. http://dx.doi.org/10.1016/S0885-2006(96)90009-5

NICDH Early Child Care Research Network. (2001). Child care and children's peer interaction at 24 and 36 months: The NICDH study of early child care. *Child Development, 72*, 1478-1500. http://dx.doi.org/10.1016/S0885-2006(96)90009-5

Pancsofar, N., Vernon-Feagans, L., & Family Life Project, I. (2010). Fathers' early contributions to children's language development in families from low-income rural communities. *Early Childhood Research Quarterly, 25*, 450-463. http://dx.doi.org/10.1016/j.ecresq.2010.02.001

Pasnak, R., Cooke, W. D., & Hendricks, C. (2006). Enhancing academic performance by strengthening class-inclusion reasoning. *Journal of Psychology, 140*, 603-613. http://dx.doi.org/10.3200/jrlp.140.6.603-613

Passolunghi, M. C., & Lanfranchi, S. (2012). Domain-specific and domain-general precursors of mathematical achievement: A longitudinal study from kindergarten to first grade. *British Journal of Educational Psychology, 82*, 42-63. http://dx.doi.org/10.1111/j.2044-8279.2011.02039.x

Peltenburg, M., van den Heuvel-Panhuizen, M., & Robitzsch, A. (2012). Special education students' use of indirect addition in solving subtraction problems up to 100-A proof of the didactical potential of an ignored procedure. *Educational Studies in Mathematics, 79*, 351-369. http://dx.doi.org/10.1007/s10649-011-9351-0

Peters, G., De Smedt, B., Torbeyns, J., Verschaffel, L., & Ghesquiere, P. (2014). Subtraction by addition in children with mathematical learning disabilities. *Learning and Instruction, 30*, 1-8. http://dx.doi.org/10.1016/j.learninstruc.2013.11.001

Putnam, R. D. (1995). Tuning in, tuning out - The strange disappearance of social capital in America. *Ps-Political Science & Politics, 28*, 664-683. http://dx.doi.org/10.2307/420517

Revkin, S. K., Piazza, M., Izard, V., Cohen, L., & Dehaene, S. (2008). Does subitizing reflect numerical estimation? *Psychological Science, 19*, 607-614. http://dx.doi.org/10.1111/j.1467-9280.2008.02130.x

Rousselle, L., & Noël, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs nonsymbolic number magnitude processing. *Cognition, 102*, 361-395. http://dx.doi.org/10.1016/j.cognition.2006.01.005

Sarama, J., & Clements, D. H. (2008). Mathematics in early childhood. In O. N. Saracho & B. Spodek (Series Eds.) & O. N. Saracho & B. Spodek (Vol. Eds.) *Contemporary perspectives in early childhood education: Mathematics in early childhood education* (pp. 67-94). Greenwich, CT: Information Age Publishing.

Sameroff., A. J. (1975). Transactional models in early social relations. *Human Development, 18*, 65-79. http://dx.doi.org/10.1159/000271476

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52*, piii-viii, 3-162.

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental science, 14,* 280-291. http://dx.doi.org/10.1111/j.1467-7687.2010.00976.x

Shalev, R. S., Manor, O., Kerem, B., Ayali, M., Badichi, N., Friedlander, Y., & Gross-Tsur, V. (2001). Developmental dyscalculia is a familial learning disability. *Journal of Learning Disabilities, 34*, 59-65. doi: 10.1177/002221940103400105

Simms, V., Cragg, L., Gilmore, C., Marlow, N., & Johnson, S. (2013). Mathematics difficulties in children born very preterm: current research and future directions. *Archives of Disease in Childhood-Fetal and Neonatal Edition, 98*, F457-F463. doi: 10.1136/archdischild-2013-303777

Simms, V., Gilmore, C., Cragg, L., Marlow, N., Wolke, D., & Johnson, S. (2013). Mathematics difficulties in extremely preterm children: evidence of a specific deficit in basic mathematics processing. *Pediatric Research, 73*, 236-244. doi: 10.1038/pr.2012.157

Skwarchuk, S. L. (2009). How do parents support preschoolers' numeracy learning experiences at home? *Early Childhood Educational Journal, 37*, 189-197. http://dx.doi.org/10.1007/s10643-009-0340-1

Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children’s early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology, 121*, 63-84. http://dx.doi.org/10.1016/j.jecp.2013.11.006

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*, 99-120. http://dx.doi.org/10.1016/j.ecresq.2004.01.002

Starr, A., Libertus, M. E., & Brannon, E. M. (2013a). Infants show ratio-dependent number discrimination regardless of set size. *Infancy*, *18*, 927–941. http://dx.doi.org/10.1111/infa.12008

Starr, A., Libertus, M. E., & Brannon, E. M. (2013b). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences of the United States of America (PNAS), 110,* 18116-18120. http://dx.doi.org/10.1073/pnas.1302751110

Stock, P., Desoete, A., & Roeyers, H. (2009). Mastery of the counting principles in toddlers: A crucial step in the development of budding arithmetic abilities? *Learning and Individual Differences, 19*, 419-422. http://dx.doi.org/10.1016/j.lindif.2009.03.002

Stock, P., Desoete, A., & Roeyers, H. (2010). Detecting children with arithmetic disabilities from kindergarten: Evidence from a 3-year longitudinal study on the role of reparatory arithmetic abilities. *Journal of Learning Disabilities, 43*, 250-268. htt://dx.doi.org/10.1177/0022219409345011

Trick, L. M. & Pylyshyn, Z. W. (1993). What enumeration studies can show us about spatial attention – Evidence for limited capacity preattentive processing. *Journal of Experimental Psychology-Human Perception and Performance, 19*, 331-351. http://dx.doi.org/10.1037/0096-1523.19.2.331

Tudge, J. R. H. (2009). Methods of assessment of young children’s informal mathematical experiences. *Encyclopedia of Language and Literacy Development.* Retrieved from http://literacyencyclopedia.ca/index.php?fa=items.show&topicId=269

Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19*, 21-39. http://dx.doi.org/10.1016/j.ecresq.2004.01.007

Tudge, J. R. H., Li, L., & Stanley, K. (2008). The impact of method on assessing young children’s everyday mathematics experiences. In O.N. Saracho & B. Spodek (Series Eds.) & O.N. Saracho & B. Spodek (Vol. Eds.), *Contemporary perspectives in early childhood education: Mathematics in early childhood education* (pp. 187-214). Greenwich, CT: Information Age Publishing.

United States Department of Labor. (1994). *Handbook on women workers: Trends & issues.* Washington, DC: United States Government Printing Office.

Verschaffel, L., Torbeyns, J., De Smedt, B., Luwel, K., & Van Dooren, W. (2007). Strategic flexibility in children with low achievement in mathematics. *Educational & Child Psychology, 24*, 16-27.

Wu, S. S., Meyer, M. L., Maeda, U., Salimpoor, V., Tomiyama, S., Geary, D. C., & Menon, V. (2008). Standardized assessment of strategy use and working memory in early mental arithmetic performance. *Developmental Neuropsychology, 33*, 365-393. http://dx.doi.org/10.1080/87565640801982445

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25. http://dx.doi.org/10.1016/S0010-0277(03)00050-7

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology, 25*, 103-108. http://dx.doi.org/10.1348/026151005x90704

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, B1-B11. http://dx.doi.org/10.1016/S0010-0277(99)00066-9

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8*, 88-101. http://dx.doi.org/10.1111/j.1467-7687.2005.00395.x

NEDERLANDSTALIGE SAMENVATTING

'Number sense' (getalgevoeligheid) is een construct met vele benamingen en definities dat verwijst naar de bouwstenen van rekenen (Powell & Fuchs, 2012). Volgens het 'lage orde' perspectief is getalgevoeligheid een aangeboren perceptueel besef van hoeveelheid bij niet-symbolische stimuli (Butterworth, 1999; Dehaene, 1997; Jordan & Levine, 2009). Het ontwikkelt zich zonder of mits geringe verbale input en vormt de basis van een 'hoger (in) orde' eerder verworven getalgevoeligheid die vatbaar is voor externe ervaring (Berch, 2005, Jordan & Levine, 2009) wanneer kinderen over taal beschikken.

Subitizeren (een term afkomstig van Kaufman, Lord, Reese, en Volkmann (1949) die duidt op de snelle en accurate benadering van kleine hoeveelheden) is een van de aspecten die in de literatuur wordt beschreven onder de term getalgevoeligheid. Echter, of subitizeren aanleunt bij het lage orde of hoge orde perspectief hangt af van de operationalisering ervan. Subitizeren wordt immers traditioneel ingevuld als een perceptueel-verbale vaardigheid waarbij getalwoorden worden gebruikt om hoeveelheden uit te drukken (Benoit, Lehalle, & Jouen, 2004). Omdat getalwoorden een symbolische betekenis hebben, leunt deze invulling aan bij het hoge orde perspectief op getalgevoeligheid. De invulling van subitizeren als een perceptueel-preverbale vaardigheid waarbij benoemen niet vereist is, leunt aan bij het lage orde perspectief.

Deze laatste invulling van subitizeren kan herkend worden in het non-verbaal discrimineren van kleine (< 4; Feigenson, Dehaene, & Spelke, 2004) hoeveelheden (*getaldiscriminatie*). Dit is een vaardigheid die reeds aanwezig is vanaf babyleeftijd (e.g., Xu, 2003) en tevens kan worden beschouwd als een voorloper van het traditioneel *verbaal subitizeren.* Op zijn beurt wordt verbaal subitizeren in de literatuur dan weer gelinkt met kleutervaardigheden zoals tellen, kennen van kardinaliteit en rekenoperaties (Baroody, Bajwa, & Eiland, 2009; Benoit et al., 2004; Le Corre & Carey, 2007), die ook vallen onder hoge orde getalgevoeligheid. De predictieve waarde van deze *numerieke vaardigheden op kleuterleeftijd* werd aangetoond als gevolg van het feit dat individuele verschillen in getalgevoeligheid onderzoekers motiveerden dit construct te onderzoeken als voorspeller van latere typische en ook atypische rekenontwikkeling (e.g., Jordan, Kaplan, Ramineni, & Locuniak, 2009; Stock, Desoete, & Roeyers, 2010).

Er bestaan echter ook individuele verschillen in getaldiscriminatie op babyleeftijd (Libertus & Brannon, 2010). Bovendien werd in navolging hiervan de predictieve waarde van deze individuele verschillen in getaldiscriminatie van grote hoeveelheden (> 3; Feigenson et al., 2004) bewezen voor later rekenen (Starr, Libertus, & Brannon, 2013). Er is echter nog niet bekend of getaldiscriminatie van kleinere hoeveelheden vanaf babyleeftijd dezelfde belofte in zich heeft. Vanuit deze beweging van onderzoek naar voorspellers van rekenen op (nog) jongere leeftijd is het interessant om na te gaan of *numerieke moeder-kind interactie*, als omgevingsfactor naast eigen kindvaardigheden zoals getaldiscriminatie, ook reeds vroeg een rol speelt in de rekenontwikkeling.

DOEL VAN HET DOCTORAATSONDERZOEK

Het voornaamste doel van dit doctoraatsonderzoek was het uitbreiden van kennis over de waarde van getaldiscriminatie van kleine hoeveelheden en numerieke moeder-kind interactie voor huidig en later numeriek functioneren. Binnen dit kader werden drie specifieke hoofdonderzoeksdoelen geformuleerd die werden opgenomen in vier studies. Twee studies waren cross-sectioneel in opzet en de overige twee waren longitudinaal.

In functie van een eerste onderzoeksdoel werden beide invullingen van subitizeren (d.i., verbaal subitizeren en getaldiscriminatie van kleine hoeveelheden) verkend in atypische en typische rekenontwikkeling. Ter bevestiging van voorgaand onderzoek (e.g., Fischer, Gebhart, & Hartnegg, 2008), werd nagegaan of jongeren met een rekenstoornis tekorten vertonen in verbaal subitizeren in vergelijking met leeftijdsgenoten zonder een rekenstoornis. Vervolgens werd, in de aanloop van onderzoek naar getaldiscriminatie van kleine hoeveelheden als voorspeller van latere rekenontwikkeling bij typisch ontwikkelende kinderen, verkend of baby's in staat waren om een set kleine hoeveelheden bestaande uit drie items versus één item van elkaar te onderscheiden. Een succes op groepsniveau zou erop wijzen dat genoeg kinderen deze set kunnen discrimineren met ruimte voor detectie van kinderen die geen succes kenden, wat naadloos leidt tot het volgende doel. Dit tweede onderzoeksdoel betrof de vraag of getaldiscriminatie met kleine hoeveelheden als een voorspeller van latere rekenvaardigheden kan worden beschouwd. Heel concreet werd in longitudinaal verband onderzocht of een getaldiscriminatietaak met kleine hoeveelheden op baby- (8 maanden) of peuterleeftijd (24 maanden) kon differentiëren tussen niveaus van rekenvaardigheden op kleuterleeftijd (48 maanden). Bijkomend werd onderzocht of de getaldiscriminatieprestaties op 8 maanden en 24 maanden onderling correleerden. Een laatste onderzoeksdoel was gelijkaardig aan het voorgaande maar had betrekking op een omgevingsfactor. Vanuit deze optiek werd de rol van numerieke moeder-kind interactie voor aspecten van getalgevoeligheid onder de loep genomen. Er werd nagegaan of numerieke moeder-kind interactie op peuterleeftijd in verband stond met getaldiscriminatie op peuterleeftijd. Tevens werd de relatie nagegaan tussen numerieke moeder-kind interactie op peuterleeftijd en latere getalgevoeligheid in de vorm van de eerste rekenvaardigheden op kleuterleeftijd. Via de combinatie van de twee laatste onderzoeksdoelen wou het doctoraatsonderzoek een blik werpen op zowel een kindvaardigheid als een omgevingsinvloed in kader van numerieke latere vaardigheden.

VOORNAAMSTE ONDERZOEKSRESULTATEN

Overeenkomstig vorig onderzoek waarbij geen evidentie gevonden werd voor een tekort in verbaal subitizeren (e.g., De Smedt & Gilmore, 2011; Rousselle & Noël, 2007), werd ook in *Hoofdstuk 2* geen afwijkende subitizeerprestatie gevonden wat betreft accuraatheid of reactietijd bij jongeren met een rekenstoornis. Een combinatie van snelheid en accuraatheid wees echter wel op het volgende: Terwijl jongeren zonder een rekenstoornis blijk gaven van een 'strategiewissel' afhankelijk van kleine of grote hoeveelheden items, kon deze strategiewissel niet worden opgemerkt bij jongeren met een rekenstoornis. In de controlegroep leek het alsof participanten bij grotere hoeveelheden terugvielen op een snellere schattingsstrategie (dan tellen), wat ze niet deden bij de kleine hoeveelheden (d.i., hoeveelheden tot en met vier voor deze studie). Dit werd geïllustreerd door een positieve correlatie tussen snelheid en accuraatheid voor het benoemen van grotere hoeveelheden. Hoewel er geen afwijkingen werden geregistreerd tussen de groepen jongeren wat betreft accuraatheid en snelheid tijdens verbaal subitizeren, lijkt dit resultaat te wijzen op een verschillende aanpak van kleine ten opzichte van grote hoeveelheden tussen jongeren met en zonder een rekenstoornis.

De combinatie van één ten opzichte van drie items bleek een set te zijn die kinderen (op groepsniveau) reeds op babyleeftijd non-verbaal konden onderscheiden van elkaar (*Hoofdstuk 3)*. In tegenstelling echter tot getaldiscriminatie met grotere aantallen werd aangetoond in *Hoofdstuk 4* dat deze getaldiscriminatie met kleine(re) aantallen geen voorspellende waarde had voor de eerste rekenvaardigheden (d.i., voor deze studie: telvaardigheden, begrip hebben van de kardinaliteitswaarde van cijfers en het uitvoeren van eenvoudige rekenoperaties) op kleuterleeftijd.

De gecombineerde resultaten van de studies die beschreven werden in *Hoofdstuk 4* en *5* wierpen licht op de voorspellende waarde van getaldiscriminatie (van kleine hoeveelheden) op peuterleeftijd (*Hoofdstuk 4)* en van de numerieke moeder-kind interactie op diezelfde leeftijd (*Hoofdstuk* 5) voor later rekenen. Beide studies zijn een aanvulling op vorig onderzoek omdat ze de leeftijdsfocus vervroegen van kleuter- (e.g., Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Jordan et al., 2009) naar peuterleeftijd. Deze bevindingen stroken met de idee dat baby's aangeboren vaardigheden hebben die overvloeien in latere numerieke of rekenvaardigheden, maar ook gaandeweg (bijvoorbeeld op peuterleeftijd) leren via (externe) ervaring zoals numerieke ouder-kind interactie.

Gebaseerd op de gegevens van het huidige doctoraatsonderzoek kan alvast besloten worden dat numerieke interactie tussen ouder en peuter en de peuters eigen getaldiscriminatievaardigheid elk – in enige mate – hun eigen bijdrage leveren ten aanzien van numerieke vaardigheden op kleuterleeftijd. Op het eerste zicht was er aanwijzing voor het feit dat beide voorspellers de score op conceptuele telkennnis voorspelden op kleuterleeftijd en dat numerieke moeder-kind interactie op peuterleeftijd bijkomend ook een voorspellende waarde had voor latere rekenoperaties. Aanvullende analyse waarbij beide predictoren in eenzelfde model werden opgenomen, wees echter uit dat getaldiscriminatie vooral kon gelinkt worden aan conceptuele telkennis en numerieke moeder-kind interactie vooral aan rekenoperaties. De eigen complexiteit van de betreffende rekenaspecten, zou een verklaring kunnen zijn. Conceptuele telkennis kan meer als basis worden beschouwd in tegenstelling tot meer complexe rekenoperaties, net zoals van de op zichzelf staande kindvaardigheid getaldiscriminatie verondersteld kan worden dat deze minder complex is dan een dyadische numerieke interactie. Niet enkel betrof het voorspellend verband van de predictoren naar numerieke vaardigheden een ander aspect, ook was het verband op zichzelf van een andere aard. Een lineair verband werd aangetroffen in het geval van getaldiscriminatie, terwijl voor numerieke moeder-kind interactie een kwadratisch verband werd waargenomen. Het is dan ook niet verwonderlijk dat er geen onderling significant verband werd gevonden tussen de gemeende voorspellende constructen binnen de huidige dataset. Gezien getaldiscriminatie tevens verondersteld wordt een ander niveau van vaardigheden te vereisen dan deze die nodig zijn voor het uitvoeren van rekenoperaties, had numerieke moeder-kind interactie mogelijks wel gerelateerd met een taak op peuterleeftijd van een gelijkaardig complexiteitsniveau dan rekenoperaties. Wanneer bovendien ook nog eens een meer strikte benadering werd gevolgd van (succesvolle) getaldiscriminatie, kon voorgenoemd voorspellend (lineair) verband tussen conceptuele telkennis niet meer bevestigd worden voor getaldiscriminatie. Vanuit het oogpunt dat het onderzoeken van getaldiscriminatie op zeer vroege leeftijd nog in de kinderschoenen staat, is het te begrijpen dat er nog geen ‘gouden standaard’ voorhanden is met betrekking tot een operationalisatie van ‘succes’. Meer nog, onderzoekers moeten worden aangemoedigd een verkenning uit te voeren van verschillende benaderingen van getaldiscriminatie als voorspeller voor later rekenen. Toch zal het belangrijk zijn uiteindelijk op zijn minst één gelijke benadering te hanteren (naast andere), in functie van het vergelijken en veralgemenen van resultaten.

PRAKTISCHE IMPLICATIES

Vanuit de bevindingen resulterend uit het huidige doctoraatsonderzoek kunnen enkele praktische implicaties geformuleerd worden, zowel met betrekking tot de typische rekenontwikkeling als de atypische (problematische) rekenontwikkeling. De belangrijkste implicaties worden in wat volgt in een notendop samengevat.

Typische ontwikkeling

Uit het onderzoek in functie van dit doctoraat is het belang gebleken van de ‘peuterleeftijd’ (24 maanden) als scharnierleeftijd voor het voorspellen van later rekenen, meer dan voor de ‘babyleeftijd’ (8 maanden). Zowel bij getaldiscriminatie (van kleine hoeveelheden) als numerieke moeder-kind interactie werd immers op deze peuterleeftijd een voorspellend verband aangetoond met aspecten van kleuterrekenen (48 maanden). Momenteel is onze maatschappij reeds ruim geïnformeerd en overtuigd van de meerwaarde van vroege blootstelling aan talige activiteiten en voorbereidende leesactiviteiten via ouder-kind interactie (zoals aangehaald in Skwarchuk, 2009). De huidige bevindingen moedigen echter ook aan om in navolging van Skwarchuk (2009) het imago van *gecijferdheid* (naast *geletterdheid*) op te krikken. Dit wordt geïllustreerd doordat specifieke numerieke moeder-kind interactie relateert met conceptuele telkennis en complexe rekenoperaties. Het lijkt dus de moeite om afgestemde numerieke input via moeder-kind interactie bij peuters te sensibiliseren. Instanties die instaan voor opvoedingsondersteuning (van baby’s en peuters) kunnen hierin een rol spelen.

Het is wel belangrijk om in dit verband te beklemtonen dat de focus in het geval van rekenoperaties eerder dient te liggen op de mogelijkheid van numerieke stimulatie, dan wel op een werkelijke noodzakelijkheid hiervan op peuterleeftijd. De bevinding die deze suggestie ondersteunt, had immers betrekking op een kwadratisch en dus geen lineair verband zoals deze wel werd waargenomen voor de gelijkaardige numerieke interactie-maat op kleuterleeftijd met hetzelfde rekenaspect (in dit geval dus rekenoperaties). Dit betekent dat, in tegenstelling tot een lineair verband waarbij meer stimulatie zou samengaan met een grotere vaardigheidsbeheersing minder blootstelling (d.i., gedoseerd) beter zou zijn dan een overvloed, maar dat enige blootstelling nog altijd wel meer loont dan geen enkele stimulatie. Zelfs als het kind er niet direct de vruchten van zou (lijken te) plukken, kunnen de voordelen toch reeds sluimerend aanwezig zonder reeds op uitgesproken wijze tot uiting te komen op peuterleeftijd.

Atypische ontwikkeling

Met oog op het verminderen van de impact van latere rekenproblemen is het aangewezen zo vroeg mogelijk individuele verschillen in de numerieke ontwikkeling op te sporen (Coleman, Buysse, & Neitzel, 2006; Fuchs et al., 2007; Gersten, Jordan & Flojo, 2005; Pasnak, Cooke, & Hendricks, 2006; Passolunghi & Lafranchi, 2012).

Wat getaldiscriminatie van kleine hoeveelheden op peuterleeftijd betreft, zou het belangrijk kunnen zijn om deze kinderen op te volgen die ofwel zeer slecht (ver onder het gemiddelde) ofwel net zeer goed (ver boven het gemiddelde) presteerden. Zulke prestaties zouden kunnen differentiëren tussen kinderen die een typisch of atypisch (verstoord) rekenontwikkelingspatroon volgen op latere leeftijd, maar ook onderzoek mogelijk maken naar eventuele protectieve factoren. Zeker voor kinderen met een verhoogd risico op een rekenstoornis kunnen dergelijke sensitieve maten nuttig zijn. Verder onderzoek is echter nodig om deze denkpiste te ondersteunen.

Ook al was dit niet de hoofdfocus van het doctoraatsonderzoek, toch kan er ook met betrekking tot jongeren met een rekenstoornis voorzichtig een implicatie geformuleerd worden. Een van de studies in dit proefschrift toonde immers aan dat het samen onderzoeken van zowel de verwerking van kleine als grote hoeveelheden met bijzondere focus op bijhorend strategiegebruik kan wijzen op een verschil in aanpak tussen typisch en atypisch ontwikkelende jongeren op vlak van rekenen.

CONCLUSIE

De studies in dit doctoraatsproefschrift hebben meer inzicht opgeleverd met betrekking tot vroege voorspellers van getalgevoeligheid met aandacht voor zowel kind- als omgevingsfactoren. In de zijlijn werd extra informatie ingewonnen over de verwerking van kleine en grote hoeveelheden door jongeren met een rekenstoornis in vergelijking met typisch ontwikkelende leeftijdsgenoten. Tijdens de uitvoering van dit doctoraatsonderzoek kwamen enkele aandachtspunten aan het licht omtrent meten, definiëren en analyseren van constructen en data.

Door het aantonen van de voorspellende waarde van getaldiscriminatie en numerieke moeder-kind interactie op peuterleeftijd voor de eerste rekenvaardigheden op kleuterleeftijd, draagt het huidige onderzoek enerzijds bij aan de opkomende literatuur rond getaldiscriminatie als voorspeller van later rekenen, anderzijds breidt het ook het bestaande onderzoeksveld uit over het verband tussen numerieke ouder-kind interactie en numerieke vaardigheden van kinderen. Het huidige doctoraatsonderzoek hoopt toekomstig onderzoek te inspireren, omdat onderzoek naar deze (zeer) vroege voorspellers nog in de kinderschoenen staat. Er is nog een lange weg te gaan via vele mogelijke paden om tot een sluitstuk te komen.

REFERENTIES

Aunola, K., Leskinen, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology, 96*, 699-713. http://dx.doi.org/10.1037/0022-0663.96.4.699

Baroody, A. J., Bajwa, N. P., & Eiland, M. (2009). Why can't Johnny remember the basic facts? *Developmental Disabilities Research Reviews, 15*, 69-79. http://dx.doi.org/10.1002/ddrr.45. [PubMed: 19213010]

Benoit, L., Lehalle, H., & Jouen, F. (2004). Do young children acquire number words through subitizing or counting? *Cognitive Development, 19*, 291-307. http://dx.doi.org/10.1016/j.cogdev.2004.03.005

Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities, 38*, 333-339. http://dx.doi.org/10.1177/00222194050380040901

Butterworth, B. (1999). *The mathematical brain.* London: Macmillan.

Coleman, M. R., Buysse, V., & Neitzel, J. (2006). Recognition and response. An early intervening system for young children at-risk for learning disabilities. *Research synthesis and recommendations.* UNIC FPG Child Development Institute.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics.* New York: Oxford University Press.

De Smedt, B., & Gilmore, C. K. (2011). Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties. *Journal of Experimental Child Psychology, 108,* 278-292. doi: 10.1016/j.jecp.2010.09.003

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences, 8*, 307-314. http://dx.doi.org/10.1016/j.tics.2004.05.002

Fischer, B., Gebhardt, C., & Hartnegg, K. (2008). Subitizing and visual counting in children with problems acquiring basic arithmetic skills. *Optometry and Vision*

Development, *39*, 24-29. Retrieved from http://www.covd.org/Portals/0/24-29SubitizingVisualCounting.pdf

Fuchs, L. S., Fuchs, D., Compton, D. L., Bryant, J. D., Hamlett, C. L., & Seethaler, P. M. (2007). Mathematics screening and progress monitoring at first grade: Implications for responsiveness to intervention. *Exceptional Children, 73*, 311-330.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*, 293-304. http://dx.doi.org/10.1177/00222194050380040301

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45*, 850-867. http://dx.doi.org/10.1037/a0014939.

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews, 15*, 60-68. http://dx.doi.org/10.1002/ddrr.46

Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology, 62*, 498-525. http://dx.doi.org/10.2307/1418556

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition, 105*, 395–438. http://dx.doi.org/10.1016/j.cognition.2006.10.005

Libertus, M. E., & Brannon, E. M. (2010). Stable individual differences in number discrimination in infancy. *Developmental Science, 13*, 900-906. http://dx.doi.org/10.1111/j.1467-7687.2009.00948.x

Pasnak, R., Cooke, W. D., & Hendricks, C. (2006). Enhancing academic performance by strengthening class-inclusion reasoning. *Journal of Psychology, 140*, 603-613. http://dx.doi.org/10.3200/jrlp.140.6.603-613

Passolunghi, M. C., & Lanfranchi, S. (2012). Domain-specific and domain-general precursors of mathematical achievement: A longitudinal study from kindergarten to first grade. *British Journal of Educational Psychology, 82*, 42-63. http://dx.doi.org/10.1111/j.2044-8279.2011.02039.x

Powell, S. R., & Fuchs, L. S. (2012). Early Numerical Competencies and Students with Mathematics Difficulty. *Focus on Exceptional Children, 44*, 1-16. http://dx.doi.org/10.1111/j.1540-5826.2011.00345.x

Rousselle, L., & Noel, M. P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition, 102*, 361-395. http://dx.doi.org/10.1016/j.cognition.2006.01.005

Skwarchuk, S. L. (2009). How do parents support preschoolers' numeracy learning experiences at home? *Early Childhood Educational Journal, 37*, 189-197. http://dx.doi.org/10.1007/s10643-009-0340-1

Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. *Proceedings of the National Academy of Sciences of the United States of America (PNAS), 110,* 18116-18120. http://dx.doi.org/10.1073/pnas.1302751110

Stock, P., Desoete, A., & Roeyers, H. (2010). Detecting children with arithmetic disabilities from kindergarten: Evidence from a 3-year longitudinal study on the role of reparatory arithmetic abilities. *Journal of Learning Disabilities, 43*, 250-268. doi: 10.1177/0022219409345011

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representations. *Cognition, 89*, B15-B25. http://dx.doi.org/10.1016/S0010-0277(03)00050-7