MATHEMATICAL CLASSIFICATION OF PHYSICAL QUANTITIES AND PHYSICS RELATIONS

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3 Preface

4 Introduction

Equations in physics are in most cases represented in the form $F(x_1, x_2, \dots, x_n) = 0$ where the x_i represent physical quantities. In any case, the equation has to be dimensionally homogeneous to be able to add two terms together. If we consider an equation of state like $PV = nRT$, which represents the equation of state of an ideal gas, one may wonder if there is any underlying mathematical structure in these equations . This type of question was discussed by James Clerck Maxwell in his presentation to the London Mathematical Society (Maxwell, 1874). He made clear in his presentation that this problem should be investigated to increase the efficiency in finding new laws. Maxwell addressed clearly the problem when discussing the quantity “velocity”. We quote(Maxwell, 1874):

> Thus, in the ordinary theory of fluids, in which the only motion considered is that which we can directly perceive, we may define the velocity equally well in two different ways. We may define it with reference to unit of length, as the number of such units described by a particle in unit of time; or we may define it with reference to unit of area, as the volume of the fluid which passes through unit of area in unit of time. If defined in the first way, it belongs to the category of forces; if defined in the second way, to the category of fluxes.

Can we define categories for the physical quantities on a mathematical basis and develop a formalism for their relations? Is it possible to consider the product of physical quantities as a chemical formula which represents chemical bonds between atoms and deduce for the physical quantities a “Table of Mendeleyev” based on their respective relations?

It is the purpose of this research to find an answer to the above questions which could result in a classification of the physical quantities and their respective relations.

The simplification of the equation $F(x_1, x_2, \dots, x_n) = 0$ in physics, where the x_i represents dimensional physical quantities, has been studied by Buckingham resulting in the famous Buckingham π-theorem.

The foundations of dimensional analysis were worked out episodically by Bertrand, Vaschy, Buckingham, Campbell, Bridgman, O’Rahilly, Palacios, Birkhoff and many others(Roche, 1998). Many famous physicists have used the technique of dimensional analysis as tool to find “their” equation. Among these physicist we find names as Fourier, Maxwell, Rayleigh, Reynolds, Bohr(Roche, 1998). Rayleigh applied the dimensional analysis on the problem of the effect of temperature on the viscosity of a gas(Rayleigh, 1899-1900).

Unanswered questions of dimensional analysis according to Barenblatt (Roche, 1998) are typically:

“Which cluster of related quantities fixed the value of the quantity being investigated, and did not under or over-determine it?”

“Which variables, material constants and universal constants should appear multiplied together in a trail law?”

“Which quantities should be excluded?”

It is the hope of this research that the mathematical classification of the physical quantities and their relations will give an answer to the above questions.

5 Definitions

A **quantity in the general sense** is a property ascribed to phenomena, bodies, or substances that can be quantified for, or assigned to, a particular phenomenon, body, or substance. Examples are mass and electric charge.

A **quantity in the particular sense** is a quantifiable or assignable property ascribed to a particular phenomenon, body, or substance. Examples are the mass of the moon and the electric charge of the proton.

A **physical quantity** is a quantity that can be used in the mathematical equations of science and technology.

A **unit** is a particular physical quantity, defined and adopted by convention, with which other particular quantities of the same kind are compared to express their value.

The **value of a physical quantity** is the quantitative expression of a particular physical quantity as the product of a number and a unit, the number being its numerical value. Thus, the numerical value of a particular physical quantity depends on the unit in which it is expressed.

6 Base units in physics

Physical quantities are used to describe physical processes. The physical quantities are expressed in units. The units are used according to international convention which is the "Système International" (SI). The SI units recognize seven base units. The base units are defined as meter, kilogram, second, Ampere, Kelvin, mole, candela.

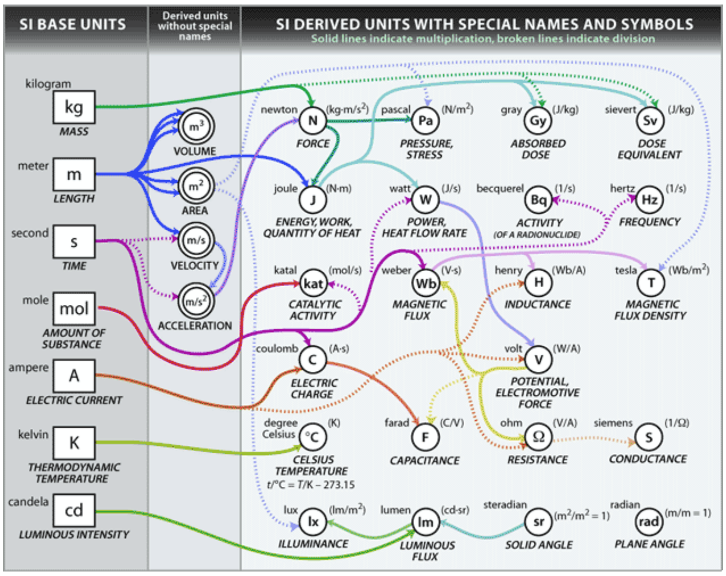


Figure 6.1-1 SI base units

These SI base units correspond respectively to the base quantities of length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity. So, each physical quantity can be described by a n-tuple. In the case of the SI base units the value of n = 7. A 7-tuple is also called septuple.

6.1 Impact of the number of base units on physics

We can consider the evolution of the number of base units as an historical variable. How more we know about physics, how better the description and so how more in detail we have to model the reality. This process of increase of base units can also be modelled as a growth and evolution of graphs. Several physicists have studied the impact of the base units on the equations and quantities of physics(Roche, 1998)(Uzan & Leclercq, 2005).

6.1.1 Description with one unit resulting in the graph v_0v_1.

In this case we work in the space $\boldsymbol{Z}, +$

6.1.2 Description with two units resulting in the graph $v_0v_1v_2$.

In this case we work in the space $\boldsymbol{Z}^2, +$

6.1.3 Description with three units resulting in the graph $v_0v_1v_2v_3$.

In this case we work in the space $\boldsymbol{Z}^3, +$

In the past a lot of physics equations have been expressed in this space. This space is also referred to as Gaussian units. An example of a conversion table(Grant & Phillips, 1979) is given in Figure 6.1-1.

CONVERSION FACTORS BETWEEN SI AND GAUSSIAN UNITS

Quantity	SI unit	Dimensions of SI unit	Gaussian unit	No. of Gaussian units in one SI unit
Force	newton	$[MLT^{-2}]$	dyne	10^5 (i.e. 10^5 dyne = 1 newton)
Energy	joule	$[ML^2T^{-2}]$	erg	10^7
Power	watt	$[ML^2T^{-3}]$	erg/s.	10^7
Charge	coulomb	$[Q]$	e.s.u.	3×10^9
Current	ampere	$[T^{-1}Q]$	e.s.u./s.	3×10^9
Potential	volt	$[ML^2T^{-2}Q^{-1}]$	statvolt	1/300
Capacitance	farad	$[M^{-1}L^{-2}T^2Q^2]$		9×10^{11}
Resistance	ohm	$[ML^2T^{-1}Q^{-2}]$		$1/(9 \times 10^{11})$
Inductance	henry	$[ML^2Q^{-2}]$		$1/(9 \times 10^{11})$
E	volt/metre	$[MLT^{-2}Q^{-1}]$	statvolt/cm.	$1/(3 \times 10^4)$
D	coulomb/sq. metre	$[L^{-2}Q]$	statvolt/cm.	$12\pi \times 10^5$
B	tesla	$[MT^{-1}Q^{-1}]$	gauss	10^4
H	ampere/metre	$[L^{-1}T^{-1}Q]$	oersted	$4\pi \times 10^{-3}$

In this table c has been taken as 3×10^8 m/sec.

Figure 6.1-1 Conversion factors to SI units

6.1.4 Description with four units resulting in the graph $v_0v_1v_2v_3v_4$.

In this case we work in the space $\boldsymbol{Z}^4, +$

6.1.5 Description with five units resulting in the graph $v_0v_1v_2v_3v_4v_5$

In this case we work in the space $\boldsymbol{Z}^5, +$

6.1.6 Description with six units resulting in the graph $v_0v_1v_2v_3v_4v_5v_6$

In this case we work in the space $\boldsymbol{Z^6}, +$

6.1.7 Description with seven units resulting in the graph $v_0v_1v_2v_3v_4v_5v_6v_7$

This description is at the basis of the present study where we work in $\boldsymbol{Z^7}$.

6.1.8 Description with *n* units resulting in the graph $v_0v_1v_2v_3v_4v_5v_6 \dots v_\mathrm{n}$

In this case we work in the space $\boldsymbol{Z^n}, +$

7 Properties of Z^7 for mathematical modelling of physics equations

7.1 General mathematical properties and definitions applicable for Z^7

$\boldsymbol{Z}^7, +$ is an Abelian group.

$\boldsymbol{Z}^7$ is a **Z** module if $\boldsymbol{Z}^7$ is an additive Abelian group and there exist a mapping $\boldsymbol{Z} \times \boldsymbol{Z}^7 \rightarrow \boldsymbol{Z}^7$ **where** the following axioms are valid: For any vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \boldsymbol{Z}^7$ and any integer $i, j \in \boldsymbol{Z}$:

$$j(\boldsymbol{a} + \boldsymbol{b}) = j\boldsymbol{a} + j\boldsymbol{b}$$

$$(i + j)\boldsymbol{a} = i\boldsymbol{a} + j\boldsymbol{a}$$

$$(ij)\boldsymbol{a} = i(j\boldsymbol{a})$$

$$1\boldsymbol{a} = \boldsymbol{a}$$

Quotient set $\boldsymbol{Z}^7/\boldsymbol{R}$ which is the set of equivalence classes under the equivalence relation R on the set $\boldsymbol{Z}^7$**.** We will see in §7.4 the application of such an equivalence relation on $\boldsymbol{Z}^7$**.**

V_1= set of vectors of $\boldsymbol{Z}^7$ having norm = 1. The number of elements in this set is 7.

V_r= set of vectors of $\boldsymbol{Z}^7$ having norm = r. The number of elements in this set is ?

GL(7,***Z***) is the general linear group of dimension 7 on the field ***Z*** (Penrose, 2005). It is the multiplicative group of 7 x 7 non-singular matrices.

SL(7,***Z***) is the normal subgroup of dimension 7 on the field ***Z*** (Penrose, 2005). It is the multiplicative group of 7 x 7 non-singular matrices having their determinant = 1.

7.2 Z^7 integer lattice as representation of physical quantities

The physical quantities represented by 7-tuples are elements of the set $\boldsymbol{Z}^7$**.**

So, each coordinate of the 7-tuple(septuple) is an integer.

A physical quantity **a** can be considered as a vector in a 7 dimensional integer lattice say

$$\boldsymbol{a} = (a_1, a_2, \dots, a_7)$$

where $a_i \in \boldsymbol{Z}$ are the coordinates of **a**.

The coordinate a_1 represents the exponent of the unit meter.

The coordinate a_2 represents the exponent of the unit kilogram.

The coordinate a_3 represents the exponent of the unit second.

The coordinate a_4 represents the exponent of the unit Ampere.

The coordinate a_5 represents the exponent of the unit Kelvin.

The coordinate a_6 represents the exponent of the unit mole.

The coordinate a_7 represents the exponent of the unit candela.

7.3 Vector operations in Z^7

The seven SI base units represent an ortho-normal basis for $\boldsymbol{Z}^7$. One has the following base vectors

$$\boldsymbol{e_1} = (1,0,0,0,0,0,0)$$

$$\boldsymbol{e_2} = (0,1,0,0,0,0,0)$$

$$\boldsymbol{e_3} = (0,0,1,0,0,0,0)$$

$$\boldsymbol{e_4} = (0,0,0,1,0,0,0)$$

$$\boldsymbol{e_5} = (0,0,0,0,1,0,0)$$

$$\boldsymbol{e_6} = (0,0,0,0,0,1,0)$$

$$\boldsymbol{e_7} = (0,0,0,0,0,0,1)$$

We follow the classical treatment from linear algebra(Lipshutz, 1968) but applied to the special case $\boldsymbol{Z}^7$.

The representation of two physical quantities **a** and **b** is equal, **a** = **b** , if they have the same number of coordinates and if the corresponding coordinates are equal.

7.3.1 Addition in Z^7

The sum of **a** and **b**, written **a** + **b** is obtained by adding the corresponding components:

$$\boldsymbol{a} + \boldsymbol{b} = (a_1 + b_1, a_2 + b_2, \dots, a_7 + b_7)$$

The sum of the representation of two physical quantities corresponds to the multiplication of these physical quantities in the equations used in physics theories.

Consider the representation of the quantity pressure **P** and the quantity volume **V** then the product of PV has the dimension of the quantity energy E=PV.

In $\boldsymbol{Z}^7$ this is represented by the sum of the vectors **P** and **V** :

P = (-1,1,-2,0,0,0,0) and **V**=(3,0,0,0,0,0,0)

P + **V** = **E** = (2,1,-2,0,0,0,0)

The product of an integer number k by the vector **a**, written k**a** , is the vector obtained by multiplying each coordinate of **a** by k:

$$\boldsymbol{ka} = (ka_1, ka_2, \dots, ka_7)$$

The vector **0** = (0,0,…,0) in $\boldsymbol{Z}^7$ is called the zero vector . We will see later that the zero vector represents a dimensionless product of physical quantities.

For any vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \boldsymbol{Z}^{7}$ and any integer $i, j \in \boldsymbol{Z}$:

$$(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c}=\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})$$

$$\boldsymbol{a}+\mathbf{0}=\boldsymbol{a}$$

$$\boldsymbol{a}+(-\boldsymbol{a})=\mathbf{0}$$

$$\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{b}+\boldsymbol{a}$$

$$j(\boldsymbol{a}+\boldsymbol{b})=j\boldsymbol{a}+j\boldsymbol{b}$$

$$(i+j)\boldsymbol{a}=i\boldsymbol{a}+j\boldsymbol{a}$$

$$(ij)\boldsymbol{a}=i(j\boldsymbol{a})$$

$$1\boldsymbol{a}=\boldsymbol{a}$$

7.3.2 Dot product in Z^{7}

A “dot product” can be defined between the physical quantities **a** and **b.**

$$\boldsymbol{a}\cdot\boldsymbol{b}=a_{1}b_{1}+a_{2}b_{2}+\cdots+a_{7}b_{7}$$

The vectors a and b are orthogonal when $\boldsymbol{a}\cdot\boldsymbol{b}=0$.

The vector **F**, representing a force, is given by **F**=(1,1,-2,0,0,0,0) and the vector **v**, representing a velocity, is given by **v**=(1,0,-1,0,0,0,0)

The dot product between force **F** and velocity **v** is $\boldsymbol{F}\cdot\boldsymbol{v}=3$

Let the speed of light be represented by **c =**(1,0,-1,0,0,0,0)

Let the Newton gravitation constant **G** be represented by **G =**(3,-1,-2,0,0,0,0)

Let the Planck constant be represented by **h =**(2,1,-1,0,0,0,0)

The dot product between **c** and **G** is $\boldsymbol{c}\cdot\boldsymbol{G}=5$

The dot product between **c** and **h** is $\boldsymbol{c}\cdot\boldsymbol{h}=3$

The dot product between **G** and **h** is $\boldsymbol{G}\cdot\boldsymbol{h}=7$

The vector representation of the physical constants c, G and h are ***not orthogonal.***

What is the impact of the condition of orthogonality of the physical quantities on the set of possible equations $\boldsymbol{F}(\boldsymbol{x_1},\boldsymbol{x_2},\dots,\boldsymbol{x_n})=\mathbf{0}$ describing the physical system?

We can calculate all vectors **x** which are perpendicular to the vector representing the physical quantity energy **E**.

So, we want to solve the equation $\boldsymbol{E}\cdot\boldsymbol{x}=0$ where $\boldsymbol{x} \in \boldsymbol{Z}^{7}$. This equation represents a hyperplane in $\boldsymbol{Z}^{7}$.

The equation becomes : $2x_{1}+x_{2}-2x_{3}+0x_{4}+0x_{5}+0x_{6}+0x_{7}=0 \ and\ x_{i} \in \boldsymbol{Z}$

A sample of the solution space expressed in (x_1, x_2, x_3) is given in the table below:

x1	**-10**	**-9**	**-8**	**-7**	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
x2																					
-10	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
-8	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-6	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
-4	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
-2	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
0	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
2	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
4	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
6	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
8	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
10	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

It can be seen from this table that the quantity mass (m) and the square of the velocity (c^2) are not elements of this table. Checking all the variables and constants reported in this article shows that no quantity fulfils the above relation.

It is also clear that all quantities which are not containing the units m, kg and s will be orthogonal to the vector **E.**

Conjecture "Exclusion principle": A physical quantity represented by its vector **a** will form no meaningful physical relation with a physical quantity **b** if and only if $\boldsymbol{a} \cdot \boldsymbol{b} = 0$.

So, the set {**G**, **h**, **c}** is a meaningful set of physical quantities because they are not orthogonal to each other.

For any vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in \boldsymbol{Z}^7$ and any integer $i, j \in \boldsymbol{Z}$:

$$(\boldsymbol{a} + \boldsymbol{b}) \cdot \boldsymbol{c} = \boldsymbol{a} \cdot \boldsymbol{c} + \boldsymbol{b} \cdot \boldsymbol{c}$$

$$(j\boldsymbol{a}) \cdot \boldsymbol{b} = j(\boldsymbol{a} \cdot \boldsymbol{b})$$

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{b} \cdot \boldsymbol{a}$$

$$\boldsymbol{a} \cdot \boldsymbol{a} \geq 0 \; and \; \boldsymbol{a} \cdot \boldsymbol{a} = 0 \; if \; \boldsymbol{a} = \boldsymbol{0}$$

The coordinates of the physical quantities represent points in a **7 dimensional integer lattice**. We want to define the distance $d(\boldsymbol{a}, \boldsymbol{b})$ between two lattice points in the following way:

$$d(\boldsymbol{a}, \boldsymbol{b}) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \cdots + (a_7 - b_7)^2}$$

The distance between force **F** and velocity **v** is $d(\boldsymbol{F}, \boldsymbol{v}) = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$

The norm of the vector **a** **,** written $||\boldsymbol{a}||$ is defined as the nonnegative square root of $\boldsymbol{a} \cdot \boldsymbol{a}$

$$||\boldsymbol{a}|| = \sqrt{a_1^2 + a_2^2 + \cdots + a_7^2}$$

$$d(\boldsymbol{a}, \boldsymbol{b}) = ||\boldsymbol{a} - \boldsymbol{b}||$$

$$d(\boldsymbol{a}, \boldsymbol{0}) = ||\boldsymbol{a}||$$

Consider now a 6-sphere in $\boldsymbol{Z}^{\boldsymbol{7}}$, then one can calculate how many lattice points are elements of the 6-sphere surface for each possible distance of a lattice point to the origin. The creation of finite subsets based on $d(\boldsymbol{a}, \boldsymbol{0}) = ||\boldsymbol{a}||$ is a first step in the classification of the physical quantities.

The unit 7-dimensional hypercube has $2^7 = 128$ lattice points(Banchoff, 1996).

It is interesting to observe that the vector **E** has a norm $||\boldsymbol{E}|| = 3$ which is a large value with respect to the other physical quantities.

The norm of the force F is $||\boldsymbol{F}|| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{6}$

The norm of the velocity v is $||\boldsymbol{v}|| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

A vector **u** is a unit vector if its norm is 1.

The Cauchy-Schwarz theorem applies for any vector $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{Z}^{\boldsymbol{7}}$: $|\boldsymbol{a} \cdot \boldsymbol{b}| \leq ||\boldsymbol{a}|| \, ||\boldsymbol{b}||$

The angle θ between any two nonzero vectors $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{Z}^{\boldsymbol{7}}$ is defined by

$$\cos\theta = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{||\boldsymbol{a}|| \, ||\boldsymbol{b}||}$$

The angle θ between force **F** and velocity **v** is given by:

$$\cos\theta = \frac{\boldsymbol{F} \cdot \boldsymbol{v}}{||\boldsymbol{F}|| \, ||\boldsymbol{v}||} = \frac{\boldsymbol{3}}{\sqrt{\boldsymbol{6}}\,\sqrt{\boldsymbol{2}}} = \frac{\sqrt{\boldsymbol{3}}}{\boldsymbol{2}}$$

$$\theta = 30°$$

7.4 Map SUM()

Define SUM() as the mapping $SUM\colon \boldsymbol{Z}^{\boldsymbol{7}} \rightarrow \boldsymbol{Z}$, written as SUM(**a**), as the “sum of the coordinates” of the vector **a**

$$SUM(\boldsymbol{a}) = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{7}} ai$$

We will see further in §7.15 that the “sum of coordinates” is used by Coxeter(Banchoff, 1996) to classify polytopes in an integer lattice.

The sum of the force **F** is given by $SUM(\boldsymbol{F}) = 1 + 1 - 2 = 0$

The sum of the velocity **v** is given by $SUM(\boldsymbol{v}) = 1 - 1 = 0$

Consider the Cartesian product $\boldsymbol{Z}^{7} \times \boldsymbol{Z}$ of the two sets $\boldsymbol{Z}^{7}$ and $\boldsymbol{Z}$ having a propositional function (Lipshutz, 1964, pp. 81-87) $P(\boldsymbol{a}, j) = "SUM(\boldsymbol{a})\ is\ equal\ to\ j"$

Let's call S a relation from $\boldsymbol{Z}^{7}$ to $\boldsymbol{Z}$ and denote it by $S = (\boldsymbol{Z}^{7}, \boldsymbol{Z}, P(\boldsymbol{a}, j))$.

If $P(\boldsymbol{a}, j))$ is true then we write "vector **a** has the sum j" as

aSj

If $P(\boldsymbol{a}, j))$ is false then we write "vector **a** has not the sum j" as

aNSj

Let $S = \left(\boldsymbol{Z}^{7}, \boldsymbol{Z}, P(\boldsymbol{a}, j)\right)$ be a relation. The solution set S* of the relation S consists of the elements (**a**,j) in $\boldsymbol{Z}^{7} \times \boldsymbol{Z}$ for which P(**a**,j) is true.

$$S^{*} = \{(\boldsymbol{a}, j) | \boldsymbol{a} \in Z^{7}, j \in \boldsymbol{Z}, P(\boldsymbol{a}, j)\ is\ true\}$$

S* is a subset of $\boldsymbol{Z}^{7} \times \boldsymbol{Z}$. We can rephrase P(**a**,j) as "the ordered pair (**a**,j) belongs to S*".

7.5 Subgroups of Z^{7} for classification of physical quantities

One can form subgroups of $\boldsymbol{Z}^{7}$ relevant for classification of physical quantities in the following way.

Let the set $\boldsymbol{CL}_{(7-\boldsymbol{n},\boldsymbol{n})}$ be defined as
$\boldsymbol{CL}_{(7-\boldsymbol{n},\boldsymbol{n})} = \{(a_1, a_2, \ldots, a_7) |\ a_i \neq 0\ for\ i \leq 7-n,\ a_i = 0\ for\ i > 7-n\ and\ i, n = 1,2,\ldots,7\}$

So, we have the set $\boldsymbol{CL}_{\boldsymbol{6},\boldsymbol{1}} = \{\boldsymbol{a} | \boldsymbol{a} \in \boldsymbol{Z}^{7}\ and\ \ a_7 = 0\}$ with forms an Abelian group $CL_{6,1}, +$.

The same steps can be repeated for the subgroups: $\boldsymbol{CL}_{\boldsymbol{5},\boldsymbol{2}}, +$; $\boldsymbol{CL}_{\boldsymbol{4},\boldsymbol{3}}, +$; $\boldsymbol{CL}_{\boldsymbol{3},\boldsymbol{4}}, +$; $\boldsymbol{CL}_{\boldsymbol{2},\boldsymbol{5}}, +$; $\boldsymbol{CL}_{\boldsymbol{1},\boldsymbol{6}}, +$; $\boldsymbol{CL}_{\boldsymbol{0},\boldsymbol{7}}, +$.

The subgroup $\boldsymbol{CL}_{\boldsymbol{0},\boldsymbol{7}}, +$ contains all the physical quantities that are dimensionless products.

The subgroup $\boldsymbol{CL}_{\boldsymbol{1},\boldsymbol{6}}, +$ contains all the multiples of the physical quantity *length* (1,0,0,0,0,0,0).

The subgroup $\boldsymbol{CL}_{\boldsymbol{2},\boldsymbol{5}}, +$ contains all the physical quantities that are formed by combinations of *length* (1,0,0,0,0,0,0) and *mass* (0,1,0,0,0,0,0).

The subgroup $\boldsymbol{CL}_{\boldsymbol{3},\boldsymbol{4}}, +$ contains all the physical quantities that are formed by combinations of *length* (1,0,0,0,0,0,0) , *mass* (0,1,0,0,0,0,0) and *time* (0,0,1,0,0,0,0).

The subgroup $\boldsymbol{CL}_{\boldsymbol{4},\boldsymbol{3}}, +$ contains all the physical quantities that are formed by combinations of *length* (1,0,0,0,0,0,0) , *mass* (0,1,0,0,0,0,0) , *time* (0,0,1,0,0,0,0) and *current* (0,0,0,1,0,0,0).

The subgroup $\boldsymbol{CL}_{\boldsymbol{5},\boldsymbol{2}}, +$ contains all the physical quantities that are formed by combinations of *length* (1,0,0,0,0,0,0) , *mass* (0,1,0,0,0,0,0) , *time* (0,0,1,0,0,0,0), *current* (0,0,0,1,0,0,0) and *temperature* (0,0,0,0,1,0,0).

The subgroup $\boldsymbol{CL}_{\boldsymbol{6},\boldsymbol{1}}, +$ contains all the physical quantities that are formed by combinations of *length* (1,0,0,0,0,0,0) , *mass* (0,1,0,0,0,0,0) , *time* (0,0,1,0,0,0,0), *current* (0,0,0,1,0,0,0), *temperature* (0,0,0,0,1,0,0) and *amount of substance*(0,0,0,0,0,1,0).

7.6 Equivalence relation R as a classifier

Let $R = (\boldsymbol{Z}^7, \boldsymbol{Z}^7, P(\boldsymbol{a}, \boldsymbol{b}))$ be a relation in the set $\boldsymbol{Z}^7$. The propositional function is defined as $P(\boldsymbol{a}, \boldsymbol{b})$ = "$\boldsymbol{a}$ *and* $\boldsymbol{b}$ *are equivalent if* $SUM(\boldsymbol{a}) = SUM(\boldsymbol{b})$".

The relation R is reflexive if $(\boldsymbol{a}, \boldsymbol{a}) \in R$

The relation R is symmetric if $(\boldsymbol{a}, \boldsymbol{b}) \in R$ *implies* $(\boldsymbol{b}, \boldsymbol{a}) \in R$

The relation R is transitive if $(\boldsymbol{a}, \boldsymbol{b}) \in R$ *and* $(\boldsymbol{b}, \boldsymbol{c}) \in R$ *implies* $(\boldsymbol{a}, \boldsymbol{c}) \in R$

The relation R is an equivalence relation in the set Z^7.

The equivalence relation R in the set $\boldsymbol{Z}^7$ partitions the set $\boldsymbol{Z}^7$ by putting the physical quantities which are related to each other in the same equivalence class.

The set $C_\alpha = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{\alpha}) \in R\}$ is the equivalence class determined by **α**.

The set of equivalence classes is denoted $\boldsymbol{Z}^7/R$ and called the quotient set.

The set $C_0 = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{0}) \in R\}$ is the equivalence class determined by **0**.

The vector **L =** (1,0,0,0,0,0,0), representing the quantity length, is selected to represent the set $C_{\boldsymbol{L}} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{L}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{\boldsymbol{L}}$ so that SUM(**a)** = 1.

The vector **A =** (2,0,0,0,0,0,0), representing the quantity area, is selected to represent the set

$C_{\boldsymbol{A}} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{A}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{\boldsymbol{A}}$ so that SUM(**a)** = 2.

The vector **V =** (3,0,0,0,0,0,0), representing the quantity volume, is selected to represent the set

$C_{\boldsymbol{V}} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{V}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{\boldsymbol{V}}$ so that SUM(**a)** = 3.

The vector **k =** (-1,0,0,0,0,0,0), representing the quantity wave vector, is selected to represent the set

$C_{\boldsymbol{k}} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{k}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{\boldsymbol{k}}$ so that SUM(**a)** = -1.

The vector **P =** (-1,1,-2,0,0,0,0), representing the quantity pressure, is selected to represent the set

$C_{\boldsymbol{P}} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{P}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{\boldsymbol{P}}$ so that SUM(**a)** = -2.

The vector **σ =** (0,1,-3,0,-4,0,0), representing the quantity Stefan-Boltzmann constant, is selected to represent the set

$C_{\sigma} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{\sigma}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{\sigma}$ so that SUM(**a) =** -6.

$C_{QM} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{QM}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{QM}$ so that SUM(**a) = 4**.

$C_{EP} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{EP}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{EP}$ so that SUM(**a) = 5**.

$C_{1HP} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{1HP}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{1HP}$ so that SUM(**a) = 7**.

$C_{2HP} = \{\boldsymbol{x} \mid (\boldsymbol{x}, \boldsymbol{2HP}) \in R\}$.

In this equivalence class one has $\boldsymbol{a} \in C_{2HP}$ so that SUM(**a) = 9**.

Classification of physical quantities

SUM	CL07	CL16	CL25	CL34	CL43	CL52	CL61	Z7
-6						Stefan-Boltzmann constant		
-5								
-4								
-3		Loschmidt constant			atomic unit of electric field gradient			
-2		Space-time curvature	Newtonian constant of gravitation over h-bar c	Energy density, Pressure, Energy-momentum tensor, Fermi coupling constant	Magnetic flux density, Magnetic constant, Electrical resistance, Characteristic Impedance of vacuum, von Klitzing constant		Amount of substance concentration	
-1		Wave number		Frequency, activity, Acceleration, vorticity	Inductance, Electrical potential difference	Current density	Avogadro constant, Molar gas constant, alpha particle molar mass	Luminance
0	Plane angle, solid angle			Velocity, Mass frequency, Force, Absorbed dose, Dose equivalent, Specific energy, Newtonian constant of gravitation, Power	Magnetic field, Charge surface density, Electrical polarisation, Magnetic induction, Magnetic moment, Specific resistance, Josephson constant, mag. flux quantum, elementary charge over h	Entropy, Specific heat, Boltzmann constant	Catalytic activity	
1		Length	Mass	Time, Linear momentum, Diffusion constant, Energy, vorticity flux	Electric current	Thermodynamic temperature	Faraday constant	Luminous flux
2		Area, Thomson cross section		Planck constant, Specific volume, first radiation constant, angular momentum	Electric charge, Electric constant, conductance quantum	second radiation constant	molar volume of ideal gas	
3		Volume			Electrical capacitance, atomic unit of electric dipole moment, Bohr magneton			
4					Atomic unit of electric quadrupole moment			
5					Atomic unit of electric polarizablity, atomic unit of magnetizability			
6								
7					Atomic unit of 1st hyperpolarizability			
8								
9					Atomic unit of 2^{nd} hyperpolarizability			

7.7 Classification using Principal Component Analysis (PCA)

We will apply the PCA-method (or equivalently the Singular Value Decomposition method) on the integer lattice $\boldsymbol{Z}^7$ and project the coordinates of the physical quantities on 3D. For this purpose we will use the program VisuMap. VisuMap implements PCA to project high dimensional dataset to 3D space.

PCA is the simplest of the true eigenvector-based multivariate analyses. Often, its operation can be thought of as revealing the internal structure of the data in a way which best explains the variance in the data. If a multivariate dataset is visualised as a set of coordinates in a high-dimensional data space (1 axis per variable), PCA supplies the user with a lower-dimensional picture, a "shadow" of this object when viewed from its (in some sense) most informative viewpoint.

The result of the projection on the XY-plane is given in Figure 7.7-1.

One can recognize a grid of straight lines grouping physical quantities.

The eigenvalues and eigenvectors are

Eigenvalues	C1	C2	C3	C4	C5	C6	C7
8.74003	-0.380847	-0.245838	0.827484	0.32897	0.0387645	0.00725913	0.00364446
1.70506	0.919703	-0.17726	0.317277	0.132191	0.0297565	-0.0523389	-0.0309959
0.449004	-0.0110769	-0.0620704	0.101363	-0.421437	0.896498	0.0635429	0.0198704
0.191312	0.0352802	0.388944	0.436249	-0.722168	-0.355369	-0.0934721	0.0247813
0.152376	-0.0456714	-0.837368	-0.0969575	-0.410073	-0.257398	0.214653	0.0826978
0.0727539	-0.061773	-0.209711	-0.0668717	-0.0447139	0.0360651	-0.960526	0.147729
0.0364027	-0.0428044	-0.0885934	-0.0120772	-0.0707795	-0.00785344	-0.125489	-0.984558

The PCA view can also be rotated which results in interesting information as shown in Figure 7.7-2.

One can remark that by overlapping points A and B of physical quantities in the map and observing the changes close to the origin reveals that any quantity C that coincides in projection with the origin forms a set of dimensionless products. This is demonstrated in Figure 7.7-4 where the overlapping of E(energy) and m(mass), which automatically creates an overlap with p(linear momentum), moves the quantity v(velocity) to overlap with the origin. So, the quantities (E,m,p,v) form a dimensionless set. One can conclude that the PCA 3D view of the physical quantities shows graphically which sets of physical quantities from dimensionless products. The missing quantities in the set are these points that move in projection to the origin. If we try this for the electric constant and the magnetic constant which was done in Figure 7.7-3 we find no overlap with the origin. This is because the square of the velocity is not an element of the dataset. The electric constant, the magnetic constant and the square of the velocity form a dimensionless product.

One can recognize 7 classes visible in Figure 7.7-5,Figure 7.7-6 and Figure 7.7-7.

In the Figure 7.7-8 one can observe the overlap process for the quantity time as it approaches the origin. We can recognize the following dimensionless product sets using the graphical method:

Parameter **t** = { (G, specific volume, t), (Volume, c^3, t), (c^2, Diffusion constant, area, t), (Power, energy, h, t), (Potential difference, B, t), (Vacuum impedance, Inductance , t), (acceleration, velocity, length, t), (force, linear momentum , t), (current, e, t), (frequency, t), (mass frequency, t)} (see Figure 7.7-8.

Parameter **e** = {(space-time curvature, charge surface density, e), (magnetic flux density, mass frequency, e), (area, atomic unit of electric quadrupole moment, e), (vacuum impedance, B, h, e), (Potential difference, energy, e)} (see Figure 7.7-9)

Parameter **T** = {(Current density, space-time curvature, T), (energy, entropy, T)}

Parameter **H** = {(Length, current, H), (B, force, H), (Magnetic constant, Magnetic flux density, energy density, H), (Specific resistance, Power, H)}

Parameter **capacitance** = {(area, atomic unit of electric polarisation, capacitance), (vacuum impedance, t, capacitance), (wave number, electric constant, capacitance), (potential difference, e, capacitance)}

Parameter **magnetic constant** = {(length, inductance, magnetic constant), (velocity, vacuum impedance, magnetic constant), (H, magnetic flux density, magnetic constant)}

Parameter **mass** = {(t, mass frequency, mass), (acceleration, force, mass), (c^2, energy, mass), (velocity, linear momentum, mass), (diffusion constant, h, mass), (volume, specific volume, mass)}

Parameter **force** = {(space-time curvature, energy-density, force), (t, linear momentum, force), (velocity, power, force), (length, energy, force), (G, c^4, force)}

Parameter **linear momentum** = {(wave number, mass frequency, linear momentum), (force, frequency, linear momentum), (power, acceleration, linear momentum), (velocity, energy, linear momentum), (length, h, linear momentum)}

Parameter **potential difference** = {(atomic unit of electric quadrupole moment, atomic unit of electric polarisation, potential difference), (energy, e, capacitance, potential difference), (B, t, potential difference), (power, current, potential difference)}

Parameter **energy** ={(force, wave number, energy), (h, t, energy), (power, frequency, energy)}

Parameter **h** = {(space-time curvature, mass frequency, h), (linear momentum, wave number, h), (energy, frequency, h)}

Parameter **energy-density** = {(force, area, energy-density), (energy, volume, energy-density)}

Parameter **G** = {(mass-frequency, c^3, G), (force, c^4, G)}

Parameter **diffusion constant** = {(h, mass, diffusion constant), (velocity, wave number, diffusion constant), (c^2, frequency, space-time curvature, diffusion constant), (energy, mass frequency, diffusion constant), (area, t, diffusion constant), (current, charge surface density, diffusion constant), (potential difference, magnetic flux density, diffusion constant), (acceleration, c^3, diffusion constant)}

Parameter **velocity** = {(diffusion constant, length, t, velocity), (H, charge surface density, velocity), (power, force, mass frequency, velocity), (energy, linear momentum, mass, velocity), (specific resistance, B, velocity), (vacuum impedance, magnetic constant, velocity)}

Parameter **atomic unit of electric polarisation** = {(capacitance, space-time curvature, atomic unit of electric polarisation), (potential difference, atomic unit of electric quadrupole moment, atomic unit of electrical polarisation)}

Parameter **acceleration** = {(c^4, area, acceleration), (c^3, diffusion constant, acceleration), (c^2, length, acceleration), (velocity, t, acceleration), (linear momentum, power, acceleration), (force, mass, acceleration)}

Parameter **current** = {(e, t, current), (H, wave number, current), (force, B, inductance, current), (power, potential difference, vacuum impedance, current)}

Parameter **length** = {(electric constant, capacitance, atomic unit of electric polarisation, length), (charge surface density, e, atomic unit of electric quadrupole moment, length), (H, current, length), (current density, temperature, length), (frequency, velocity, diffusion constant, length), (mass frequency, linear momentum, h, length), (acceleration, c^2, length), (energy density, force, energy, length), (magnetic flux density, B, length), (magnetic constant, inductance, length), (potential difference, specific resistance, length)} (see Figure 7.7-10)

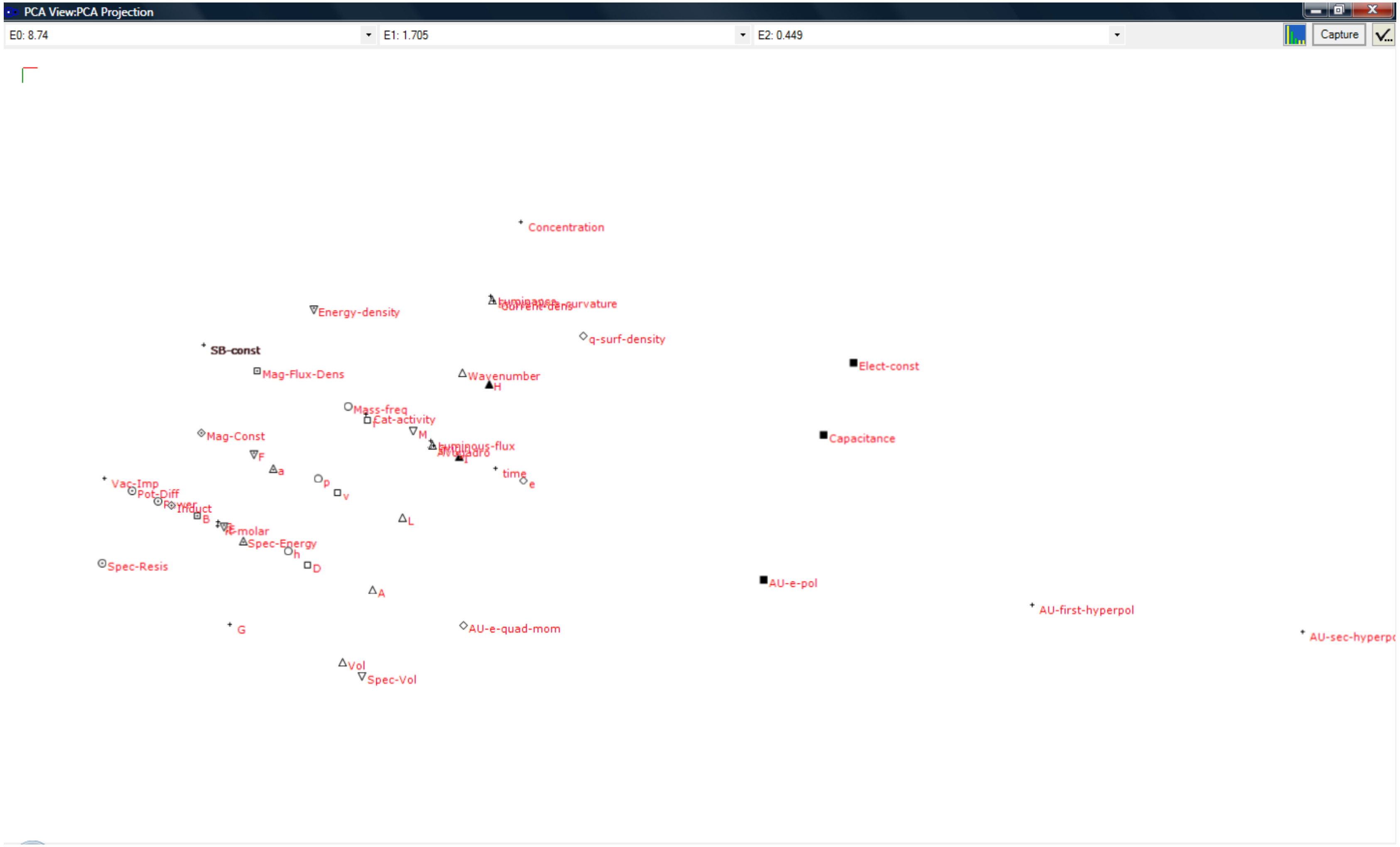


Figure 7.7-1 PCA projection of physical quantities on 3D

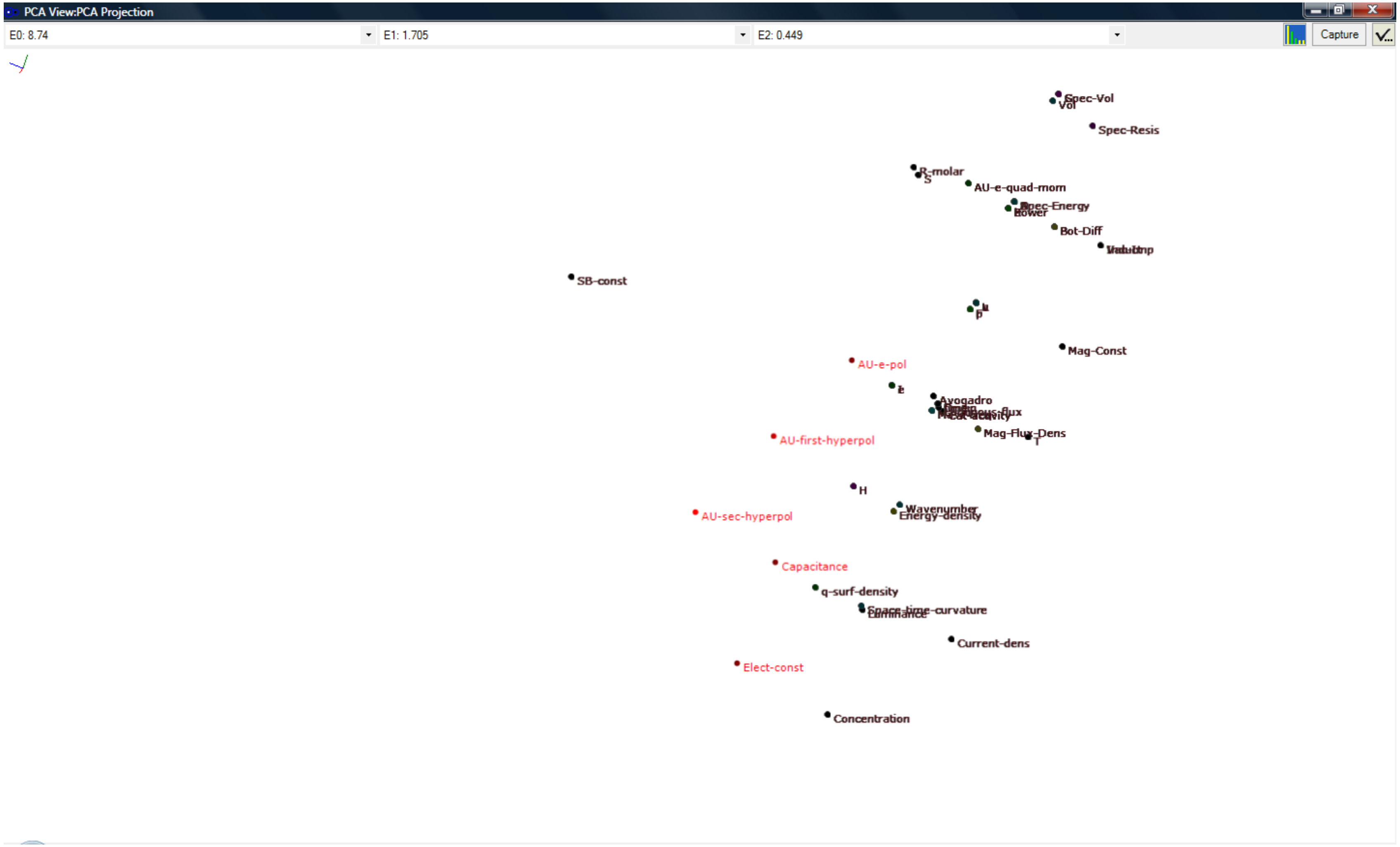


Figure 7.7-2 Rotated view of PCA projection of physical quantities on 3D

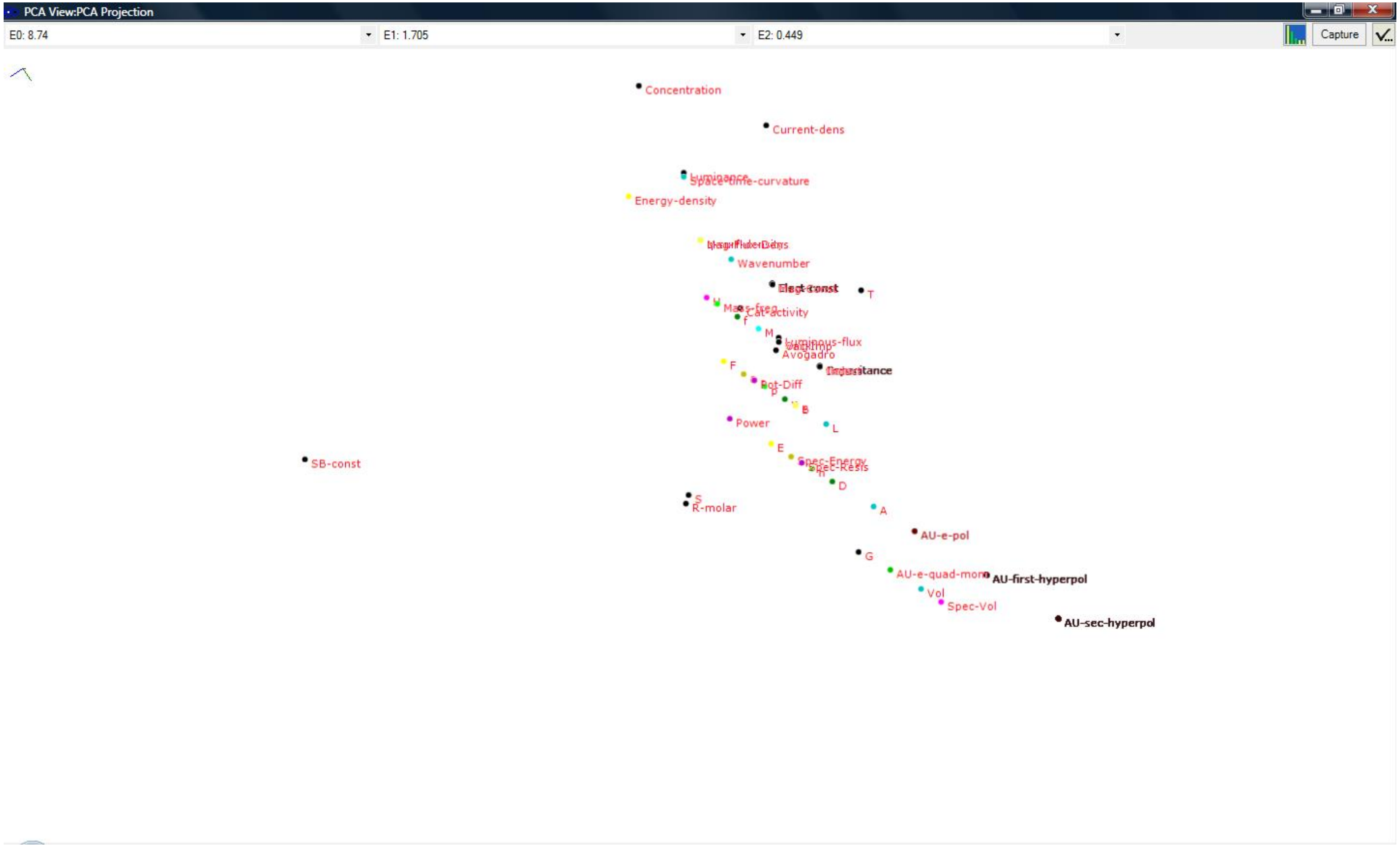


Figure 7.7-3 Rotated view of PCA projection of physical quantities on 3D with overlap of electric constant and magnetic constant

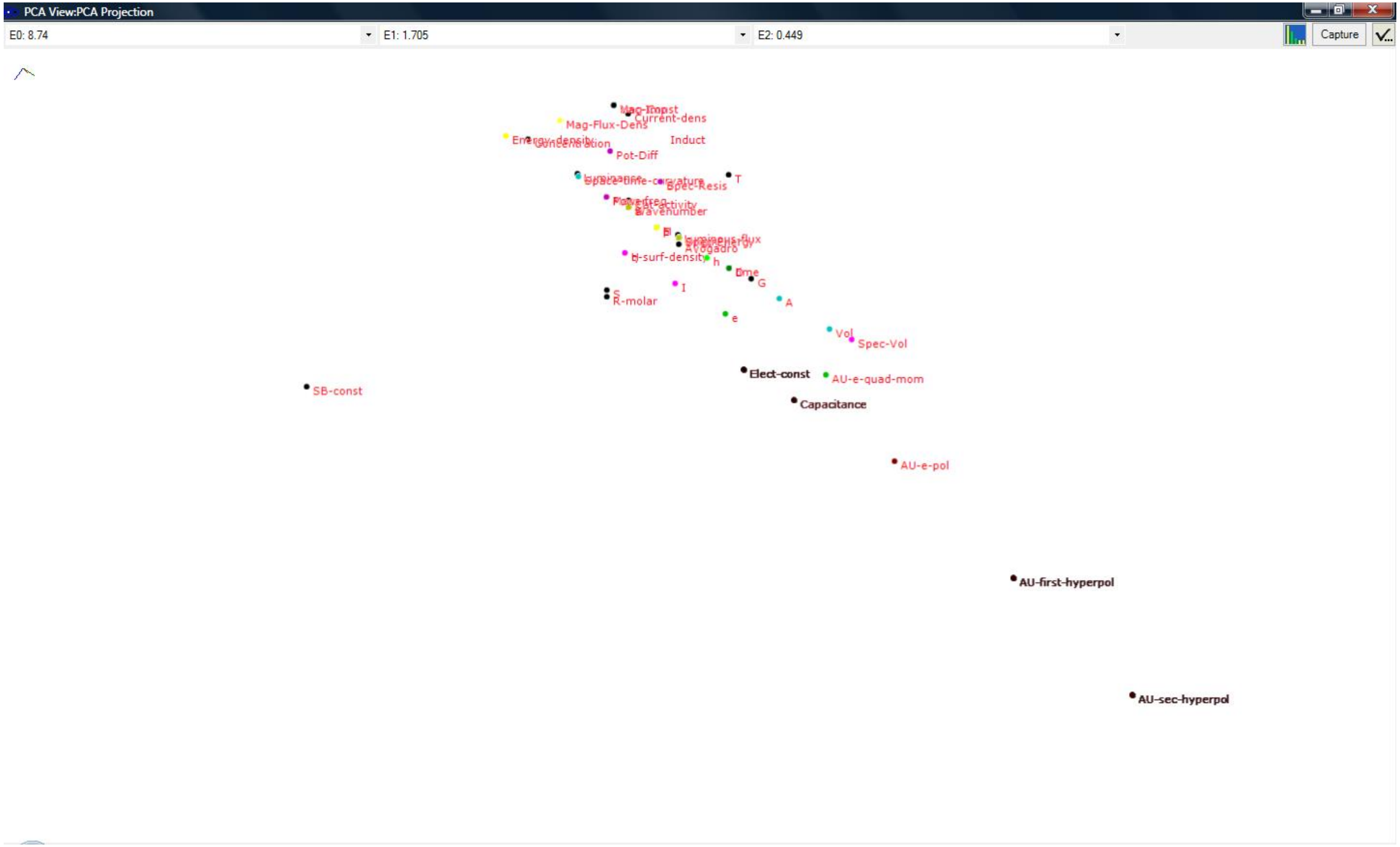


Figure 7.7-4 Rotated view of PCA projection of physical quantities on 3D with overlap of energy, mass and linear momentum

Figure 7.7-5 Rotated view of PCA projection of physical quantities on 3D showing the 7 classes as horizontal layers

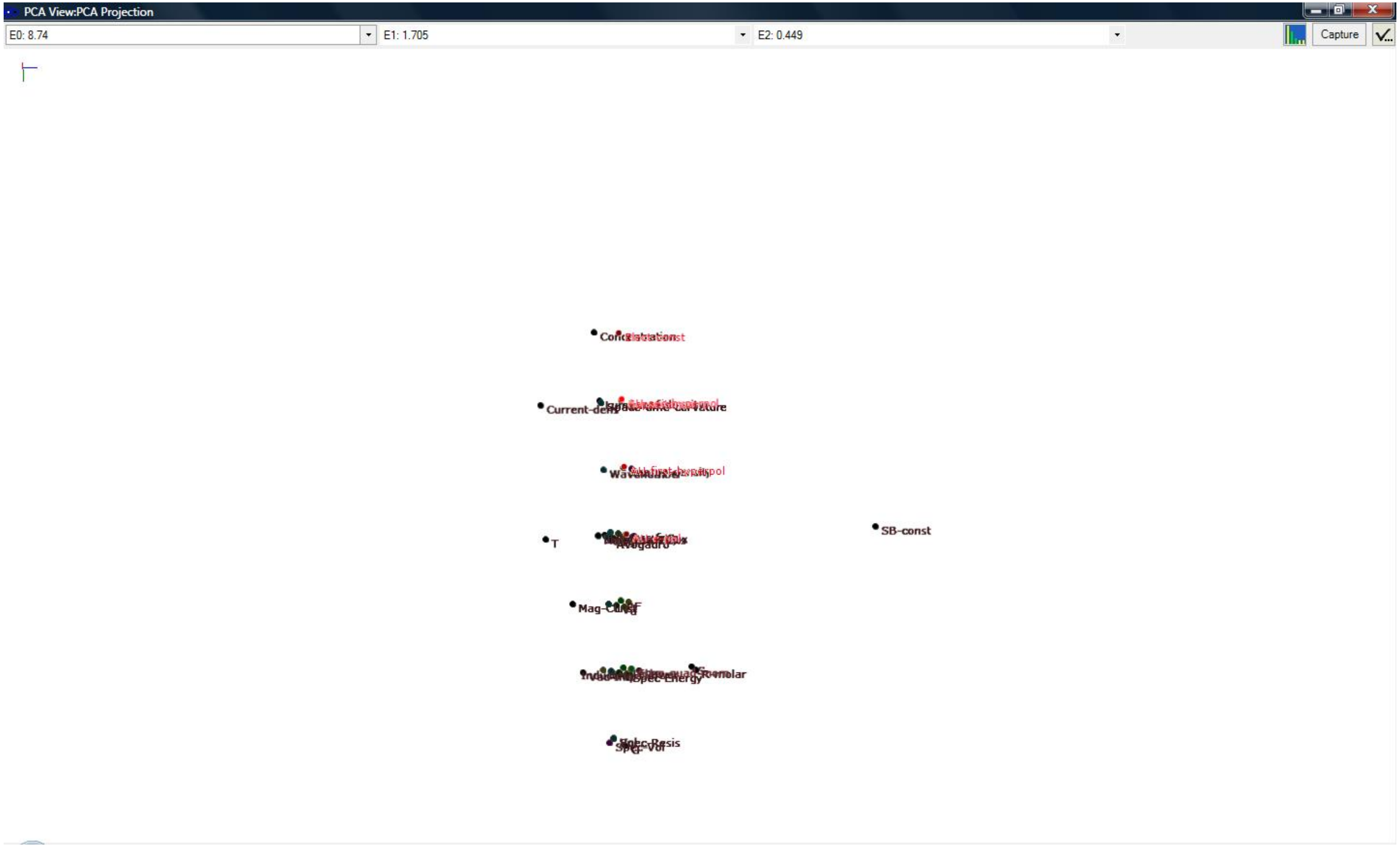


Figure 7.7-6 Rotated view of PCA projection of physical quantities on 3D showing the 7 classes as horizontal layers compacted

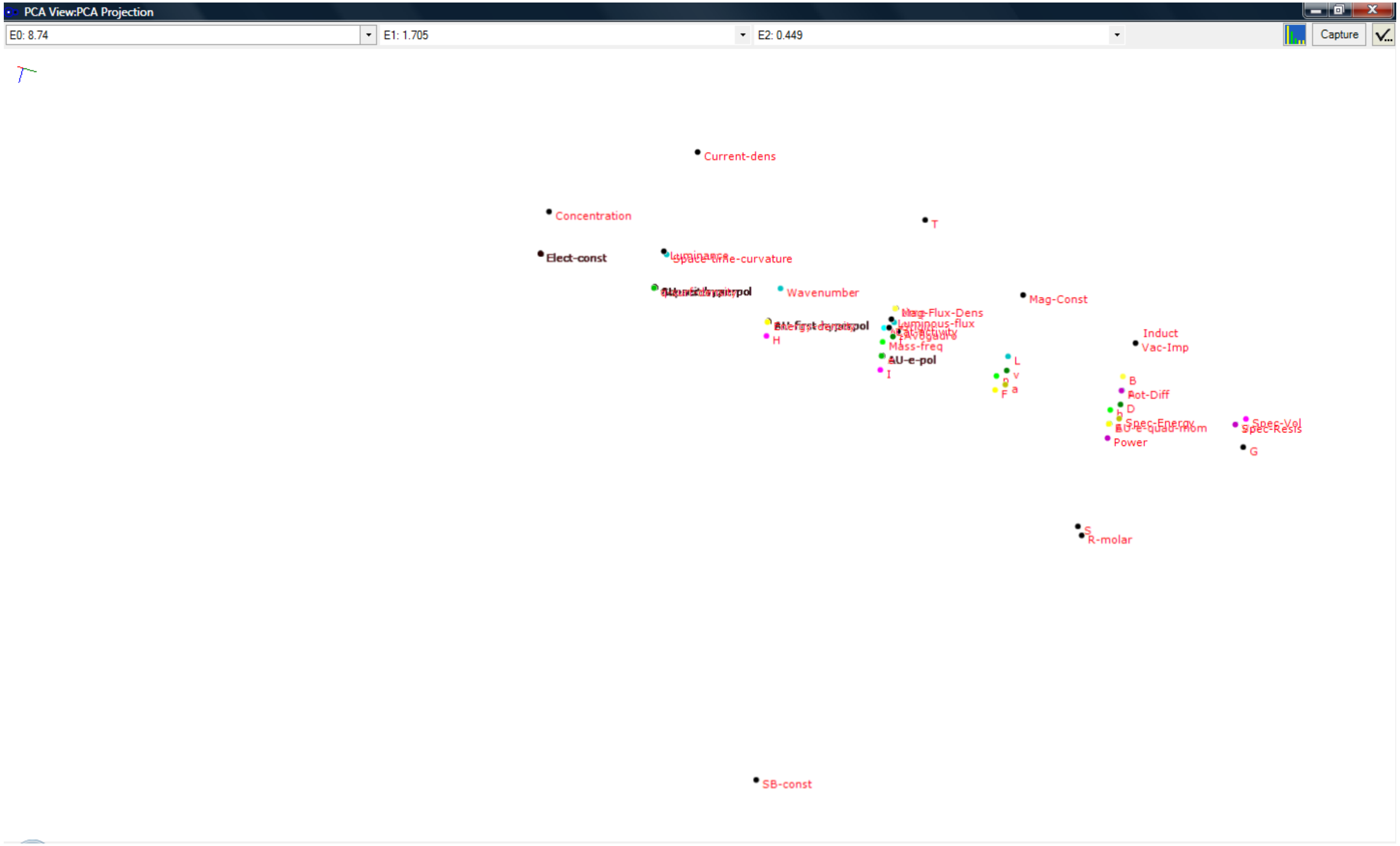


Figure 7.7-7 View of PCA projection of physical quantities on 3D showing the 7 classes

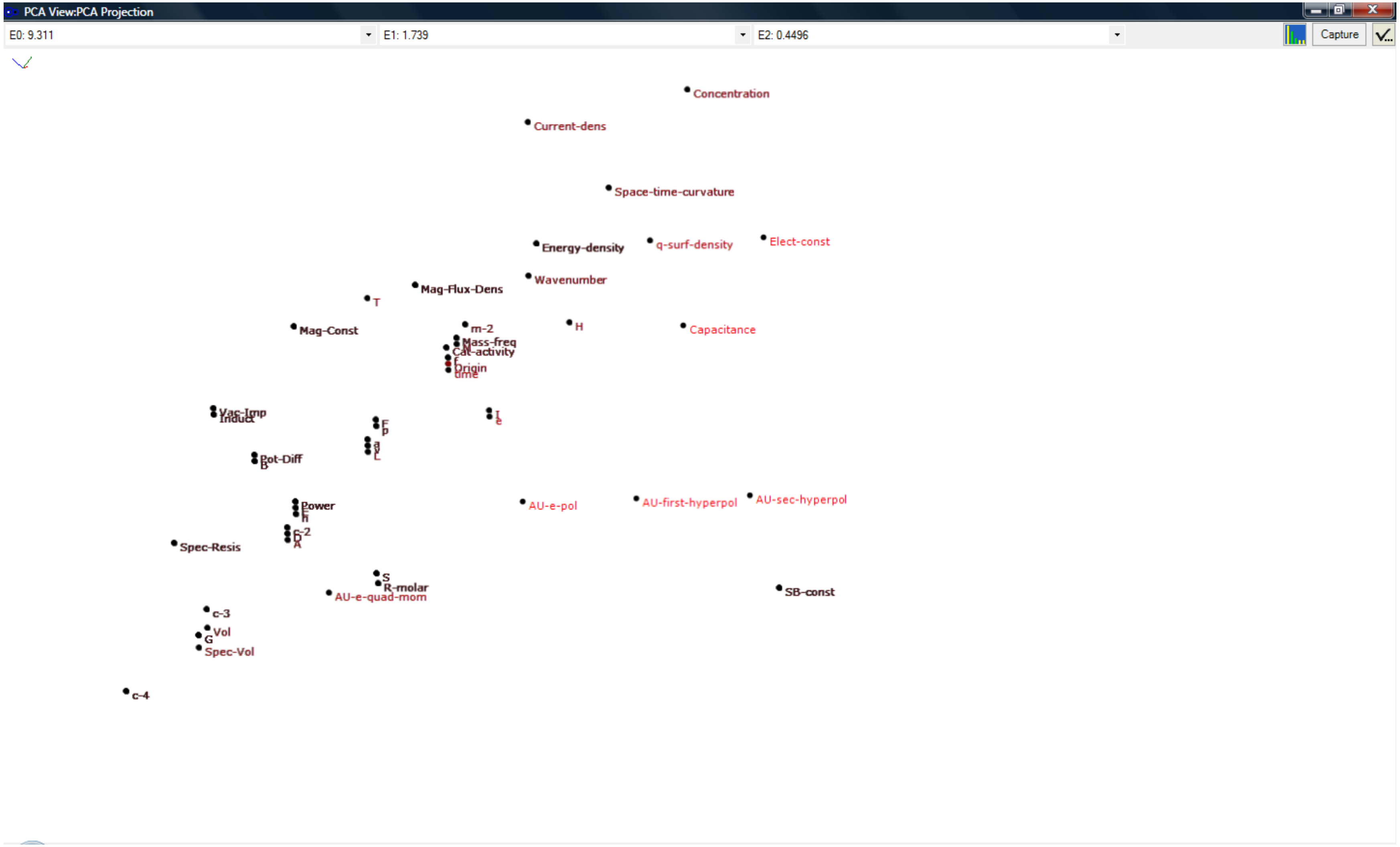


Figure 7.7-8 View of PCA projection of physical quantities on 3D with overlap of time quantity

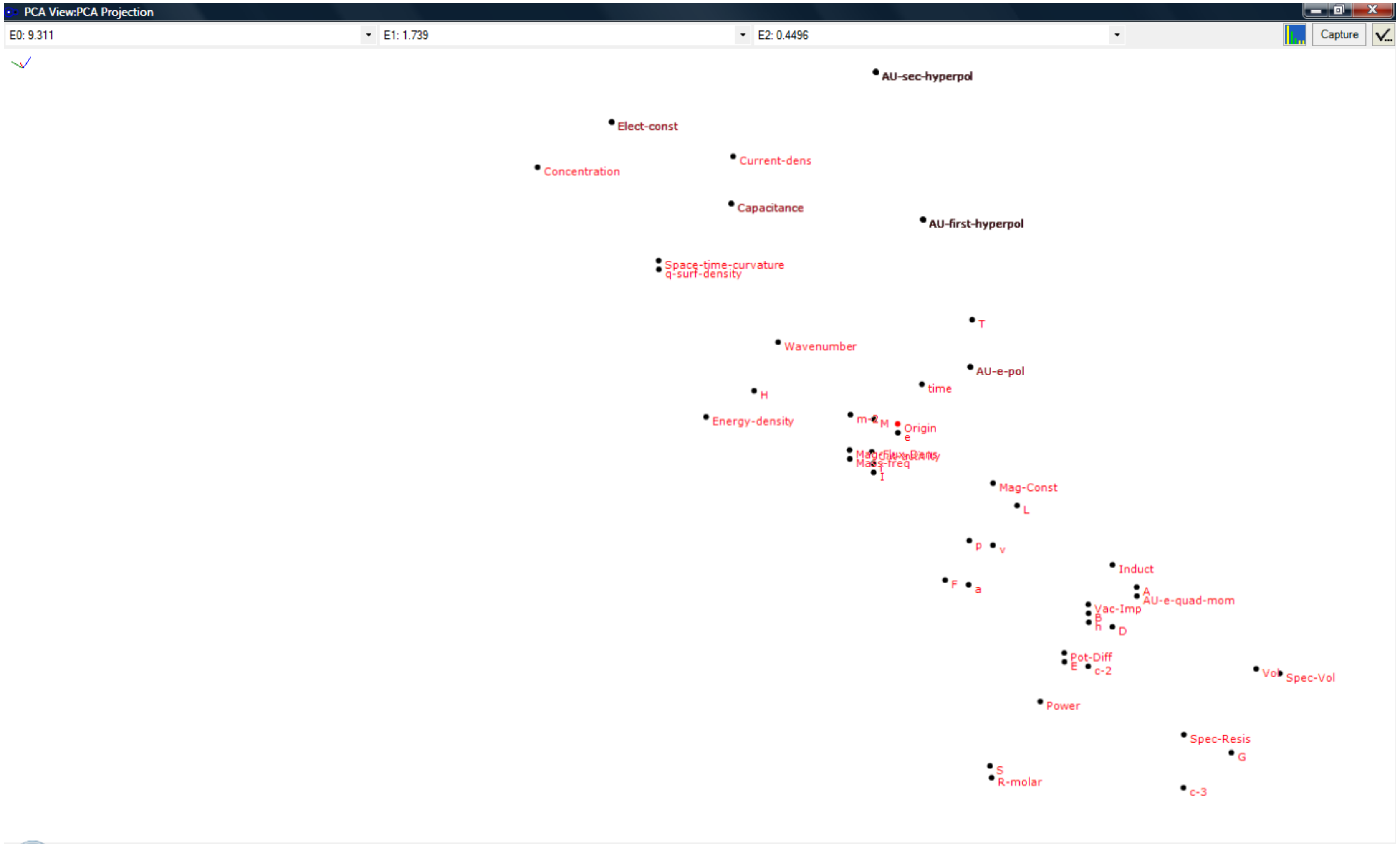


Figure 7.7-9 View of PCA projection of physical quantities on 3D with overlap of charge quantity

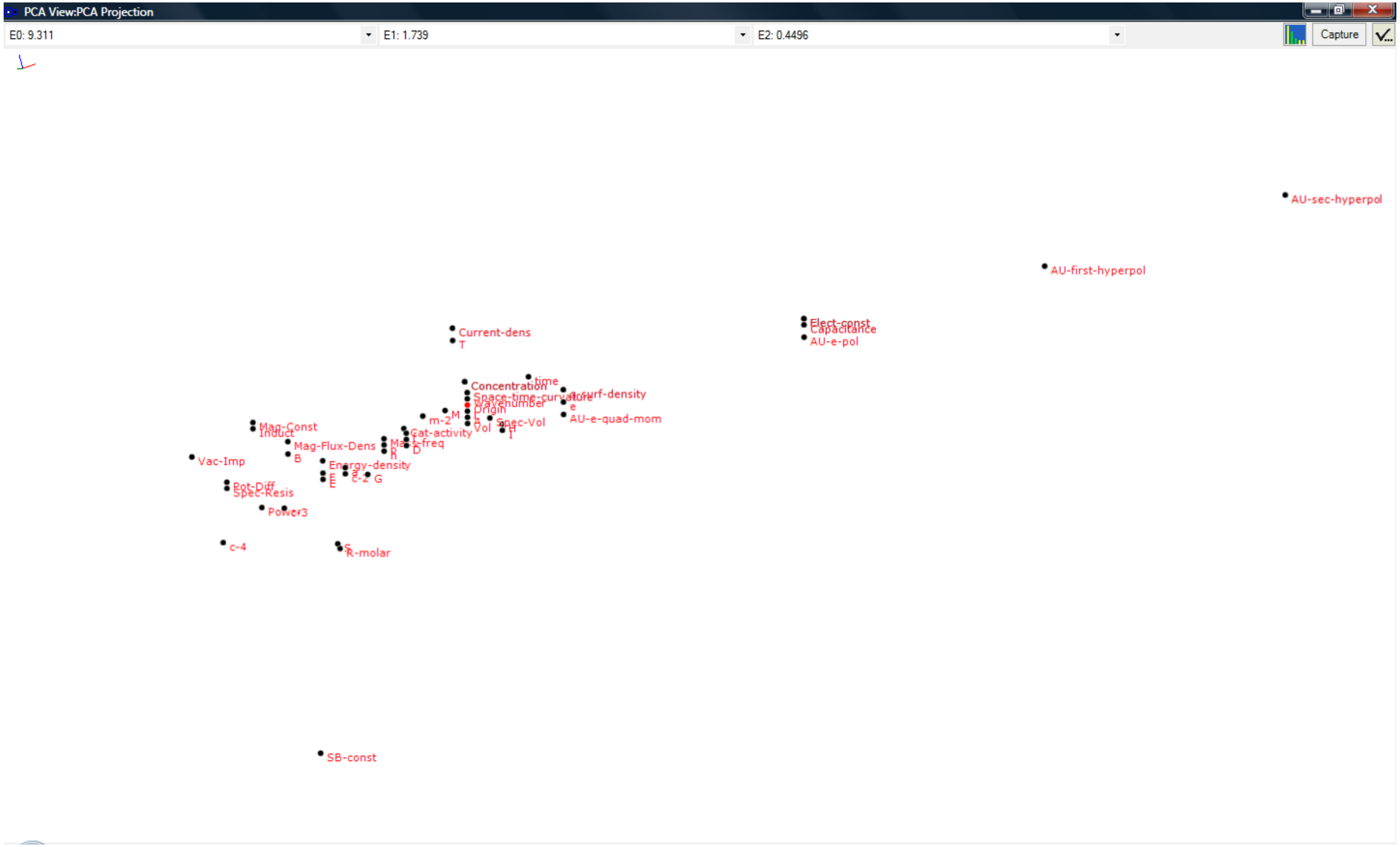


Figure 7.7-10 View of PCA projection of physical quantities on 3D with overlap of length quantity

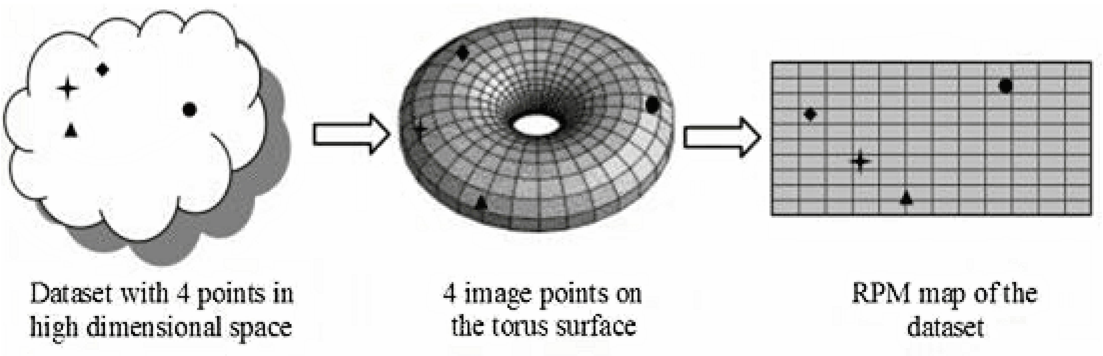
7.8 Classification using RPM mapping

We will apply the RPM-method on the integer lattice $\boldsymbol{Z}^7$ and project the coordinates of the physical quantities on 3D-torus. For this purpose we will also use the program VisuMap.

Relational perspective map (RPM), developed by James X. Li(Li, 2004), is a general purpose method to visualize distance information of data points in high dimensional spaces.

The starting point of the RPM algorithm is a set of data point s_i, i=1,...,N, and a distance matrix δ_{ij}. The matrix δ_{ij}, called the relational distance, is the numeric representation of a relationship between the data points. The goal of the RPM algorithm is to map the data points s_i into a two or three dimensional map so that Euclidean distances d_{ij} between the image points visually approaches δ_{ij} as much as possible. The resulting lower dimensional map is called relational perspective map, the matrix d_{ij} is called the image distance matrix. From geometric point of view, a RPM map attempts to preserve as much as possible distance information of the original dataset.

The following picture shows the RPM algorithm works to create 2D maps: it first maps data points to the surface of a torus, then unfolds the torus surface by a vertical and a horizontal cut. The second step is more or less straightforward, so the RPM algorithm focus on how to map the data points to the torus surface so that the distances between the image points resembles the distances between data points.



In order the find the best mapping RPM algorithm defines an energy function as follows:

$$E_P := \sum_{i<j} \frac{\delta_{ij}}{p d_{ij}^{P}} \quad \text{with} \quad E_0 := -\sum_{i<j} \delta_{ij} \ln(d_{ij})$$

where p is an algorithm parameter called the rigidity, d_{ij} is the geodesic block distance between two image points on the torus surface. The RPM algorithm then uses gradient descent optimization method to find a configuration with minimum energy. The rigidity parameter, which is normally a value between -1 and +1, alters the energy landscape in a global manner, so that the resulting RPM maps have different characteristics.

To better understand the RPM algorithm it is helpful to consider the image points on the torus as a force directed multi-particle system with mutual repulsive forces between them; and consider the energy E_p as a kind of total potential energy. According to physical law the repulsive force is characterized by following form:

$$\frac{\partial E_p}{\partial d_{ij}} = -\frac{\delta_{ij}}{d_{ij}^{p+1}}$$

Above form says that the repulsive force between two points is proportional to their relational distance. Thus the process to minimize the energy E_p is actually a process that simulates the dynamic system directed by the force defined by above form. Since points with larger relational distances between them correspond to larger repulsive force on the torus, their image points on the torus should be further apart from each other.

The key idea of RPM algorithm, that distinguish it from other known algorithms like those listed in the next section, is that RPM successfully exploited the property of closed manifold (the torus) to keep the configuration in balance. Whereas other non-linear methods apply, directly or indirectly, attractive force to map closely related points to closely located positions, RPM algorithm maps closely related points to closely location area by the collective repulsive force of all points. This characteristics make RPM the true, and the only (as far as we know), global mapping algorithm.

It should be noted here that RPM algorithm made a significant relaxation to the original problem setting: the resulting map is not a normal rectangle map, but a map on the torus. That means the opposite edges of the map should be considered as stuck with each other.

The result of the projection is given in Figure 7.8-1.

This method doesn’t seem to reveal structure.

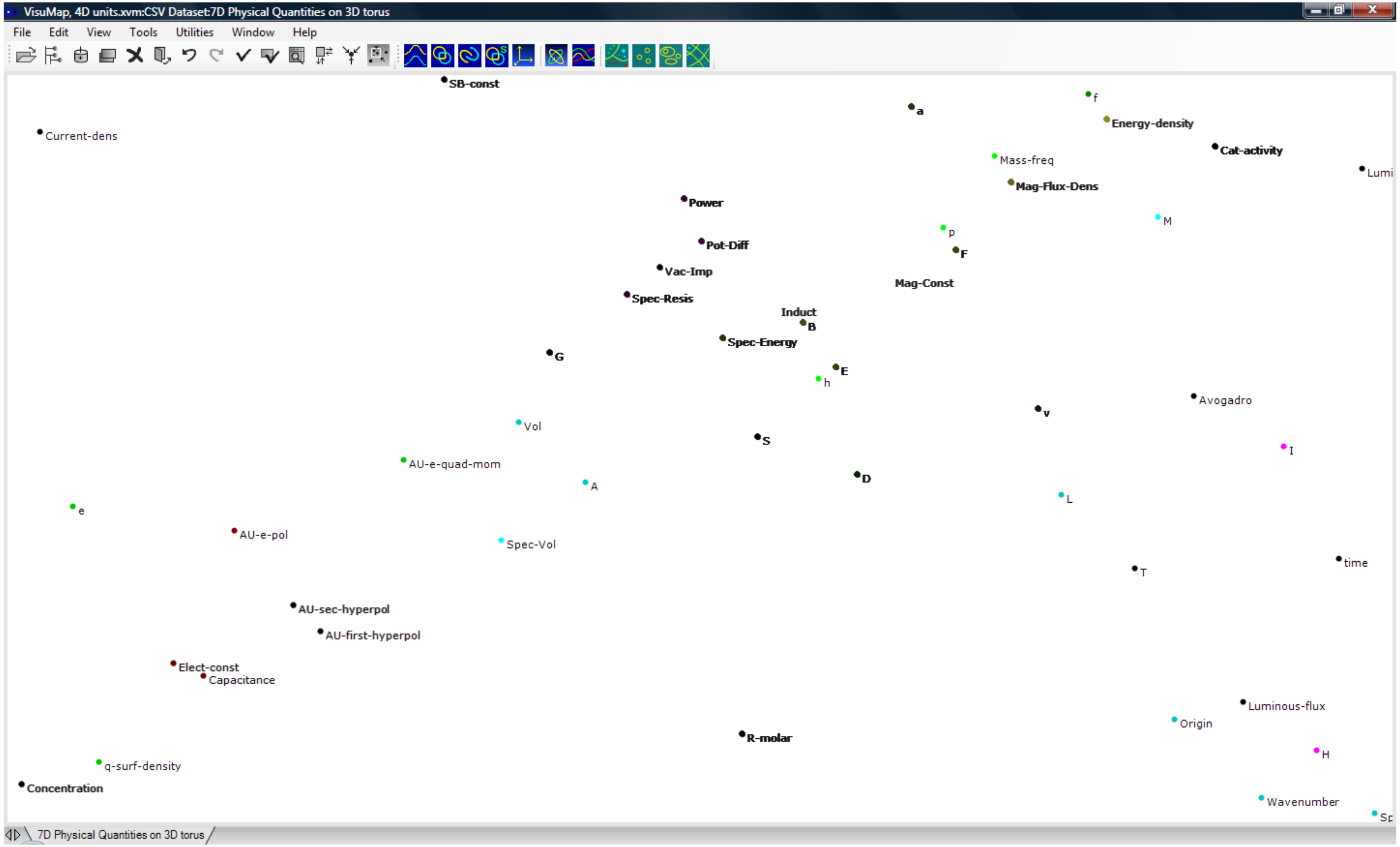


Figure 7.8-1 Projection of physical quantities on 3D-torus

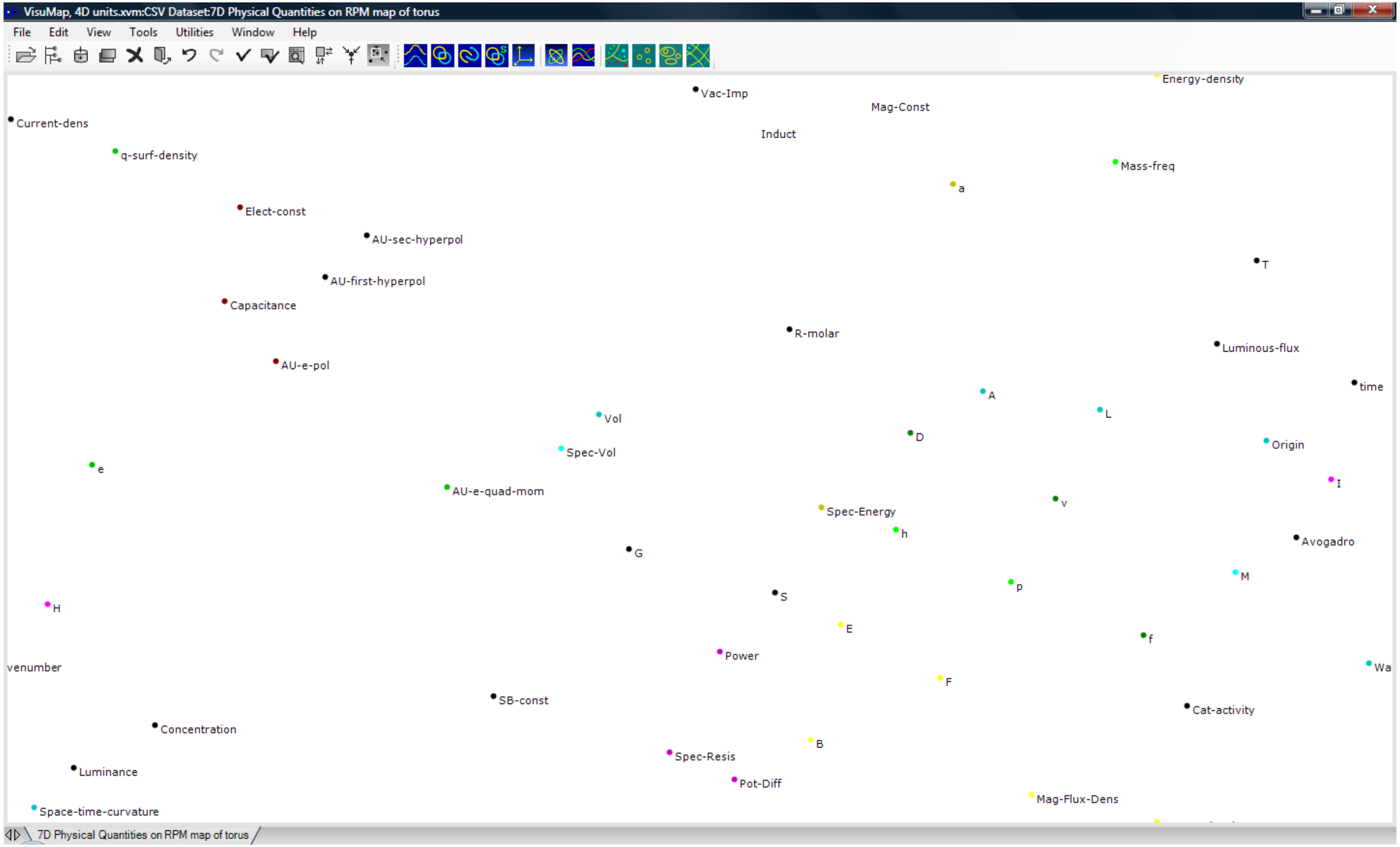


Figure 7.8-2 Projection of physical quantities on torus(unfolded)

7.9 Classification using Sammon map

We will apply the Sammon(Sammon, 1969) map on the integer lattice $\boldsymbol{Z}^7$ and project the coordinates of the physical quantities. For this purpose we will also use the program VisuMap.

The result of the projection on a rectangle is given in Figure 7.9-1.

The result of the projection on a cube is given in Figure 7.9-2.

This method reveals some structure. One can observe also a grid of straight lines.

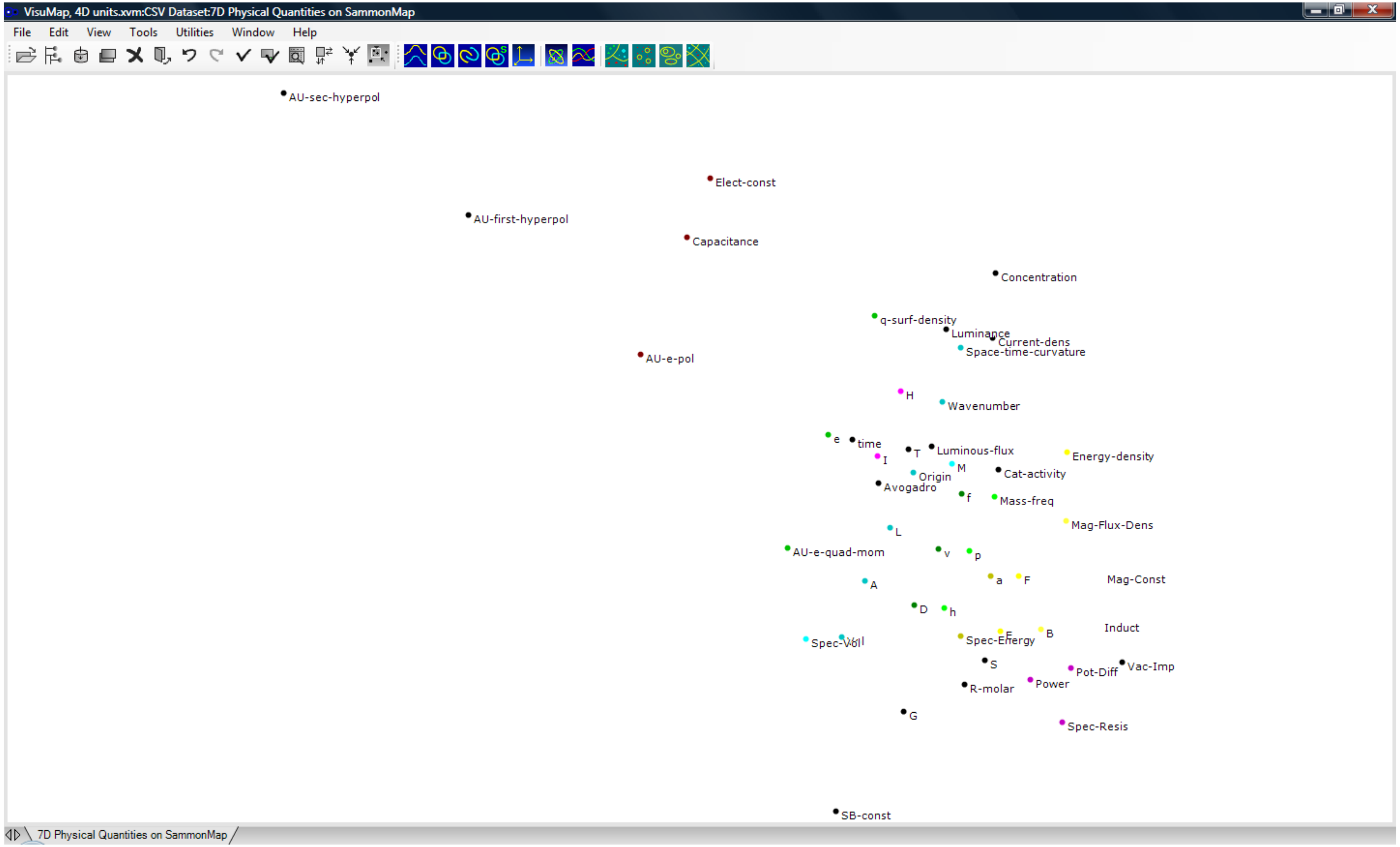


Figure 7.9-1 Projection of physical quantities on Sammon map

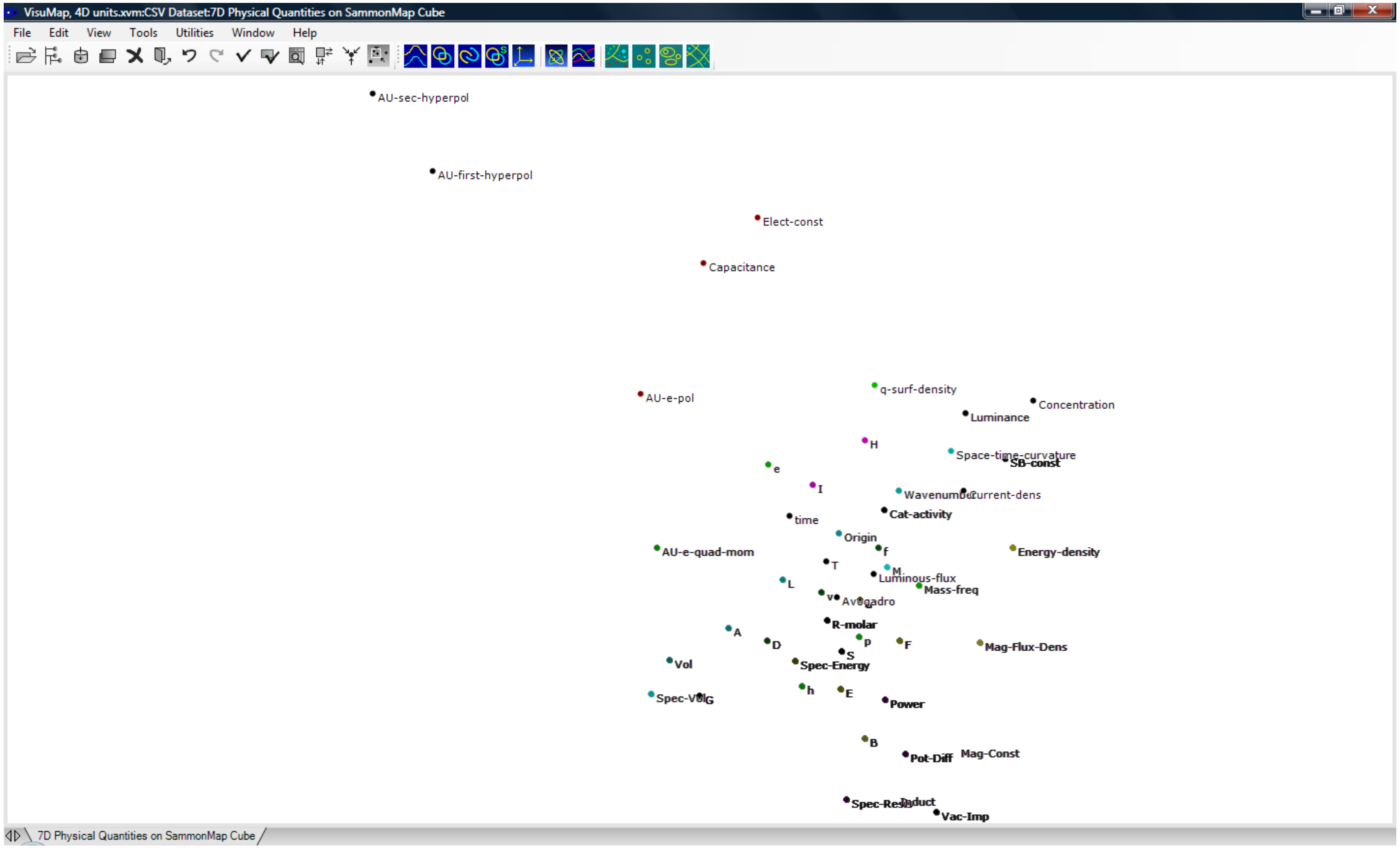


Figure 7.9-2 Projection of physical quantities on Sammon map on a cube

7.10 Classification using Curvilinear Component Analysis

We will apply the Curvilinear Component Analysis (CCA) on the integer lattice $\boldsymbol{Z}^7$ and project the coordinates of the physical quantities. For this purpose we will also use the program VisuMap.

Curvilinear Component Analysis (CCA) algorithm is a variation of the Sammon algorithm that tries to preserve more short distance information (local information) while relaxing the constraints posed by long distance information. Mathematically, CCA uses the gradient descent algorithm to minimize the following stress function:

$$E := \sum_{i<j} (\delta_{ij} - d_{ij})^2 F(d_{ij}, \lambda_t) \quad \text{with} \quad F(d_{ij}, \lambda_t) := \begin{cases} 1 & \text{if } d_{ij} \leq \lambda_t, \\ 0 & \text{if } d_{ij} > \lambda_t, \end{cases}$$

Where δ_{ij} and d_{ij} are respectively the relational distance and image distance between two bodies i and j. λ_t is a time dependent parameter that changes from a given initial value λ_t gradually to 0 during the optimization process.

The result of the projection on a rectangle is given in Figure 7.10-1

The result of the projection on a cube is given in Figure 7.10-2

This method reveals some structure. One can observe also a grid of straight lines.

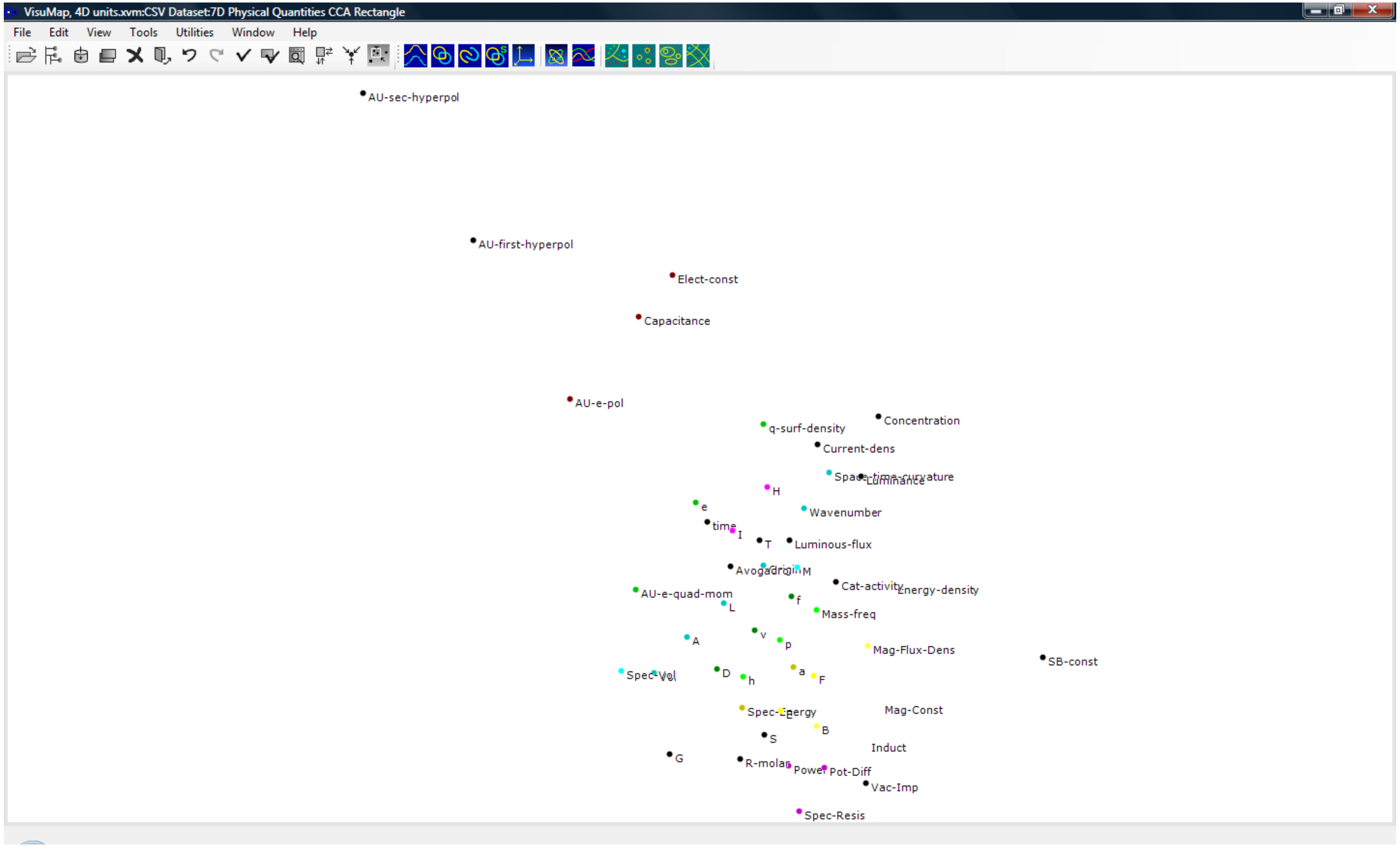


Figure 7.10-1 Projection of physical quantities on CCA rectangle

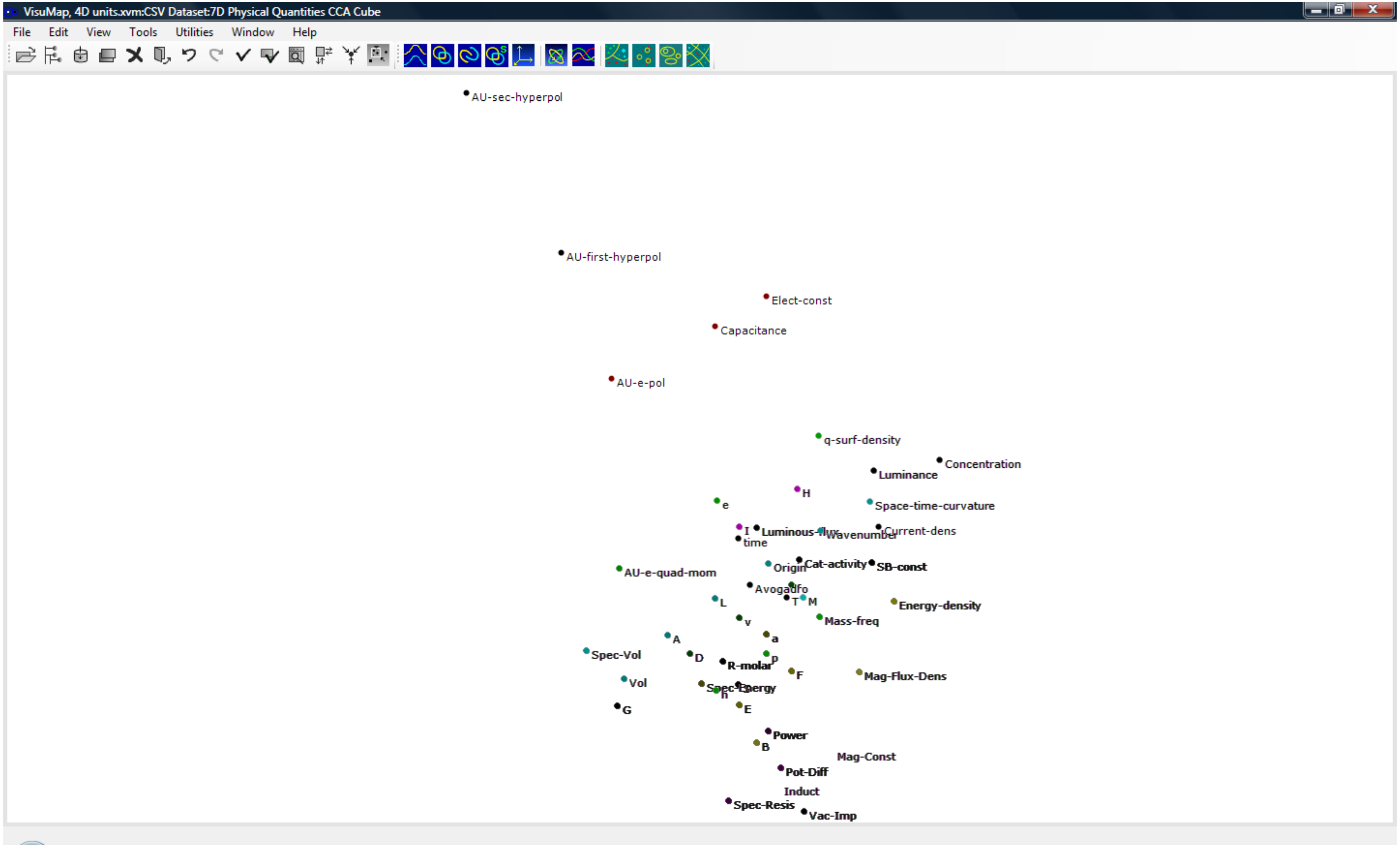


Figure 7.10-2 Projection of physical quantities on CCA cube

7.11 Graph of dimensionless products in Z^7

A dimensionless product as the **fine-structure constant**(Peter J. Mohr, 2008) $\alpha = \frac{e^2}{2\varepsilon_0 hc} =$ $7.297\ 352\ 5376 \times 10^{-3}$ can be represented by the sum of vectors in the following way:

$$\mathbf{0} = 2\boldsymbol{e} - \boldsymbol{\epsilon} - \boldsymbol{h} - \boldsymbol{c}$$

Explicitly we have the following 7-tuples:

$2\boldsymbol{e} = (0,0,2,2,0,0,0)$, $\boldsymbol{\epsilon} = (-3,-1,4,2,0,0,0)$, $\boldsymbol{h} = (2,1,-1,0,0,0,0)$, $\boldsymbol{c} = (1,0,-1,0,0,0,0)$

$\mathbf{0} = (0+3-2-1, 0+1-1-0, 2-4+1+1, 0,0,0)$

The zero vector is giving *no useful information* over the dimensionless product. To alleviate to this problem we will use graph theory(Epp, 1993)(Diestel, 2000) in the integer lattice $\boldsymbol{Z}^7$.

Graph of $\frac{e^2}{2\varepsilon_0 hc}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
2e	0	0	2	2	0	0	0		
epsilon	-3	-1	4	2	0	0	0		
h	2	1	-1	0	0	0	0		
c	1	0	-1	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
2e	0	0	2	2	0	0	0		v1
2e-epsilon	3	1	-2	0	0	0	0		v2
2e-epsilon-h	1	0	-1	0	0	0	0		v3
2e-epsilon-h-c	0	0	0	0	0	0	0		v0
Edges	**W**								
v0v1	2,828								
v1v2	5,477								
v2v3	2,449								
v3v0	1,414								
Total weight of cycle	**12,169**								
Adjacency matrix of cycle	v0	v1	v2	v3	v0				
v0	**0**	**1**	**0**	**0**	**0**				
v1	**1**	**0**	**1**	**0**	**0**				
v2	**0**	**1**	**0**	**1**	**0**				
v3	**0**	**0**	**1**	**0**	**1**				
v0	**0**	**0**	**0**	**1**	**0**				

This simple cycle $v_0 v_1 v_2 v_3 v_0$ is symmetric and has no loops. It is a weighted cycle in $\boldsymbol{Z}^7$ representing the dimensionless product α. The weight (W) of an edge is the Euclidean distance between the two vertices of the edge. In the above case the order(number of vertices) of the cycle is 5. The size(number of edges) of the cycle is 4.

Let us form the graph representing the dimensionless product $\frac{Gm_0^2}{hc}$.

Graph of $\frac{Gm_0^2}{hc}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
G	3	-1	-2	0	0	0	0		
2m	0	-2	0	0	0	0	0		
h	2	1	-1	0	0	0	0		
c	1	0	-1	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
G	3	-1	-2	0	0	0	0		v1
G+2m	3	-3	-2	0	0	0	0		v2
G+2m-h	1	-4	-1	0	0	0	0		v3
G+2m-h-c	0	-4	0	0	0	0	0		v0
Edges	**W**								
v0v1	3,742								
v1v2	2,000								
v2v3	2,449								
v3v0	1,414								
Total weight of cycle	**9,605**								
Adjacency matrix of cycle	v0	v1	v2	v3	v0				
v0	**0**	**1**	**0**	**0**	**0**				
v1	**1**	**0**	**1**	**0**	**0**				
v2	**0**	**1**	**0**	**1**	**0**				
v3	**0**	**0**	**1**	**0**	**1**				
v0	**0**	**0**	**0**	**1**	**0**				

Let us form the graph representing the dimensionless product $\frac{E}{m_0 c^2}$.

Graph of $\frac{E}{m_0 c^2}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
E	2	1	-2	0	0	0	0		
m	0	1	0	0	0	0	0		
2c	2	0	-2	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
E	2	1	-2	0	0	0	0		v1
E-m	2	0	-2	0	0	0	0		v2
E-m-2c	0	0	0	0	0	0	0		v0
Edges	**W**								
v0v1	3,000								
v1v2	1,000								
v2v0	2,828								
Total weight of cycle	**6,828**								
Adjacency matrix of cycle	v0	v1	v2	v0					
v0	**0**	**1**	**0**	**0**					
v1	**1**	**0**	**1**	**0**					
v2	**0**	**1**	**0**	**1**					
v0	**0**	**0**	**1**	**0**					

Let us form the graph representing the dimensionless product $\frac{FG}{c^4}$.

Graph of $\frac{FG}{c^4}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
F	1	1	-2	0	0	0	0		
G	3	-1	-2	0	0	0	0		
4c	4	0	-4	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
F	1	1	-2	0	0	0	0		v1
F+G	4	0	-4	0	0	0	0		v2
F+G-4c	0	0	0	0	0	0	0		v0
Edges	**W**								
v0v1	2,449								
v1v2	3,742								
v2v0	5,657								
Total weight of cycle	**11,848**								
Adjacency matrix of cycle	v0	v1	v2	v0					
v0	**0**	**1**	**0**	**0**					
v1	**1**	**0**	**1**	**0**					
v2	**0**	**1**	**0**	**1**					
v0	**0**	**0**	**1**	**0**					

Let us form the graph representing the dimensionless product $\frac{p}{m_0 c}$.

Graph of $\frac{p}{m_0 c}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
p	1	1	-1	0	0	0	0		
m	0	1	0	0	0	0	0		
c	1	0	-1	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
p	1	1	-1	0	0	0	0		v1
p-m	1	0	-1	0	0	0	0		v2
p-m-c	0	0	0	0	0	0	0		v0
Edges	**W**								
v0v1	1,732								
v1v2	1,000								
v2v0	1,414								
Total weight of cycle	**4,146**								
Adjacency matrix of cycle	v0	v1	v2	v0					
v0	**0**	**1**	**0**	**0**					
v1	**1**	**0**	**1**	**0**					
v2	**0**	**1**	**0**	**1**					
v0	**0**	**0**	**1**	**0**					

Let us form the graph representing the dimensionless product $\frac{q^2}{G\epsilon_0 m_0^2}$

Graph of $\frac{q^2}{G\epsilon_0 m_0^2}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
2e	0	0	2	2	0	0	0		
G	-3	-1	-2	0	0	0	0		
epsilon	2	1	-1	0	0	0	0		
2m	0	2	0	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
2e	0	0	2	2	0	0	0		v1
2e-G	3	1	4	2	0	0	0		v2
2e-G-epsilon	1	0	5	2	0	0	0		v3
2e-G-epsilon-2m	1	-2	5	2	0	0	0		v0
Edges	**W**								
v0v1	2,828								
v1v2	3,742								
v2v3	2,449								
v3v0	2,000								
Total weight of cycle	**11,020**								
Adjacency matrix of cycle	v0	v1	v2	v3	v0				
v0	**0**	**1**	**0**	**0**	**0**				
v1	**1**	**0**	**1**	**0**	**0**				
v2	**0**	**1**	**0**	**1**	**0**				
v3	**0**	**0**	**1**	**0**	**1**				
v0	**0**	**0**	**0**	**1**	**0**				

Let us form the graph representing the dimensionless product $\frac{tc^3}{Gm_0}$.

Graph of $\frac{tc^3}{Gm_0}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
t	0	0	1	0	0	0	0		
3c	3	0	-3	0	0	0	0		
G	3	-1	-2	0	0	0	0		
m	0	1	0	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
t	0	0	1	0	0	0	0		v1
t + 3c	3	0	-2	0	0	0	0		v2
t+3c-G	0	1	0	0	0	0	0		v3
t+3c-G-m	0	0	0	0	0	0	0		v0
Edges	**W**								
v0v1	1,000								
v1v2	4,243								
v2v3	3,742								
v3v0	1,000								
Total weight of cycle	**9,984**								
Adjacency matrix of cycle	v0	v1	v2	v3	v0				
v0	**0**	**1**	**0**	**0**	**0**				
v1	**1**	**0**	**1**	**0**	**0**				
v2	**0**	**1**	**0**	**1**	**0**				
v3	**0**	**0**	**1**	**0**	**1**				
v0	**0**	**0**	**0**	**1**	**0**				

Let us form the graph representing the dimensionless product $\frac{2\pi E}{h\omega}$.

Graph of $\frac{2\pi E}{h\omega}$									
Vectors	**m**	**kg**	**s**	**A**	**K**	**mole**	**Cd**		
E	2	1	-2	0	0	0	0		
h	2	1	-1	0	0	0	0		
f	0	0	-1	0	0	0	0		
Vertices									
O	0	0	0	0	0	0	0		v0
E	2	1	-2	0	0	0	0		v1
E-m	0	0	-1	0	0	0	0		v2
E-m-2c	0	0	0	0	0	0	0		v0
Edges	**W**								
v0v1	3,000								
v1v2	2,449								
v2v0	1,000								
Total weight of cycle	**6,449**								
Adjacency matrix of cycle	v0	v1	v2	v0					
v0	**0**	**1**	**0**	**0**					
v1	**1**	**0**	**1**	**0**					
v2	**0**	**1**	**0**	**1**					
v0	**0**	**0**	**1**	**0**					

Weighted cycles in the lattice formed by the 7-tuples of $\boldsymbol{Z}^7$ can be formed for each dimensionless product of physical quantities.

It is the purpose of this study to create a catalogue of all the dimensionless products and to classify these dimensionless products.

Graph Order	Dimensionless product	Weight
4	$\frac{p}{m_0 c}$	**4,146**
4	$\frac{2\pi E}{h\omega}$	**6,449**
4	$\frac{E}{m_0 c^2}$	**6,828**
4	$\frac{FG}{c^4}$	**11,848**
5	$\frac{Gm_0^2}{hc}$	**9,605**
5	$\frac{tc^3}{Gm_0}$	**9,984**
5	$\frac{q^2}{G\epsilon_0 m_0^2}$	**11,020**
5	$\frac{e^2}{2\varepsilon_0 hc}$	**12,169**

A straightforward classification is based on the *order of the graph* and on the *weight of the graph*.

All physical equations can be reduced to dimensionless products.

The equation $E = mc^2$ can be rewritten as dimensionless product $\frac{E}{mc^2} = 1$ and the equation $E = \frac{h\omega}{2\pi}$ can also be transformed to the dimensionless product $\frac{2\pi E}{h\omega} = 1.$

These dimensionless products represent different weighted cycles of order 4 and each weighted cycle has its weight(W) and size(S).

7.12 Graph spectrum and graph eigenvalues

To find the graph spectrum we will form the Laplacian matrix of the graph G.

The definition of the Laplacian matrix is: $L = D - A$

- L is the Laplacian matrix (n x n symmetric matrix)
- D is the degree matrix of the graph, which is the diagonal matrix formed from the vertex degrees;
- A is the adjacency matrix.

A graphical partitioning can be performed based on the eigenvalues and eigenvectors of the Laplacian matrix.

7.13 Minimal spanning tree of a graph representing the physical quantity "force" in Z^7

Consider the graph G consisting of the vertices V={(0,0,0,0,0,0,0),(1,1,-2,0,0,0,0)} and the edge e=[(0,0,0,0,0,0,0),(1,1,-2,0,0,0,0)]. This graph represents the physical quantity force **F.**

We will now study the variation of **F** by considering the "*closest paths*" between the vertex **0** and the vertex **F.** We will restrict the description of the paths to the 1-2-3 coordinate hyper plane of Z^7.

Path 0: {(0,0,0),(1,1,-2)} (order = 2)(which is the shortest path, trivial solution)

Path 1:{(0,0,0),(1,1,-1)},{(1,1,-1),(1,1,-2)} (order = 3)

Path 2:{(0,0,0),(0,0,-1)},{(0,0,-1),(1,1,-2)} (order = 3)

Path 3:{(0,0,0),(0,1,-1)},{(0,1,-1),(1,1,-2)} (order = 3)

Path 4:{(0,0,0),(0,1,-2)},{(0,1,-2),(1,1,-2)} (order = 3)

Path 5:{(0,0,0),(1,0,-2)},{(1,0,-2),(1,1,-2)} (order = 3)

Path 6:{(0,0,0),(1,0,-1)},{(1,0,-1),(1,1,-2)} (order = 3)

Path 7:{(0,0,0),(0,0,-2)},{(0,0,-2),(1,1,-2)} (order = 3)

Path 8:{(0,0,0),(1,0,0)},{(1,0,0),(1,1,-2)} (order = 3)

Path 9:{(0,0,0),(0,1,0)},{(0,1,0),(1,1,-2)} (order = 3)

Path 10:{(0,0,0),(1,0,-1)},{(1,0,-1),(1,0,-2)},{ (1,0,-2),(1,1,-2)} (order= 4)

Path 11:{(0,0,0),(0,0,-1)},{(0,0,-1),(0,0,-2)},{ (0,0,-2),(1,1,-2)} (order= 4)

Path 12:{(0,0,0),(0,1,-1)},{(0,1,-1),(0,1,-2)},{ (0,1,-2),(1,1,-2)} (order= 4)

Path 13:{(0,0,0),(1,1,0)},{(1,1,0),(1,1,-1)},{ (1,1,-1),(1,1,-2)} (order= 4)

Path 14:{(0,0,0),(1,0,0)},{(1,0,0),(1,0,-1)},{ (1,0,-1),(1,0,-2)},{ (1,0,-2),(1,1,-2)} (order= 5)

Path 15:{(0,0,0),(0,1,0)},{(0,1,0),(0,1,-1)},{ (0,1,-1),(0,1,-2)},{ (0,1,-2),(1,1,-2)} (order= 5)

The paths are weighted with the Euclidean distance and so for each path the distance can be calculated. The values are in the next table.

Path weight for representations of the physical quantity "force"

				Weight of Edge	Weight of total path
ORDER 2					
Path 0	0	0	0	0,000	2,449
	1	1	-2	2,449	
ORDER 3					
Path 1	0	0	0	0,000	2,732
	1	1	-1	1,732	
	1	1	-2	1,000	
Path 2	0	0	0	0,000	2,732
	0	0	-1	1,000	
	1	1	-2	1,732	
Path 3	0	0	0	0,000	2,828
	0	1	-1	1,414	
	1	1	-2	1,414	
Path 4	0	0	0	0,000	3,236
	0	1	-2	2,236	
	1	1	-2	1,000	
Path 5	0	0	0	0,000	3,236
	1	0	-2	2,236	
	1	1	-2	1,000	
Path 6	0	0	0	0,000	2,828
	1	0	-1	1,414	
	1	1	-2	1,414	
Path 7	0	0	0	0,000	3,414
	0	0	-2	2,000	
	1	1	-2	1,414	
Path 8	0	0	0	0,000	3,236
	1	0	0	1,000	
	1	1	-2	2,236	
Path 9	0	0	0	0,000	3,236
	0	1	0	1,000	
	1	1	-2	2,236	
ORDER 4					
Path 10	0	0	0	0,000	3,414
	1	0	-1	1,414	
	1	0	-2	1,000	
	1	1	-2	1,000	
Path 11	0	0	0	0,000	3,414
	0	0	-1	1,000	

	0	0	-2	1,000	
	1	1	-2	1,414	
Path 12	0	0	0	0,000	3,414
	0	1	-1	1,414	
	0	1	-2	1,000	
	1	1	-2	1,000	
Path 13	0	0	0	0,000	3,414
	1	1	0	1,414	
	1	1	-1	1,000	
	1	1	-2	1,000	
ORDER 5					
Path 14	0	0	0	0,000	4,000
	1	0	0	1,000	
	1	0	-1	1,000	
	1	0	-2	1,000	
	1	1	-2	1,000	
Path 15	0	0	0	0,000	4,000
	0	1	0	1,000	
	0	1	-1	1,000	
	0	1	-2	1,000	
	1	1	-2	1,000	

Classification of the paths based on the minimum weight of the total path and on the order of the path results in the table below:

Order	Path	Weight of total path	Dimensional Equation
2	0	2.449	$F = F$
3	1	2.732	$F = (\frac{d}{dt})p$
3	2	2.732	$F = pf$
4	10	3.414	$F = m(\frac{d}{dt})v$
4	11	3.414	$F = mx(\frac{d}{dt})f$
4	12	3.414	$F = x\left(\frac{d}{dt}\right)(\frac{m}{t})$
4	13	3.414	$F = \left(\frac{d}{dt}\right)\left(\frac{d}{dt}\right)(mx)$
5	14	4.000	$F = x(\frac{d}{dt})(\frac{d}{dt})m$
5	15	4.000	$F = m(\frac{d}{dt})(\frac{d}{dt})x$

We find that the well known formula $F = \frac{dp}{dt}$ has the lowest weight(W=2.732), except the trivial solution $F = F$.

The method of minimizing the weight of the total path, for a certain order of the path, will allow for a classification of the representation of the physical quantity in $\boldsymbol{Z}^7$. It can be compared to variation analysis where a minimum for the cost functional is searched for.

In some cases the weight of the total path will be identical as we can see for some paths in the above table. It will mean that the graphs G and G' are isomorphic because the property "simple cycle of length *k*" is an isomorphic invariant(Epp, 1993).

7.14 Syntax and semantics of "energy equations"

We see also from the previous minimal spanning tree calculation that if the path is considered as a directed graph a ***syntax*** is generated for the representation of the "dimensional equation". It is also valid for dimensionless products which as we have seen previously are represented by a cycle.

We should explore in this study both the ***syntax*** and the ***semantics*** of the physical equations to find the rules.

Let us consider as case the physical quantity energy represented by the vector **E**.

The vector **E** can has the coordinates **E**=(2,1,-2,0,0,0,0).

We can also represent the ***syntax*** of the vector **E** using M,L,T labels respectively for Mass, Length and Time. We find the following ratio: $E = \frac{M^1\, L^1\, L^1}{T^1\, T^1}$ which we will represent as the code $\{\frac{1\,1\,1}{1\,1}\}$. We will now form all possible codes of the physical quantity energy and obtain the following table:

	SYNTAX OF QUANTITY ENERGY	
Syntax N°	**Codification**	**Representation**
1	$\{\frac{0\,0\,0}{0\,0}\}$	C_1
2	$\{\frac{1\,0\,0}{0\,0}\}$	C_2 x $(Mass)$
3	$\{\frac{1\,0\,1}{0\,0}\};\{\frac{1\,1\,0}{0\,0}\}$	C_3 x $(Mass\ Length)$
4	$\{\frac{1\,1\,1}{0\,0}\}$	C_4 x $(Mass\ Length^2)$
5	$\{\frac{1\,1\,1}{0\,1}\};\{\frac{1\,1\,1}{1\,0}\}$	C_5 x $(\frac{Mass\ Length^2}{Time})$
6	$\{\frac{1\,1\,1}{1\,1}\}$	C_6 x $(\frac{Mass\ Length^2}{Time^2})$
7	$\{\frac{0\,0\,1}{0\,0}\};\{\frac{0\,1\,0}{0\,0}\}$	C_7 x $(Length)$
8	$\{\frac{0\,1\,1}{0\,0}\}$	C_8 x $(Length^2)$

9	$\{\frac{0\,1\,1}{1\,0}\};\{\frac{0\,1\,1}{0\,1}\}$	$C_9 \times (\frac{Length^2}{Time})$
10	$\{\frac{0\,1\,1}{1\,1}\}$	$C_{10} \times (\frac{Length^2}{Time^2})$
11	$\{\frac{0\,0\,1}{1\,0}\};\{\frac{0\,1\,0}{1\,0}\};\{\frac{0\,1\,0}{0\,1}\};\{\frac{0\,0\,1}{0\,1}\}$	$C_{11} \times (\frac{Length}{Time})$
12	$\{\frac{0\,1\,0}{1\,1}\}$	$C_{12} \times (\frac{Length}{Time^2})$
13	$\{\frac{0\,0\,0}{0\,1}\};\{\frac{0\,0\,0}{1\,0}\}$	$C_{13} \times (\frac{1}{Time})$
14	$\{\frac{0\,0\,0}{1\,1}\}$	$C_{14} \times (\frac{1}{Time^2})$
15	$\{\frac{1\,0\,0}{1\,1}\}$	$C_{15} \times (\frac{Mass}{Time^2})$
16	$\{\frac{1\,1\,0}{1\,1}\};\{\frac{1\,0\,1}{1\,1}\}$	$C_{16} \times (\frac{Mass\ Length}{Time^2})$
17	$\{\frac{1\,1\,0}{1\,0}\};\{\frac{1\,1\,0}{0\,1}\};\{\frac{1\,0\,1}{1\,0}\};\{\frac{1\,0\,1}{0\,1}\}$	$C_{17} \times (\frac{Mass\ Length}{Time})$
18	$\{\frac{1\,0\,0}{1\,0}\};\{\frac{1\,0\,0}{0\,1}\}$	$C_{18} \times (\frac{Mass}{Time})$

The C_i 's represents proportionality constants.

If we assume that all the syntaxes represents "real" equations and that the constants C_i can be constructed from the following set $C = \{G, e, h, \mu_0, \varepsilon_0, N_A, R\}$ of physical constants then one finds a set of "interesting" equations by solving a system of linear equations in the dimensions of the physical constants.

7.14.1 Energy syntax N°2

The solution is $E = (\mu_0\varepsilon_0)^{-1} \times mass = c_0^2 \times mass$. The constant C_2 represents the square of the speed of light in vacuum.

7.14.2 Energy syntax N°3

The solution is $E = (\frac{c_0^7}{Gh})^{\frac{1}{2}} \times (Mass\ Length)$. The constant C_3 is "unknown" and has dimension $m\ s^{-2}$. The constant C_3 represents a constant acceleration.

7.14.3 Energy syntax N°5

The solution is $E = (\frac{c_0^5}{Gh})^{\frac{1}{2}} \times (\frac{Mass\ Length^2}{Time})$. The constant C_5 is $\frac{1}{t_P}$ where t_P is the Planck time.

7.14.4 Energy syntax N°7

The solution is $E = (\frac{c_0^4}{G}) \times (Length)$. The constant C_7 is proportional to the string tension.

7.14.5 Energy syntax N°8

The solution is $E = (\frac{c_0^{11}}{G^3h})^{\frac{1}{2}} \times (Length^2)$. The constant C_8 is "unknown" and has dimension $kg\ s^{-2}$.

7.14.6 Energy syntax N°9

The solution is $E = \frac{c_0^3}{G} \times (\frac{Length^2}{Time})$. The constant C_9 is proportional to a mass frequency.

7.14.7 Energy syntax N°10

The solution is $E = (\frac{hc_0}{G})^{\frac{1}{2}} \times (\frac{Length^2}{Time^2})$. The constant C_{10} is proportional to the Planck mass.

7.14.8 Energy syntax N°13

The solution is $E = h \times (\frac{1}{Time})$. The constant C_{13} is the Planck constant.

7.14.9 Energy syntax N°14

The solution is $E = (\frac{Gh^3}{c_0^5})^{\frac{1}{2}} \times (\frac{1}{Time^2})$. The constant C_{14} is "unknown" and has dimension $kg\ m^2$.

7.14.10 Energy syntax N°15

The solution is $E = (\frac{Gh}{c_0^3}) \times (\frac{Mass}{Time^2})$. The constant C_{15} is proportional to the Planck area which occurs in the Hawking-Bekenstein radiation.

7.14.11 Energy syntax N°16

The solution is $E = (\frac{Gh}{c_0^3})^{\frac{1}{2}} \times (\frac{Mass\ Length}{Time^2})$. The constant C_{16} is proportional to the Planck length.

7.14.12 Energy syntax N°17

The solution is $E = c_0 \times (\frac{Mass\ Length}{Time})$. The constant C_{17} is the speed of light in vacuum.

7.14.13 Energy syntax N°18

The solution is $E = \sqrt{\frac{Gh}{c_0}} \times (\frac{Mass}{Time})$. The constant C_{18} represents a diffusion constant or a flux of vorticity.

7.15 n-polytopes in Z^7

Conjecture: The theory of polytopes created by H. Coxeter(Banchoff, 1996) could be considered as framework for classification of the physical quantities.

The study of the n-polytopes and especially their lattice points should reveal symmetries that could be associated to the laws relating physical quantities.

It is known that sections of a unit cube in 3 dimensions (which is applicable for the case $\boldsymbol{Z^3}$ where we have 3 units to describe the physical quantities e.g. length, mass and time) by a plane perpendicular on the long diagonal starting at (0,0,0) up to (1,1,1) generates the following series of lattice points:

Lattice points in the unit cube	Number of lattice points	Sum of coordinates
(0,0,0)	1	0
(1,0,0);(0,1,0);(0,0,1)	3	1

(0,1,1);(1,0,1);(0,1,1)	3	2
(1,1,1)	1	3

It is known that sections of a unit cube in 4 dimensions (which is applicable for the case $\boldsymbol{Z}^{\boldsymbol{4}}$ where we have 4 units to describe the physical quantities e.g. length, mass, time and current) by a hyper plane perpendicular to the long diagonal starting at (0,0,0,0) up to (1,1,1,1) generates the following series of lattice points(Banchoff, 1996):

Lattice points in the unit hypercube	Number of lattice points	Sum of coordinates
(0,0,0,0)	1	0
(1,0,0,0);(0,1,0,0);(0,0,1,0);(0,0,0,1)	4	1
(1,1,0,0);(1,0,1,0);(1,0,0,1);(0,1,1,0);(0,1,0,1);(0,0,1,1)	6	2
(1,1,1,0);(1,1,0,1);(1,0,1,1);(0,1,1,1)	4	3
(1,1,1,1)	1	4

The number of lattice points in the unit hypercube of dimension *n* with sum *k* is given by

$$C(n,k) = \frac{n!}{k!\,(n-k)!}$$

which are the binomial coefficients(Abramowitz & Stegun, 1972).

Applied to $\boldsymbol{Z}^{7}$ we find:

Number of lattice points in the unit hypercube in 7 dimensions	Sum of coordinates
1	0
7	1
21	2
35	3
35	4
21	5
7	6

1	7

We see from the above table that we can start classifying the physical quantities which are elements of the unit hypercube in $\boldsymbol{Z}^7$. Based on their sum. The "intuitive" equivalence class generated by the function SUM() is hereby founded on a more solid mathematical ground. However, it was applied on the complete lattice instead of the unit hypercube.

The n-simplex is the smallest shape that contains n+1 points in the n dimensional space and that are not part of a lower dimensional space(Banchoff, 1996).

Dimension of n-simplex	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
# 0-simplex	1	2	3	4	5	6	7	8
# 1-simplex	0	1	3	6	10	15	21	28
# 2-simplex	0	0	1	4	10	20	35	56
# 3-simplex	0	0	0	1	5	15	35	70
# 4-simplex	0	0	0	0	1	6	21	56
# 5-simplex	0	0	0	0	0	1	7	28
# 6-simplex	0	0	0	0	0	0	1	8
# 7-simplex	0	0	0	0	0	0	0	1
Sum of k-simplices	1	3	7	15	31	63	127	255

The number of k-dimensional simplices in a n dimensional simplex is given by(Banchoff, 1996):

$$C(n+1, k+1) = \frac{(n+1)!}{(k+1)!\,(n-k)!}$$

Let us describe by Q(*n*,*k*) the quantity of *k*-cubes in a *n*-cube. The formula for Q(*n*,*k*) is(Banchoff, 1996):

$$Q(n,k) = C(n,k)2^{n-k}$$

Dimension of n-cube	0	1	2	3	4	5	6	7
# 0-cube	1	2	4	8	16	32	64	128
# 1-cube	0	1	4	12	32	80	192	448
# 2-cube	0	0	1	6	24	80	240	672
# 3-cube	0	0	0	1	8	40	160	560
# 4-cube	0	0	0	0	1	10	60	280
# 5-cube	0	0	0	0	0	1	12	84
# 6-cube	0	0	0	0	0	0	1	14
# 7-cube	0	0	0	0	0	0	0	1
Sum of k-cubes	**1**	**3**	**9**	**27**	**81**	**243**	**729**	**2187**

One can remark that the number of all *k*-cubes for a *n*-cube is given by 3^n.

We know that in $\boldsymbol{Z}^7$ there are three kinds of regular 7-dimensional polytopes(Erich W. Ellers, 2003):

- simplex
- cube
- cross-polytope

7.16 Closest neighbour lattice points in Z^7 for the quantity "energy "

The physical quantity energy represented by the vector **E** in $\boldsymbol{Z}^7$ has closest lattice points. We will now investigate these dimensional relations:

Dimensional relation 1: $E = h\left(\frac{1}{t}\right)$ where h is the Planck constant.

Dimensional relation 2: $E = v^2(m)$ where m is the mass and v the velocity.

Dimensional relation 3: $E = F\,(l)$ where l is the length.

Dimensional relation 4: $E = p^2\left(\frac{1}{m}\right)$ where p^2 is the square of the linear momentum. This form is used in the Hamiltonian.

Dimensional relation 5: $E = W\left(\frac{1}{t}\right)$ where W represents the power.

Dimensional relation 6: $E = FA\left(\frac{1}{l}\right)$ where F represents the force and A the surface area.

The dimensional relation 6 suggests the existence of an equation $E = \frac{d}{dx}(m\frac{d}{dt}\frac{dV}{dt})$ where V is the volume.

The product $FA = hc$ or $FA = Gm_0^2$ can be considered as a constant. In that case the energy is inversely proportional to the length l . For $FA = hc$ we find the known Planck relation between energy and wavelength of an electromagnetic wave.

The form $FA = hc$ suggests a possible quantisation in the form $\iint F(x,y)dxdy = nhc$. If we put $F(x,y) = \frac{m_0^2c^3}{h}$ then we have $\iint dxdy = \frac{nhc}{\frac{m_0^2c^3}{h}} = n\left(\frac{h}{m_0c}\right)^2$. This can be interpreted as a force acting along the normal on a surface with as area the square of the Compton wavelength.

The total number of closest lattice points of **E** in $\boldsymbol{Z}^7$ is ?

Test the compression algorithm on ***E*** in $\boldsymbol{Z}^7$ and obtain the graph(Delahaye, 2006).

Test minimal spanning tree algorithms to classify ***E*** in $\boldsymbol{Z}^7$.

7.17 Grassmann algebra of Z^7 over Z for classification of physical quantities

We denote $\Lambda(Z^7)$ as the Grassmann algebra of Z^7 over $\boldsymbol{Z}$.

We have $\Lambda: \Lambda(Z^7) \times \Lambda(Z^7) \rightarrow \Lambda(Z^7): (\boldsymbol{a}, \boldsymbol{b}) \rightarrow \boldsymbol{a} \wedge \boldsymbol{b}$

The basic properties of the “wedge product” are(G.Grosche, Zeidler, Ziegler, & Ziegler, 2003):

$a \wedge a = 0$ for all $a \in Z^7$

$a \wedge b = -b \wedge a$ for all $a, b \in Z^7$

$v_1 \wedge v_2 \wedge \ldots \wedge v_k = 0$ whenever $v_1, v_2, \ldots v_k \in Z^7$ are linearly dependent.

$$e^a = 1 + a + \frac{1}{2!}a^2 + \cdots = 1 + a$$

The most general function is $f(a) = m + na$ with m and n elements from $\boldsymbol{Z}$.

The derivative of a function is $Df(a) = n$

The integral is defined as $\int (m + na)\, d\, a = n$

The integral is insensitive for addition of a constant $\int f(a)\, d\, a = \int f(a + c)\, d\, a$

The Grassmann algebra is a graded-algebra in the sense that it contains *r*th-order elements (where *r* is the number of $\boldsymbol{v_i}'\boldsymbol{s}$ that are ‘wedge-producted’ together within the expression)(Penrose, 2005). The number *r* (where r = 0, 1, 2, …, 7) is called the *grade* of the element of the Grassmann algebra.

We give as example 1: the wedge product between **h** and **v** :

$$\mathbf{h} = 2e_1 + e_2 - e_3$$

$$v = -e_3$$

$$\mathbf{h} \wedge \mathbf{v} = -2(e_1 \wedge e_3) - (e_2 \wedge e_3)$$

The wedge product $e_1 \wedge e_3$ is a 2-order element representing a signed area spanned by the vectors e_1 and e_3. So, the absolute value of the coefficient "-2" represents the area of the parallelogram.

We can allocate to $\mathbf{h} \wedge \mathbf{v}$ a measure representing the sum of the absolute values of the coefficients of the r-order element. For $\mathbf{h} \wedge \mathbf{v}$ we find a value of 3.

We give as example 2: the wedge product between **m** and **c²** :

$$\mathbf{m} = e_2$$

$$\mathbf{c}^2 = -2e_1 - 2\,e_3$$

$$\mathbf{m} \wedge \mathbf{c}^2 = -2(e_1 \wedge e_2) - 2(e_2 \wedge e_3)$$

For $\mathbf{m} \wedge \mathbf{c}^2$ we find a value of 4.

Further analysis of the equations in the Grassmann algebra should lead to a classification of relations between physical quantities. Is this classification related to the Coxeter polytopes?

8 Dimensional exploration of physics equations using dimensionless products

8.1 Dimensional analysis

The most famous of the dimensionless numbers is the number $\pi = \frac{P}{D}$ where P is the perimeter of a circle and D is the diameter of the circle.

Background examples: Rayleigh scattering(Silverman, 1998) , onset of thermal instability in fluids(Bénard cells) (Chandrasekhar, 1961).

8.2 Buckingham π theorem

"If an equation is dimensionally homogeneous, it can be reduced to a relationship among a complete set of dimensionless products" (Buckingham, 1914)

"A set of dimensionless products of given variables is complete, if each product in the set is independently of the others, and every other dimensionless of the variables is a product of powers of dimensionless products in the set."(Langhaar, 1951)

"The number of dimensionless products in a complete set is equal to the total number of variables minus the maximum number of these variables that will not form a dimensionless product."(Driest, March 1946)

"The number of dimensionless products in a complete set is equal to the total number of variables minus the rank of their dimensional matrix."(Langhaar, 1951)

8.3 Dimensional matrix for SI units

The dimensional matrices that will be considered will be 7 x n matrices where n represents the number of physical quantities used in the modelling of the physics. The dimensional matrix will be build up by the transposed vectors of the representation of the physical quantities.

The unit 7 x 7 dimensional matrix(Langhaar, 1951) build up from the SI base units is represented by

$$[I] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

8.4 Dimensional exploration of "a particle in a box "

We will now illustrate the technique of "dimensional exploration"(Roche, 1998) by searching a universal relation expressing the force F (or equivalent the tension τ) as function of "important" physical quantities. It can be considered as a "one particle in a box"-problem.

Today physics describes all known processes based on 4 interactions: electromagnetic interaction, weak interaction, strong interaction and the gravitational interaction.

We also know "absolute" conservation laws which are valid for the following physical quantities:

- Energy **E**
- Linear momentum **p**
- Total angular momentum **J**
- Baryon number **A**
- Electric charge **e**

All interactions between particles comply with these conservation laws. It is straightforward that we want to express the force F as function of physical quantities occurring in the conservation laws.

A dimensional matrix representing the physical quantities $(F, E, p, J, T, q, t, m_0, G, \mu_0, \epsilon_0, k)$ is given below

$$[\mathrm{M}] = \begin{bmatrix} 1 & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 3 & 1 & -3 & 2 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & -1 & 1 & -1 & 1 \\ -2 & -2 & -1 & -1 & 0 & 1 & 1 & 0 & -2 & -2 & 4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By inspection one can see that the columns in the matrix represent the coordinates of the representation of the physical quantity in SI units which is an integer lattice in $\boldsymbol{Z}^7$.

This 7 x 12 matrix has rank[M]= 5.

The above 7 x 12 dimensional matrix results in 12 – 5 = 7 dimensionless products.

Buckingham has pointed out that we obtain the maximum amount of experimental control over the dimensionless variables if the original variables that can be regulated each occur in only one dimensionless product. The dependent variable of the problem must also be considered. It is desired to know how this variable depends on the other variables. The dependent variable consequently should not occur in more than one dimensionless product. The following rule should be applied

"In the dimensional matrix, let the first variable be the dependent variable. Let the second variable be that which is easiest to regulate experimentally. Let the third variable be that which is next easiest to regulate experimentally, and so on." (Langhaar, 1951)

Conjecture: The dot product of two columns from the dimensional matrix should be different from zero to have a physically meaningful dimensional matrix.

The above 7 x 12 matrix is not in agreement with the above proposition. It means that solving this dimensional matrix will result in finding dimensionless products that are not related to the physical system under study. Inspection of the matrix suggests removing column 5 which represents the temperature T, from this dimensional matrix. As column 5 is removed, one can immediately see that column 12 can also be removed because it is in a one-to-one relation with column 5. Using the above conjecture the dimensional matrix can be made more meaningful and results in a 7 x 10 matrix of rank 4, which has 6 dimensionless products as solutions.

The dimensionless products are:

$\pi^1 = \frac{FG}{c^4}$, $\pi^2 = \frac{E}{m_0 c^2}$, $\pi^3 = \frac{p}{m_0 c}$, $\pi^4 = \frac{Jc}{Gm_0^2}$, $\pi^5 = \frac{q^2}{G\epsilon_0 m_0^2}$, $\pi^6 = \frac{tc^3}{Gm_0}$

These dimensionless products are the "natural" form of the relevant variables in the framework of SI units. It means that theory if physics should be best expressed in terms of these dimensionless products.

So, the equation $\varphi(F, E, p, J, q, t, m_0, G, \mu_0, \varepsilon_0) = 0$ could be reduced in complexity to an equation of the form: $\omega(\pi^1, \pi^2, \pi^3, \pi^4, \pi^5, \pi^6) = 0$. We use here the Einstein notation for the π^i coordinates of the manifold(G.Grosche, Zeidler, Ziegler, & Ziegler, 2003).

The configuration space CS of our "one particle in a box"-problem is reduced to a non-Euclidean 6-manifold *M* having 6 degrees of freedom, where ω is a smooth function defined on *M*.

These dimensionless products should all play an important role in a "theory of everything". To see the importance of them a short qualitative discussion follows:

8.4.1 π^1

This dimensionless product $\pi^1 = \frac{FG}{c^4}$ represents the ratio of a force to the string tension up to a constant factor.

8.4.2 π^2

This dimensionless product $\pi^2 = \frac{E}{m_0 c^2}$ represents the ratio of the total energy of a system to the rest-mass energy of a particle of the system.

8.4.3 π^3

This dimensionless product $\pi^3 = \frac{p}{m_0 c}$ can be transformed using the "de Broglie" relation $\lambda_{dB} = \frac{h}{p}$ for a wave-packet to the ratio of the Compton wavelength to the "de Broglie" wavelength of a particle of rest-mass m_0. It is interesting to remark that $\left(\frac{Gm_0}{c^2}\right) = \left(\frac{Gm_0^2}{hc}\right)\frac{h}{m_0 c}$ where the factor $\left(\frac{Gm_0^2}{hc}\right)$ can be considered as an dilution coefficient because it reduces the Compton wavelength.

It is also interesting to see the following dimensionless product $\pi^2\pi^3 = \frac{Ep}{m_0^2 c^3} = \left(\frac{\frac{Ep}{h}}{\frac{m_0^2 c^3}{h}}\right)$ which represents the ratio of two forces.

8.4.4 π^4

This dimensionless product $\pi^4 = \frac{Jc}{Gm_0^2}$ can be transformed using $J = n\frac{h}{2}$, with n= quantum number, for a wave-packet to the ratio of the atomic interaction to the gravitational interaction of a particle of rest-mass m_0.

8.4.5 π^5

This dimensionless product $\pi^5 = \frac{q^2}{G\epsilon_0 m_0^2}$ represents the ratio of the electromagnetic interaction to the gravitational interaction of a particle of rest-mass m_0.

8.4.6 π^6

This dimensionless product $\pi^6 = \frac{tc^3}{Gm_0}$ can be represented as $\pi^6 = \frac{ct}{\frac{Gm_0}{c^2}}$ which is the ratio of the distance travelled by an electromagnetic wave in vacuum to a term proportional to the Schwarzschild radius $\left(\frac{Gm_0}{c^2}\right)$ of a black hole of rest-mass m_0. The exact Schwarzschild radius is $r_S = 2\left(\frac{Gm_0}{c^2}\right)$. The quantity t is considered to be the proper time.

8.4.7 Approximation of $\omega(\pi^1, \pi^2, \pi^3, \pi^4, \pi^5, \pi^6) = 0$

The equation $\omega(\pi^1, \pi^2, \pi^3, \pi^4, \pi^5, \pi^6) = 0$ can further be transformed to

$$F = \frac{c^4}{G}\theta(\pi^2, \pi^3, \pi^4, \pi^5, \pi^6)$$

Based on the magnitude of $\frac{c^4}{G} = 1.21 \times 10^{44}\ N$ one expects that the function $\theta(\pi^2, \pi^3, \pi^4, \pi^5, \pi^6) \leq 1$ and that it rapidly decreases as time increases.

We assume that $\pi^2 \in [1, \infty[$ and that $\pi^3 \in [0, \infty[$.

We also know that for a free particle one can write $E^2 = c^2p^2 + m_0^2c^4$. This can be rewritten in two ways using dimensionless products:

Type 1:

$\left(\frac{m_0c^2}{E}\right)^2 + \left(\frac{cp}{E}\right)^2 = 1$ or with dimensionless notation $\left(\frac{1}{\pi^2}\right)^2 + \left(\frac{\pi^3}{\pi^2}\right)^2 = 1$

It represents a unit circle $x^2 + y^2 = 1$ where (x, y) represent the dimensionless products $\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}\right)$.

Type 2:

$\left(\frac{E}{m_0c^2}\right)^2 - \left(\frac{cp}{m_0c^2}\right)^2 = 1$ or with dimensionless notation $(\pi^2)^2 - (\pi^3)^2 = 1$

It represents a hyperbola $x^2 - y^2 = 1$ where (x, y) represent the dimensionless products (π^2, π^3).

The type 2 equation is the most used in quantum electrodynamics theory. In the absence of an external field the Dirac equation possesses only continuum solutions with energies in agreement with type 2 equation. The vacuum state is determined by the requirement that all positive energy states are empty and all negative energy states are occupied. Therefore, the physically observable vacuum (without electromagnetic field) is free of particles and is electrically neutral (Greiner & Reinhardt, 1992, 1994).

The type 1 equation is useful if one realizes that the inner circle represents a region where excited states of the free particle can occur. If the total energy of the system rises then the particle state will move towards (0,0). This picture can be in an elegant way being treated with holomorphic functions and can also be treated in respect to the z-transform used in the modelling of discrete systems.

In the present discussion of the function $\theta(\pi^2, \pi^3, \pi^4, \pi^5, \pi^6)$ it is more appropriate to consider the type 1 equation which helps us to transform the function to $\theta_1\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \pi^5, \pi^6\right)$ so that its variables are <1. It is also interesting to remark that the ratio $\frac{\pi^5}{\pi^4} = \frac{q^2}{Jc\epsilon_0} = \frac{2\mu_0 e^2}{2h(\frac{n}{2})} = \frac{2\alpha}{\frac{n}{2}}$ *and* $\alpha = \frac{1}{137.035999070}$. The constant α is nothing else than the fine-structure constant if q = e. It should also be remarked that the form $\frac{\pi^5}{\pi^4} = \frac{q^2}{Jc\epsilon_0}$ could also be used for expressing the weak interaction. If one interprets q as the "effective charge" and J as the "effective total angular momentum", one could write $\frac{\pi^5}{\pi^4} = \frac{1}{2hc}(\frac{4e^2}{n\epsilon_0} + g^2)$ where electromagnetic and weak interactions are combined. To have a general treatment of the problem one could represent $J = n\frac{h}{2}$ where $n \in \boldsymbol{Z}$ so that fermions and bosons can be treated by the function $\theta_1\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \pi^5, \pi^6\right)$. So, let us change π^5 by the ratio $\frac{\pi^5}{\pi^4}$.

We have now the equation $\theta_2\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \frac{\pi^5}{\pi^4}, \pi^6\right)$ in which 3 coordinates are < 1. We still have to find a suitable transformation for π^4 and π^6 so that we can perform a MacLaurin series expansion on the function θ_2 where we assume that the first term of the expansion is equal to 0.

We need initial conditions for this equation. So, we define that at $t = t_0$ the force $F = F_0$. This force F_0 is a constant force. A good candidate, based on dimensional analysis, for this constant force is $F_0 = \frac{m_0^2 c^3}{h}$. It represents the ratio of the rest-mass energy to the Compton wavelength of the particle with rest-mass m_0.

We can consider now the function $\frac{F - F_0}{\frac{c^4}{G} - F_0} = \theta_2\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \frac{\pi^5}{\pi^4}, \pi^6\right)$.

The variable t in π_6 represents the time interval in which the force changes from F_0 to F. So, Δt is the time interval in which the force F changes from F_0 to $F_0 + \Delta F$.

Let us assume that the dynamics are determined by π^6 ,so in first crude approximation we can state that the equation $\frac{dF}{\frac{c^4}{G} - F} = \frac{\theta_3\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \frac{\pi^5}{\pi^4}\right) c\,dt}{\frac{Gm_0}{c^2}}$ describes the dynamics of the system in which $\theta_3\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \frac{\pi^5}{\pi^4}\right)$ is a function of the π^i.

Integration of the equation results in $\ln\left(F - \frac{C^4}{G}\right) = -\frac{\theta_3\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \frac{\pi^5}{\pi^4}\right) ct}{\frac{Gm_0}{c^2}} + constant.$

The constant must be chosen to render the equation dimensionally homogeneous and to satisfy the initial condition that at $t = t_0$ the force $F = F_0$.

We find now $\frac{\theta_3\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \frac{\pi^5}{\pi^4}\right) ct}{\frac{Gm_0}{c^2}} = ln\left(\frac{\frac{c^4}{G} - F_0}{\frac{c^4}{G} - F}\right)$. This equation determines the time t in which the force changes from F_0 to F.

We can also write that $\frac{F - F_0}{\frac{c^4}{G} - F_0} = 1 - exp\left(\frac{-\theta_3\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}, \pi^4, \frac{\pi^5}{\pi^4}\right) ct}{\frac{Gm_0}{c^2}}\right)$

This type of equation is typical for diffusion phenomena. The function $\theta_3\left(\frac{1}{\pi^2},\frac{\pi^3}{\pi^2},\pi^4,\frac{\pi^5}{\pi^4}\right)$ has still to be further investigated.

Instead of using π^4 it is better to use the reciprocal because $\frac{1}{\pi^4} < 1$. So, we transform $\theta_3\left(\frac{1}{\pi^2},\frac{\pi^3}{\pi^2},\pi^4,\frac{\pi^5}{\pi^4}\right)$ to $\theta_4\left(\frac{1}{\pi^2},\frac{\pi^3}{\pi^2},\frac{1}{\pi^4},\frac{\pi^5}{\pi^4}\right)$.

Conjecture : one trial function for $\theta_4\left(\frac{1}{\pi^2},\frac{\pi^3}{\pi^2},\frac{1}{\pi^4},\frac{\pi^5}{\pi^4}\right) = \left[(\frac{1}{\pi^2})^2 + (\frac{\pi^3}{\pi^2})^2 - (1-\alpha_{QCD} - \frac{\pi^5}{\pi^4} - \frac{1}{\pi^4})^2\right]$.

This dimensional exploration/analysis results in the equation :

$$\frac{F - F_0}{\frac{c^4}{G} - F_0} = 1 - exp\left(\frac{-\left[(\frac{1}{\pi^2})^2+(\frac{\pi^3}{\pi^2})^2 - (1-\alpha_{QCD} - \frac{\pi^5}{\pi^4} - \frac{1}{\pi^4})^2\right] ct}{\frac{Gm_0}{c^2}}\right) \quad \textbf{[1]}$$

Conjecture: The above equation describes the time evolution of the total force(or tension τ) in a particle with rest-mass m_0.

It can be seen that for a free particle the equation reduces to $F - F_0 = 0$ because $\left[(\frac{1}{\pi^2})^2 + (\frac{\pi^3}{\pi^2})^2 - (1-\alpha_{QCD} - \frac{\pi^5}{\pi^4} - \frac{1}{\pi^4})^2\right] = 0$. According to classical mechanics one would suggest that $F_0 = 0 \; at \; t = 0$. However, in quantum mechanics one can derive a term known as "Zitterbewegung"(Merzbacher, 1970) $(frequency = \frac{m_0c^2}{h} \; and \; amplitude = \frac{h}{m_0c})$ that is probably related to the constant force F_0.

Conjecture: Any particle with rest-mass m_0 experiences a constant force $F_0 = \frac{m_0^2c^3}{h}$.

The force $\frac{m_0^2c^3}{h}$ appears in QED in relation to the *critical field strength* $E_{cr} \sim \frac{2m_0c^2}{\left(\frac{eh}{2\pi m_0 c}\right)} = \frac{4\pi m_0^2c^3}{eh}$, so we have $F_0 \sim qE_{cr}$. This force is also related to the concept of *maximal acceleration* $\frac{m_0c^3}{h}$ (see E.R. Caianiello (Fradkin, I.A.Batalin, C.J.Isham, & Vilkovisky, 1987).

A rough estimate tells us that pair production becomes considerable if the potential changes by a value of two rest-masses over a characteristic length scale which is set by the Compton wavelength of the particle(Greiner & Reinhardt, 1992, 1994).

The magnitude of this force for $m_0 = m_e$ is $F_0 = 3.37 \times 10^{-2}\,N$.

One can also remark that $\frac{m_0^2c^3}{h} = \left(\frac{Gm_0^2}{hc}\right)\frac{c^4}{G}$ where the factor $\left(\frac{Gm_0^2}{hc}\right)$ can be considered as an "dilution coefficient" because it weakens the constant force or tension $\frac{c^4}{G}$.

The magnitude of this force for $m_0 = m_P$ is $F_0 = \frac{c^4}{G}\,N$. The equation [1] is in the limit for $m_0 \rightarrow m_P$ also valid.

It can be verified that $m_0c^2 = \int_0^{\frac{h}{m_0c}} F_0 ds$ where $F_0 = \frac{m_0^2c^3}{h}$ .This would mean that the rest-mass energy is created by a constant force acting over a distance equal to the Compton wavelength of the particle.

This view can also be interpreted in another way.

Consider the energy $E_1 = \left(\frac{c^4}{G}\right)\left(\frac{h}{m_0c}\right) = \frac{hc^3}{Gm_0}$ and $E_2 = \left(\frac{c^4}{G}\right)\left(\frac{Gm_0}{c^2}\right) = m_0c^2$. Performing the ratio $\frac{E_2}{E_1} = \frac{Gm_0^2}{hc} = \left(\frac{m_0}{m_P}\right)^2$ results in the "dilution coefficient" which for a proton has a value of approximately 10^{-38}. It can be seen that the "dilution coefficient" is the square of the ratio of the rest-mass of the particle to the Planck mass.

Conjecture: We suspect that the equation [1] is valid for dimensions ~ Compton wavelength.

Could the equation [1] be experimentally verified by probing an electron for detection of a constant force or constant tension of magnitude $F_0 = 3.37 \times 10^{-2}\ N$? This will be difficult especially considering already the problems that have been encountered with the Lamb shift measurement and the electron self-force problem(Venkataraman, 1994).

A "curious" coincidence is:

$$m_0c^2 = \left(\frac{c^4}{G}\right)\left(\frac{Gm_0}{c^2}\right) = \left(\frac{m_0^2c^3}{h}\right)\left(\frac{h}{m_0c}\right)$$

The first equality $m_0c^2 = \left(\frac{c^4}{G}\right)\left(\frac{Gm_0}{c^2}\right)$ is clearly related to the general relativity theory.

The second equality $m_0c^2 = \left(\frac{m_0^2c^3}{h}\right)\left(\frac{h}{m_0c}\right)$ is clearly related to quantum mechanics.

Both equations express the relation "energy = force x length" or $E = \int F ds$

If we want to combine both equations we could form the geometric average which results in

$$m_0c^2 = \sqrt{\left(\frac{c^4}{G}\right)^2\left(\frac{Gm_0}{c^2}\right)^2 + \left(\frac{m_0^2c^3}{h}\right)^2\left(\frac{h}{m_0c}\right)^2}$$

which should be applicable for quantum gravity.

8.4.8 Relations involving the dilution coefficient

Energy relation: $m_0c^2 = \left(\frac{Gm_0^2}{hc}\right)\frac{hc^3}{Gm_0} = \left(\frac{m_0}{m_P}\right)^2\frac{hc^3}{Gm_0} = \left(\frac{m_0}{m_P}\right)^2\left(\frac{2hc}{r_s}\right)$

Force relation: $\frac{m_0^2c^3}{h} = \left(\frac{Gm_0^2}{hc}\right)\frac{c^4}{G} = \left(\frac{m_0}{m_P}\right)^2\frac{c^4}{G}$

Length relation: $\left(\frac{Gm_0}{c^2}\right) = \left(\frac{Gm_0^2}{hc}\right)\frac{h}{m_0c} = \left(\frac{m_0}{m_P}\right)^2\frac{h}{m_0c}$

From dimensional point of view one can write "energy = force x length". If we apply this on the above relations we get :

$$\frac{m_0^2 c^3}{h}\left(\frac{Gm_0}{c^2}\right) = \frac{Gm_0^3 c}{h} = \left(\frac{Gm_0^2}{hc}\right) m_0 c^2 = \left(\frac{m_0}{m_P}\right)^4 \left(\frac{c^4}{G}\right) \left(\frac{h}{m_0 c}\right)$$

A "dilution coefficient" is natural in a **hyperbolic geometry**(Delahaye, 2006). On a Poincaré disc the size of an object becomes smaller when it moves away from the centre. This is a dilution effect. Consider now $m_0 = m_P$,as a geometric entity, at the centre of the disc then m_0 will be diluted by the factor $\frac{Gm_0^2}{hc}$ when it moves toward the outer circle of the Poincaré disc.

Another hypothetical explanation for the "dilution coefficient" is to consider the factor $\left(\frac{Gm_0^2}{hc}\right)$ as a canonical projection(Penrose, 2005) from the fibre bundle *B* to the manifold *M* which collapses each entire fibre *V* down to a particular point of *M*. The product space of *M* with *V* contains pairs of elements (a,b) where *a* belongs to *M* and *b* belongs to *V*. Here we have the following pairs:

$\left(\frac{m_0^2 c^3}{h}, \frac{c^4}{G}\right)$; $\left(\frac{Gm_0}{c^2}, \frac{h}{m_0 c}\right)$ and $\left(m_0 c^2, \frac{hc^3}{Gm_0}\right)$.

8.4.9 Dimensional exploration of total angular momentum *J*

8.4.9.1 Present status

"*The elementary particles are grouped according to one of their fundamental properties, namely spin*(Veltman, 2003)."

Particles related to matter(electrons,…) have all spin ½.

Particles associated to the electromagnetic force (photon), weak force(W and Z vector-bosons) and strong force(gluon) have spin 1.

The particle associated with the gravitational force has spin 2.

The hypothetical particle Higgs-boson has probably spin 0.

The interaction strength of the 4 interaction forces are different functions of the energy E of the particle and the interaction strength is not a basis for classification of the interaction forces(Veltman, 2003).

From previous calculations we know that the dimensionless product $\pi^4 = \frac{Jc}{Gm_0^2}$ is related to the total angular momentum of the particle with rest-mass m_0.

It means that we have the relation $J = \frac{\pi^4 \mathrm{G}\, \mathrm{m}_0^2}{\mathrm{c}} = \left(\frac{\pi^4 \mathrm{G}}{\mathrm{c}}\right) \mathrm{m}_0^2$ where $J = n\frac{h}{2}$ and $n \in \boldsymbol{Z}$.

This relationship is a straight line, *in the assumption that* π^4 *is a constant*, between the total angular momentum and the square of the rest-mass of the particle. This straight line is known as the Regge trajectory (Veltman, 2003) and is at the cradle of the string theory.

8.4.9.2 What if we don't know the Planck constant?

It is interesting to ask the question "***What if we don't know the Planck constant?***" It can be shown that solving the system of equations for a constant with dimension $kg\ m^2\ s^{-1}$ from the set of constants $C = \{G, e, \mu_0, \varepsilon_0\}$ results in an constant angular momentum of $J_N = e^2\sqrt{\frac{\mu_0}{\varepsilon_0}}$ which could have been considered as the basic "natural" form of the angular momentum quantum. It could have been derived before the year 1900. The following correspondence exists $h = 2\alpha J_N$. One could have objected, based on the magnitude of J_N, that J_N is more fundamental.

Conjecture: The parameter $\frac{\pi^2\pi^6}{\pi^4} = x$ is proportional to a quantum number.

We show by inspection that $\frac{\pi^2\pi^6}{\pi^4} = x$ with $x \in \boldsymbol{Q}$. We have $\left(\frac{E}{m_0c^2}\right)\left(\frac{tc^3}{Gm_0}\right)\left(\frac{Gm_0^2}{Jc}\right) = \left(\frac{Et}{J}\right) \sim \left(\frac{E}{h\omega}\right)$.

8.4.10 Interaction of external fields on free particles

Dimensional analysis on the equation $\varphi(F, E, p, J, q, t, m_0, G, \mu_0, \varepsilon_0) = 0$ has shown that the known relation for free particles could be written in function of dimensionless products:

$\left(\frac{m_0c^2}{E}\right)^2 + \left(\frac{cp}{E}\right)^2 = 1$ or with dimensionless notation $\left(\frac{1}{\pi^2}\right)^2 + \left(\frac{\pi^3}{\pi^2}\right)^2 = 1$

The equation represents a unit circle in the $\left(\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}\right)$–plane.

We can now consider the switching on of an external field on the state of a free particle. We assume that the particle is subjected to a field which increases the total energy E of the system under consideration. In that case the coordinates of the particle in the $\frac{1}{\pi^2}, \frac{\pi^3}{\pi^2}$–plane will move towards the origin. It means that these states will be described by a circle with a radius < 1.

Conjecture: We propose to describe the perturbation of free particles by a dimensionless equation of the following type:

$$\left(\frac{1}{\pi^2}\right)^2 + \left(\frac{\pi^3}{\pi^2}\right)^2 = \left(1 - \alpha_{QCD} - \frac{\pi^5}{\pi^4} - \frac{1}{\pi^4}\right)^2$$

where the mathematics is performed using holomorphic functions in the complex plane **C** (see § 9).

Where $\alpha_w = \frac{g^2}{2hc} = \frac{1}{32}$ representing the weak interactions, $\alpha_{QCD} \sim 1$ representing the strong interactions at low energy, $\alpha_{EM} = \frac{1}{137{,}04}$ representing the electromagnetic interaction(Veltman, 2003). The fourth interaction, the gravitational interaction, is represented by $\frac{1}{\pi^4} \sim 10^{-38}$ for a rest-mass like the electron and proton a becomes 1 for the Planck mass.

We represent $\frac{\pi^5}{\pi^4} = \frac{1}{2hc}\left(\frac{4e^2}{n\epsilon_0} + g^2\right)$ as the combined weak and electromagnetic interaction.

8.5 Dimensional exploration of “vacuum states”

8.6 Dimensional exploration of “a box containing charged particles”

Let us consider the following physical quantities:

- l = characteristic size of the box
- m_0 = mass of the particle
- Ze = total charge of the particle
- ε_0 = electric constant
- μ_0 = magnetic constant

We assume the following equation $f(l, m_0, Ze, \varepsilon_0, \mu_0) = 0$ to be valid.

The dimensional matrix M becomes:

$$M = \begin{bmatrix} 1 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The solution after some calculus is $\pi^1 = \frac{\mu_0 (Ze)^2}{l m_0}$.

One can see that it is more appropriate to consider $\frac{1}{\pi^1} = \frac{l m_0}{\mu_0 (Ze)^2}$ if we want to use series expansions because then $l m_0 \to 0$ will not create infinities. So, we have now $f_1\left(\frac{l m_0}{\mu_0 (Ze)^2}\right) = 0$. Expansion in MacLaurin series gives $f_1\left(\frac{l m_0}{\mu_0 (Ze)^2}\right) = f_1(0) + f_1'(0)\left(\frac{l m_0}{\mu_0 (Ze)^2}\right) + \cdots$.

So we have $\left(\frac{l m_0}{\mu_0 (Ze)^2}\right) = \frac{-f_1(0)}{f_1'(0)} = constant = A$.

We recall that the fine-structure constant is given by $\alpha = \frac{e^2}{2\varepsilon_0 hc}$ and $\mu_0 \varepsilon_0 c^2 = 1$.

Reworking the terms we find $1 = \frac{A\mu_0 (Ze)^2}{l m_0} = \frac{A (Ze)^2}{l m_0 \varepsilon_0 c^2} = \frac{2hA(Z)^2}{l m_0 c} \frac{e^2}{2\varepsilon_0 hc} = \frac{\left(\frac{h}{m_0 c}\right)}{\left(\frac{l}{2}\right)} A Z^2 \alpha$.

So, we can write $\left(\frac{l}{2}\right) = AZ^2 \alpha \left(\frac{h}{m_0 c}\right)$. If the box is considered as a sphere then $\frac{l}{2}$ is the radius of the sphere.

If we consider that Z=1, then for one electron in a box we find $\frac{A\left(\frac{h}{m_0 c}\right)}{\left(\frac{l}{2}\right)} = \frac{1}{\alpha}$. The geometric interpretation is that $\frac{1}{\alpha}$ represents the ratio of the diameter of a “fibre” to the length of the “fibre”. The “fibre” represents the “*volume*” occupied by the particle. So, we can define a characteristic angle

$\theta_e = \arctan\left(\frac{1}{\alpha}\right) \sim 89{,}581°$. It means that the volume of the "fibre" resembles to a very thin flat disc propagating at a speed v along the axis of the "fibre".

Within the box we have the quantum condition $l = n\lambda_{dB}$ with $n \in \boldsymbol{Z}$.

So, we have finally $\alpha = \frac{n}{2}\frac{(\lambda_{dB})}{(\lambda_C)}\frac{1}{AZ^2}$. We will see in §9.1 that this can be written as $\frac{1}{\alpha} = \frac{2}{n}\frac{\left(\frac{cp}{E}\right)}{\left(\frac{m_0c^2}{E}\right)} AZ^2$.

We know that $\frac{1}{\alpha}$ is a constant which means that we have for Z=1 a straight line defined by

$\left(\frac{cp}{E}\right) = \frac{1}{\alpha}\frac{n}{2}\frac{1}{AZ^2}\left(\frac{m_0c^2}{E}\right)$ with $n \in \boldsymbol{Z}$. This equation is valid for all m_0.

8.7 Dimensional exploration of "Casimir force"

8.8 Dimensional exploration of "a rotating body"

Let us consider the following physical quantities $f(J, r, c_0, G) = 0$ where we have

- J = angular momentum of the object
- r = radius of the rotating object
- c_0 = speed of light in vacuum
- G = Newtonian constant of gravitation

Let us form the dimensional matrix M. So, we find:

$$M = \begin{bmatrix} 2 & 1 & 1 & 3 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & -1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We can find after some calculus that $\pi^1 = \frac{JG}{r^2c_0^3}$

As J is normally a vector quantity we can write $\pi^1 = \frac{\|\vec{J} \times \vec{r}\|G}{r^3c_0^3}$. The dimensionless product π^1should play an important role in all phenomena involving rotations. Is this also true for the total angular momentum of an elementary particle?

If yes then we put $J = n\frac{h}{2}$ and $r = \frac{h}{m_0c_0}$ then we obtain $\pi^1 = \left(\frac{n}{2}\right)\left(\frac{Gm_0^2}{hc_0}\right)$ with $n \in \boldsymbol{Z}$. This result is in accordance with§ 8.4.4.

8.9 Parametric "dimensional exploration" of physics equations

We have seen that dimensional analysis is looking for dimensionless products. This technique can be modified by solving the system of equations for a dimensional quantity.

A question could be ***"Could you find a quantity with dimension length build up from the following constants G, h, c?"***.

It was this question, that I found as exercise in a book of physics(Borowitz & Beiser, 1971), that triggered in 1978, during my first year at the University of Ghent, my search for structure in the physical quantities.

The answer to the question is $L = \sqrt{\frac{Gh}{c^3}}$ which is proportional to the Planck length.

8.9.1 Solution for the quantity "Length" as function of the parametric set $\{h, c_0, m_0, G\}$

Solve the equation for a physical quantity with dimension length L.

The solution L ,as function of the parameter r , is $L = h^{\left(\frac{1-r}{2}\right)} c_0^{-\left(\frac{3+r}{2}\right)} m_0^r G^{\left(\frac{1+r}{2}\right)}$.

It is interesting to explore the equation by entering a value for the parameter r, so that the exponent of the variable/constant vanishes.

8.9.1.1 Length independent of h

We have the condition $\left(\frac{1-r}{2}\right) = 0$ so we have *r* = 1.

We find the equation $L = \frac{Gm_0}{c_0^2}$ which is proportional to the gravitational radius of an object (Landau & Lifchitz, Physique Statistique, 1967). It is described by Landau in relation to the equilibrium of a neutron star.

8.9.1.2 Length independent of c_0

We have the condition $-\left(\frac{3+r}{2}\right) = 0$ so we have *r* = -3

We find the equation $L = \frac{h^2}{Gm_0^3}$ which is "unknown". For a proton one finds a length having the value $L = 1{,}406 \times 10^{24}\ m$ which is equal to $1{,}487 \times 10^8 ly$. The radius of the universe is estimated to be not smaller than 24 Gpc(Cornish, Spergel, Starkman, & Komatsu, 2003) which is approximately $78 \times 10^9\ ly$. So, a proton fits in that universe. What now with an electron?

For an electron we find the value $L = 8{,}702 \times 10^{33}\ m$ which is equal to $9{,}205 \times 10^{17} ly$. This is much larger than the 24 Gpc. Is it possible that we should consider an ellipse for the topology of the universe based on the maximum wavelength of an electron and a proton? Is their any anisotropy to be found in the behaviour of an electron or a proton when the particle is rotated?

The "unknown" length can be represented as $L = \frac{h}{m_0 c}\left(\frac{hc}{Gm_0^2}\right) = \frac{h}{m_0 c}\left(\frac{m_P}{m_0}\right)^2$.

We could interpret this length as the largest wavelength of the particle. So, it creates a cut-off for the wavelength of the particle. Using the standard quantum mechanical description of a particle in a box we can conclude that the basic cavity allowing this wavelength has the size of half the wavelength. The cavity length L_c becomes $L_c = \frac{h}{2m_0 c}\left(\frac{m_P}{m_0}\right)^2$.

Is it possible that $\left(\frac{m_P}{m_0}\right)^2$ always has to be an integer?

Conjecture: The "de Broglie" wavelength of an elementary particle is bounded by the following values : $\lambda_{dB} \in \left[\frac{h}{m_0 c}, \frac{h^2}{G m_0^3}\right]$.

So, the ***minimum linear momentum*** of a particle with rest mass m_0 is $p_{min} = \frac{h}{\lambda_{dB}} = \frac{G m_0^3}{h}$ and the ***maximum linear momentum*** of a particle with rest mass m_0 is $p_{max} = m_0 c$.

8.9.1.3 Length independent of m_0

We have the condition $r = 0$ so we have *r* = 0

We find the equation $L = \sqrt{\frac{Gh}{c_0^3}}$ which is proportional to the Planck length.

8.9.1.4 Length independent of G

We have the condition $\left(\frac{1+r}{2}\right) = 0$ so we have *r* = -1

We find the equation $L = \frac{h}{m_0 c}$ which is the Compton wavelength.

8.9.2 Solution for the quantity "Energy" as function of the parametric set $\{h, c_0, m_0, G\}$

Solve the equation for a physical quantity with dimension energy E.

The solution *E* ,as function of the parameter *r* , is $E = h^{\left(\frac{1-r}{2}\right)} c_0^{\left(\frac{5-r}{2}\right)} m_0^r G^{\left(\frac{r-1}{2}\right)}$.

8.9.2.1 Energy independent of h

We have the condition $\left(\frac{1-r}{2}\right) = 0$ so we have *r* = 1.

We find the equation $E = m_0 c_0^2$ which is the equation of Einstein for the rest-energy.

8.9.2.2 Energy independent of c_0

We have the condition $\left(\frac{5-r}{2}\right) = 0$ so we have *r* = 5

We find the equation $E = \frac{m_0^5 G^2}{h^2}$ which is "unknown".

8.9.2.3 Energy independent of m_0

We have the condition $r = 0$ so we have *r* = 0

We find the equation $E = \sqrt{\frac{h c_0^5}{G}}$ which is proportional to the Planck energy.

8.9.2.4 Energy independent of G

We have the condition $\left(\frac{r-1}{2}\right) = 0$ so we have *r* = 1.

We find the equation $E = m_0 c_0^2$ which is the equation of Einstein for the rest-energy.

What is the relation between all these equations. It can be shown that one has as function of the parameter r the following equation:

$$\frac{E}{m_0 c_0^2} = \left(\sqrt{\frac{G m_0^2}{h c_0}}\right)^{r-1} \quad \text{with } r \in \boldsymbol{N}$$

8.9.3 Solution for the quantity "Force" as function of the parametric set $\{h, c_0, m_0, G\}$

Solve the equation for a physical quantity with dimension force F.

The solution F ,as function of the parameter r , is $F = h^{\left(\frac{-r}{2}\right)} c_0^{\left(\frac{8-r}{2}\right)} m_0^r \, G^{\left(\frac{r-2}{2}\right)}$.

8.9.3.1 Force independent of h

We have the condition $\left(\frac{-r}{2}\right) = 0$ so we have *r* = 0.

We find the equation $F = \frac{c_0^4}{G}$ which is appearing in the string theory as the tension of a string and also appears in the field equations of Einstein. The fact that this force is independent of h suggest according to the "old" Copenhagen school that this force is mainly relevant for the non-quantum world. We will see however in §8.9.3.4 that this "constant" force can be derived from another force relation by considering the rest-mass m_0 as a variable.

Conjecture: replace the term $\frac{c_0^4}{G}$ by $\frac{m_0^2 c_0^3}{h}$ in the field equations of Einstein when the mass $m_0 \leq m_{\mathrm{P}}$.

The magnitude of the force $F = \frac{c_0^4}{G}$ is $F \cong 12 \times 10^{43}\ N$ which is a huge force(tension).

8.9.3.2 Force independent of c_0

We have the condition $\left(\frac{8-r}{2}\right) = 0$ so we have *r* = 8

We find the equation $F = \frac{m_0^8 G^3}{h^4}$ which is "unknown". The magnitude of this force for a proton is $F \cong 9 \times 10^{-113}\ N$ which is **extremely small**. However if we substitute for m_0 the Planck mass m_{P} , then we obtain the value $F = \frac{c_0^4}{G}$. We therefore assume that this force is only of importance for very heavy elementary particles of a mass similar to the Planck mass.

8.9.3.3 Force independent of m_0

We have the condition $r = 0$ so we have *r* = 0

We find the equation $F = \frac{c_0^4}{G}$ which is appearing in the string theory as the tension of a string. This force is related to the force $F = \frac{m_0^2 c_0^3}{h}$ through the Planck mass. Subsituting the Planck mass in the equation results in the force $F = \frac{c_0^4}{G}$.

8.9.3.4 Force independent of G

We have the condition $\left(\frac{r-2}{2}\right) = 0$ so we have *r* = 2.

We find the equation $F = \frac{m_0^2 c_0^3}{h}$ which is the force related to the critical field for pair production.

The equation suggest the existence of an equation $\frac{d\vec{p}}{dt} = mc_0^2\vec{k}$ where $\vec{p}$ is the linear momentum and $\vec{k}$ is the wave vector of the particle.

8.9.3.5 Force linearly dependent on m_0 (Newton's Law)

We observe that in the case of *r* = 1 we have a situation as in the law of Newton $\boldsymbol{F} = m\boldsymbol{a}$ but where here the acceleration is a constant based on fundamental physical constants. We find a constant acceleration $a = \left(\frac{c_0^{13}}{Gh}\right)^{\frac{1}{8}}$ which has a magnitude $a = 1.563 \times 10^{19}\ ms^{-2}$.

Conjecture: Is there any relation of this constant acceleration with the inflation theory (Guth, 1997)?

Constant acceleration relation to inflation theory

Parameter	Value
t1 (start inflation) [s]	1,000E-37
t2 (end inflation) [s]	1,000E-35
dt	9,900E-36
a [m/s²]	1,563E+19
dv=a dt [m/s]	1,547E-16
dx= a (dt)²[m]	7,659E-52

In chapter 10 of his book "The inflationary Universe", Alan H. Guth gives a rough value of $x = 10^{-52}\ m$ (Guth, 1997) for the size of the observed universe which seems to be roughly in agreement with the naïve calculation in the above table, which assumes the existence in the universe of a constant acceleration $a = \left(\frac{c_0^{13}}{Gh}\right)^{\frac{1}{8}}$.

What is the relation between all these equations. It can be shown that one has as function of the parameter r the following equation:

$$\frac{F}{\left(\frac{c_0^4}{G}\right)} = \left(\frac{m_0}{\sqrt{\frac{hc_0}{G}}}\right)^r \quad \text{with } r \in \boldsymbol{N}$$

If we combine now the generic equation for energy and the generic equation for the force we could derive a generic equation for length l which represents the ratio of energy to force. The calculus results in the equation: $\frac{l}{\left(\frac{Gm_0}{c_0^2}\right)} = \left(\frac{\sqrt{\frac{hc_0}{G}}}{m_0}\right)$ which is **independent of the parameter r**. From the relation one can find the value of l which is $l = \sqrt{\frac{Gh}{c_0^3}} = l_P$,nothing else than the Planck length.

We derive from this relation the following invariant $\frac{m_P}{l_P} = \frac{m_0}{\left(\frac{Gm_0}{c_0^2}\right)} = \frac{c_0^2}{G}$ having dimension $kg\ m^{-1}$.

We have seen that the parametric dimensional exploration of physical quantities can give a hint about the relations and constants to be considered. One should ask the question if the results of this parametric exploration are biased by the set $\boldsymbol{UC} = \{\boldsymbol{h}, \boldsymbol{c_0}, \boldsymbol{m_0}, \boldsymbol{G}\}$ and if this set UC is the correct choice.

We know that $c_0^2\varepsilon_0\mu_0 = 1$ so it is more appropriate to select as basic constants ε_0 and μ_0 as characteristics of vacuum.

We have also seen that the Planck constant could have been defined as $J_N = e^2\sqrt{\frac{\mu_0}{\varepsilon_0}}$.

It can also be proven that the vector representing the physical quantity **G** which is **G** =(3,-1,-2,0,0,0,0) *cannot be written as a linear combination* of the vectors $\boldsymbol{\mu_0} = (1,1,-2,-2,0,0,0)$ and $\boldsymbol{\varepsilon_0} = (-3,-1,4,2,0,0,0)$.

It can also be proven that the vector representing the physical quantity **e** which is **e** =(0,0,1,1,0,0,0) *cannot be written as a linear combination* of the vectors $\boldsymbol{\mu_0} = (1,1,-2,-2,0,0,0)$ and $\boldsymbol{\varepsilon_0} = (-3,-1,4,2,0,0,0)$.

Let us now consider a new set $\boldsymbol{UCN} = \{\boldsymbol{\mu_0}, \boldsymbol{\varepsilon_0}, \mathbf{e}, \mathbf{G}\}$ of 4 "fundamental constants". We have deliberately eliminated the rest-mass to obtain constants that are totally independent of the physical quantity mass. These constants should be "universal".

8.9.4 Solution for the quantity "Length" as function of the parametric set $\{\mu_0, \varepsilon_0, \mathrm{e}, \mathrm{G}\}$

Solve the equation for a physical quantity with dimension length L.

One finds $L = e\mu_0\sqrt{G\varepsilon_0}$ which is smaller than the Planck length and could thus be more fundamental. The magnitude of this constant length is $L = 4.8935 \times 10^{-36}\, m$.

8.9.5 Solution for the quantity "Frequency" as function of the parametric set $\{\mu_0, \varepsilon_0, \mathrm{e}, \mathrm{G}\}$

Solve the equation for a physical quantity with dimension frequency f.

One finds $f = \frac{1}{\varepsilon_0 e\sqrt{G\mu_0^3}}$. The magnitude of this constant frequency is $f = 6.125 \times 10^{43}\, s^{-1}$

Is this constant frequency related to the vibrations of the "vacuum-state"?

8.9.6 Solution for the quantity "Energy" as function of the parametric set $\{\mu_0, \varepsilon_0, \mathrm{e}, \mathrm{G}\}$

Solve the equation for a physical quantity with dimension energy E.

One finds $E = ec_0^2\sqrt{\frac{1}{G\varepsilon_0}}$. This constant energy has the magnitude $E = 5.923 \times 10^{+8}\, J$.

We compare this energy with the rest energy of an electron which is $E_e = 8.187 \times 10^{-14}\, J$.

If the constant energy would correspond to an elementary particle then this elementary particle has a rest-mass $m_0 = 6.591 \times 10^{-9}\, kg$.

If the constant energy would correspond to annihilation with generation of photons with this energy, then the wavelength is $\lambda = 3.3454 \times 10^{-34}\, m$ and the thermodynamic temperature $T = 4.290 \times 10^{31}\, K$.

What is special to this constant energy? Can we detect the remnant of these photons? Which value for the red shift? Is it the temperature at which symmetry breaking occurs between gravitational interaction and electromagnetic interaction?

The total energy of the observable universe is $10^{68}\ J$ (Davies, Superforce - The search for a grand unified theory of nature, 1995). Can we divide this energy by the Planck energy $\sqrt{\frac{\hbar c_0^5}{G}}$ to obtain the quantity of Planck particles at the Planck time and consider a decay process as generator of the elementary particles? The total quantity of Planck particles would be 2×10^{58}? The total quantity of elementary particles in the observable universe is estimated to 10^{80} (Delahaye, 2006).

8.9.7 Solution for the quantity "Magnetic moment" as function of the parametric set $\{\mu_0, \varepsilon_0, e, G\}$

Solve the equation for a physical quantity with dimension magnetic moment [A m^2].

One finds $\mu_m = e^2\sqrt{G\mu_0}$. This constant magnetic moment has the magnitude $\mu_m = 2.3505 \times 10^{-46}\ Am^2$.

What is special to this constant magnetic moment?

8.9.8 Solution for the quantity "mass/magnetic moment" as function of the parametric set $\{\mu_0, \varepsilon_0, e, G\}$

One finds the equation $\varepsilon_0\mu_0^2$ with as numerical value $1.398 \times 10^{-23}\ kg\ \ A^{-1}\ m^{-2}$

8.9.9 Listing of constants as function of the parametric set $\{G, h, c_0, \mathrm{e}\}$

We give a list of constants that can be calculated in the same way as explained above.

Physical quantity	Equation	Value	Units
Length	$\sqrt{\frac{Gh}{c_0^3}}$	4.05×10^{-35}	m
Time	$\sqrt{\frac{Gh}{c_0^5}}$	1.35×10^{-43}	s
Mass	$\sqrt{\frac{hc_0}{G}}$	5.456×10^{-8}	kg
Mass density	$\frac{c_0^5}{hG^2}$	8.21×10^{95}	$kg\ m^{-3}$
Velocity	c_0	2.997×10^{8}	$m\ s^{-1}$
Acceleration	$\left(\frac{c_0^{13}}{Gh}\right)^{\frac{1}{8}}$	1.563×10^{19}	$m\ s^{-2}$
Length x Time	$\frac{Gh}{c_0^4}$	5.47×10^{-78}	$m\ s$
Linear momentum	$\sqrt{\frac{hc_0^3}{G}}$	16.358	$kg\ m\ s^{-1}$
Energy	$\sqrt{\frac{hc_0^5}{G}}$	4.9×10^{9}	J
Pressure	$\frac{h^5 c_0}{G^2}$	859×10^{-140}	$N\ m^{-2}$
Force(Tension)	$\frac{c_0^4}{G}$	1.21×10^{44}	N
Power	$\frac{c_0^5}{G}$	3.62×10^{52}	W
Frequency	$\sqrt{\frac{c_0^5}{Gh}}$	7.401×10^{42}	Hz
Electrical Charge	e	1.602×10^{-19}	$A\ s$
Electrical potential	$\sqrt{\frac{hc_0^5}{Ge^2}}$	3.060×10^{28}	V
Electrical capacitance	$\sqrt{\frac{Ge^4}{hc_0^5}}$	5.234×10^{-48}	F

Electrical resistance	$\frac{h}{e^2}$	25 812.82	Ω
Magnetic flux	$\frac{h}{e}$	4.1357×10^{-15}	
Magnetic induction	$\frac{c_0^3}{eG}$	2.52×10^{54}	
Diffusion constant	$\sqrt{\frac{Gh}{c_0}}$	$3.84 \times 10^{-(\frac{53}{2})}$	$m^2\, s^{-1}$
Induction	$\sqrt{\frac{Gh^3}{c_0^5 e^4}}$	3.487×10^{-39}	
Mass per length unit	$\frac{c_0^2}{G}$	1.347×10^{27}	$kg\, m^{-1}$

8.10 Some relativistic invariant equations

Dimensionless products are by definition relativistic invariant which makes them “pure” mathematical objects..

8.10.1 Relation mass, temperature and entropy

It is known(Menzel, 1960) that for the mass *m* we have:

$m = \frac{m_0}{\sqrt{1-\beta^2}}$ and $\beta = \frac{v}{c_0}$.

It is also known(Menzel, 1960) for the temperature *T* that we have:

$T = T_0\sqrt{1-\beta^2}$ and $\beta = \frac{v}{c_0}$.

It is also known(Menzel, 1960) for the entropy S that we have:

$S = S_0$.

If we consider the three physical quantities then we can write: $m\, T\, S = m_0\, T_0\, S_0$.

The product $m\, T\, S$ has the dimension of the “square of the linear momentum”.

So we could consider that $m\, T\, S = m_0\, T_0\, S_0 \sim \frac{hc_0^3}{G}$

8.10.2 Relation mass and temperature

From the above equations we have also $m\, T = m_0\, T_0$ as relativistic invariant. It represents a hyperbolic relation which is typically scale invariant.

It is also known(Menzel, 1960) that the ratio $\frac{kc_0}{h}$ is relativistic invariant.

We can derive now that $\frac{mc_0}{h}\,kT = \frac{m_0 c_0}{h}\,kT_0$. This equation has the *dimension of a force*. The value of this force is relativistic invariant!

In the case that $kT_0 = m_0 c_0^2$ which is valid for a relativistic plasma then the force can be written as: $F_0 = \frac{m_0^2\,c_0^3}{h}$ which occurs in the pair creation process.

A "phase diagram"(Weinberg, 1977) can be created based on the hyperbolic relation $m\,T = constant$.

The following dimensionless equation $\pi^2 = \pi^4$ where $E = kT$ results in the hyperbolic relation $mT = \frac{hc_0^3}{Gk}$.

The hyperbola $mT = \frac{hc_0^3}{Gk}$ crosses the straight line $T = \frac{c_0^2}{k} m_0$ in the coordinates $(m_\mathrm{P}, T_\mathrm{P})$.

The hyperbolic relation, derived from dimensional exploration, is fully in agreement with the temperature T_H of the Bekenstein-Hawking radiation of a black hole given by: $m_{BH} T_H = \frac{1}{8\pi}\left(\frac{hc_0^3}{Gk}\right)$

where m_{BH} represents the mass of the black hole. The black hole with the smallest rest-mass is when $m_{BH} = m_\mathrm{P}$.

The constant $\frac{hc_0^3}{Gk}$ can also be dived from the following:

Consider relativistic baryonic mass $m_b = \frac{k}{c_0^2}\,T_{WMAP}$ and its "conjugate black hole mass" $m_{BH} = \frac{constant}{T_{WMAP}}$.

We want to determine the "*constant*". We know that at the crossing of the straight line and the hyperbola we have the Planck conditions, so we suppose (see page 125 of(Hawking & Penrose, 1994)) that we can write $m_b m_{BH} = \frac{k}{c_0^2}\,T_{WMAP}\frac{constant}{T_{WMAP}} = \frac{k\;constant}{c_0^2} = \frac{hc_0}{G}$, which results in $constant = \frac{hc_0^3}{Gk}$.

The (m, T) phase diagram showing the straight line $T = \frac{c_0^2}{k} m_0$, the hyperbola $mT = \frac{hc_0^3}{Gk}$, the line $T = T_\mathrm{P}$ (isotherm) and $m = m_\mathrm{P}$ is made up from 8 regions.

The physical significance of these 8 regions is still unclear and should be investigated. It is possible that T_P should be considered as a critical point as well as m_P. The graph should then be expressed as $\frac{m_0}{m_\mathrm{P}}$ and $\frac{T_0}{T_\mathrm{P}}$ where at the critical point the ratios are equal to 1.

8.11 Dimensionless products in characteristic polynomials

Consider the following dimensionless products of the problem $f(a_{11}, a_{22}) = 0$ where

$a_{11} = \left(\frac{1}{\pi_2}\right)^2$ and $a_{22} = (\frac{\pi_3}{\pi_2})^2$ and form the characteristic polynomial $\begin{vmatrix} x - a_{11} & 0 \\ 0 & x - a_{22} \end{vmatrix} = 0$ which results in $(x - a_{11})(x - a_{22}) = 0$.

Explicitly we find $\left(x-\frac{m_0^2c_0^4}{E^2}\right)\left(x-\frac{c_0^2p^2}{E^2}\right)=0$. Some calculus results in the characteristic polynomial $x^2-(\frac{m_0^2c_0^4}{E^2}+\frac{c_0^2p^2}{E^2})\,x+\frac{m_0^2c_0^4}{E^2}\frac{c_0^2p^2}{E^2}=0$. We known that for elementary symmetric function one has $x^2-\sigma_1 x+\sigma_2=0$. Identifying the terms we have: $\sigma_1=a_{11}+a_{22}$ and $\sigma_2=a_{11}a_{22}$.

The function $y=x^2-(\frac{m_0^2c_0^4}{E^2}+\frac{c_0^2p^2}{E^2})\,x+\frac{m_0^2c_0^4}{E^2}\frac{c_0^2p^2}{E^2}$ represents parabola trajectories where the parameters are (E,p,m_0). For free particles we have $\left(\frac{m_0^2c_0^4}{E^2}+\frac{c_0^2p^2}{E^2}\right)=1$.

It could be interesting to study the roots of the general equation.

The method described above could be applied to the problem of § 8.4.7. Could this method of characteristic polynomials be generalized and under which conditions is this true?

Can we assume that each solution of a dimensional analysis can be written in the form:

$\prod_{i=1}^{n}(\pi^i-x)=0$ being the characteristic polynomial of $\det(\boldsymbol{T}-\lambda\boldsymbol{I})=0$?

So, we have as "trial function" for the problem of § 8.4.7 a polynomial equation in x of degree 6:

$$\left(\frac{FG}{c^4}-x\right)\left(\frac{E}{m_0c^2}-x\right)\left(\frac{p}{m_0c}-x\right)\left(\frac{Jc}{Gm_0^2}-x\right)\left(\frac{q^2}{G\epsilon_0m_0^2}-x\right)\left(\frac{tc^3}{Gm_0}-x\right)=0$$

8.12 Heisenberg uncertainty principle applied to dimensionless products

Let us consider the following dimensionless products:

$\pi^2=\frac{E}{m_0c^2}$ and $\pi^6=\frac{tc^3}{Gm_0}$.

We can form the product: $\pi^2\pi^6=\frac{\Delta E}{m_0c^2}\frac{\Delta tc^3}{Gm_0}=\frac{\Delta E\Delta t}{\left(\frac{Gm_0^2}{c}\right)}\geq\frac{\left(\frac{h}{4\pi}\right)}{\left(\frac{Gm_0^2}{c}\right)}$

8.13 Planck era parameters and relations

At the Planck era it is assumed that we have the following parameters:

Planck mass $m_{\mathrm{P}}=\sqrt{\frac{hc}{G}}$

Planck length $L_{\mathrm{P}}=\sqrt{\frac{Gh}{c^3}}=\frac{h}{m_{\mathrm{P}}c}=\lambda_{c,\mathrm{P}}$

Planck area of the Planck sphere $A_{\mathrm{P}}=4\pi\frac{Gh}{c^3}$

Volume of the Planck sphere $V_{\mathrm{P}}=\frac{4}{3}\pi\left(\sqrt{\frac{Gh}{c^3}}\right)^3$

Planck energy $E_{\mathrm{P}}=m_{\mathrm{P}}c^2=kT_{\mathrm{P}}=\sqrt{\frac{hc^5}{G}}=\frac{c^4}{G}\frac{r_{\mathrm{S}}}{2}$

Schwarzschild radius of a Planck particle "maximon" : $r_{\mathrm{S}}=2\,L_{\mathrm{P}}$

The quantum-mechanical "cavity" would have a size equal to $\frac{L_{\mathrm{P}}}{2}$.

Let us follow Bohr's correspondence principle and consider constructive interference of de Broglie waves in the "cavity" containing the Planck particle resulting in allowed stable orbits. We put $\oint pds = nh$ and consider $p = \frac{m_0^2 c^3}{h} t$ to be the linear momentum of the particle with restmass m_0 within the cavity. We assume that the particle is executing a "circular motion" that results in the existence of a total angular momentum J. We also consider that $vt = 2\pi r$ with r the radius of the orbit of the particle.

We have now to calculate for the circular path the integral $\oint pds = \frac{m_0^2 c^3}{h} \int t\, ds = \frac{4\pi^2 m_0^2 c^3 r^2}{hv} = nh$.

The orbits for the "cavity" are given by $r_n = \frac{1}{2\pi}\left(\frac{h}{m_0 c}\right)\sqrt{\frac{nv}{c}}$ with $n \in \boldsymbol{N_0}$.

For the "Planck particle cavity" we have $m_0 = m_{\mathrm{P}}$ and so $r_n = \frac{L_{\mathrm{P}}}{2\pi}\sqrt{\frac{nv}{c}}$. If "Zitterbewegung"(Merzbacher, 1970) is applicable then we can assume $r_n = \frac{L_{\mathrm{P}}}{2\pi}\sqrt{n}$, so the smallest orbit is $r_1 = \frac{L_{\mathrm{P}}}{2\pi}$.

The energy of each "orbit" could be given by $E_{\mathrm{n,P}} = \frac{c^4}{G}\frac{L_{\mathrm{P}}}{2\pi}\sqrt{n}$ with $n \in \boldsymbol{N_0}$.

These "energy levels" could be compared to string theory or quantum loop gravity.

8.14 Flux of vorticity

One can find that a constant with dimension $m^2 s^{-1}$ and based on G,h and c exists (see §8.9.9).

It could be interpreted as a "flux of vorticity" and expressed as $\int \boldsymbol{\omega} \cdot d\boldsymbol{S_1} = \int \boldsymbol{\omega} \cdot d\boldsymbol{S_2} = \sqrt{\frac{Gh}{c}}$

This property of "space" could be interpreted as a vortex tube connecting one area $d\boldsymbol{S_1}$ of "inner space" with another area $d\boldsymbol{S_2}$ of "inner space" while passing the vortex tube through "outer space" making the "energy/mass" visible to "outer space". Inner space is filled with a scalar field having magnitude $\frac{c^4}{G}$.

One can remark that $\sqrt{\frac{Gh}{c}} = cL_{\mathrm{P}}$. An area equal to $\frac{Gh}{c^3}$ is swept by the line element $L_P = \sqrt{\frac{Gh}{c^3}}$ moved over the distance $c\Delta t = c\sqrt{\frac{Gh}{c^5}}$. We see that the area $A_{\mathrm{P}} = \frac{Gh}{c^3} = \Delta t \int \boldsymbol{\omega} \cdot d\boldsymbol{S_1}$.

8.15 Mass frequency

We expect the following relation to be valid: $J = \frac{c^3}{G} A$ where *A* is the area of the "surface" and *J* the total angular momentum. The constant $\frac{c^3}{G}$ represents a constant mass frequency $\frac{dm}{dt}$ or rate of change of mass.

8.16 Geometric representations of an elementary particle

8.16.1 Torus model

Let us consider a torus with radii:

$R_1 = \frac{h}{m_0 c}$ being the radius from the centre of the tube to the centre of the torus;

$R_2 = \frac{G m_0}{c^2}$ being the radius of the tube.

The equation in Cartesian coordinates for a torus radially symmetric around the z-axis is:

$$\left(x^2 + y^2 + z^2 + {R_1}^2 - {R_2}^2\right)^2 = 4{R_1}^2(x^2 + y^2)$$

The surface area of the torus is $A = (2\pi R_1)(2\pi R_2) = 4\pi^2\left(\frac{Gh}{c^3}\right) = 4\pi^2 L_{\mathrm{P}}^2$ which is **independent** from the rest mass m_0 of a particle!

The interior volume of the torus is $V = (\pi R_2^2)(2\pi R_1) = 2\pi^2 \frac{G^2 h m_0}{c^5}$.

The ratio of volume to area is $\frac{V}{A} = \frac{1}{2}\frac{G m_0}{c^2}$.

8.16.2 Torus-sphere model

Let us consider a torus with cross-sectional diameter of $L = \frac{h}{2 m_0 c}$ and a sphere concentric with the axis of torus.

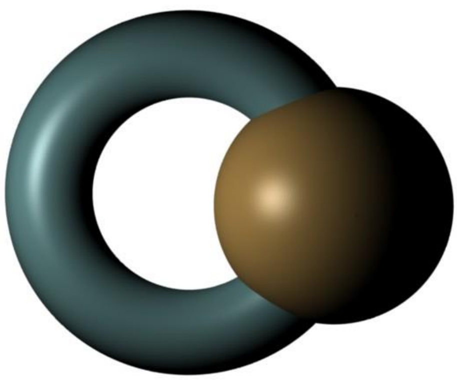


Figure 8.16-1 Torus-sphere model

Let us assume that the following equation is valid:

$\iiint_V \frac{E}{V}\, dV = \iiint_{V'} \frac{F}{S'}\, dV'$ and let the scalar fields $\frac{E}{V} = div\, \boldsymbol{X}$ and $\frac{F}{S'} = div\, \boldsymbol{Y}$, where V is the outer space volume and V' the inner space volume. The inner space volume is filled with virtual particles.

We have now $\iiint_V \, div\, \boldsymbol{X}\, dV \;=\; \iiint_{V'} div\, \boldsymbol{Y}\, dV'$ and using the divergence theorem results in in $\iint_{\partial V} \boldsymbol{e_N} \cdot \boldsymbol{X}\, dS = \; \iint_{\partial V'} \boldsymbol{e_N} \cdot \boldsymbol{Y}\, dS'$.

The outer surface of the cylinder is S_1 and the inner surface of the cylinder is S_2.

We have now $\iint_{S_1} \boldsymbol{n_1^u} \cdot \boldsymbol{X}\, dS + \; \iint_{S_2} \boldsymbol{n_2^u} \cdot \boldsymbol{X}\, dS = \; \iint_{S_2} \boldsymbol{n_2^u} \cdot \boldsymbol{Y}\, dS$ which results in

$\iint_{S_1} \boldsymbol{n_1^u} \cdot \boldsymbol{X}\, dS = \iint_{S_2} \boldsymbol{n_2^u} \cdot (\boldsymbol{Y} - \boldsymbol{X})\, dS$

Let now

- $div\, \boldsymbol{X} = \frac{E}{V} = \frac{m_0 c^2}{V} = \frac{m_0 c^2}{\frac{4}{3}\pi(r^3 - \frac{h^3}{64 m_0^3 c^3})}$ where V is approximated by the difference of volume of two spheres, one sphere V with a variable radius r and the other internal sphere V’ with radius $\frac{h}{4m_0 c}$ and the speed of the particle is $v = 0$.
- $div\, \boldsymbol{Y} = \frac{F}{S'} = \frac{m_0^2 c^3}{h} \frac{1}{4\pi \frac{h^2}{16 m_0^2 c^2}} = \frac{16 m_0^4 c^5}{4\pi h^3}$ for a sphere inside the torus where $F = \frac{m_0^2 c^3}{h}$ and $S' = 4\pi \frac{h^2}{16 m_0^2 c^2}$

We have now approximatively:

$$\iiint_V \frac{m_0 c^2}{\frac{4}{3}\pi(r^3 - \frac{h^3}{64 m_0^3 c^3})}\, dV \;=\; \frac{16 m_0^4 c^5}{4\pi h^3} \iiint_{V'} dV' = \frac{16 m_0^4 c^5}{4\pi h^3} \frac{4}{3}\pi \frac{h^3}{64 m_0^3 c^3} = \frac{m_0 c^2}{4}$$

which means that $\iiint_V \frac{3}{\pi(r^3 - \frac{h^3}{64 m_0^3 c^3})}\, dV = 1$

Let us try to make a better approximation for the volume V and V’ by considering the cross-section of a cylinder and a sphere of radius R.

We will calculate the left side of the equation where we need to know the volume V in spherical coordinates and we put:

$r sin\varphi = \frac{h}{4m_0 c}$ and $0 < \theta < \frac{\pi}{2}$ and $\arcsin(\frac{h}{4Rm_0 c}) < \varphi < \frac{\pi}{2}$ and $\frac{h}{4m_0 c} < r < R$

$$\iiint_V \frac{E}{V}\, dV \;= 8\left\{ \frac{3E}{4\pi} \int_0^{\frac{\pi}{2}} d\theta \int_{\arcsin(\frac{h}{4Rm_0 c})}^{\frac{\pi}{2}} sin\varphi d\varphi \int_{\frac{h}{4m_0 c}}^{R} \frac{r^2 dr}{\left[r^3 - \frac{1}{4}\pi \left(\frac{h}{4m_0 c}\right)^2 R cos\varphi \right]} \right\}$$

$$\iiint_V \frac{E}{V}\, dV \;= 8\left\{ \frac{E}{4\pi} \int_0^{\frac{\pi}{2}} d\theta \int_{\arcsin(\frac{h}{4Rm_0 c})}^{\frac{\pi}{2}} sin\varphi d\varphi \int_{\frac{h}{4m_0 c}}^{R} \frac{dr^3}{\left[r^3 - \frac{h^2 \pi R cos\varphi}{64 m_0^2 c^2} \right]} \right\}$$

Put $x = \; r^3$ and $dr^3 = 3r^2 dr$ and we have $\int \frac{dx}{(x-a)} = ln|x - a|$ + constant. So, we find

$$\int_{\sqrt[3]{\frac{h}{4m_0c}}}^{\sqrt[3]{R}} \frac{dx}{\left[x-\frac{h^2\pi R\cos\varphi}{64m_0^2c^2}\right]} = \ln\left(\sqrt[3]{R}-\frac{h^2\pi R\cos\varphi}{64m_0^2c^2}\right) - \ln\left(\sqrt[3]{\frac{h}{4m_0c}}-\frac{h^2\pi R\cos\varphi}{64m_0^2c^2}\right))$$

$$\iiint_V \frac{E}{V}\,dV = 8\left\{\frac{E}{4\pi}\int_0^{\frac{\pi}{2}} d\theta \int_{\arcsin(\frac{h}{4Rm_0c})}^{\frac{\pi}{2}} \left[\ln\left(\sqrt[3]{R}-\frac{h^2\pi R\cos\varphi}{64m_0^2c^2}\right) - \ln\left(\sqrt[3]{\frac{h}{4m_0c}}-\frac{h^2\pi R\cos\varphi}{64m_0^2c^2}\right)\right]\sin\varphi d\varphi\right\}$$

Solution in Maple gives:

$$-\frac{1}{12}E\left(-6144\,m_0^2c^2R\ln(2)+768\,m_0^2c^2\ln\left(-\frac{-256\,R^{\left(\frac{1}{3}\right)}m_0^2c^2+h^2\pi R\sqrt{\frac{16R^2m_0^2c^2-h^2}{R^2m_0^2c^2}}}{m_0^2c^2}\right)R+2816\,m_0^2c^2\,2^{\left(\frac{1}{3}\right)}\left(\frac{h}{m_0c}\right)^{\left(\frac{1}{3}\right)}R^{\left(\frac{2}{3}\right)}\ln(2)\right.$$
$$-384\,m_0^2c^2\,2^{\left(\frac{1}{3}\right)}\left(\frac{h}{m_0c}\right)^{\left(\frac{1}{3}\right)}\ln\left(-\frac{-128\,2^{\left(\frac{1}{3}\right)}\left(\frac{h}{m_0c}\right)^{\left(\frac{1}{3}\right)}m_0^2c^2+h^2\pi R\sqrt{\frac{16R^2m_0^2c^2-h^2}{R^2m_0^2c^2}}}{m_0^2c^2}\right)R^{\left(\frac{2}{3}\right)}$$
$$-3\sqrt{\frac{16R^2m_0^2c^2-h^2}{R^2m_0^2c^2}}\ln\left(-\frac{-256\,R^{\left(\frac{1}{3}\right)}m_0^2c^2+h^2\pi R\sqrt{\frac{16R^2m_0^2c^2-h^2}{R^2m_0^2c^2}}}{m_0^2c^2}\right)R^{\left(\frac{5}{3}\right)}h^2\pi$$
$$\left.+3\sqrt{\frac{16R^2m_0^2c^2-h^2}{R^2m_0^2c^2}}\ln\left(-\frac{-128\,2^{\left(\frac{1}{3}\right)}\left(\frac{h}{m_0c}\right)^{\left(\frac{1}{3}\right)}m_0^2c^2+h^2\pi R\sqrt{\frac{16R^2m_0^2c^2-h^2}{R^2m_0^2c^2}}}{m_0^2c^2}\right)R^{\left(\frac{5}{3}\right)}h^2\pi-256\,m_0^2c^2\ln(R)R+128\,m_0^2c^2\,2^{\left(\frac{1}{3}\right)}\left(\frac{h}{m_0c}\right)^{\left(\frac{1}{3}\right)}R^{\left(\frac{2}{3}\right)}\ln\left(\frac{h}{m_0c}\right)\right)$$
$$\Big/\left(\pi R^{\left(\frac{5}{3}\right)}h^2\right)$$

When we calculate the right hand side for the volume V' we have

$$\iiint_{V'} \frac{F}{S'}\,dV' = \frac{8m_0^2c^3}{h}\left\{\int_0^{\frac{\pi}{2}} d\theta \int_{\arcsin(\frac{h}{4Rm_0c})}^{\frac{\pi}{2}} \sin\varphi d\varphi \int_{\frac{h}{4m_0c}}^{R} \frac{r^2dr}{\left[\frac{1}{4}2\pi\frac{h}{4m_0c}R\cos\varphi\right]}\right\}$$

This results in $\iiint_{V'} \frac{F}{S'}\,dV' = \frac{4m_0^2c^3}{h}\left\{\int_0^{\frac{\pi}{2}} d\theta \int_{\arcsin(\frac{h}{4Rm_0c})}^{\arcsin(\pi-\frac{h}{4Rm_0c})} \sin\varphi d\varphi \int_{\frac{h}{4m_0c}}^{R} \frac{r^2dr}{\left[\frac{1}{4}2\pi\frac{h}{4m_0c}R\cos\varphi\right]}\right\} = \frac{32m_0^3c^4}{3\pi Rh^2}\left(R^3-\frac{h^3}{64m_0^3c^3}\right)\frac{\pi}{2}\int_{\arcsin(\frac{h}{4Rm_0c})}^{\arcsin(\pi-\frac{h}{4Rm_0c})} \tan\varphi d\varphi = \frac{32m_0^3c^4}{6Rh^2}\left(R^3-\frac{h^3}{64m_0^3c^3}\right)\left\{\ln(\sec\left(\arcsin\left(\pi-\frac{h}{4Rm_0c}\right)\right) - \ln(\sec(\arcsin\left(\frac{h}{4Rm_0c}\right))\right\}$

In Maple we find the solution:

$$vgl2 := \frac{32}{3} \frac{m_0^3 c^4 \left(R^3 - \frac{1}{64} \frac{h^3}{m_0^3 c^3}\right) \left(\frac{1}{2} \ln\left(-\frac{4}{\sqrt{16 - \frac{h^2}{R^2 m_0^2 c^2}}}\right) \pi - \frac{1}{2} \ln\left(\frac{4}{\sqrt{16 - \frac{h^2}{R^2 m_0^2 c^2}}}\right) \pi\right)}{h^2 \pi R}$$

If we create now $\iiint_V \frac{E}{V}\, dV \; - \; \iiint_{V\prime} \frac{F}{S\prime}\, dV' = 0$ we get an equation in *R* as function of the parameters (m_0, E). So, we have $f(R; m_0, E) = 0$.

The equation in Maple is then:

```
> oplossing:=vgl1-vgl2=0;
```

$$oplossing := \frac{1}{12} E \Bigg(6144\, m_0^2 c^2 R \ln(2) - 768\, m_0^2 c^2 \ln\left(-\frac{-256 R^{\left(\frac{1}{3}\right)} m_0^2 c^2 + h^2 \pi R \sqrt{\frac{16 R^2 m_0^2 c^2 - h^2}{R^2 m_0^2 c^2}}}{m_0^2 c^2}\right) R - 2816\, 2^{\left(\frac{1}{3}\right)} \left(\frac{h}{m_0 c}\right)^{\left(\frac{1}{3}\right)} m_0^2 c^2 R^{\left(\frac{2}{3}\right)} \ln(2)$$

$$+ 384\, 2^{\left(\frac{1}{3}\right)} \left(\frac{h}{m_0 c}\right)^{\left(\frac{1}{3}\right)} m_0^2 c^2 \ln\left(-\frac{-128\, 2^{\left(\frac{1}{3}\right)} \left(\frac{h}{m_0 c}\right)^{\left(\frac{1}{3}\right)} m_0^2 c^2 + h^2 \pi R \sqrt{\frac{16 R^2 m_0^2 c^2 - h^2}{R^2 m_0^2 c^2}}}{m_0^2 c^2}\right) R^{\left(\frac{2}{3}\right)} + 3 \sqrt{\frac{16 R^2 m_0^2 c^2 - h^2}{R^2 m_0^2 c^2}} \ln\left(-\frac{-256 R^{\left(\frac{1}{3}\right)} m_0^2 c^2 + h^2 \pi R \sqrt{\frac{16 R^2 m_0^2 c^2 - h^2}{R^2 m_0^2 c^2}}}{m_0^2 c^2}\right) h^2 \pi R^{\left(\frac{5}{3}\right)}$$

$$- 3 \sqrt{\frac{16 R^2 m_0^2 c^2 - h^2}{R^2 m_0^2 c^2}} \ln\left(-\frac{-128\, 2^{\left(\frac{1}{3}\right)} \left(\frac{h}{m_0 c}\right)^{\left(\frac{1}{3}\right)} m_0^2 c^2 + h^2 \pi R \sqrt{\frac{16 R^2 m_0^2 c^2 - h^2}{R^2 m_0^2 c^2}}}{m_0^2 c^2}\right) h^2 \pi R^{\left(\frac{5}{3}\right)} + 256\, m_0^2 c^2 \ln(R) R - 128\, 2^{\left(\frac{1}{3}\right)} \left(\frac{h}{m_0 c}\right)^{\left(\frac{1}{3}\right)} m_0^2 c^2 R^{\left(\frac{2}{3}\right)} \ln\left(\frac{h}{m_0 c}\right) \Bigg) \Bigg/ \left(\pi h^2 R^{\left(\frac{5}{3}\right)}\right)$$

$$- \frac{32}{3} \frac{m_0^3 c^4 \left(R^3 - \frac{1}{64} \frac{h^3}{m_0^3 c^3}\right) \left(\frac{1}{2} \ln\left(-\frac{4}{\sqrt{16 - \frac{h^2}{R^2 m_0^2 c^2}}}\right) \pi - \frac{1}{2} \ln\left(\frac{4}{\sqrt{16 - \frac{h^2}{R^2 m_0^2 c^2}}}\right) \pi\right)}{h^2 \pi R} = 0$$

Solving for a set of parameters (m_0, E) one can find the "radius" of a particle according to the proposed geometrical model of an elementary particle. We tried to solve the equation in Maple but Maple fails to find a solution. (To do: find solution for $f(R; m_0, E) = 0$)

If the particle is at rest we have $E = m_0 c^2$ and $f(R; m_0, E) = g(R; m_0) = 0$

It is known from topology that this "torus-sphere model" has a total curvature $2\pi\chi(s) = 2\pi 2(n-1) = 0$ because $n = 1$ with *n* = genus of the orientable surface. The genus *n* is the number of handles(tori) on the sphere.

9 Mathematical modelling of physics in $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane

9.1 Representation of particles in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane

The following equation $E^2 - p^2c^2 = m_0^2c^4$ (Feynman, 1963) is valid for any free particle.

We can rewrite this equation in terms of dimensionless products: $1 - \left(\frac{cp}{E}\right)^2 = \left(\frac{m_0c^2}{E}\right)^2$.

We define the **dimensionless products:** $x = \left(\frac{m_0c^2}{E}\right)$ and $y = \left(\frac{cp}{E}\right)$.

So, the equation becomes: $1 - y^2 = x^2$. One recognizes the equation of the unit circle $x^2 + y^2 = 1$.

So, the unit circle is the set of all possible states that a free particle can occupy in a system with total energy E. It is clear that it is a continuum of states in accordance with classical physics.

We now see that each particle with rest-mass m_0 and linear momentum *p* for a system with total energy E can be represented in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane. The states of particles are in most cases not lying on the unit circle because the particles are subject to "varying" forces. Any deviation in position from the unit circle brings the particle in a "*force field*".

We can form the complex number $z = x + iy$,so that all mathematical formalism for the complex plane can be used in helping to describe the states of particles and how these states evolve with time $z(t) = x(t) + iy(t)$. We will be interested in paths between points in the complex plane $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane .

The system under study can be anything (a bubble chamber, the universe, …) as long as it can be characterized by the configurational space variables $(m_0, E, \boldsymbol{p})$ where $p^2 = \boldsymbol{p} \cdot \boldsymbol{p}$.

The points in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane are "**universal" states** once the system is defined.

The free particles on the unit circle can be represented using the Euler formalism by $z = e^{i\arctan\left(\frac{p}{m_0c}\right)} = e^{i\arctan\left(\frac{\lambda_c}{\lambda_{dB}}\right)} = e^{i\arctan\left(\frac{v}{\sqrt{c^2-v^2}}\right)}$ with λ_c the Compton wavelength and λ_{dB} the "de Broglie" wavelength of the particle and v is the velocity of the particle .

When $\frac{\lambda_c}{\lambda_{dB}} = 1$ then the particle state is represented by a point P on the unit circle that makes an angle of 45° with the x-axis. The condition $\frac{\lambda_c}{\lambda_{dB}} = 1$ is described in QED as the boundary where "virtual" particles start playing an important role. It can be assumed that pair creation will spontaneously start when $z = 0.5e^{i\frac{\pi}{4}}$. So, the lines y=x and y=-x should be considered as special boundaries because the condition $\lambda_{dB} = \lambda_c$ is fulfilled.

The condition $\frac{\lambda_c}{\lambda_{dB}} = 1$ divides the unit disc in 4 regions:

“Region 1” contains all states where $re^{-i\frac{\pi}{4}} \leq z \leq re^{i\frac{\pi}{4}}$ and $r \in \boldsymbol{R}^{+}$

“Region 2” contains all states where $re^{i\frac{\pi}{4}} \leq z \leq re^{i\frac{3\pi}{4}}$ and $r \in \boldsymbol{R}^{+}$

“Region 3” contains all states where $re^{i\frac{3\pi}{4}} \leq z \leq re^{i\frac{5\pi}{4}}$ and $r \in \boldsymbol{R}^{+}$

“Region 4“ contains all states where $re^{i\frac{5\pi}{4}} \leq z \leq re^{i\frac{7\pi}{4}}$ and $r \in \boldsymbol{R}^{+}$

Region 1 represents the states of “real” particle states

Region 3 represents the states of “real” anti-particle states in line with the Dirac sea formalism.

Region 2 and region 4 represent “virtual” particle states

It is possible to define in the (x, y) plane two elementary symmetric functions(Siegfried Gottwald, 1995):

$$\sigma_1(x, y) = x + y \qquad \text{(straight lines)}$$

$$\sigma_2(x, y) = xy \qquad \text{(hyperbola)}$$

The equation $x^2 + y^2 = 1$ is equivalent to $\sigma_1^2 - 2\,\sigma_2 = 1$. It can be important to write the equations in function of their symmetric functions, because symmetry and symmetry-breaking are interesting properties in physics.

The symmetric form $\sigma_2(x, y) = xy$ is very interesting because it express a hyperbolic relation that is scale invariant. Several equations like the Heisenberg uncertainty principle have that form(Schroeder, 1990). It is also found in a lot of dimensionless products.

The half plane where x<0 is to be considered as anti-matter particles. The axis x=0 is the set of all particles with rest-mass $m_0 = 0$. So, photons are always located on the y-axis. The photon is the prime responsible for the electromagnetic interaction. We define a photon to be located in the coordinate (0,1) and the anti-photon in the coordinate (0,-1).

A point with coordinates $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ has a distance to the origin given by $d = \frac{1}{E}\sqrt{m_0^2c^4 + c^2p^2}$ where d=1 for free particles. From the equation one can see that if d decreases that E increases.

Consider as example the case of plasmas having a total energy E = kT and $m_0 = m_e$,the rest-mass of an electron, then all the points where $\frac{m_0c^2}{E} > 1$ are classified as non-relativistic plasmas (see Figure 9.1-1).

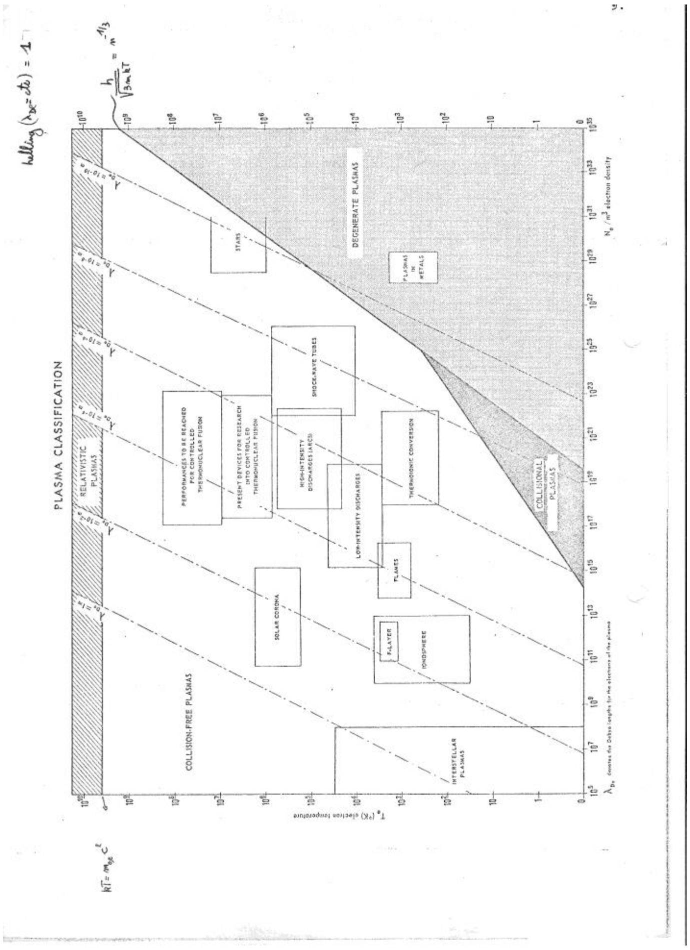


Figure 9.1-1 Plasma classification

If one puts a particle with rest-mass m_0 in a box at energy E_1 and then one increases the total energy to E_2 then the locus of the particle states will be a circle with a radius r < 1.

Suppose that the energy increase is ΔE then the (x, y)-coordinates will transform in the following way:

$$\frac{m_0c^2}{E} \rightarrow \frac{m_0c^2}{E+\Delta E} = \frac{m_0c^2}{E}\frac{1}{(1+\frac{\Delta E}{E})}$$

$$\frac{cp}{E} \rightarrow \frac{cp}{E+\Delta E} = \frac{cp}{E}\frac{1}{(1+\frac{\Delta E}{E})}$$

One can see that an increase in energy results in the application of a scaling factor of $\frac{1}{(1+\frac{\Delta E}{E})}$ to the original coordinates. This is equivalent to a homothetic at the origin with factor $r = \frac{1}{(1+\frac{\Delta E}{E})}$.

As $\frac{\Delta E}{E} \ll 1$ then we have $\frac{1}{(1+\frac{\Delta E}{E})} \approx 1 + \frac{\Delta E}{E} + \left(\frac{\Delta E}{E}\right)^2 + \left(\frac{\Delta E}{E}\right)^3 + \cdots$

We have defined $z = \left(\frac{m_0c^2}{E}\right) + i\left(\frac{cp}{E}\right)$.

We consider now the holomorphic function that images $z \rightarrow f(z) = z$

We know that $\int_k zdz = F(z_{2)} - F(z_1)$ where $f(z) = z = \frac{dF(z)}{dz} = \frac{d}{dz}(\frac{1}{2}z^2)$ and k is the path from z_1 to z_2.

So, we find $\int_k zdz = F(z_{2)} - F(z_1) = \frac{1}{2}(z_2^2 - z_1^2)$

Consider now the function $f: z \rightarrow \frac{1}{(1+\frac{\Delta E}{E})}z$. This function results in a mapping of all the points of $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane according to a homothetic with centre 0 and factor $\left|\frac{1}{(1+\frac{\Delta E}{E})}\right|$. There is no rotation because $\frac{1}{\left(1+\frac{\Delta E}{E}\right)} \in \boldsymbol{R}^+ \Rightarrow Arg\left(\frac{1}{\left(1+\frac{\Delta E}{E}\right)}\right) = 0.$

The function $f: z \rightarrow e^z$ has the property to transform the imaginary axis y in an unit circle. We have $i\theta \rightarrow e^{i\vartheta} = \cos(\theta) + i\sin(\vartheta)$. This means that the function e^z can be considered as a “creator of free particles” starting from particles with rest-mass $m_0 = 0$.

Which function represents the annihilator?

What is the representation of the interaction of 2 particles or n-particles?

9.2 Lyapunov’s stability of states in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane

We will discuss Lyapunov’s stability criterion(DiStefano, Stubberud, & Williams, 1967) applied to $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane. We recall that we have $z(t) = x(t) + iy(t)$ and $x = \left(\frac{m_0c^2}{E}\right)$ and $y = \left(\frac{cp}{E}\right)$

Let us consider the following equations:

$\frac{d\left(\frac{m_0c^2}{E}\right)}{dt} = f_1\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ and $\frac{d\left(\frac{cp}{E}\right)}{dt} = f_2\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$

If we eliminate time as the independent variable, we obtain the single equation

$$\frac{d\left(\frac{m_0c^2}{E}\right)}{d\left(\frac{cp}{E}\right)} = \frac{f_1\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)}{f_2\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)}$$

Whose solution describes a trajectory in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane.

A point $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane simultaneously satisfying the two equations $f_1\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right) = 0$ and $f_2\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right) = 0$ is called a **singular point**.

We can let the origin (0,0) be a singular point. The origin is said to be **stable** if, for any circular region S(A) in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane centred at (0,0), having radius A, there exists a circular region S(B), centred at the origin with radius $B \leq A$, in which any trajectory beginning in S(B) remains in S(A) ever after.

The origin is **asymptotically stable** if it is stable and all trajectories tend to the origin as time goes to infinity.

Lyapunov's stability criterion states that if the origin is a singular point, then it is stable if a function $V\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ can be found such that:

a) $V\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ is positive for all values of $\frac{m_0c^2}{E}$ and $\frac{cp}{E}$, except that it may be zero for $\frac{m_0c^2}{E} = \frac{cp}{E} = 0$; and
b) $\frac{dV}{dt} < 0$ (multi-dimensional one has $\frac{dV(\boldsymbol{x})}{dt} = \dot{\boldsymbol{x}} \cdot \boldsymbol{\nabla} V < 0$ which means that $\mathbf{cos}(\boldsymbol{\theta}) < 0$)

If $\frac{dV}{dt} = 0$ at the origin then the origin is asymptotically stable.

Consider the function $V = \left(\frac{m_0c^2}{E}\right)^2 + \left(\frac{cp}{E}\right)^2$ which is positive for all $\frac{m_0c^2}{E}$ and $\frac{cp}{E}$ except for $\frac{m_0c^2}{E} = \frac{cp}{E} = 0$ where V=0. The selected function $V\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ represents concentric circles around the origin.

The derivative $\frac{dV}{dt} = 2\left(\frac{m_0c^2}{E}\right)\frac{d\left(\frac{m_0c^2}{E}\right)}{dt} + 2\left(\frac{cp}{E}\right)\frac{d\left(\frac{cp}{E}\right)}{dt} < 0$ to have a stable origin.

Let us assume that we have now the pair of equations

$$\frac{d\left(\frac{m_0c^2}{E}\right)}{dt} = a_1\left(\frac{m_0c^2}{E}\right) + a_2\left(\frac{cp}{E}\right)$$

$$\frac{d\left(\frac{cp}{E}\right)}{dt} = a_3\left(\frac{m_0c^2}{E}\right) + a_4\left(\frac{cp}{E}\right)$$

A sufficient condition for asymptotic stability of the origin is(DiStefano, Stubberud, & Williams, 1967):

$a_1 < 0,\ a_4 < 0$ and $a_2 = -a_3$

If we want a "stable" universe we will have to request that as time passes the circles must inevitably shrink. Time is implicit in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ state plane

9.3 Maximum linear momentum per time unit in the $\left(\frac{m_0c^2}{E},\frac{cp}{E}\right)$ plane

Let us consider the following equation $\frac{\Delta p}{\Delta t} = \frac{m_0^2c^3}{h}$ which express Newton's law where the acceleration is replaced by the maximum acceleration $\frac{m_0c^3}{h}$. Let us now assume that the time interval is the shortest time difference that we can define, which is the Planck time $\Delta t = \sqrt{\frac{Gh}{c^5}}$.

If the linear momentum of the particle is 0 at time t=0, then we find that the maximum change in linear momentum is $p_{max} = m_0^2\sqrt{\frac{Gc}{h}}$.

The trajectory in the $\left(\frac{m_0c^2}{E},\frac{cp}{E}\right)$ plane formed by all points where $p = p_{max}$ is found in the following way:

The points have the coordinates $\left(\frac{m_0c^2}{E},\frac{m_0^2c}{E}\sqrt{\frac{Gc}{h}}\right)$ so we search for:

$\frac{m_0^2c}{E}\sqrt{\frac{Gc}{h}} = f\left(\frac{m_0c^2}{E}\right)$. We assume that $f\left(\frac{m_0c^2}{E}\right) = a(m_0)\frac{m_0c^2}{E}$ is a good choice where $a(m_0)$ is a proportionality factor that is only function of the rest mass m_0.

After some calculus we find $a(m_0) = \frac{m_0}{m_P}$.

The equation of the trajectory becomes $\left(\frac{cp}{E}\right) = \left(\frac{m_0}{m_P}\right)\left(\frac{m_0c^2}{E}\right)$.

If $\left(\frac{cp}{E}\right) = 1$,which is a physical boundary for the particles, then we have the relation $E = m_0^2\sqrt{\frac{Gc^3}{h}}$.

9.4 Variational principles in the $\left(\frac{m_0c^2}{E},\frac{cp}{E}\right)$ state plane

We will discuss the "maximum principle" of Pontryagin(Elgerd, 1967) applied to the $\left(\frac{m_0c^2}{E},\frac{cp}{E}\right)$ plane.

9.5 Model of Planck era in the $\left(\frac{m_0c^2}{E},\frac{cp}{E}\right)$ state plane

We seek now a set of nonlinear differential equations representing the dynamics of the universe in the Planck era.

We consider a point with coordinates $\left(\frac{m_0c^2}{E},\frac{cp}{E}\right)$. We consider the coordinates to depend on the variable *t* representing time. We calculate the time derivative of each coordinate.

We have now the system of first order differential equations:

$$\frac{dx}{dt} = \frac{d\left(\frac{m_0c^2}{E}\right)}{dt} = c^2\left\{\frac{dm_0}{Edt} - \frac{m_0dE}{E^2dt}\right\} = x\left\{\frac{dm_0}{m_0dt} - \frac{dE}{Edt}\right\}$$

$$\frac{dy}{dt} = \frac{d\left(\frac{cp}{E}\right)}{dt} = c\left\{\frac{dp}{Edt} - \frac{pdE}{E^2dt}\right\} = y\left\{\frac{dp}{pdt} - \frac{dE}{Edt}\right\}$$

We will now estimate $\frac{dm_0}{dt}, \frac{dE}{dt}, \frac{dp}{dt}$ so that the system can be solved.

For a free particle we have an equation where $E = f(p, m_0)$ so we will seek $\frac{dE}{dt} = g(\frac{dp}{dt}, \frac{dm_0}{dt})$

Consider $E^2 = m_0^2c^4 + c^2p^2$ and differentiate it. Then $2EdE = 2m_0dm_0c^4 + 2c^2pdp$ can be transformed to $\frac{dE}{E} = \frac{m_0c^4dm_0}{E^2} + \frac{c^2pdp}{E^2} = \left(\frac{c^2dm_0}{E}\right)x + \left(\frac{cdp}{E}\right)y$

Substituting this result in the system of first order differential equations results in:

$$\frac{dx}{dt} = \left(\frac{dm_0}{m_0dt}\right)x - \left(\frac{c^2dm_0}{Edt}\right)x^2 - \left(\frac{cdp}{Edt}\right)xy = a_4x - a_2x^2 - a_3xy$$

$$\frac{dy}{dt} = \left(\frac{dp}{pdt}\right)y - \left(\frac{c^2dm_0}{Edt}\right)xy - \left(\frac{cdp}{Edt}\right)y^2 = a_1y - a_2xy - a_3y^2$$

We have as parameters:

$a_1 = \left(\frac{dp}{pdt}\right), a_2 = \left(\frac{c^2dm_0}{Edt}\right), a_3 = \left(\frac{cdp}{Edt}\right), a_4 = \left(\frac{dm_0}{m_0dt}\right)$

We make the following ***assumptions*** :

- $\frac{dm_0}{dt} = \frac{1}{2}m_0$,which means that we assume that the restmass is divided by two for each "time step". The consequence is that $a_2 = \frac{x}{2}$ and $a_4 = \frac{1}{2}$.
- $\frac{dp}{dt} = \frac{m_0^2c^3}{h}$,which means that we assume that the "maximum acceleration" is acting on a particle of restmass m_0. We find $a_1 = \left(\frac{dp}{pdt}\right) = \frac{m_0^2c^3}{hp} = \frac{m_0^2c^4}{hcp} = \frac{m_0^2c^4}{Eh\left(\frac{cp}{E}\right)} = \frac{m_0^2c^4E}{E^2h\left(\frac{cp}{E}\right)} = \left(\frac{E}{h}\right)\frac{x^2}{y}$ and $a_3 = \left(\frac{cdp}{Edt}\right) = \frac{m_0^2c^4}{hE} = \left(\frac{E}{h}\right)x^2$

We find as special case the following system of differential equations:

$$\frac{dx}{dt} = \frac{1}{2}x - \frac{1}{2}x^3 - \left(\frac{E}{h}\right)x^3y = \frac{1}{2}x - \left\{\frac{1}{2} + \left(\frac{E}{h}\right)y\right\}x^3$$

$$\frac{dy}{dt} = \left(\frac{E}{h}\right)x^2 - \frac{1}{2}x^2y - \left(\frac{E}{h}\right)x^2y^2 = x^2\left\{\left(\frac{E}{h}\right) - \frac{1}{2}y - \left(\frac{E}{h}\right)y^2\right\}$$

The parameter $\left(\frac{E}{h}\right)$ has the dimension of s^{-1}

The system of differential equations was solved using the MATLAB "phase portrait" Java applet *pplane* of John Polking.

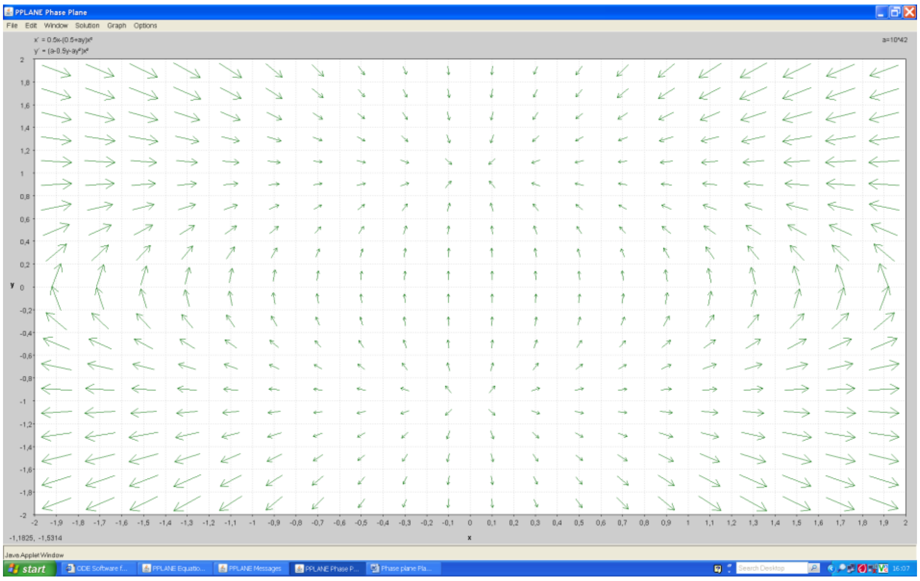


Figure 9.5-1 Parameter $\left(\frac{E}{h}\right) = 10^{42}\ s^{-1}$ (corresponds to the Planck frequency $\sqrt{\frac{c_0^5}{Gh}}$)

All trajectories in Figure 9.5-1 ,where y > -1, are ending in the point (0,1). The point (0,1) represents the state of a "mass less" free particle moving at the speed of light.

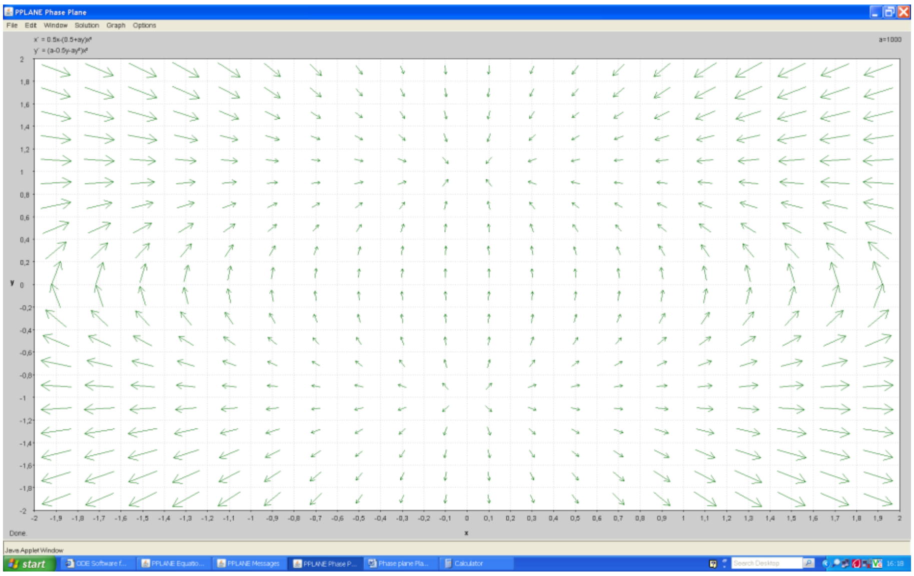


Figure 9.5-2 Parameter $\left(\frac{E}{h}\right) = 10^{3}\ s^{-1}$

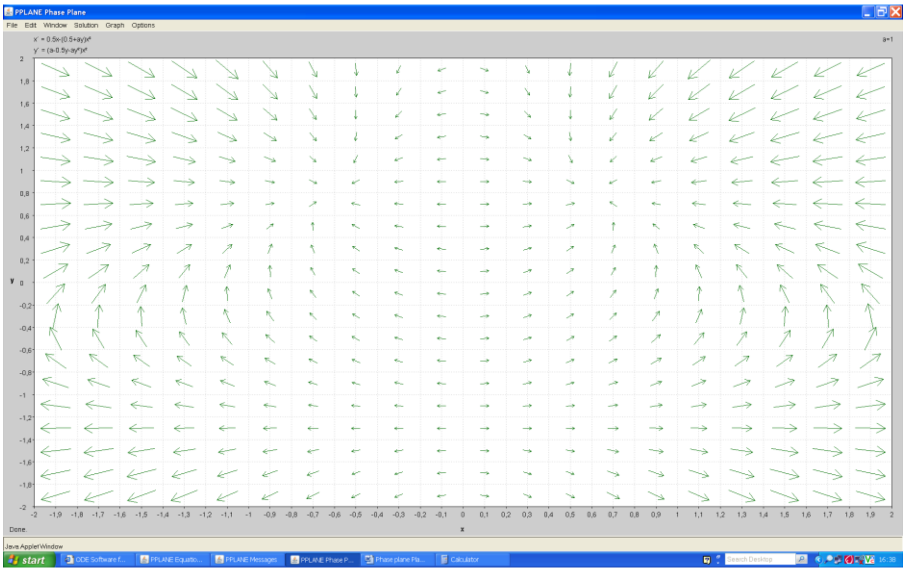


Figure 9.5-3 Parameter $\left(\frac{E}{h}\right) = 10^{0}\ s^{-1}$

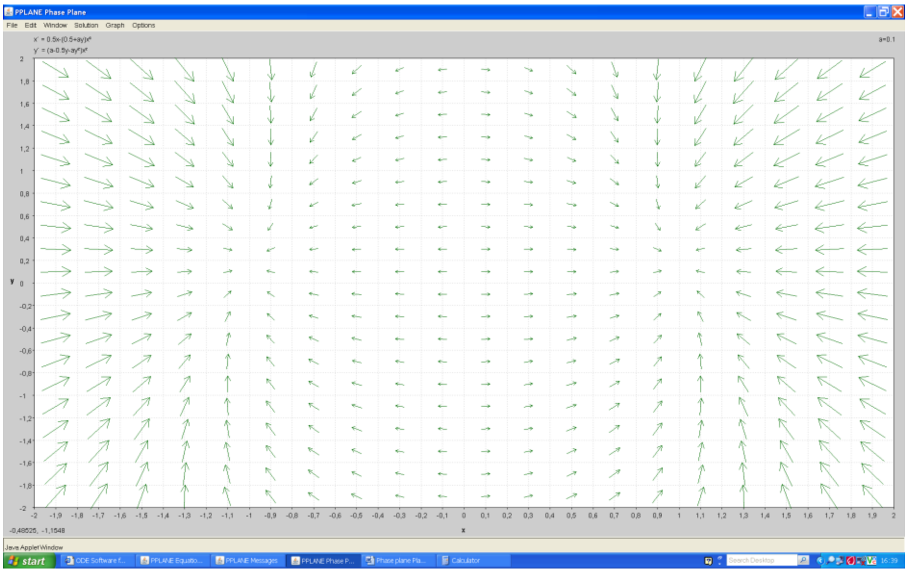


Figure 9.5-4 Parameter $\left(\frac{E}{h}\right) = 10^{-1}\ s^{-1}$

Figure 9.5-4 shows two states located on the unit circle representing two free particles moving at constant velocity.

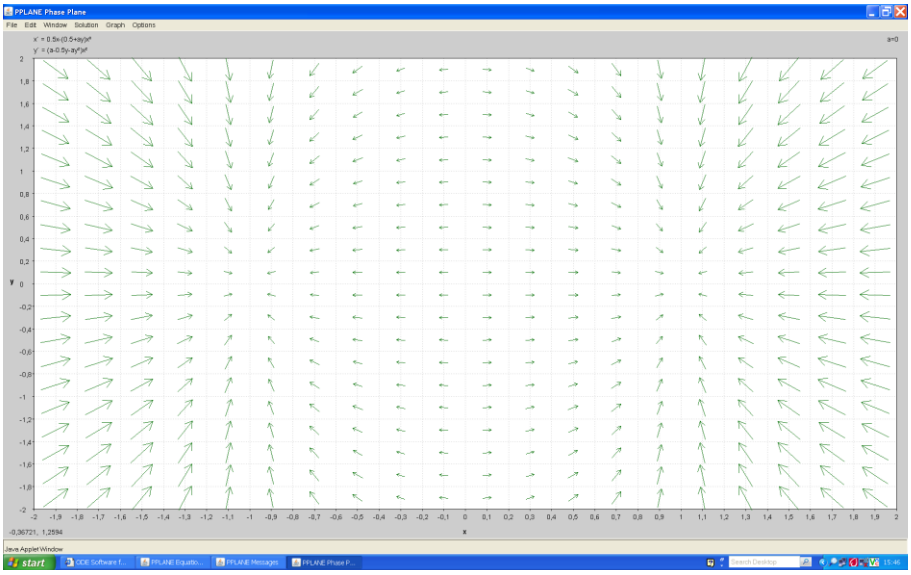


Figure 9.5-5 Parameter $\left(\frac{E}{h}\right) = 0\ s^{-1}$

Figure 9.5-5 shows two states (-1,0) and (1,0) representing 2 particles at rest.

9.6 Particle interaction representation in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane

Create within the unit disk the following processes:

Pair annihilation $e^- + e^+ \rightarrow \gamma + \gamma$

Solution: Consider the points with coordinates (a,b) and (-a, -b) on the unit circle. Move both points to the coordinates (0, 1) and (0, -1) so that they are on the imaginary axis.

Compton scattering $e^- + \gamma \rightarrow \gamma + e^-$

Solution: Consider the points with coordinates (a, b) and (0, 1) on the unit circle. Move both points to the coordinates (0, -1) and (a, -b).

Pair creation $\gamma + \gamma \rightarrow e^+ + e^-$

Solution: Consider the points with coordinates (0, 1) and (0, -1) on the unit circle. Move both points, by physically increasing the external field, to the coordinates (0, 0.5) and (0, -0.5) so that they are on the circle $|z| = 0.5e^{i\theta}$ where the energy E has reached the critical value for pair production. Perform a clockwise rotation of $\frac{\pi}{2}$ which gives the coordinates (0.5, 0) and (-0.5, 0). Decrease physically the external field, so that the points move to coordinates (1, 0) and (-1, 0).

9.7 Evolution of Planck mass in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane

Info to be investigated:

- Study the period(Compton wavelength, de Broglie wavelength) doubling cascade and model in function of Feigenbaum number (4.669)(Stewart, 1995)(Davies, 1995)?
- Bifurcation process for $z = 0.5e^{i\theta}$?

9.8 Evolution of masses of elementary particles in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane

In this analysis we consider *E* as the total energy of the universe which is a constant. It is assumed that the elementary particles are a result of a "pair creation" process. So, we expect that this process can be visualized in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane as jumps from one point to another point as time evolves.

According to J. Rosen, one is not able to visualize this evolution process at the scale of the universe (Rosen, 1995).

However, according to the ideas of I. Prigogine, the laws of physics should include the parameter "evolution" in their equation (Prigogine & Stengers, 1984).

The number of particles in the universe is not relevant in our picture, only the number of *types of particles* is relevant. So, we seek solutions for m_0 **"variable"**.

We expect to find "*evolving symmetry*" in the solution.

We expect to find the solutions on the unit circle or in the close vicinity of the unit circle.

A potential solution could be a "simple" equation of the type $z^m = 1$ in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane.

What could be the value of the exponent m?

Four basic interactions(strong, weak, electromagnetic and gravitation) can be observed. We could choose a base-4 (0,1,2,3) and identify the elementary particles as number strings of these 4 coding numbers in a similar way as genes and amino acids. This scenario will give us at time level n=1 a number of $4^1 = 4$ observable particles, at time level n = 2 we have $4^2 = 16$ new "observable" particles resulting in 4+16= 20 particles.

This scheme is different from the combinatorial hierarchy scheme designed by Frederick Parker-Rhodes and the variant from Pierre Noyes presented in his Bit-String Physics model(Noyes, 1994).

That scheme is similar to the "*creation of the amino acids*"(Raeymaekers, 2001) from the 4 bases (G,T,C,A).

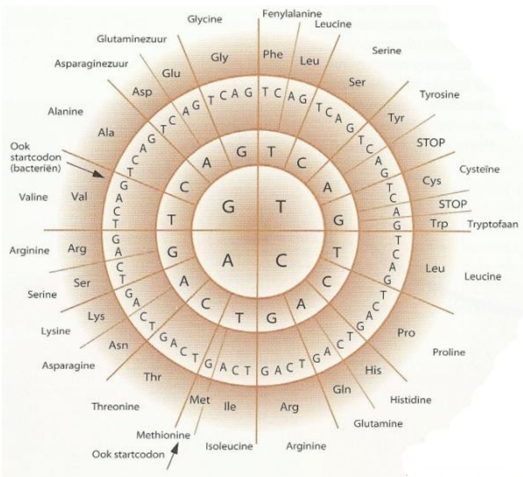


Figure 9.8-1 Genetic code

This scenario could then be in line with the “Anthropic Principle”(Barrow & Tipler, 1986).

Hypothetical elementary particle distribution				
Time Level	1	2	3	4
Quantity of particles in each time level	4	16	64	256
Cumulative number of particles	4	20	84	340

According to Veltman (Veltman, 2003) we can observe 61 elementary particles including the graviton.

According to Christiansen(Christiansen, 2003) we count in the Standard Model the following number of particles:

- 12 leptons including their respective anti-particles
- 12 quarks including their respective anti-particles
- 10 “spin 3/2” baryons (sss,dss,uss,dds,uds,uus,ddd,udd,uud,uuu)
- 7 “spin 1/2” baryons (dss,uss,dds,uds,uus,udd,udd(neutron),uud(proton))
- 4 “spin 0” mesons
- 6 “spin 1” mesons
- 3 vector bosons(W^+, W^-, Z_0)
- 8 gluons
- 1 Higgs boson (not yet found)

Where is the photon in the list of Christiansen?

To observe the particles of time level 1,2 we need much more energy.

So, the unit circle at time level n=1 will have the equation $z^{4^1} = 1$

The unit circle at time level n=2 will have the equation $z^{4^2} = 1$

The unit circle at time level n=3 will have the equation $z^{4^3} = 1$. As observers today we probably live in time level n=3.

The masses of the particles will decrease as function of the length of the “string” code and the length of the code increases as function of the time level.

The fundamental hypothesis is that the number of observable elementary particles is generated as the law 4^n , where *n* is the time level of the observer.

Further info to be investigated:

- Unit circle partitioning according to the “Pizza theorem” of L. Upton in 8, 12, 16, 20, 24, … “equal parts”(Delahaye, 2006).
- Fractal structure based on §9.7. Bifurcation is expected. What is the equation of the fractal?
- “A mathematical model of genes” from George C. Nelson, University of Iowa.

9.9 General particle states represented in the $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane

It is possible to define in the (x,y) plane two elementary symmetric functions(Siegfried Gottwald, 1995):

$$\sigma_1(x,y) = x + y \qquad \text{(straight lines)}$$

$$\sigma_2(x,y) = xy \qquad \text{(hyperbola)}$$

We have already seen that the equation $x^2 + y^2 = 1$, representing the locus of the free particle states, is equivalent to $\sigma_1^2 - 2\,\sigma_2 = 1$.

We are looking for an expression representing the bounded states. We expect that bounded states will be represented by points in $\left(\frac{m_0c^2}{E}, \frac{cp}{E}\right)$ plane fulfilling the condition $|z| > 1$, which are all the points outside the unit disc.

Where are the loci of the bounded states?

Let us consider the product $\sigma_2(x,y) = xy = \frac{m_0c^2}{E}\frac{cp}{E} = \frac{m_0c^3p}{E^2} = constant1$.

We can form an orthogonal trajectory(Ayres, 1972) to $\sigma_2(x,y)$ giving the equation $\left(\frac{m_0c^2}{E}\right)^2 - \left(\frac{cp}{E}\right)^2 = constant2$.

We choose the $constant2$ so that we have $\left(\frac{m_0c^2}{E}\right)^2 - \left(\frac{1}{b}\right)^2\left(\frac{cp}{E}\right)^2 = 1$ the equation of a family of hyperbolas, where b is a parameter to be further defined.

When p = 0 then we have $\left(\frac{m_0c^2}{E}\right)^2 = 1$ which is in line with the condition of a free particle.

We have now a general equation $\left(\frac{m_0c^2}{E}\right) = \pm\sqrt{1 + \left(\frac{1}{b}\right)^2\left(\frac{cp}{E}\right)^2}$ for bounded states .

10 Appendices

10.1 Appendix 1: Detailed classification parameters of physical quantities

To get a better understanding of the elements of the different equivalence classes a non-exhaustive list of common physical quantities is given in the next table.

Subgroup	Class	Quantity	7-tuple	SUM	NORM	PATH LENGTH
$CL_{0,7}$	C_0	Plane angle	(0,0,0,0,0,0,0)	0	0	0
$CL_{0,7}$	C_0	Solid angle	(0,0,0,0,0,0,0)	0	0	0
$CL_{1,6}$	C_P	Space-time curvature(R_{ij})	(-2,0,0,0,0,0,0)	-2	2	2
$CL_{1,6}$	C_k	Wave number	(-1,0,0,0,0,0,0)	-1	1	1
$CL_{1,6}$	C_L	Length (L)	(1,0,0,0,0,0,0)	1	1	1
$CL_{1,6}$	C_L	Planck length (l_P)	(1,0,0,0,0,0,0)	1	1	1
$CL_{1,6}$	C_A	Area (A)	(2,0,0,0,0,0,0)	2	2	2
$CL_{1,6}$	C_V	Volume (V)	(3,0,0,0,0,0,0)	3	3	3
$CL_{2,5}$	C_L	Mass (m)	(0,1,0,0,0,0,0)	1	1	1
$CL_{2,5}$	C_L	Planck mass (m_P)	(0,1,0,0,0,0,0)	1	1	1
$CL_{3,4}$	C_P	Energy density	(-1,1,-2,0,0,0,0)	-2	2.449	4
$CL_{3,4}$	C_P	Pressure (P)	(-1,1,-2,0,0,0,0)	-2	2.449	4
$CL_{3,4}$	C_P	Energy-momentum tensor (T_{ij})	(-1,1,-2,0,0,0,0)	-2	2.449	4
$CL_{3,4}$	C_k	Frequency (v)	(0,0,-1,0,0,0,0)	-1	1	1
$CL_{3,4}$	C_k	activity	(0,0,-1,0,0,0,0)	-1	1	1
$CL_{3,4}$	C_k	Acceleration (a)	(1,0,-2,0,0,0,0)	-1	2.236	3
$CL_{3,4}$	C_0	Velocity (v)	(1,0,-1,0,0,0,0)	0	1.414	2
$CL_{3,4}$	C_0	Speed of light in vacuum (c)	(1,0,-1,0,0,0,0)	0	1.414	2
$CL_{3,4}$	C_0	Mass frequency	(0,1,-1,0,0,0,0)	0	1.414	2

$CL_{3,4}$	C_0	Force (F)	(1,1,-2,0,0,0,0)	0	2.449	4
$CL_{3,4}$	C_0	Absorbed dose	(2,0,-2,0,0,0,0)	0	2.828	4
$CL_{3,4}$	C_0	Dose equivalent	(2,0,-2,0,0,0,0)	0	2.828	4
$CL_{3,4}$	C_0	Specific energy	(2,0,-2,0,0,0,0)	0	2.828	4
$CL_{3,4}$	C_0	Newtonian constant of gravitation (G)	(3,-1,-2,0,0,0,0)	0	3.742	6
$CL_{3,4}$	C_0	Power	(2,1,-3,0,0,0,0)	0	3.742	6
$CL_{3,4}$	C_L	Time (t)	(0,0,1,0,0,0,0)	1	1	1
$CL_{3,4}$	C_L	Planck time (t_{P})	(0,0,1,0,0,0,0)	1	1	1
$CL_{3,4}$	C_L	Linear momentum (p)	(1,1,-1,0,0,0,0)	1	1.732	3
$CL_{3,4}$	C_L	Diffusion constant (D)	(2,0,-1,0,0,0,0)	1	2.236	3
$CL_{3,4}$	C_L	Energy (E)	(2,1,-2,0,0,0,0)	1	3	5
$CL_{3,4}$	C_A	Planck constant (h)	(2,1,-1,0,0,0,0)	2	2.449	4
$CL_{3,4}$	C_A	Specific volume	(3,-1,0,0,0,0,0)	2	3.162	4
$CL_{4,3}$	C_P	Magnetic flux density	(0,1,-2,-1,0,0,0)	-2	2.449	4
$CL_{4,3}$	C_P	Magnetic constant (μ_0)	(1,1,-2,-2,0,0,0)	-2	3.162	6
$CL_{4,3}$	C_P	Electrical resistance (h/e^2,R)	(2,1,-3,-2,0,0,0)	-2	4.243	8
$CL_{4,3}$	C_P	Characteristic Impedance of vacuum (Z_0)	(2,1,-3,-2,0,0,0)	-2	4.243	8
$CL_{4,3}$	C_k	Inductance	(2,1,-2,-2,0,0,0)	-1	3.606	7
$CL_{4,3}$	C_k	Electrical potential difference(V)	(2,1,-3,-1,0,0,0)	-1	3.873	7
$CL_{4,3}$	C_0	Magnetic field (H)	(-1,0,0,1,0,0,0)	0	1.414	2
$CL_{4,3}$	C_0	Charge surface density	(-2,0,1,1,0,0,0)	0	2.449	4
$CL_{4,3}$	C_0	Electrical polarisation	(-2,0,1,1,0,0,0)	0	2.449	4
$CL_{4,3}$	C_0	Magnetic induction (B)	(2,1,-2,-1,0,0,0)	0	3.162	6
$CL_{4,3}$	C_0	Magnetic moment	(2,1,-2,-1,0,0,0)	0	3.162	6

$CL_{4,3}$	$C_{\mathbf{0}}$	Specific resistance (ρ)	(3,1,-3,-1,0,0,0)	0	4.472	8
$CL_{4,3}$	$C_{\boldsymbol{L}}$	Electric current (I)	(0,0,0,1,0,0,0)	1	1	1
$CL_{4,3}$	$C_{\boldsymbol{A}}$	Electric charge (e)	(0,0,1,1,0,0,0)	2	1.414	2
$CL_{4,3}$	$C_{\boldsymbol{A}}$	Electric constant(ϵ_0)	(-3,-1,4,2,0,0,0)	2	5.477	10
$CL_{4,3}$	$C_{\boldsymbol{V}}$	Electrical capacitance	(-2,-1,4,2,0,0,0)	3	5	9
$CL_{4,3}$	$C_{\boldsymbol{QM}}$	Atomic unit of electric quadrupole moment	(2,0,1,1,0,0,0)	4	2.449	4
$CL_{4,3}$	$C_{\boldsymbol{EP}}$	Atomic unit of electric polarizablity	(0,-1,4,2,0,0,0)	5	4.583	7
$CL_{4,3}$	$C_{\boldsymbol{1HP}}$	Atomic unit of 1st hyperpolarizability	(-1,-2,7,3,0,0,0)	7	7.937	13
$CL_{4,3}$	$C_{\boldsymbol{2HP}}$	Atomic unit of 2nd hyperpolarizability	(-2,-3,10,4,0,0,0)	9	11.358	19
$CL_{5,2}$	$C_{\boldsymbol{\sigma}}$	Stefan-Boltzmann constant(σ)	(0,1,-3,0,-4,0,0)	-6	5.099	8
$CL_{5,2}$	$C_{\boldsymbol{k}}$	Current density	(-2,0,0,0,1,0,0)	-1	2.236	3
$CL_{5,2}$	$C_{\mathbf{0}}$	Entropy (S)	(2,1,-2,0,-1,0,0)	0	3.162	6
$CL_{5,2}$	$C_{\mathbf{0}}$	Specific heat	(2,1,-2,0,-1,0,0)	0	3.162	6
$CL_{5,2}$	$C_{\mathbf{0}}$	Boltzmann constant (k)	(2,1,-2,0,-1,0,0)	0	3.162	6
$CL_{5,2}$	$C_{\boldsymbol{L}}$	Thermodynamic temperature (T)	(0,0,0,0,1,0,0)	1	1	1
$CL_{5,2}$	$C_{\boldsymbol{L}}$	Planck temperature (T_{P})	(0,0,0,0,1,0,0)	1	1	1
$CL_{6,1}$	$C_{\boldsymbol{P}}$	Amount of substance concentration	(-3,0,0,0,0,1,0)	-2	3.162	4
$CL_{6,1}$	$C_{\boldsymbol{k}}$	Avogadro constant (N_{A})	(0,0,0,0,0,-1,0)	-1	1	1
$CL_{6,1}$	$C_{\boldsymbol{k}}$	Molar gas constant (R)	(2,1,-2,0,-1,-1,0)	-1	3.317	7
$CL_{6,1}$	$C_{\mathbf{0}}$	Catalytic activity	(0,0,-1,0,0,1,0)	0	1.414	2
$\boldsymbol{Z}^{\mathbf{7}}$	$C_{\boldsymbol{k}}$	Luminance	(-2,0,0,0,0,0,1)	-1	2.236	3
$\boldsymbol{Z}^{\mathbf{7}}$	$C_{\boldsymbol{L}}$	Luminous flux	(0,0,0,0,0,0,1)	1	1	1

10.2 Appendix 2: Table with SI constants

The table below has been modified and classified starting from the table created by M. R. Sheppard (Sheppard, 2008) which was based on the NIST table of Physical constants.

Quantity	Units	Exponents for Units							SUM	NORM
		m	kg	s	A	K	mol	cd		
Stefan-Boltzmann constant	W m^-2 K^-4		1	-3		-4			-6	4,12
atomic unit of electric field gradient	V m^-2		1	-3	-1				-3	1,41
Loschmidt constant (273.15 K, 101.325 kPa)	m^-3	-3							-3	3,00
Boltzmann constant in Hz/K	Hz K^-1			-1		-1			-2	1,00
Wien frequency displacement law constant	Hz K^-1			-1		-1			-2	1,00
Boltzmann constant in inverse meters per Kelvin	m^-1 K^-1	-1				-1			-2	1,41
standard atmosphere	Pa	-1	1	-2					-2	1,41
atomic unit of mag. flux density	T		1	-2	-1				-2	1,41
atomic unit of electric field	V m^-1	1	1	-3	-1				-2	1,73
Newtonian constant of gravitation over h-bar c	(GeV/c^2)^-2		-2						-2	2,00
mag. constant	N A^-2	1	1	-2	-2				-2	2,45
inverse of conductance quantum	ohm	2	1	-3	-2				-2	3,00
von Klitzing constant	ohm	2	1	-3	-2				-2	3,00
characteristic impedance of vacuum	ohm	2	1	-3	-2				-2	3,00
conventional value of von Klitzing constant	ohm	2	1	-3	-2				-2	3,00
Fermi coupling constant	GeV^-2	-4	-2	4					-2	4,47
Avogadro constant	mol^-1						-1		-1	0,00
atomic mass unit-hertz relationship	Hz			-1					-1	0,00
electron volt-hertz relationship	Hz			-1					-1	0,00
hartree-hertz relationship	Hz			-1					-1	0,00
joule-hertz relationship	Hz			-1					-1	0,00
Kelvin-hertz relationship	Hz			-1					-1	0,00
kilogram-hertz relationship	Hz			-1					-1	0,00
Rydberg constant times c in Hz	Hz			-1					-1	0,00
inverse meter-hertz relationship	Hz			-1					-1	0,00
standard acceleration of gravity	m s^-2	1		-2					-1	1,00
atomic mass unit-inverse meter relationship	m^-1	-1							-1	1,00
electron volt-inverse meter relationship	m^-1	-1							-1	1,00
hartree-inverse meter relationship	m^-1	-1							-1	1,00
joule-inverse meter relationship	m^-1	-1							-1	1,00
Kelvin-inverse meter relationship	m^-1	-1							-1	1,00
kilogram-inverse meter relationship	m^-1	-1							-1	1,00
Rydberg constant	m^-1	-1							-1	1,00
hertz-inverse meter relationship	m^-1	-1							-1	1,00
molar gas constant	J mol^-1 K^-1	2	1	-2		-1	-1		-1	2,45
atomic unit of electric potential	V	2	1	-3	-1				-1	2,45
atomic unit of charge density	C m^-3	-3		1	1				-1	3,16
tau-electron mass ratio									0	0,00
proton mag. shielding correction									0	0,00
tau-muon mass ratio									0	0,00
alpha particle-electron mass ratio									0	0,00
alpha particle-proton mass ratio									0	0,00
deuteron-electron mag. mom. ratio									0	0,00
deuteron-electron mass ratio									0	0,00
deuteron g factor									0	0,00
deuteron mag. mom. to Bohr magneton ratio									0	0,00
deuteron mag. mom. to nuclear magneton ratio									0	0,00
deuteron-neutron mag. mom. ratio									0	0,00

deuteron-proton mag. mom. ratio									0	0,00
deuteron-proton mass ratio									0	0,00
electron-deuteron mag. mom. ratio									0	0,00
electron-deuteron mass ratio									0	0,00
electron g factor									0	0,00
electron mag. mom. anomaly									0	0,00
electron mag. mom. to Bohr magneton ratio									0	0,00
electron mag. mom. to nuclear magneton ratio									0	0,00
electron-muon mag. mom. ratio									0	0,00
electron-muon mass ratio									0	0,00
electron-neutron mag. mom. ratio									0	0,00
electron-neutron mass ratio									0	0,00
electron-proton mag. mom. ratio									0	0,00
electron-proton mass ratio									0	0,00
electron-tau mass ratio									0	0,00
electron to alpha particle mass ratio									0	0,00
Electron to shielded helion mag. mom. ratio									0	0,00
Electron to shielded proton mag. mom. ratio									0	0,00
fine-structure constant									0	0,00
helion-electron mass ratio									0	0,00
helion-proton mass ratio									0	0,00
inverse fine-structure constant									0	0,00
muon-electron mass ratio									0	0,00
muon g factor									0	0,00
muon mag. mom. anomaly									0	0,00
muon mag. mom. to Bohr magneton ratio									0	0,00
muon mag. mom. to nuclear magneton ratio									0	0,00
muon-neutron mass ratio									0	0,00
muon-proton mag. mom. ratio									0	0,00
muon-proton mass ratio									0	0,00
muon-tau mass ratio									0	0,00
neutron-electron mag. mom. ratio									0	0,00
neutron-electron mass ratio									0	0,00
neutron g factor									0	0,00
neutron mag. mom. to Bohr magneton ratio									0	0,00
neutron mag. mom. to nuclear magneton ratio									0	0,00
neutron-muon mass ratio									0	0,00
neutron-proton mag. mom. ratio									0	0,00
neutron-proton mass ratio									0	0,00
neutron-tau mass ratio									0	0,00
neutron to shielded proton mag. mom. ratio									0	0,00
proton-electron mass ratio									0	0,00
proton g factor									0	0,00
proton mag. mom. to Bohr magneton ratio									0	0,00
proton mag. mom. to nuclear magneton ratio									0	0,00
proton-muon mass ratio									0	0,00
proton-neutron mag. mom. ratio									0	0,00
proton-neutron mass ratio									0	0,00
proton-tau mass ratio									0	0,00
Sackur-Tetrode constant (1 K, 100 kPa)									0	0,00
Sackur-Tetrode constant (1 K, 101.325 kPa)									0	0,00
shielded helion mag. mom. to Bohr magneton ratio									0	0,00
shielded helion mag. mom. to nuclear magneton ratio									0	0,00
shielded helion to proton mag. mom. ratio									0	0,00
shielded helion to shielded proton mag. mom. Ratio									0	0,00
shielded proton mag. mom. to Bohr magneton ratio									0	0,00

shielded proton mag. mom. to nuclear magneton ratio									0	0,00
tau-neutron mass ratio									0	0,00
tau-proton mass ratio									0	0,00
triton-electron mag. mom. ratio									0	0,00
triton-electron mass ratio									0	0,00
triton g factor									0	0,00
triton mag. mom. to Bohr magneton ratio									0	0,00
triton mag. mom. to nuclear magneton ratio									0	0,00
triton-neutron mag. mom. ratio									0	0,00
triton-proton mag. mom. ratio									0	0,00
triton-proton mass ratio									0	0,00
weak mixing angle									0	0,00
alpha particle molar mass	kg mol^-1		1				-1		0	1,00
deuteron molar mass	kg mol^-1		1				-1		0	1,00
electron molar mass	kg mol^-1		1				-1		0	1,00
helion molar mass	kg mol^-1		1				-1		0	1,00
muon molar mass	kg mol^-1		1				-1		0	1,00
neutron molar mass	kg mol^-1		1				-1		0	1,00
proton molar mass	kg mol^-1		1				-1		0	1,00
tau molar mass	kg mol^-1		1				-1		0	1,00
triton molar mass	kg mol^-1		1				-1		0	1,00
molar mass constant	kg mol^-1		1				-1		0	1,00
molar mass of carbon-12	kg mol^-1		1				-1		0	1,00
atomic unit of velocity	m s^-1	1		-1					0	1,00
natural unit of velocity	m s^-1	1		-1					0	1,00
speed of light in vacuum	m s^-1	1		-1					0	1,00
atomic unit of force	N	1	1	-2					0	1,41
mag. flux quantum	Wb	2	1	-2	-1				0	2,45
Josephson constant	Hz V^-1	-2	-1	2	1				0	2,45
elementary charge over h	A J^-1	-2	-1	2	1				0	2,45
conventional value of Josephson constant	Hz V^-1	-2	-1	2	1				0	2,45
Boltzmann constant in eV/K	eV K^-1	2	1	-2		-1			0	2,45
Boltzmann constant	J K^-1	2	1	-2		-1			0	2,45
Newtonian constant of gravitation	m^3 kg^-1 s^-2	3	-1	-2					0	3,16
atomic unit of time	7 s			1					1	0,00
natural unit of time	s			1					1	0,00
Planck time	s			1					1	0,00
Faraday constant	C mol^-1			1	1		-1		1	1,00
alpha particle mass	kg		1						1	1,00
atomic mass constant	kg		1						1	1,00
atomic mass unit-kilogram relationship	kg		1						1	1,00
atomic unit of mass	kg		1						1	1,00
deuteron mass	kg		1						1	1,00
electron mass	kg		1						1	1,00
electron volt-kilogram relationship	kg		1						1	1,00
hartree-kilogram relationship	kg		1						1	1,00
helion mass	kg		1						1	1,00
hertz-kilogram relationship	kg		1						1	1,00
inverse meter-kilogram relationship	kg		1						1	1,00
Kelvin-kilogram relationship	kg		1						1	1,00
muon mass	kg		1						1	1,00
natural unit of mass	kg		1						1	1,00
neutron mass	kg		1						1	1,00
Planck mass	kg		1						1	1,00
proton mass	kg		1						1	1,00
tau mass	kg		1						1	1,00
triton mass	kg		1						1	1,00
unified atomic mass unit	kg		1						1	1,00

alpha particle mass in u	u		1						1	1,00
deuteron mass in u	u		1						1	1,00
electron mass in u	u		1						1	1,00
electron volt-atomic mass unit relationship	u		1						1	1,00
hartree-atomic mass unit relationship	u		1						1	1,00
helion mass in u	u		1						1	1,00
hertz-atomic mass unit relationship	u		1						1	1,00
inverse meter-atomic mass unit relationship	u		1						1	1,00
joule-atomic mass unit relationship	u		1						1	1,00
Kelvin-atomic mass unit relationship	u		1						1	1,00
kilogram-atomic mass unit relationship	u		1						1	1,00
muon mass in u	u		1						1	1,00
neutron mass in u	u		1						1	1,00
proton mass in u	u		1						1	1,00
tau mass in u	u		1						1	1,00
triton mass in u	u		1						1	1,00
joule-kilogram relationship	kg		1						1	1,00
proton rms charge radius	m	1							1	1,00
deuteron rms charge radius	m	1							1	1,00
{220} lattice spacing of silicon	m	1							1	1,00
Angstrom star	m	1							1	1,00
atomic unit of length	m	1							1	1,00
Bohr radius	m	1							1	1,00
classical electron radius	m	1							1	1,00
Compton wavelength	m	1							1	1,00
Compton wavelength over 2 pi	m	1							1	1,00
Cu x unit	m	1							1	1,00
lattice parameter of silicon	m	1							1	1,00
Mo x unit	m	1							1	1,00
muon Compton wavelength	m	1							1	1,00
muon Compton wavelength over 2 pi	m	1							1	1,00
natural unit of length	m	1							1	1,00
neutron Compton wavelength	m	1							1	1,00
neutron Compton wavelength over 2 pi	m	1							1	1,00
Planck length	m	1							1	1,00
proton Compton wavelength	m	1							1	1,00
proton Compton wavelength over 2 pi	m	1							1	1,00
tau Compton wavelength	m	1							1	1,00
tau Compton wavelength over 2 pi	m	1							1	1,00
atomic mass unit-Kelvin relationship	K					1			1	1,00
electron volt-Kelvin relationship	K					1			1	1,00
hartree-Kelvin relationship	K					1			1	1,00
hertz-Kelvin relationship	K					1			1	1,00
inverse meter-Kelvin relationship	K					1			1	1,00
joule-Kelvin relationship	K					1			1	1,00
kilogram-Kelvin relationship	K					1			1	1,00
Planck temperature	K					1			1	1,00
atomic unit of current	A				1				1	1,00
Bohr magneton in Hz/T	Hz T^-1		-1	1	1				1	1,41
electron gyromag. ratio over 2 pi	MHz T^-1		-1	1	1				1	1,41
neutron gyromag. ratio over 2 pi	MHz T^-1		-1	1	1				1	1,41
nuclear magneton in MHz/T	MHz T^-1		-1	1	1				1	1,41
proton gyromag. ratio over 2 pi	MHz T^-1		-1	1	1				1	1,41
shielded helion gyromag. ratio over 2 pi	MHz T^-1		-1	1	1				1	1,41
shielded proton gyromag. ratio over 2 pi	MHz T^-1		-1	1	1				1	1,41
electron gyromag. ratio	s^-1 T^-1		-1	1	1				1	1,41
neutron gyromag. ratio	s^-1 T^-1		-1	1	1				1	1,41
proton gyromag. ratio	s^-1 T^-1		-1	1	1				1	1,41
shielded helion gyromag. ratio	s^-1 T^-1		-1	1	1				1	1,41

shielded proton gyromag. ratio	s^-1 T^-1		-1	1	1				1	1,41
electron charge to mass quotient	C kg^-1		-1	1	1				1	1,41
proton charge to mass quotient	C kg^-1		-1	1	1				1	1,41
natural unit of momentum in MeV/c	MeV/c	1	1	-1					1	1,41
atomic unit of momentum	kg m s^-1	1	1	-1					1	1,41
natural unit of momentum	kg m s^-1	1	1	-1					1	1,41
Bohr magneton in inverse meters per tesla	m^-1 T^-1	-1	-1	2	1				1	1,73
nuclear magneton in inverse meters per tesla	m^-1 T^-1	-1	-1	2	1				1	1,73
quantum of circulation	m^2 s^-1	2		-1					1	2,00
quantum of circulation times 2	m^2 s^-1	2		-1					1	2,00
inverse meter-hartree relationship	E_h	2	1	-2					1	2,24
joule-hartree relationship	E_h	2	1	-2					1	2,24
Kelvin-hartree relationship	E_h	2	1	-2					1	2,24
kilogram-hartree relationship	E_h	2	1	-2					1	2,24
atomic mass unit-electron volt relationship	eV	2	1	-2					1	2,24
hartree-electron volt relationship	eV	2	1	-2					1	2,24
Hartree energy in eV	eV	2	1	-2					1	2,24
hertz-electron volt relationship	eV	2	1	-2					1	2,24
inverse meter-electron volt relationship	eV	2	1	-2					1	2,24
joule-electron volt relationship	eV	2	1	-2					1	2,24
Kelvin-electron volt relationship	eV	2	1	-2					1	2,24
kilogram-electron volt relationship	eV	2	1	-2					1	2,24
Rydberg constant times hc in eV	eV	2	1	-2					1	2,24
alpha particle mass energy equivalent	J	2	1	-2					1	2,24
atomic mass constant energy equivalent	J	2	1	-2					1	2,24
atomic mass unit-joule relationship	J	2	1	-2					1	2,24
atomic unit of energy	J	2	1	-2					1	2,24
deuteron mass energy equivalent	J	2	1	-2					1	2,24
electron mass energy equivalent	J	2	1	-2					1	2,24
electron volt	J	2	1	-2					1	2,24
electron volt-joule relationship	J	2	1	-2					1	2,24
Hartree energy	J	2	1	-2					1	2,24
hartree-joule relationship	J	2	1	-2					1	2,24
helion mass energy equivalent	J	2	1	-2					1	2,24
hertz-joule relationship	J	2	1	-2					1	2,24
inverse meter-joule relationship	J	2	1	-2					1	2,24
Kelvin-joule relationship	J	2	1	-2					1	2,24
muon mass energy equivalent	J	2	1	-2					1	2,24
natural unit of energy	J	2	1	-2					1	2,24
neutron mass energy equivalent	J	2	1	-2					1	2,24
proton mass energy equivalent	J	2	1	-2					1	2,24
Rydberg constant times hc in J	J	2	1	-2					1	2,24
tau mass energy equivalent	J	2	1	-2					1	2,24
triton mass energy equivalent	J	2	1	-2					1	2,24
kilogram-joule relationship	J	2	1	-2					1	2,24
tau mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
hertz-hartree relationship	E_h	2	1	-2					1	2,24
Planck mass energy equivalent in GeV	GeV	2	1	-2					1	2,24
alpha particle mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
atomic mass constant energy equivalent in MeV	MeV	2	1	-2					1	2,24
deuteron mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
electron mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
helion mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
muon mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
natural unit of energy in MeV	MeV	2	1	-2					1	2,24
neutron mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
proton mass energy equivalent in MeV	MeV	2	1	-2					1	2,24
triton mass energy equivalent in MeV	MeV	2	1	-2					1	2,24

atomic mass unit-hartree relationship	E_h	2	1	-2					1	2,24
electron volt-hartree relationship	E_h	2	1	-2					1	2,24
molar Planck constant times c	J m mol^-1	3	1	-2			-1		1	3,16
atomic unit of charge	C			1	1				2	1,00
elementary charge	C			1	1				2	1,00
second radiation constant	m K	1				1			2	1,41
Wien wavelength displacement law constant	m K	1				1			2	1,41
Thomson cross section	m^2	2							2	2,00
natural unit of action in eV s	eV s	2	1	-1					2	2,24
Planck constant in eV s	eV s	2	1	-1					2	2,24
Planck constant over 2 pi in eV s	eV s	2	1	-1					2	2,24
atomic unit of action	J s	2	1	-1					2	2,24
natural unit of action	J s	2	1	-1					2	2,24
Planck constant	J s	2	1	-1					2	2,24
Planck constant over 2 pi	J s	2	1	-1					2	2,24
molar Planck constant	J s mol^-1	2	1	-1					2	2,24
conductance quantum	S	-2	-1	3	2				2	3,00
molar volume of ideal gas (273.15 K, 100 kPa)	m^3 mol^-1	3					-1		2	3,00
molar volume of ideal gas (273.15 K, 101.325 kP	m^3 mol^-1	3					-1		2	3,00
molar volume of silicon	m^3 mol^-1	3					-1		2	3,00
Planck constant over 2 pi times c in MeV fm	MeV fm	3	1	-2					2	3,16
atomic unit of permittivity	F m^-1	-3	-1	4	2				2	3,74
electric constant	F m^-1	-3	-1	4	2				2	3,74
first radiation constant	W m^2	4	1	-3					2	4,12
first radiation constant for spectral radiance	W m^2 sr^-1	4	1	-3					2	4,12
atomic unit of electric dipole mom.	C m	1		1	1				3	1,41
Bohr magneton in K/T	K T^-1		-1	2	1	1			3	1,73
nuclear magneton in K/T	K T^-1		-1	2	1	1			3	1,73
Bohr magneton in eV/T	eV T^-1	2			1				3	2,24
nuclear magneton in eV/T	eV T^-1	2			1				3	2,24
atomic unit of mag. dipole mom.	J T^-1	2			1				3	2,24
Bohr magneton	J T^-1	2			1				3	2,24
deuteron mag. mom.	J T^-1	2			1				3	2,24
electron mag. mom.	J T^-1	2			1				3	2,24
muon mag. mom.	J T^-1	2			1				3	2,24
neutron mag. mom.	J T^-1	2			1				3	2,24
nuclear magneton	J T^-1	2			1				3	2,24
proton mag. mom.	J T^-1	2			1				3	2,24
shielded helion mag. mom.	J T^-1	2			1				3	2,24
shielded proton mag. mom.	J T^-1	2			1				3	2,24
triton mag. mom.	J T^-1	2			1				3	2,24
atomic unit of electric quadrupole mom.	C m^2	2		1	1				4	2,24
atomic unit of electric polarizablity	C^2 m^2 J^-1		-1	4	2				5	2,24
atomic unit of magnetizability	J T^-2	2	-1	2	2				5	3,00
atomic unit of 1st hyperpolarizablity	C^3 m^3 J^-2	-1	-2	7	3				7	3,74
atomic unit of 2nd hyperpolarizablity	C^4 m^4 J^-3	-2	-3	10	4				9	5,39

11 References

Abramowitz, M., & Stegun, I. A. (1972). *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.* New York: Dover Publications.

Anderson, J. D., Laing, P. A., Lau, E. L., Liu, A. S., Nieto, M. M., & Turyshev, S. G. (1998). Indication, from Pioneer 10/11, Galileo, and Ulysses data, of an apparent anomalous, weak, long-range acceleration. *Phys. Rev. Lett. 81* , 2858 - 2861.

Ayres, F. (1972). *Theory and problems of differential equations in SI metric units.* New York: McGraw-Hill International Book Company.

B.Baumslag, B. C. (1968). *Theory and problems of Group theory.* McGraw-Hill book company.

Banchoff, T. F. (1996). *BEYOND THE THIRD DIMENSION, Geometry, Computer Graphics, and Higher Dimensions.* (V. Magazines, Trans.) Scientific American Library.

Barrow, J. D., & Tipler, F. J. (1986). *The Anthropic Cosmological Principle.* New York: Oxford University Press Inc.

Borowitz, S., & Beiser, A. (1971). *Essentials of Physics.* Addison-Wesley Publishing Co.

Buckingham, E. (1914). On Physically similar systems; Illustrations of the Use of Dimensional Equations. *Phys. Rev., Vol IV, no. 4* , 345.

Cartan, E. (1946). *Leçons sur la Géométrie des espaces de Riemann, deuxième edidition.* Paris: Jacques Gabay, Guathier-Villars.

Chandrasekhar, S. (1961). *Hydrodynamic and Hydromagnetic stability.* London: Oxford University Press.

Christiansen, P. V. (2003). The semiotic flora of elementary particles. In *Semiotiske Unders0gelser.* Glydendal: Thellefsen andDinesen.

Cornish, N. J., Spergel, D. N., Starkman, G. D., & Komatsu, E. (2003). Constraining the Topology of the Universe. *arXiv:astro-ph0310233v1* .

Davies, P. (1995). *Superforce - The search for a grand unified theory of nature.* London: Penguin Books.

Davies, P. (1995). *The Cosmic Blueprint.* Harmondsworth: Pengiun Books Ltd.

Delahaye, J.-P. (2006). *Complexités - Aux limites de mathématiques et de l'informatique.* (P. Smitt, & A. Frehan, Trans.) Belin.

Diestel, R. (2000). *Graph Theory 2nd edition .* New York: Springer-Verlag.

DiStefano, J. J., Stubberud, A. R., & Williams, I. J. (1967). *Feedback and Control Systems.* Las Angeles: McGraw-Hill.

Dittus, H. J., & Lämmerzahl, C. (2006). Is Physics in the Solar System really understood? Firenze.

Driest, E. V. (March 1946). On Dimensional Analysis and the Presentation of Data in Fluid Flow Problems. *Journal Applied Mechanics, Vol . 13, N° 1* , A-34.

Elgerd, O. I. (1967). *Control Systems Theory.* Tokyo: McGraw-Hill Kogakusha.

Epp, S. (1993). *Discrete Mathematics with applications.* PWS Publishing Company.

Erich W. Ellers, B. G. (2003). H.S.M. Coxeter (1907-2003). *Notices of the AMS, Volume50, Number 10* , 1234-1240.

Feynman, R. P. (1963). *Six not-so-easy pieces : Einstein's relativity, symmetry, and space-time.* New York: Addison-Wesley Publishing Company, Inc.

Fradkin, E., I.A.Batalin, C.J.Isham, & Vilkovisky, G. (1987). *Quantum Field Theory and Quantum Statistics.* Bristol: Adam Hilger.

G.Grosche, Zeidler, E., Ziegler, D., & Ziegler, V. (2003). *Teubner-Taschenbuch der Mathematik - Teil II.* Wiesbaden: B.G. Teubner Verlag.

Grant, I., & Phillips, W. (1979). *Electromagnetism.* Bristol: John Wiley & Sons.

Greiner, W., & Reinhardt, J. (1992, 1994). *Quantum Electrodynamics 2nd Edition.* Berlin Heidelberg: Springer-Verlag.

Guth, A. H. (1997). *The Inflationary Universe.* (U. Contact, Trans.)

Hawking, S., & Penrose, R. (1994). *The nature of space and time.* Oxford: Oxford University Press.

Landau, L., & Lifchitz, E. (1967). *Physique Statistique.* Moscou: Editions MIR.

Landau, L., & Lifchitz, E. (1970). *Théorie des Champs.* Moscou: Editions MIR.

Langhaar, H. L. (1951). *Dimensional Analysis and Theory of Models.* Wiley.

Li, J. X. (2004). Visualization of High Dimensional data with Relational Perspective Map. *Information Visualization, Vol3 No.1* , 49-59.

Lipschutz, M. M. (1969). *Theory and problems of Differential Geometry.* New york: McGraw-Hill.

Lipshutz, S. (1968). *Theory and problems of Linear Algebra.* McGraw-Hill book company.

Lipshutz, S. (1964). *Theory and problems of Set theory and related topics.* McGraw-Hill book company.

Maxwell, J. C. (1874). On the Mathematical Classification of Physical Quantities. *Proceedings of the London Mathematical Society* , 258-266.

Menzel, D. H. (1960). *Fundamental formulas of physics.* New York: Dover Publications Inc.

Merzbacher, E. (1970). *Quantum Mechanics second edition .* New York: John Wiley & Sons .

Noyes, P. H. (1994). Bit-String Physics: a Novel "Theory of Everything". *SLAC-PUB-6509* .

Penrose, R. (2005). *The road to reality - A Complete Guide to the Laws of the Universe.* London: Vintage Books.

Peter J. Mohr, B. N. (2008). CODATA recommended values of the fundamental physical constants:2006. *J.Phys.Chem. Ref.Data, Vol. 37, No. 3* , 1187-1284.

Prigogine, I., & Stengers, I. (1984). *Order out of Chaos.* Bert Bakker.

Raeymaekers, P. (2001). *Genen en gezondheid.* Amsterdam: Veen Magazines.

Rayleigh, L. (1899-1900). On the Viscosity of Argon as affected by Temperature. *Proc. Roy. Soc. London, Vol LXVI* , 68-74.

Roche, J. (1998). *The mathematics of measurement.* The Athlone Press.

Rosen, J. (1995). *Symmetry in science: an introduction to the general theory .* New York: Springer-Verlag.

Sammon, J. (1969). A nonlinear mapping algorithm for data structure analysis. *IEEE Transaction on Computers, C18* , 401-409.

Schroeder, M. R. (1990). *Fractals, chaos, power laws: minutes from an infinite paradise.* New York: W.H. Freeman and Company.

Sheppard, M. R. (2008, April 28). *Systematic Search for Expressions of Dimensionless Constants using the NIST database of Physical constants*. Retrieved from https://www.msu.edu/~sheppa28/constants/constants.html

Siegfried Gottwald, H. K. (1995). *Meyers Kleine Enzyklopädie Mathematik.* Mannheim: Bibiographisches Institut & F.A. Brockhaus AG.

Silverman, M. P. (1998). *Waves and Grains: reflections on light and learning.* Princeton: Princeton University Press.

Stewart, I. (1995). *Nature's numbers: the unreal reality of mathematics.* New York: BasicBooks.

Uzan, J.-P., & Leclercq, B. (2005). *De l'importance d'être une constante.* (B. Mizon, Trans.) Paris: Dunod.

Veltman, M. (2003). *Feiten en mysteries in de deeltjesfysica.* Amsterdam: Veen Magazines.

Venkataraman, G. (1994). *Qunatum Revolution II: QED: the jewel of physics.* Hyderabad: University Press (india) Private Limited.

Weinberg, S. (1977). *The first three minutes - A modern view of the origin of the universe.* (H. v. Herk-Kluyver, Trans.) New York: Basic Books Inc.