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Chapter 3

Teachers' views of curriculum programs in Flanders: does it (not) matter which mathematics textbook series schools choose?

Chapter 3

Teachers' views of curriculum programs in Flanders: does it (not) matter which mathematics curriculum program schools choose?[3]

Abstract

The debate on the differential effects of mathematics curriculum programs is a recurrent topic in the research literature. Research results remain inconclusive, pointing to a lack of evidence to decide on the relevance of the selection by schools of a mathematics curriculum programs. Studies also point to difficulties in comparing curriculum programs. Recently, in order to examine the influence of mathematics curriculum programs on student learning, the need to take into account mediating variables between the mathematics curriculum program and the enacted curriculum is stressed. This paper focuses on one such mediating variable: teachers' views of mathematics curriculum programs. Views of mathematics curriculum programs of 814 teachers and mathematics performance results of 1579 students were analyzed. The results point out that with regard to teachers' views of curriculum programs, the question 'Does it really matter which curriculum program schools choose' has to be answered positively. Implications of the findings are discussed.

[3] Based on:
Van Steenbrugge, H., Valcke, M., & Desoete, A. (in press). Teachers' views of mathematics textbook series in Flanders: does it (not) matter which mathematics textbook series schools choose? *Journal of curriculum studies*

1. Introduction

One can hardly overemphasize the importance of mathematical literacy in our society (Dowker, 2005; Swanson, Jerman, & Zheng, 2009). Basic skills in mathematics are needed to operate effectively in today's world (Grégoire & Desoete, 2009; NCTM, 2000; OECD, 2010). As a result, mathematics generally figures as an important curriculum domain in education (Buckley, 2010; Keijzer & Terwel, 2003).

A large number of variables and processes affect mathematics learning outcomes: student characteristics, class climate, teacher characteristics, teaching approaches, … to name just a few. In this context, mathematics curriculum programs also play a role in both the teaching and learning processes that affects learning outcomes (Bryant et al., 2008; Nathan, Long, & Alibali, 2002). In the current study, the term "curriculum program" refers to the printed and published resources designed to be used by teachers and students before, during and after mathematics instruction. On the one hand, they are considered to be sources of explanations and exercises for students to complete and on the other hand, they refer to the instructional guides for teachers that highlight the how and the what of teaching (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997; Stein, Remillard, & Smith, 2007). In addition, we also refer to additional materials that are mentioned or included in the instructional guides for teachers or in the exercises for the students like additional software, coins, calculator, … This does not include other materials that are not mentioned or included in the instructional guides like videos, internet resources, and other books but on which teachers may rely when teaching mathematics.

This research consists of two studies that both focus on curriculum programs: the first study analyzes whether teachers' views of curriculum programs differ depending on the curriculum program; the second study analyzes whether students' performance results differ between curriculum programs.

2. Curriculum programs in Flemish elementary school and elsewhere

This study focuses on mathematics curriculum programs used in Flanders (the Dutch-speaking part of Belgium) and as such narrows down to a particular location with its own peculiarities. However, there

are similarities with curriculum programs in other regions. To illustrate this, we describe the situation in Flanders and highlight the situation in some other regions.

In Flanders, the choice of a curriculum program is an autonomous school-decision. Most schools adopt one commercial curriculum program throughout all grades. Five curriculum programs dominate the elementary school market: Eurobasis, Kompas, Zo gezegd, zo gerekend, Nieuwe tal-rijk, and Pluspunt (Van Steenbrugge, Valcke, & Desoete, 2010). A detailed description of the five mathematics curriculum programs is provided in Appendix. The curriculum programs consist of 2 main parts: the explanations and exercises for the students, and the educational guidelines for the teachers that explain how to teach the contents, how to organize the lessons in such a way that they build on each other, how to use other didactical materials, etc. The basic principles underlying each curriculum program are shared by all: all curriculum programs are curriculum-based, cluster lessons in a week, a block or a theme addressing the main content domains of mathematics education (numbers and calculations, measurement, geometry). The specific content of the domains are in accordance with the three most frequently used curricula in Flanders (see Appendix). These curricula specify at each grade level detailed the content to be mastered by the specific students. The curriculum programs address these curricula by means of instruction and exercises for all students that focus on mastering the specific content, and by means of additional exercises that aim to differentiate according to students' needs. The curriculum programs typically provide exercises for students to work on after the teacher explained initial examples.

To summarize, it can be stated that they are largely equivalent. Two curriculum programs stand out: Pluspunt and Nieuwe tal-rijk. Pluspunt incorporates explicit student-centred lessons, formulates rather general directions for teaching and the "courses" address more than one mathematics content domain. Nieuwe tal-rijk on the other hand, gives the teacher more additional tools and materials, provides a far more detailed description of each course, provides additional didactical suggestions and mathematical background knowledge for the teacher and provides suggestions to implement learning paths, helping the teacher to maintain control.

In the Netherlands, the same picture emerges as in Flanders: curriculum programs are curriculum-based, chosen by the school team, consist also of a guide for teachers and materials for the learners,

and within one school, the curriculum programs of one commercial series are used throughout all grades (Bruin-Muurling, 2011; O'Donnell, Sargent, Byrne, White, & Gray, 2010; van Zanten, 2011). In France, the government prescribes the content and format and approves the curriculum programs – which are all commercial– for use in schools. The choice for a curriculum program in elementary school is decided at the class level by the teacher and as a result, within a single school, mathematics curriculum programs of several commercial series can be used throughout all grades (Gratrice, 2011; O'Donnell et al., 2010). In England, all curriculum programs are commercial (Hodgen, 2011; O'Donnell et al., 2010). The extent to which the curriculum programs are used as a primary basis to teach mathematics in elementary school is lower as compared to many other countries (Mullis, Martin, & Foy, 2008). Curriculum programs are viewed as one of the many resources that teachers use in their classrooms (Askew, Hodgen, Hossain, & Bretscher, 2010; Pepin, Haggarty, & Keynes, 2001). Instead of using one single curriculum program as a primary basis for lessons, teachers are encouraged to use different resources, such as internet resources and books as lesson starters (Department for Education, 2011). Still, nearly 80% of the elementary school teachers in England make at least some use of curriculum programs to teach mathematics (Mullis et al., 2008). Curriculum programs also contain a guide for teachers but teachers mainly build on the 'mathematics framework' provided by the Department for Education (Hodgen, Küchemann, & Brown, 2010). In China, the government approves the curriculum programs and local authorities decide for each single grade which curriculum programs schools should use, resulting in the use of mat several commercial curriculum programs throughout all grades in one school (Ministry of Education in P.R.China, 2011). The curriculum programs also contain a guide for teachers.

As illustrated above, there are differences between regions considering curriculum programs for elementary school. Nevertheless, it can be concluded that mathematics curriculum programs are predominant in elementary school. Moreover, mathematics curriculum programs are often the primary resource for teachers and students in the classroom (Elsaleh, 2010; Grouws, Smith, & Sztajn, 2004; Kauffman, Johnson, Kardos, Liu, & Peske, 2002; Mullis et al., 2008; Pepin et al., 2001; Schug, Western, & Enochs, 1997).

3. The current study

Despite the recognized prominent position of mathematics curriculum programs in the teaching and learning process, there is no agreement on its differential impact on students' performance results. Slavin and Lake (2008), for instance, stress that there is a lack of evidence to conclude or not that it matters which mathematics curriculum programs schools adopt. It is difficult to judge or compare the efficacy or efficiency of curriculum programs (Deinum & Harskamp, 1995; Gravemeijer et al., 1993; Janssen, Van der Schoot, Hemker, & Verhelst, 1999). Slavin and Lake (2008) and Chval et al. (2009) expressed the need for further research in this field especially involving large numbers of students and teachers, and set in a variety of school settings. To examine the influence of curriculum programs on student learning, research recently stresses the need to take into account factors that mediate between the written and the enacted curriculum (Atkin, 1998; Ball & Cohen, 1996; Christou, Eliophotou-Menon, & Philippou, 2004; Lloyd, Remillard, & Herbel-Eisenman, 2009; Macnab, 2003; Remillard, 1999; Sherin & Drake, 2009; Verschaffel, Greer, & de Corte, 2007). Stein et al. (2007) propose a conceptual model that takes into account several mediating variables between the written curriculum (e.g. the curriculum program), the intended curriculum, and the curriculum as enacted in the classroom: teacher beliefs and knowledge, teachers' orientations toward the curriculum, teachers' professional identity, teacher professional communities, organizational and policy contexts, and classroom structures and norms. Moreover, Remillard (2005) highlights the need to focus on teachers' orientations toward the curriculum as a guiding principle for future research. Teachers' orientations toward the curriculum are described as a frame that influences how teachers engage with the materials and use them in teaching (Remillard & Bryans, 2004). These reflect the teachers' ideas about mathematics teaching and learning, teachers' views of curriculum materials in general, and teachers' views of the particular curriculum they are working with. Whereas the study pointed out that the unique combination of these ideas and views of teachers influenced the way they used the curriculum, the study also revealed that the ideas about mathematics teaching and learning and views of curriculum materials in general and of the particular curriculum they are working with separately also proved to be a mediating variable (Remillard and Bryans, 2004). Information about these mediating variables was obtained through semi-structured interviews with the eight participants (Remillard and

Bryans, 2004). In the present study, we focus on teachers' views of the particular curriculum they are working with (i.e. the mathematics curriculum programs they are using), and we do so by building on the experiences of teachers with the curriculum programs (Elsaleh, 2010) related to how they perceive that these materials impact student mathematics performance. In addition, and given the lack of agreement on the differential impact of mathematics curriculum programs on students' performance results, we also study whether the performance results of the students taught by the teachers in this study differ significantly based on the curriculum program used in the classroom. The latter will enable us to analyze if possible differences in teachers' views of curriculum programs are related to differences in students' performance results.

As such, this study aims at contributing to the curriculum programs discussion by using a large sample and by asking the question whether it really matters what mathematics curriculum programs schools adopt.

The following research questions are put forward directing our study:

- Do teachers' views of mathematics curriculum programs vary depending on the mathematics curriculum program being adopted?
- Do students' performance results vary between mathematics curriculum programs?

With regard to these questions, two studies have been set up. Each study focused on a particular research question.

4. Methodology

4.1. Respondents

The research project was announced via the media: via the national education journal, the official electronic newsletter for teachers and principals distributed by the Department of Education, an internet site, via the communication channels of the Learner Support Centres, via the communication channels of the different educational networks and the teacher unions. When respondents showed interest, they could contact the researcher for more information. This approach resulted in a large sample of 918 teachers from 243 schools. Only respondents using one of the five most frequently used mathematics curriculum programs were included in this study, resulting in a sample of 814 teachers

from 201 schools. Teaching experience of the teachers included in the present study ranged from 0 to 46 years *(Mean: 16.77)*. Experience of 80% of these teachers ranged from 4 to 30 years; 90% of the respondents had at least 4 years teaching experience. Of these teachers, 132 (16%) taught in the first grade, 133 (16%) in second grade, 130 (16%) in third grade, 125 (15%) in fourth grade, 135 (17%) in fifth grade, 110 (14%) in sixth grade, 12 (1%) in both first and second grade, 16 (2%) in both third and fourth grade, and 21 (3%) in both fifth and sixth grade.

For the second study, a sample of 90 elementary school teachers (11%) was selected at random to participate in the second study. We ended up with 89 teachers (11%) from the original sample of 814 teachers. The teachers from the former sub-sample provided us the completed tests for mathematics of the Flemish Student Monitoring System of the students in their classroom (n = 1579). Performance data resulted from the systematic administration of standardized tests incorporated in the Flemish Student Monitoring System (see 'Instruments'). Considering the 1579 elementary school children, 234 respondents (15%) were first grade students, 405 (26%) were second grade students, 253 (16%) were third grade students, 278 (18%) were fourth grade students, 255 (16%) were fifth grade students, and 154 (10%) were sixth grade students. Teaching experience of the teachers in the second study ranged from 1 to 37 years *(Mean: 16.21)*. Experience of 80% of these teachers ranged from 4 to 31 years; also 90% of the respondents in the second study had at least 4 years teaching experience.

Table 1 presents an overview of the distribution of curriculum programs as adopted by the schools in our sample.

Table 2. Distribution of mathematics curriculum programs in the sample

	Study 1		Study 2	
Mathematics curriculum program	Number of schools	%	Number of schools	%
Eurobasis [EB]	40	19.90	3	10.34
Kompas [KP]	4	1.99	2	6.90
Zo gezegd, zo gerekend	47	23.38	7	24.14
Nieuwe tal-rijk	27	13.43	5	17.24
Pluspunt	22	10.95	3	10.34
Combination of EB & KP	50	24.88	9	31.03
Another combination	11	5.47	/	/
Total	201	100	29	100

It is to be noted that Kompas is an updated version of Eurobasis. At the time of this study, no version was yet available of Kompas for the 4th, 5th and 6th grade. Most schools had implemented Eurobasis, Kompas, or a combination of Eurobasis and Kompas: 47% of the schools in the first study and 48% in the second study (see table 1). Table 1 also reveals that a minority of the schools combined multiple mathematics curriculum programs: 5% of the schools in the first study and none in the second study. This is not surprising, since the choice for a specific mathematics curriculum program in Flanders is a school-based decision.

4.2. Instruments

In order to study teachers' views of mathematics curriculum programs, we built on teachers' experiences with these materials. This was done on the base of a newly developed self-report questionnaire. At the content level, teachers' views of mathematics curriculum programs was studied in relation to the learning goals pursued within three dominant mathematics content domains in each mathematics curriculum program: numbers and calculations, measurement and geometry, and in accordance with the learning goals pursued in three curricula that are predominant in Flemish elementary school (see '2. Curriculum programs in Flemish elementary school and elsewhere'). In relation to each mathematics domain, items asked to judge on a 5-point Likert scale if 'The way the mathematics curriculum program supports this learning goal, causes difficulties in student learning' (1= 'totally disagree' and 5= 'totally agree'). Specific versions of the questionnaire were presented to first and second grade teachers, third and fourth grade teachers and fifth and sixth grade teachers. This helped to align the instrument precisely with the learning objectives that are central in the domains at each grade level. Next to information about the mathematics curriculum programs being adopted by the teachers in their school, respondents were also asked to indicate the number of years of teaching experience.

The questionnaires were tried out in the context of a pilot study. As can be derived from table 2, the internal consistency of the different subsections of the questionnaire was high, with Cronbachs' α-values between and .83 and .94.

Table 2. Internal consistency of the different subsections in the questionnaire for teachers

	Numbers and calculations		Measurement		Geometry	
	α	*n*	*α*	*n*	*α*	*n*
First and second grade	.83	15	.89	8	.83	5
Third and fourth grade	.92	25	.89	11	.87	10
Fifth and sixth grade	.94	26	.93	14	.86	9

With regard to the second study, mathematics achievement was assessed by means of curriculum-based standardized achievement tests for mathematics included in the Flemish Student Monitoring System (Dudal 2001). This student monitoring system is widely used in the Flemish elementary educational landscape and provides every grade, apart from the sixth grade, with three tests. A first test is provided at the beginning of a specific grade, another at the middle and a last one at the end of the school grade (Dudal, 2001). In the present research, only the middle grade tests were used. All tests were administered between February 1 and 15. Test administration is strictly protocolled. The assessment is spread over two consecutive morning sessions and teachers are provided with a sheet containing all the information with regard to test completion, classroom setting and clarifications for students. Teachers are provided with a sheet containing word by word the sentences they are expected to pronounce in view of the test administration.

Tests consist of 60 items covering the mathematics domains numbers and calculations, measurement, and geometry. The test items are geared to the mathematics curriculum of the specific grade. Given that in the Flemish elementary school mathematics curriculum most goals focus on numbers and calculations, most test items focused on this domain.

For example, the test in the third grade contains 45 items measuring performance in the domain numbers and calculations (e.g. Sasha has 120 stamps. Milan has half of the amount. How many stamps do they possess together?), 10 items measuring performance in the domain measurement (e.g. our postman is fat nor skinny, tall nor short. What could be his weight? 25kg – 40kg – 75kg – 110kg – 125 kg?), and 5 items measuring performance in the geometry domain (e.g. A door has the shape of a: square – triangle – circle – rectangle – hexagon?).

In addition to the 60 test items, from the second grade on, students needed to complete a grade specific test assessing students' knowledge of basic operations. By means of mental arithmetic, students need

to solve sums (e.g. 55+25 = ...), subtractions (e.g. 87-25 = ...), multiplications (e.g. 4x3 = ...) and divisions (e.g. 9:3 = ...). Time for solving these exercises was limited. The latter test items were used to measure students' mathematical basic knowledge.

4.3. Data analysis

The data in the present research reflect an inherent hierarchical structure, i.e. teachers are nested in schools (study 1) and students are nested in classes (study 2). As such, the assumption of independence of observations - inherent to ordinary least squares regressions - was violated. Ordinary least squares regressions rely heavily on the assumption of independence of observations: they assume that each observation is independent of every other single observation. Or: all the observations have nothing in common. For instance, in the first study we analyzed for 814 teachers from 201 schools their views of the curriculum program they use. Ordinary least squares regressions would consider this as 814 independent observations: all the observations have nothing in common. In reality, this is not the case. Teachers teaching in the same school are not independent of each other and do have things in common: they dialogue, they exchange ideas, they share the curriculum programs, they teach students from equal social classes, they live in the same neighborhoods, … Ordinary least squares regressions do not take into account the fact that teachers are nested in schools. This has an impact on the degree of error: it results in an increase in the possibility that observed significant differences are due to coincidence (and not due to the fact that they relate to different mathematics curriculum programs).

In contrast, multilevel modeling does take into account that not all observations are independent of each other (Goldstein & Silver, 1989; Maas & Hox, 2005). It takes into account that teachers are nested in schools (study 1) and that students are nested at classroom level (study 2). This results in a reduced degree of error: it results in a decrease of the possibility that the observed significant differences are due to coincidence. This explains why we applied multilevel modeling techniques instead of applying ordinary regression models.

Model 1 in Tables 3, 4, and 5 reveals that schools differed significantly from each other: or that teachers within the same school are related more to each other than they do to teachers in other schools. Model 1 in Table 7 also reveals that classes differed significantly from each other: or that

students within the same class are more related to each other than they do to students in other classes. The latter provides evidence that observations are not independent of each other and that applying multilevel modeling in both studies was appropriate.

Given the three outcome measures in both studies, i.e. teachers' views related to / students' scores for numbers and calculations, teachers' views related to / students' scores for measurement, and teachers' views related to / students' scores for geometry, multivariate multilevel regression models were applied. The use of several related outcome measures results in a more complete description of what is affected by changes in the predictor variables (Hox, 2002; Tabachnick & Fidell, 1996). Multivariate response data were incorporated in the multilevel model by creating an extra level below the original level 1 units to define the multivariate structure (Hox, 2002; Rasbash, Steele, Browne, & Goldstein, 2009). This implies that in the first study, we considered teachers' views of mathematics curriculum programs for the domain numbers and calculations, teachers' views of mathematics curriculum programs for the domain measurement, and teachers' views of mathematics curriculum programs for the domain geometry (level 1) nested within teachers (level 2) who in turn are nested in schools (level 3). In the second study, we considered students' performance results for the domain numbers and calculations, students' performance results for the domain measurement, and students' performance results for the domain geometry (level 1) nested within students (level 2) who in turn are nested in classes (level 3). No level 1 variation was specified since this level only helped to define the multivariate structure (Hox, 2002; Rasbash, Steele, et al., 2009; Snijders & Bosker, 2003). Fitting a multivariate model into a multilevel framework does not require balanced data. As such, it was not necessary to have the same number of available measurements for all individuals (Hox, 2002; Maas & Snijders, 2003; Rasbash, Steele, et al., 2009; Snijders & Bosker, 2003).

In view of the first study, sum scores for each mathematics content domain (numbers and calculations, measurement, and geometry) were calculated and transformed into z-scores. A number of multilevel models have been fitted, using MLwiN 2.16 (Rasbash, Charlton, Browne, Healy, & Cameron, 2009). The best fitting model was designed in a step-by-step way (Hox 2002). First, the null model was fitted with random intercepts at the teacher level (Model 0). Next, random intercepts were allowed to vary at the school level (Model 1). In a third step, the teacher-level variable "teaching experience" expressed

in number of years, was included as a fixed effect (Model 2). In a fourth step, we included the categorical variable "curriculum program" with Pluspunt as the reference category (Model 3). Pluspunt was chosen as reference since Pluspunt deviated from the other four curriculum programs in the amount of providing hands-on support; this allowed for a comparison of Pluspunt with the other curriculum programs. Since comparisons between other combinations of curriculum programs were equally of interest, in a final step, we also analyzed pairwise comparisons between all mathematics curriculum programs.

In view of the second study, sum scores for each mathematics content domain were calculated and transformed into a scale ranging from zero to ten. Correlations between the covariate "mathematical basic knowledge" and the score on mathematics domains "numbers and calculations" ($r = .64$, $n = 1247$, $p < .001$, two-tailed), "measurement" ($r = .46$, $n=1227$, $p < .001$, two-tailed), and "geometry" ($r = .24$, $n = 1224$, $p < .001$, two-tailed) were significant after Bonferroni correction. First, the null model was fitted with random intercepts at the student level (Model 0). Next, random intercepts were allowed to vary at the class level (Model 1). In a third and fourth step, the student-level variables "mathematical basic knowledge" (Model 2) and "sex" (Model 3) were included as fixed effects. In Model 4, we included the categorical class-level variable "grade". Next, class-level variable "teaching experience" was included as a fixed effect (Model 5). In a final step, "curriculum programs" was included as a fixed categorical variable (Model 6). Additionally, model improvement was analyzed after allowing interaction between curriculum programs and grade ($\chi^2(60) = 45.621$; $p = .92$), and curriculum programs and experience ($\chi^2(12) = 15.985$; $p = .19$), but since this did not result in a significant model improvement, the results of this analysis are not reported.

The parameters of the multilevel models were estimated using Iterative Generalized Least Squares estimations (IGLS). All analyses assumed at least a 95% confidence interval.

5. Results

5.1. Study 1: Differences in teachers' views of mathematics curriculum programs?

Given the use of specific grade-level questionnaires, three sets of results are presented in table 3 to 5 (grade 1-2, grade 3-4 and grade 5-6).

Table 3 presents the results with regard to the first and the second grade. According to Model 0, variance at the teacher level was statistically significant. Allowing random intercepts at the school level (Model 1), resulted in a significant decrease in deviance indicating that inclusion of the school level was appropriate. Adding the teacher-level variable "experience" in Model 2 did not result in a significant decrease in deviance and as a consequence the variable experience was excluded from further analyses. Including the variable "curriculum programs" in Model 3, on the contrary, did result in a significant decrease in deviance. The fixed effects in Model 3 revealed that with regard to the mathematics domain measurement, teachers using Kompas or Nieuwe tal-rijk as curriculum program reported significantly less difficulties as compared to teachers using the reference curriculum program (Pluspunt). Considering the mathematics domain geometry, teachers using Kompas, Zo gezegd, zo gerekend or Nieuwe tal-rijk as curriculum program reported significantly less difficulties as compared to teachers using Pluspunt.

Table 4 presents the results with regard to the third and the fourth grade. According to Model 0, variance at the teacher level was statistically significant. Allowing random intercepts at the school level (Model 1), resulted in a significant decrease in deviance indicating that inclusion of the school level was appropriate. Adding the teacher-level variable "experience" in Model 2 did not result in a significant decrease in deviance and as a consequence this variable was excluded from further analyses. Including the variable "curriculum program" in Model 3, on the contrary, did again result in a significant decrease in deviance. A closer look at the fixed effects in Model 3 showed that with regard to the mathematics domain measurement, teachers using Nieuwe tal-rijk as curriculum program reported significantly less mathematics difficulties as compared to teachers using the reference curriculum program (Pluspunt). Considering the mathematics domain geometry, teachers using Eurobasis, Kompas, Zo gezegd, zo gerekend or Nieuwe tal-rijk as curriculum program reported significantly less difficulties as compared to teachers using Pluspunt.

Table 3. First and second grade: fixed effects estimates (top) and variance-covariance estimates (bottom)

Parameter	Model 0		Model 1		Model 2		Model 3	
Fixed effects								
$Intercept_N$	-.01	(.06)	-.01	(.06)	-.07	(.12)	.07	(.18)
$Intercept_M$	-.01	(.06)	-.03	(.07)	-.15	(.12)	.45*	(.19)
$Intercept_G$	.01	(.06)	-.01	(.07)	-.13	(.12)	.64*	(.21)
Level 2 (teacher)								
$Experience_N$					.00	(.01)		
$Experience_M$					.01	(.01)		
$Experience_G$					.01	(.01)		
Level 3 (school)								
EB_N							-.04	(.26)
KP_N							-.05	(.21)
ZG_N							-.05	(.22)
NT_N							-.34	(.26)
EB_M							-.37	(.26)
KP_M							-.65*	(.22)
ZG_M							-.26	(.22)
NT_M							-.90**	(.26)
EB_G							-.39	(.27)
KP_G							-.75*	(.24)
ZG_G							-.76*	(.24)
NT_G							-.83*	(.28)
Random parameters								
Level 2								
$Intercept_N$/ $Intercept_N$ (σ^2_{u0})	.98**	(.09)	.92**	(.10)	.91**	(.10)	.93**	(.10)
$Intercept_N$/ $Intercept_M$ (σ^2_{u0u1})	.68**	(.07)	.58**	(.08)	.58**	(.08)	.58**	(.08)
$Intercept_M$/ $Intercept_M$ (σ^2_{u1})	1.01**	(.09)	.78**	(.09)	.77**	(.09)	.79**	(.09)
$Intercept_N$/ $Intercept_G$ (σ^2_{u0u2})	.45**	(.07)	.36**	(.07)	.36**	(.07)	.36**	(.07)
$Intercept_M$/ $Intercept_G$ (σ^2_{u1u2})	.57**	(.07)	.29**	(.07)	.28**	(.07)	.28**	(.06)
$Intercept_G$/ $Intercept_G$ (σ^2_{u2})	1.00**	(.09)	.65**	(.08)	.65**	(.08)	.63**	(.08)

Table 3 continued

Parameter	Model 0	Model 1	Model 2	Model 3
Level 3				
$Intercept_N$/ $Intercept_N$ (σ^2_{v0})		.06 (.07)	.07 (.08)	.06 (.07)
$Intercept_N$/ $Intercept_M$ (σ^2_{v0v1})		.09 (.07)	.10 (.07)	.10 (.07)
$Intercept_M$/ $Intercept_M$ (σ^2_{v1})		.22* (.09)	.23* (.09)	.16* (.08)
$Intercept_N$/ $Intercept_G$ (σ^2_{v0v2})		.08 (.07)	.08 (.07)	.08 (.06)
$Intercept_M$/ $Intercept_G$ (σ^2_{v1v2})		.28** (.08)	.28** (.08)	.26** (.07)
$Intercept_G$/$Intercept_G$ (σ^2_{v2})		.34** (.10)	.34** (.10)	.34** (.09)
Model fit				
Deviance	1970.64	1932.39	1930.21	1893.34
χ^2		38.25**	2.18	39.05**
df		6	3	12
p		<.001	.541	<.001
Reference		Model 0	Model 1	Model 1

Note. Standard errors are in parentheses. There are no level 1 random parameters because level 1 exists solely to define the multivariate structure. $_N$ = numbers; $_M$ = measurement; $_G$ = geometry; EB = Eurobasis; KP = Kompas; ZG = Zo gezegd, zo gerekend; NT = Nieuwe tal-rijk.
* $p < 0.05$.
** $p < 0.001$.

Table 4. Third and fourth grade: fixed effects estimates (top) and variance-covariance estimates (bottom)

Parameter	Model 0		Model 1		Model 2		Model 3	
Fixed effects								
$Intercept_N$	.01	(.06)	-.02	(.07)	-.15	(.12)	-.01	(.20)
$Intercept_M$	-.01	(.06)	-.03	(.07)	.03	(.13)	.29	(.20)
$Intercept_G$	.01	(.06)	-.01	(.07)	-.00	(.13)	.54*	(.19)
Level 2 (teacher)								
$Experience_N$					.01	(.01)		
$Experience_M$					-.00	(.01)		
$Experience_G$					-.00	(.01)		
Level 3 (school)								
EB_N							.20	(.23)
KP_N							-.13	(.26)
ZG_N							-.13	(.24)
NT_N							-.18	(.28)
EB_M							-.22	(.23)
KP_M							-.15	(.26)
ZG_M							-.41	(.23)
NT_M							-.85*	(.27)
EB_G							-.52*	(.22)
KP_G							-.51*	(.25)
ZG_G							-.68*	(.23)
NT_G							-.82*	(.27)
Random parameters								
Level 2								
$Intercept_N$/ $Intercept_N$ (σ^2_{u0})	.96**	(.09)	.75**	(.09)	.75**	(.09)	.72**	(.09)
$Intercept_N$/ $Intercept_M$ (σ^2_{u0u1})	.67**	(.07)	.53**	(.08)	.54**	(.08)	.53**	(.08)
$Intercept_M$/ $Intercept_M$ (σ^2_{u1})	1.00**	(.09)	.83**	(.10)	.82**	(.10)	.82**	(.10)
$Intercept_N$/ $Intercept_G$ (σ^2_{u0u2})	.64**	(.07)	.57**	(.09)	.57**	(.09)	.57**	(.08)
$Intercept_M$/ $Intercept_G$ (σ^2_{u1u2})	.68**	(.08)	.55**	(.09)	.55**	(.09)	.56**	(.09)
$Intercept_G$/ $Intercept_G$ (σ^2_{u2})	1.00**	(.09)	.90**	(.11)	.90**	(.11)	.92**	(.11)

Table 4 continued

Parameter	Model 0	Model 1	Model 2	Model 3
Level 3				
$Intercept_N$/ $Intercept_N$ (σ^2_{v0})		.20* (.09)	.19* (.09)	.21* (.09)
$Intercept_N$/ $Intercept_M$ (σ^2_{v0v1})		.13 (.08)	.12 (.08)	.12 (.07)
$Intercept_M$/ $Intercept_M$ (σ^2_{v1})		.17 (.09)	.18* (.09)	.12 (.08)
$Intercept_N$/ $Intercept_G$ (σ^2_{v0v2})		.07 (.07)	.06 (.07)	.06 (.07)
$Intercept_M$/ $Intercept_G$ (σ^2_{v1v2})		.13 (.08)	.13 (.08)	.08 (.07)
$Intercept_G$/$Intercept_G$ (σ^2_{v2})		.11 (.08)	.11 (.08)	.04 (.07)
Model fit				
Deviance	1838.16	1815.82	1810.58	1781.74
χ^2		22.34*	7.05	34.08**
df		6	3	12
p		<.05	.16	<.001
Reference		Model 0	Model 1	Model 1

Note. Standard errors are in parentheses. There are no level 1 random parameters because level 1 exists solely to define the multivariate structure. $_N$ = numbers; $_M$ = measurement; $_G$ = geometry; EB = Eurobasis; KP = Kompas; ZG = Zo gezegd, zo gerekend; NT = Nieuwe tal-rijk.
* $p < 0.05$.
** $p < 0.001$.

Table 5. Fifth and sixth grade: fixed effects estimates (top) and variance-covariance estimates (bottom)

Parameter	Model 0		Model 1		Model 2		Model 3	
Fixed effects								
$Intercept_N$	-.00	(.06)	.01	(.07)	-.05	(.12)	.55*	(.19)
$Intercept_M$	-.00	(.06)	-.02	(.07)	-.10	(.12)	.50*	(.20)
$Intercept_G$	-.01	(.06)	-.02	(.08)	-.15	(.12)	.50*	(.21)
Level 2 (teacher)								
$Experience_N$					.00	(.01)		
$Experience_M$					.01	(.01)		
$Experience_G$					.01	(.01)		
Level 3 (school)								
EB_N							-.48*	(.21)
ZG_N							-.86**	(.23)
NT_N							-.69*	(.26)
EB_M							-.44*	(.22)
ZG_M							-.71*	(.24)
NT_M							-.86*	(.27)
EB_G							-.48*	(.23)
ZG_G							-.76*	(.25)
NT_G							-.67*	(.28)
Random parameters								
Level 2								
$Intercept_N$/ $Intercept_N$ (σ^2_{u0})	.99**	(.09)	.65**	(.08)	.64**	(.08)	.62**	(.08)
$Intercept_N$/ $Intercept_M$ (σ^2_{u0u1})	.73**	(.08)	.39**	(.07)	.38**	(.07)	.36**	(.06)
$Intercept_M$/ $Intercept_M$ (σ^2_{u1})	1.03**	(.09)	.59**	(.08)	.59**	(.08)	.56**	(.07)
$Intercept_N$/ $Intercept_G$ (σ^2_{u0u2})	.73**	(.08)	.38**	(.07)	.37**	(.07)	.36**	(.06)
$Intercept_M$/ $Intercept_G$ (σ^2_{u1u2})	.78**	(.08)	.38**	(.06)	.37**	(.06)	.36**	(.06)
$Intercept_G$/ $Intercept_G$ (σ^2_{u2})	1.02**	(.09)	.54**	(.07)	.53**	(.07)	.52**	(.07)

Table 5 continued

Parameter	Model 0	Model 1	Model 2	Model 3
Level 3				
$Intercept_N$/ $Intercept_N$ (σ^2_{v0})		.31* (.10)	.32** (.10)	.28* (.09)
$Intercept_N$/ $Intercept_M$ (σ^2_{v0v1})		.30** (.09)	.31** (.09)	.28** (.08)
$Intercept_M$/ $Intercept_M$ (σ^2_{v1})		.39** (.10)	.39** (.10)	.36** (.10)
$Intercept_N$/ $Intercept_G$ (σ^2_{v0v2})		.33** (.09)	.33** (.09)	.30** (.08)
$Intercept_M$/ $Intercept_G$ (σ^2_{v1v2})		.37** (.09)	.37** (.09)	.35** (.09)
$Intercept_G$/$Intercept_G$ (σ^2_{v2})		.48** (.11)	.47** (.11)	.45** (.10)
Model fit				
Deviance	1726.30	1668.71	1666.74	1648.48
χ^2		57.58**	1.97	20.23*
df		6	3	9
p		<.001	.16	<.05
Reference		Model 0	Model 1	Model 1

Note. Standard errors are in parentheses. There are no level 1 random parameters because level 1 exists solely to define the multivariate structure. $_N$ = numbers; $_M$ = measurement; $_G$ = geometry; EB = Eurobasis; KP = Kompas; ZG = Zo gezegd, zo gerekend; NT = Nieuwe tal-rijk.
* $p < 0.05$.
** $p < 0.001$.

Table 5 presents the analysis results with regard to the data of fifth and sixth grade teachers. According to Model 0, variance at the teacher level was statistically significant. Allowing random intercepts at the school level (Model 1), again resulted in a significant decrease in deviance indicating that inclusion of the school level was appropriate. Adding the teacher-level variable "experience" in Model 2 did not result in a significant decrease in deviance and as a consequence the variable was excluded from further analyses. Including the variable "curriculum program" in Model 3 again resulted in a significant decrease in deviance. Focusing on the fixed effects in Model 3, we observed that with regard to the mathematics domains numbers and calculations, measurement, and geometry, teachers using Eurobasis, Zo gezegd, zo gerekend or Nieuwe tal-rijk as curriculum program reported significantly less difficulties as compared to teachers using the reference curriculum program (Pluspunt).

Estimates for the fixed effects of the variable curriculum program (see model 4 in table 3, table 4, and table 5) only allowed for comparison with the reference category (Pluspunt). Because comparisons between other combinations of curriculum programs were also of interest, table 6 presents for grade 1-2, grade 3-4 and grade 5-6 the results of the pairwise comparisons between all curriculum programs.

Considering the first and second grade (see table 6) and with regard to the content domain *numbers and calculations*, no significant differences in teachers' views were observed. With regard to the content domain *measurement*, we did observe significant differences in teachers' views. Teachers using Nieuwe tal-rijk as their curriculum program, reported significantly less learning difficulties as compared to teachers using Zo gezegd, zo gerekend, Eurobasis or Pluspunt; teachers using Pluspunt reported significantly more difficulties in learning as compared to teachers using Kompas or Nieuwe tal-rijk. With regard to the content domain *geometry*, teachers using Pluspunt as curriculum program reported significantly more difficulties in learning as compared to teachers using Nieuwe tal-rijk, Zo gezegd, zo gerekend or Kompas.

Building on the input of third and fourth grade teachers (see table 6) and considering the content domain numbers and calculations, no significant differences in teachers' views were observed. Considering the content domain measurement, teachers using Nieuwe tal-rijk reported significantly

Table 6. t-values for differences between mathematics curriculum programs (row minus column)

		Numbers					Measurement					Geometry				
		EB	KP	ZG	NT	PP	EB	KP	ZG	NT	PP	EB	KP	ZG	NT	PP
1st and 2nd grade	EB	/	0.00 (128)	0.00 (106)	1.20 (63)	-0.14 (63)	/	1.41 (127)	-0.48 (105)	2.14* (63)	-1.43 (63)	/	1.85 (128)	1.71 (105)	1.70 (63)	-1.44 (61)
	KP		/	0.00 (174)	1.43 (131)	-0.20 (131)		/	-2.36 (172)	1.24 (130)	-3.00** (130)		/	0.10 (173)	0.37 (131)	-3.19** (129)
	ZG			/	1.38 (109)	-0.20 (109)			/	2.94** (108)	-1.19 (108)			/	0.30 (108)	-3.14** (106)
	NT				/	-1.34 (66)				/	-3.49*** (66)				/	-2.95** (64)
	PP					/					/					/
3rd and 4th grade	EB	/	1.71 (129)	1.94 (169)	1.72 (126)	0.87 (120)	/	-0.36 (128)	1.13 (169)	2.92** (126)	-1.00 (120)	/	-0.10 (128)	0.99 (169)	1.41 (125)	-2.40* (120)
	KP		/	0.00 (110)	0.20 (67)	-0.50 (61)		/	1.22 (109)	2.78** (66)	-0.60 (60)		/	0.83 (109)	1.26 (65)	-1.99 (60)
	ZG			/	0.22 (107)	-0.53 (101)			/	1.98* (107)	-1.75 (101)			/	0.66 (106)	-3.00** (101)
	NT				/	-0.64 (58)				/	-3.14** (58)				/	-3.08** (57)
	PP					/					/					/
5th and 6th grade	EB	/	/	2.30* (191)	-1.04 (153)	-2.24* (144)	/	/	1.57 (190)	2.00* (152)	-2.00* (142)	/	/	1.54 (189)	0.87 (151)	-2.06* (141)
	KP		/	/	/	/		/	/	/	/		/	/	/	/
	ZG			/	0.77 (114)	-3.74*** (106)			/	0.65 (114)	-2.98** (104)			/	-0.36 (114)	-3.03** (104)
	NT				/	-1.34** (67)				/	-3.24** (66)				/	-2.41* (66)
	PP					/					/					/

Note. Between brackets: degrees of freedom; EB = Eurobasis; KP = Kompas; ZG = Zo gezegd, zo gerekend; NT = Nieuwe tal-rijk.
* $p < 0.05$; ** $p < 0.01$; *** $p < 0.001$

less difficulties in learning as compared to teachers using Eurobasis, Zo gezegd, zo gerekend, Kompas, or Pluspunt. With regard to the content domain *geometry*, teachers using Pluspunt reported significantly more difficulties in learning as compared to teachers using Eurobasis, Zo gezegd, zo gerekend or Nieuwe tal-rijk.

Considering the fifth and sixth grade (see table 6) and in relation to the content domain *numbers and calculations*, significant differences in teachers' views were observed. Teachers using Pluspunt reported significantly more difficulties as compared to teachers using Eurobasis, Zo gezegd, zo gerekend or Nieuwe tal-rijk. Teachers using Zo gezegd, zo gerekend reported significantly less learning difficulties as compared to teachers using Eurobasis. Considering the content domain *measurement*, teachers using Nieuwe tal-rijk reported significantly more difficulties in learning as compared to teachers using Eurobasis. Teachers using Pluspunt reported significantly more difficulties as compared to teachers using Eurobasis, Nieuwe tal-rijk or Zo gezegd, zo gerekend. With regard to the content domain *geometry*, teachers using Pluspunt as curriculum program reported significantly more difficulties as compared to teachers using Eurobasis, Zo gezegd, zo gerekend or Nieuwe tal-rijk.

To sum up, the results revealed that adoption of two-level models was appropriate. Despite some dissimilarities between content domains and grades (e.g. we did not notice significant differences in teachers' views of the curriculum programs related to the content domain *numbers and calculations* in the first and second grade and in the third and fourth grade whereas we did notice significant differences in teachers' views of the mathematics curriculum programs for the content domain *numbers and calculations* in the fifth and sixth grade), the results revealed a tendency across the grades and the content domains. In general, teachers using Pluspunt reported significantly more difficulties as compared to teachers using other curriculum programs whereas teachers using Nieuwe tal-rijk reported significantly less difficulties as compared to teachers using other curriculum programs. The fact that the teacher-level variable "experience" (See Model 2 in Table 3, Table 4, Table 5) did not result in a significant decrease in deviance reveals that teachers' views of curriculum programs did not differ regarding their teaching experience. More experienced teachers did not perceive the curriculum program to impact students' mathematics performance differently as compared to teachers with less experience.

5.2. Study 2: Differences in mathematics performance results?

The results are presented in table 7. According to Model 0, all variances at the student level were statistically significant. Allowing random intercepts at the class level (Model 1) resulted in a significant decrease in deviance indicating that inclusion of this second level was appropriate. Additionally, the use of contrasts revealed that scores for the domain *measurement* were significant lower as compared to scores on the domain *numbers and calculations* ($\chi^2_{(1)} = 57.34$; $p < .001$) and as compared to scores on the domain *geometry* ($\chi^2_{(1)} = 49.417$; $p < .001$). Scores for the domain *numbers and calculations* did not differ significantly from the scores for the domain *geometry* ($\chi^2_{(1)} = 2.646$; $p = .10$).

Adding the student-level variables "mathematical basic knowledge" (Model 2) and "sex" (Model 3) resulted in significant decreases in deviance. The model revealed that boys do significantly better than girls in *numbers and calculations* and in *measurement*. Including the categorical class-level variable "grade" (reference category: first grade) in Model 4 also resulted in a significant decrease in deviance. Moreover, with regard to *numbers and calculations*, second and third graders did significantly better than first grade students; with regard to *measurement*, second, third, fourth, and fifth graders did significantly better than first grade students; and with regard to *geometry*, second, third, and fifth graders did significantly better than first grade students.

According to Model 5, inclusion of the class-level variable "experience" also resulted in a significant decrease in deviance; however, the corresponding fixed effects were not significant. Given the significant improvement of the model as compared to the previous model, we continued to include this term in further analyses. Adding the variable "curriculum program" into Model 6 (reference: Pluspunt) did not result in a significant drop in deviance indicating that overall, the curriculum program did not play a significant role in student outcomes.

Table 7. fixed effects estimates (top) and variance estimates (bottom)

Parameter	Model 0		Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
Fixed effects														
Intercept $_{N}$	7.56**	(.04)	7.51**	(.10)	7.33**	(.09)	7.23**	(.10)	6.79**	(.21)	6.63**	(.25)	6.71**	(.41)
Intercept $_{M}$	6.19**	(.07)	6.21**	(.15)	6.57**	(.09)	6.35**	(.11)	5.44**	(.22)	5.23**	(.26)	4.79**	(.39)
Intercept $_{G}$	7.41**	(.06)	7.33**	(.13)	7.42**	(.13)	7.40**	(.15)	6.70**	(.34)	6.47**	(.40)	6.91**	(.64)
Level 2 (student)														
Basic knowledge $_{N}$					1.14**	(.04)	1.13**	(.04)	1.12**	(.04)	1.10**	(.04)	1.10**	(.04)
Basic knowledge $_{M}$					1.03**	(.06)	.99**	(.06)	.99**	(.06)	.95**	(.06)	.98**	(.06)
Basic knowledge $_{G}$					.64**	(.07)	.60**	(.08)	.61**	(.07)	.62**	(.08)	.62**	(.08)
sex_male $_{N}$							.21*	(.07)	.20*	(.07)	.22*	(.07)	.22*	(.07)
sex_male $_{M}$							.37**	(.11)	.38**	(.11)	.41**	(.12)	.42**	(.12)
sex_male $_{G}$							.01	(.14)	-.02	(.14)	.05	(.15)	.06	(.15)
Level 3 (class)														
grade_2 $_{N}$									1.13**	(.25)	1.12**	(.25)	.97**	(.27)
grade_3 $_{N}$									.70*	(.26)	.79*	(.28)	.60	(.32)
grade_4 $_{N}$									.04	(.26)	.07	(.27)	.03	(.27)
grade_5 $_{N}$									-.13	(.26)	-.11	(.27)	-.13	(.27)
grade_6 $_{N}$									.00	(.00)	.00	(.00)	.00	(.00)
grade_2 $_{M}$									1.23**	(.25)	1.23**	.25)	1.04**	(.26)
grade_3 $_{M}$									1.33**	(.27)	1.44**	(.28)	1.13**	(.30)
grade_4 $_{M}$									.96**	(.28)	.95**	(.28)	.96**	(.26)
grade_5 $_{M}$									.56*	(.27)	.55*	(.27)	.54*	(.25)
grade_6 $_{M}$									.00	(.00)	.00	(.00)	.00	(.00)
grade_2 $_{G}$									.84*	(.39)	.85*	(.39)	.55	(.42)
grade_3 $_{G}$									1.41**	(.41)	1.46**	(.43)	1.16*	(.50)
grade_4 $_{G}$									-.13	(.42)	-.04	(.44)	-.12	(.43)
grade_5 $_{G}$									1.01*	(.41)	.93*	(.42)	.91*	(.41)
grade_6 $_{G}$									.00	(.00)	.00	(.00)	.00	(.00)
Experience $_{N}$.											.01	(.01)	.01	(.01)
Experience $_{M}$											.01	(.01)	.02	(.01)
Experience $_{G}$											.01	(.01)	.01	(.01)

Table 7 continued

Parameter	Model 0		Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
Fixed effects														
MTS_EB_{N}													-.21	(.34)
MTS_KP_{N}													.15	(.35)
MTS_ZG_{N}													-.02	(.31)
MTS_NT_{N}													.07	(.35)
MTS_EB_{M}													.15	(.32)
MTS_KP_{M}													.74*	(.33)
MTS_ZG_{M}													.49	(.30)
MTS_NT_{M}													.80*	(.33)
MTS_EB_{G}													-.68	(.52)
MTS_KP_{G}													-.04	(.54)
MTS_ZG_{G}													-.20	(.48)
MTS_NT_{G}													-.28	(.54)
Random Parameters														
Level 3 (class)														
$Intercept_{N}$/ $Intercept_{N\ (\sigma^2v0)}$			.76**	(.14)	.50**	(.10)	.53**	(.11)	.23**	(.06)	.24**	(.06)	.23**	(.06)
$Intercept_{N}$/ $Intercept_{M\ (\sigma^2v0v1)}$			.05	(.14)	.35**	(.09)	.37**	(.09)	.17**	(.05)	.17*	(.05)	.15*	(.05)
$Intercept_{M}$/ $Intercept_{M\ (\sigma^2v1)}$			1.68**	(.30)	.42**	(.11)	.35**	(.10)	.14*	(.06)	.13*	(.06)	.08	(.05)
$Intercept_{N}$/ $Intercept_{G\ (\sigma^2v0v2)}$			.50**	(.13)	.34*	(.11)	.31*	(.11)	.18*	(.07)	.18*	(.07)	.16*	(.07)
$Intercept_{M}$/ $Intercept_{G\ (\sigma^2v1v2)}$			.51*	(.19)	.39**	(.11)	.31*	(.11)	.16*	(.07)	.13	(.07)	.12	(.06)
$Intercept_{G}$/$Intercept_{G\ (\sigma^2v2)}$			1.08**	(.21)	.81**	(.19)	.80**	(.20)	.47**	(.14)	.46*	(.14)	.43*	(.14)
Level 2 (student)														
$Intercept_{N}$/ $Intercept_{N\ (\sigma^2u0)}$	2.93**	(.10)	2.21**	(.08)	1.30**	(.05)	1.28**	(.06)	1.29**	(.06)	1.27**	(.06)	1.27**	(.06)
$Intercept_{N}$/ $Intercept_{M\ (\sigma^2u0u1)}$	1.84**	(.12)	1.80**	(.10)	1.07**	(.07)	1.07**	(.07)	1.08**	(.07)	1.07**	(.08)	1.07**	(.08)
$Intercept_{M}$/ $Intercept_{M\ (\sigma^2u1)}$	6.91**	(.25)	5.05**	(.19)	3.15**	(.13)	3.17**	(.14)	3.18**	(.14)	3.24**	(.15)	3.23**	(.15)
$Intercept_{N}$/ $Intercept_{G\ (\sigma^2u0u2)}$	1.66**	(.11)	1.21**	(.09)	.67**	(.08)	.67**	(.08)	.68**	(.08)	.72**	(.09)	.72**	(.09)
$Intercept_{M}$/ $Intercept_{G\ (\sigma^2u1u2)}$	1.68**	(.17)	1.19**	(.13)	.85**	(.12)	.81**	(.13)	.82**	(.13)	.88**	(.14)	.88**	(.14)
$Intercept_{G}$/ $Intercept_{G\ (\sigma^2u2)}$	5.76**	(.21)	4.76**	(.18)	4.83**	(.20)	4.81**	(.22)	4.81**	(.22)	4.91**	(.23)	4.91**	(.23)
Model fit														
Deviance	19219.42		19219.42		13921.20		12075.53		11987.00		10943.74		10926.17	
χ^2			944.19		5298.22		1845.67		88.53		1043.26		17.57	
Df			6		3		3		15		3		12	
P			<.001		<.001		<.001		<.001		<.001		.13	
Reference			Model0		Model 1		Model 2		Model 3		Model 4		Model 5	

Note. Standard errors are in parentheses. There are no level 1 random parameters because level 1 exists solely to define the multivariate structure. $_{N}$ = numbers; $_{M}$ = measurement; $_{G}$ = geometry; EB = Eurobasis; ZG = Zo gezegd, zo gerekend; NT = Nieuwe tal-rijk.
* $p < 0.05$; ** $p < 0.001$

6. Discussion

Mathematics curriculum programs are often the primary resource for teachers and students in the classroom (Elsaleh, 2010; Grouws et al., 2004; Kauffman et al., 2002; Nathan et al., 2002; Schug et al., 1997). Despite their prominent role in the teaching and learning processes, there is no agreement on whether it really matters which mathematics curriculum programs schools choose (Slavin & Lake, 2008). Moreover, it is seen as a difficult endeavor to compare the efficacy or efficiency of mathematics curriculum programs (Deinum & Harskamp, 1995; Gravemeijer et al., 1993; Janssen et al., 1999).

The current study aimed at contributing to the mathematics curriculum programs discussion by analyzing whether it really matters what mathematics curriculum program schools adopt if we analyze teachers' views of mathematics curriculum programs. Teachers' views of the mathematics curriculum program they teach with is one factor that influences teachers' orientations toward the curriculum, considered to be an important focus for research in the domain of curriculum studies (Remillard 2005; Stein et al. 2007). Teachers' views of mathematics curriculum programs also on its own proved to be a mediating variable (Remillard and Bryans 2004).

Therefore, this research built on the experiences of teachers with the mathematics curriculum programs (Elsaleh, 2010) and on how teachers perceive these mathematics curriculum programs impact student mathematics performance . The research was carried out in Flanders, which has its own peculiarities. But, because of similarities with mathematics curriculum programs in other regions, the findings are not limited to Flanders and have a more general validity.

In the first study, views of 814 teachers of mathematics curriculum programs were measured building on teachers' experiences with these materials and the extent they perceive the mathematics curriculum programs affect the students' learning process. The results revealed that there are significant differences in teachers' views of mathematics curriculum programs. Moreover, we observed clear patterns in teachers' views of mathematics curriculum programs curriculum programs. Teachers' views of mathematics curriculum programs were more positive in case the mathematics curriculum programs addressed one content domain of mathematics (numbers and calculations, measurement, geometry) per lesson and provided more support for the teachers, such as providing additional

materials for the teacher, a more detailed description of the course, additional didactical suggestions and theoretical background knowledge about mathematics. On the contrary, teachers' views of mathematics curriculum programs were more negative in case the mathematics curriculum program provided less of such support for the teacher and addressed more than one content domain of mathematics education per lesson. Whereas the design of the study didn't allow to control for other variables, the results suggest that mathematics curriculum programs matter.

In the second study, building on mathematics performance of 1579 students, the results revealed that students' performance results do not vary significantly between mathematics curriculum programs. Whereas the absence of a straightforward impact of mathematics curriculum programs on performance results is in line with findings from other studies (Slavin & Lake, 2008), it also points at the following. Teachers' views of mathematics curriculum programs, is but one variable that mediates between the mathematics curriculum program and the enacted curriculum. In addition, it would be useful to analyze other mediating variables and the interplay between mediating variables such as teachers' beliefs about mathematics teaching and learning, teachers' views of curriculum materials in general, teacher knowledge, teachers' professional identity, teacher professional communities, organizational and policy contexts, and classroom structures and norms (Remillard and Bryans 2004; Stein et al. 2007). The discrepancy between the results of both studies also shed light on the need to carry out observational studies about the way teachers implement mathematics curriculum programs, since the differences between mathematics curriculum programs in teachers' views do not continue to hold with regard to students' outcomes. Observational studies could reveal if teachers are compensating teaching for anticipated difficulties in learning mathematics caused by the mathematics curriculum programs.

The current study addressed the need for more research focusing on variables that mediate between the mathematics curriculum programs and the enacted curriculum, and also the call for setting up large scale studies in this context (Chval et al., 2009; Slavin & Lake, 2008). Nevertheless, our study also reflects a number of limitations. First, though the opportunity sampling approach helped to involve a large set of schools, teachers and students, this sampling approach did not build on random selection. This implies that we cannot counter a potential sampling bias in our study as to teachers who developed already a clear and explicit views of mathematics curriculum programs. Second, in the

absence of prior measures for teachers' views of mathematics curriculum programs, applicable in studies with large sample sizes, and guided by research literature (Elsaleh, 2010), we analyzed teachers' views of mathematics curriculum programs by building on their actual experiences with the curriculum program. This study is part of a larger research project that centers on learning difficulties in mathematics. In view of this larger research project, teachers were asked to judge – based on their experiences – the extent to which the mathematics curriculum program caused difficulties in learning. Other studies could shift the focus on the strengths of each mathematics curriculum programs instead of focusing on the weaknesses. That is just one way in studying on a large scale teachers' views of mathematics curriculum programs. Third, whereas the current study took into account the structure, the learning path, the teacher plans, the availability of additional materials, and described in general lines the exercises, our data was not specific enough to reveal possible differences in the cognitive load of instruction and exercises. It could be interesting to include this factor in future research.

7. Conclusion

Up to date, there is no agreement about the differential impact of mathematics curriculum programs on students' performance results. This sounds surprising given the prominent role of mathematics curriculum programs in education. It should not be a complete surprise, though, given that it is difficult to compare the efficacy or efficiency of mathematics curriculum programs. The current study focused on one specific related aspect, teachers' views of curriculum programs, considered to be a mediating variable in the process between the written and the enacted curriculum, and influencing teachers' orientations toward the curriculum, considered to be a key variable in future curriculum research. The study revealed that, at least with regard to teachers' views of mathematics curriculum programs, it matters which mathematics curriculum programs choose. The study also suggests that future research should take into account more mediating variables and that observational studies could be carried out to analyze how teachers actually implement mathematics curriculum programs. Finally, from a practical point of view, the current research revealed that teachers are more positively oriented toward mathematics curriculum programs when the latter provide them with support such as additional materials, detailed descriptions of each "course", additional didactical suggestions and theoretical and

mathematical background knowledge and addressed one content domain. As such, inclusion of these additional resources can inspire curriculum programs developers and publishers. Presence or absence of these elements can be a criterion for teachers or a school team to choose or not to choose a certain mathematics curriculum program.

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APPENDIX: Description of each curriculum program*

	KP	ZG	NT	PP	EB
Curriculum-based					
Curriculum of the publicly funded, publicly run education	✓	✓	✓	✓	✓
Curriculum of the publicly funded, privately run education	✓	✓	✓	✓	✓
Curriculum of the education of the Flemish community	✓	✓	✓	✓	✓
Student material					
Structure	- Weekly structure: 32 weeks, 5-6 courses each week (3^{rd} - 6^{th} grade), around 7 courses each week (1^{st} - 2^{nd} grade) - Duration one course: usually 50 minutes (3^{rd} -6^{th} grade); usually 25 minutes (1^{st}-2^{nd} grade) - Each week addresses 5 domains: numbers, calculations, measurement, geometry, problem solving - Courses according to a fixed order: numbers, calculations, measurement, geometry, problem solving - Each course is situated within one domain	- Around 13 themes each year; around 12 courses each theme (1st grade: more themes, less courses each theme) - Duration one course: usually 50 minutes (2^{nd} -6^{th} grade); usually 25 minutes (1^{st} grade) - Each theme addresses 4 domains: numbers, measurement, geometry, problem solving - Courses not according to a fixed order - Each course is situated within one domain - Use of pictographs	- 10 blocks, around 20 courses each block (3^{rd} – 6^{th} grade), around 26 courses each block (1^{st} – 2^{nd} grade) - Duration one course: usually 50 minutes (3^{rd} – 6^{th} grade); 25 or 50 minutes (2^{nd} grade); usually 25 minutes (1^{st} grade) - Each block addresses 4 domains: numbers, calculations, measurement, geometry - Courses not according to a fixed order - Each course is situated within one domain - Use of pictographs: basic – extra – deepening exercises	- 13 themes, 13 courses each theme - Duration each course: 50 minutes - Each theme addresses 4 domains: numbers, calculations, measurement, geometry - Courses according to a fixed order: teacher-centered and student-centered courses - The courses address more than one domain	- Weekly structure: 32 weeks, around 7 courses each week (3^{rd} - 6^{th} grade), 10 courses each week (1^{st} - 2^{nd} grade) - Duration one course: usually 50 minutes (3^{rd} -6^{th} grade); 25 minutes (1^{st}-2^{nd} grade) - Each week addresses 5 domains: numbers, calculations, measurement, geometry, problem solving - Courses according to a fixed order: numbers, calculations, measurement, geometry, problem solving - Each course is situated within one domain

Materials	- Workbook - Memorization book - CD-rom with extra exercises - Other: number line, MAB-materials, coins, calculator, …	- Workbook - Other: number line, MAB-materials, coins, calculator, …	- Manual - Workbook - Memorization book - ‘Math journal’ for communication with the parents - Other: number line, MAB-materials, coins, calculator, …	- Manual - Workbook - Software packet - Other: number line, MAB-materials, coins, calculator, …	- Workbook - Memorization book - Software packet - Other: number line, MAB-materials, coins, calculator, …
Teacher’s guides					
Basic principles	- Curriculum-based - Realistic contexts - Horizontal and vertical connections - Use of different kinds of materials - Active learning - Differentiation - For all grades	- Curriculum-based - Realistic contexts - Active learning: interaction - Use of models, schemes, symbols and diagrams - Attention for mathematical language - Horizontal connections - Differentiation - For all grades	- Curriculum-based - Realistic contexts - Active learning - Linking content with students’ prior knowledge - A lot of attention to repetition and automation - Students’ working independently - To acquire study skills - Reflection - Remediation and differentiation - For all grades	- Curriculum-based - Realistic contexts - A critical attitude - Active learning: problem solving and meaningful - Students’ working independently - Attention for mathematical language - Differentiation - Interaction - Use of models and schemes - For all grades	- Curriculum-based - Realistic contexts - Active learning: participation and interaction - Cooperation and reflection - Horizontal and vertical connections - Attention for evaluation and differentiation - For all grades
Learning path	- Outline for the whole year: overview of and order of the subject of the courses for each domain - Outline for the whole year: overview of the learning contents for each domain - Weekly outline: overview courses	- Outline for each theme: overview of and order of the subject of the courses for each domain - Outline for each theme: overview of the learning contents for each domain	- Outline for the whole year: number of courses for each domain - Outline for the whole year: overview of and order of the subject of the courses for each domain and each block - Suggestions to draw up a learning path for the whole year - Outline for each	- Outline for the whole year: for each domain an overview of how the learning contents build on each own - Outline for each theme: overview of the learning contents for each domain - A very brief suggestion to draw up a learning path for the whole year: an overview of the	- Outline for the whole year: overview of and order of the subject of the courses for each domain - Weekly outline: overview courses

			block: overview courses	number of themes for trimester	
Teaching plans	Weekly: - Overview courses, domains, materials, duration courses Description of each course: - subject, goals, materials - Directions for each teaching phase - Use of pictographs - Blackboard outline	For each theme: - Overview of learning contents - Overview courses, domains, materials, duration courses Description of each course: - subject, goals, materials, organizational aspects, starting situation - Directions for each teaching phase - Use of pictographs - Blackboard outline	For each block: - Overview courses, domains, materials, duration courses - Comprehensive discussion of the materials - For each domain: an overview of the subject of the courses Description of each course: - subject, goals, materials, starting situation - A brief outline of the course - Additional didactical suggestions - Comprehensive directions for each teaching phase: step by step, explicit guidelines - Several teaching phases provide extra didactical suggestions and mathematical background knowledge - Blackboard outline	For each theme: - A very brief introduction links the theme with mathematics and gives an overview of the materials - Overview of learning contents for each course and for each domain - Use of pictographs: student-centered – teacher-centered course Description of each course: - Goals, material - Rather general directions for each teaching phase - Required materials for the next course	Weekly: - Overview courses, domain, subject, duration, materials Description of each course: - Subject, goals, materials - Directions for each teaching phase - Use of pictographs - Blackboard outline
Materials	- Learning path - Teaching plans - Homework stencils - Test stencils - Differentiation books - Grading keys	- Learning path - Teaching plans - Homework stencils - Test stencils - remediation - Grading keys	- Learning path - Teaching plans - Extra exercises - Exercises in preparation for tests - Test stencils	- Learning path - Teaching plans - Extra exercises - Test stencils - Additional exercises for remediation	- Learning path - Teaching plans - Homework stencils - Test stencils - Differentiation books

- An analysis of each test provides an overview of the performance on the test for the class as a whole and for each individual student - Additional exercises for differentiation - Grading keys	- Grading keys	- Grading keys

**Note*. KP = Kompas; ZG = Zo gezegd, zo gerekend; NT = Nieuwe tal-rijk; PP = Pluspunt; EB = Eurobasis.

Chapter 4

Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge?

Chapter 4

Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge?[4]

Abstract

The study of preservice elementary school teachers' knowledge of fractions is important, since the subject is known to be difficult to learn and to teach. In order to analyze the knowledge required to teach fractions effectively, we reviewed research related to students' understanding of fractions. This review helped to delineate the difficulties students encounter when learning fractions. Building on this overview, the current study addressed Flemish preservice elementary school teachers' common and specialized content knowledge of fractions. The study revealed that preservice elementary school teachers' knowledge of fractions largely mirrors critical elements of elementary school students' knowledge of fractions. Further, the study indicated that preservice teachers hardly succeed in explaining the rationale underlying fraction sub-constructs or operations with fractions. The latter is considered to be a critical kind of knowledge specific for the teaching profession. Implications of the findings are discussed.

[4] Based on:
Van Steenbrugge, H., Valcke, M., Lesage, E., Desoete, A. & Burny, E. Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge? Manuscript submitted for publication in *Journal of curriculum studies*

1. Introduction

Mathematics is generally accepted as an important curriculum domain in elementary education (Hecht, Vagi, & Torgesen, 2009; Keijzer & Terwel, 2003). Within the mathematics curriculum, fractions are considered as an essential skill for future mathematics success, but yet also as a difficult subject to learn and to teach (Hecht, Close, & Santisi, 2003; Newton, 2008; Van Steenbrugge, Valcke, & Desoete, 2010; Zhou, Peverly, & Xin, 2006).

It is a common misconception that elementary school mathematics is fully understood by teachers and that it is easy to teach (Ball, 1990; Jacobbe, 2012; NCTM, 1991; Verschaffel, Janssens, & Janssen, 2005). Already more than twenty years ago, Shulman and colleagues argued that teacher knowledge is complex and multidimensional (Shulman, 1986, 1987; Wilson, Shulman, & Richert, 1987). They drew attention to the content specific nature of teaching competencies. Consequently, Shulman (1986, 1987) concentrated on what he labeled as the missing paradigm in research on teacher knowledge: the nexus between content knowledge, pedagogical content knowledge (the blending of content and pedagogy), and curricular knowledge. Building on the work of Shulman (1986, 1987), Ball, Thames, and Phelps (2008) analyzed the mathematical knowledge needed to teach mathematics. They point at two empirically discernible domains of content knowledge: *common content knowledge* and *specialized content knowledge*. *Common content knowledge* refers to knowledge that is not unique to teaching and is applicable in a variety of settings (i.e. an understanding of the mathematics in the student curriculum). Ball et al. (2008) found that common content knowledge of mathematics plays a crucial role in the planning and carrying out of instruction; this kind of knowledge is still considered as a cornerstone of teaching for proficiency (Kilpatrick, Swafford, & Findell, 2001). *Specialized content knowledge* refers to the mathematical knowledge and skill unique to teaching: it is a kind of knowledge 'not necessarily needed for purposes other than teaching' (Ball, et al., 2008, p. 400). For instance, people with other professions need to be able to multiply two fractions, but none of them needs to explain why you multiply both the numerators and denominators.

The question 'What does effective teaching requires in terms of content knowledge' can be investigated in several ways (Ball, et al., 2008). An established approach to investigate what effective

teaching requires in terms of content knowledge, is by reviewing students' understanding to determine the mathematics difficulties encountered by students (Ball, et al., 2008; Stylianides & Ball, 2004). Therefore, in the following section, we first review literature considering students' understanding of fractions. Afterwards, we shift attention to (preservice) teachers' knowledge of fractions and present the aims of the present study.

2. Elementary school students' understanding of fractions

Fractions are difficult to learn (Akpinar & Hartley, 1996; Behr, Harel, Post, & Lesh, 1992; Behr, Wachsmuth, Post, & Lesh, 1984; Bulgar, 2003; Hecht, et al., 2003; Kilpatrick, et al., 2001; Lamon, 2007; Newton, 2008; Siegler et al., 2010). Not surprisingly, ample research focused on students' difficulties with fractions and tried to develop an understanding of the critical components of well-developed fraction knowledge (e.g.,Cramer, Post, & delMas, 2002; Keijzer & Terwel, 2003; Lamon, 2007; Mack, 1990; Siegler, Thompson, & Schneider, 2011; Stafylidou & Vosniadou, 2004). Authors agree that a main source producing difficulties in learning fractions is the interference with students' prior knowledge about natural numbers (Behr, et al., 1992; Grégoire & Meert, 2005; Stafylidou & Vosniadou, 2004). This 'whole number bias' (Ni & Zhou, 2005) results in errors and misconceptions since students' prior conceptual framework of numbers does no longer hold. It is, for example, counterintuitive that the multiplication of two fractions results in a smaller fraction (English & Halford, 1995). Students have to overcome this bias between natural numbers and fractions and therefore need to reconstruct their understanding of numbers. However, constructing a correct and clear conceptual framework is far from trouble-free because of the multifaceted nature of interpretations and representations of fractions (Baroody & Hume, 1991; Cramer, et al., 2002; English & Halford, 1995; Grégoire & Meert, 2005; Kilpatrick, et al., 2001). Research more particularly distinguishes five sub-constructs to be mastered by students in order to develop a full understanding of fractions (Charalambous & Pitta-Pantazi, 2007; Hackenberg, 2010; Kieren, 1993; Kilpatrick, et al., 2001; Lamon, 1999; Moseley, Okamoto, & Ishida, 2007). The *'part-whole'* sub-construct refers to a continuous quantity, a set or an object divided into parts of equal size (Hecht, et al., 2003; Lamon,

1999). The *'ratio'* sub-construct concerns the notion of a comparison between two quantities and as such, it is considered to be a comparative index rather than a number (Hallett, Nunes, & Bryant, 2010; Lamon, 1999). The *'operator'* sub-construct comprises the application of a function to a number, an object or a set. By means of the *'quotient'* sub-construct, a fraction is regarded as the result of a division (Charalambous & Pitta-Pantazi, 2007; Kieren, 1993). In the *'measure'* sub-construct, fractions are seen as numbers that can be ordered along a number line (Hecht, et al., 2003; Keijzer & Terwel, 2003; Kieren, 1988). As such, this sub-construct is associated with two intertwined notions (Charalambous & Pitta-Pantazi, 2007; Lamon, 2001). The number-notion refers to the quantitative aspect of fractions (how large is the fraction) while the interval-notion concerns the measure assigned to an interval.

Research of students' conceptual knowledge of fractions revealed that students are most successful in assignments regarding the part-whole sub-construct; in general, they have developed little knowledge of the other sub-constructs (Charalambous & Pitta-Pantazi, 2007; Clarke, Roche, & Mitchell, 2007; Martinie, 2007). Especially knowledge regarding the measure sub-construct is disappointing (Charalambous & Pitta-Pantazi, 2007; Clarke, et al., 2007; Hannula, 2003).

Students with an inadequate procedural knowledge level of fractions can make errors due to an incorrect implementation of the different steps needed to carry out calculations with fractions (Hecht, 1998). Students, for example, apply procedures that are applicable for specific operations with fractions, but are incorrect for the requested operation; e.g., maintaining the common denominator on a multiplication problem as in 3/7 * 2/7 = 6/7 (Hecht, 1998; Siegler, et al., 2011). There is a debate whether related procedural knowledge precedes conceptual knowledge or vice versa or whether it is an iterative process. While we do not disregard this debate, the present study accepts that both types of knowledge are critical in view of mastery of fractions (Kilpatrick, et al., 2001; NMAP, 2008; Rittle-Johnson, Siegler, & Alibali, 2001).

Several studies mention a gap between students' conceptual and procedural knowledge level of fractions; particularly students' conceptual knowledge of fractions is reported to be problematic whereas students' procedural knowledge of fractions is reported to be better (Aksu, 1997; Bulgar,

2003; Post, Cramer, Behr, Lesh, & Harel, 1993; Prediger, 2008). Some students do not develop a deep conceptual understanding resulting in a rather instrumental understanding of the procedures (Aksu, 1997; Hecht, et al., 2003; Prediger, 2008). Ma (1999) labels this as a pseudoconceptual understanding.

3. (Preservice) teachers' knowledge of fractions

Studies concerning (preservice) teachers' knowledge of fractions focused primarily on one aspect of fractions like ratio (Cai & Wang, 2006), multiplication of fractions (Isiksal & Cakiroglu, 2011; Izsak, 2008), and division of fractions (Ball, 1990; Borko et al., 1992; Ma, 1999; Tirosh, 2000). Borko et al. (1992) described the situation of a preservice middle school teacher who had taken a lot of mathematics courses. Although the teacher was able to divide fractions herself, she was not able to explain why the invert-and-multiply algorithm worked. Another study about preservice teachers' knowledge of students' conceptions revealed that preservice teachers were not aware of the main sources of students' wrong answers related to division of fractions (Tirosh, 2000). At the beginning of the mathematics course, the preservice teachers – though they were able to divide fractions – were also not able to explain the rationale behind the procedure.

Another strand of research is set up cross-cultural and compared U.S. and Asian teachers' knowledge of fractions. The rationale comes from the finding that Asian students outperform other students in the field of mathematics and teacher expertise is considered to be a possible explanation for these cross-cultural differences (Ma, 1999; Stigler & Hiebert, 1999; Zhou, et al., 2006). Studies point out that on a variety of aspects, Asian teachers do have a better understanding of fractions as compared to U.S. teachers (Cai & Wang, 2006; Moseley, et al., 2007; Zhou, et al., 2006). A cross-cultural study focusing on the division of fractions is the well-known study of Liping Ma (1999). Ma studied 23 U.S. and 72 Chinese elementary school teachers' knowledge of mathematics in four domains: subtraction with regrouping, multi-digit multiplication, division by fractions, and the relationship between perimeter and area. With regard to the fractions task, teachers were asked to indicate how they would calculate the quotient and to think of a good story or model to represent the division. Ma stated that the Chinese teachers' "way of 'doing mathematics' showed significant conceptual understanding" (Ma,

1999, p. 81) and that “one of the reasons why the U.S. teachers’ understanding of the meaning of division of fractions was not built might be that their knowledge lacked connections and links” (Ma, 1999, p. 82).

Arguing that studies of preservice teachers’ knowledge of fractions have focused primarily on division of fractions, Newton (2008) analyzed preservice teachers’ performance on all four operations with fractions. Data of 85 participants were collected at the beginning and at the end of a course in which preservice teachers were required to link the meaning of the operations with the specific algorithm. The outcomes revealed that at the end of the course, preservice teachers’ computational skill, knowledge of basic concepts, and solving word problems capacity improved. There was however no meaningfull change in flexibility and transfer was low at the post test.

Moseley, Okamoto, and Ishida (2007) studied 6 U.S. and 7 Japanese experienced fourth grade teachers’ knwoledge of all five sub-constructs of fractions. The study showed that the U.S. teachers focused strongly on the part-whole subconstruct - even when this was inapropriate - whereas the Japanese teachers focused to a larger extent on correct underlying subconstructs.

This overview illustrates that research on (preservice) teachers’ knowledge of fractions targetted participants common and specialized content knowledge, and did so for one or more sub-constructs, or for operations (mostly one operation) with fractions. However, research that addresses both (preservice) teachers’ knowledge of the four operations and the sub-constructs and did so by addressing their common and specialized content knowledge is lacking. We elaborate further on this in the next section.

4. A comprehensive overview of preservice teachers’ knowledge of fractions is lacking

The research on preservice teachers’ knowledge of fractions suggests that teacher misconceptions mirror the misconceptions of elementary school students (Newton, 2008; Silver, 1986; Tirosh, 2000). These studies however were too narrow in scope to attend to the broader range of students’ difficulties . In order to develop a more comprehensive picture about the parallels between elementary preservice teachers’ and elementary school students’ knowledge of fractions, the current study analyzes

preservice teachers' knowledge of the five fraction sub-constructs and preservice teachers' procedural knowledge of fractions. In addition, since Ball et al.(2008) underline the importance of specialized content knowledge, student teachers' capacity to explain the rationale underlying a sub-construct or operation was studied as well. Given that teacher education is a crucial period to obtain a profound understanding of fractions (Borko, et al., 1992; Ma, 1999; Newton, 2008; Toluk-Ucar, 2009; Zhou, et al., 2006), we included both first-year and last-year preservice teachers to study gains in their knowledge. Hence the present study centers on preservice teachers' common content knowledge as measured by their conceptual and procedural understanding of fractions on the one hand and on preservice teachers specialized content knowledge as measured by their skill in explaining the underlying rationale on the other hand. Two research questions are put forward:

- To what extent do preservice teachers master the procedural and conceptual knowledge of fractions (common content knowledge)?
- To what extent are preservice teachers able to explain the underlying rationale of a procedure or the underlying conceptual meaning (specialized content knowledge)?

5. Methodology

5.1. Participants

Participants were 290 preservice teachers (184 first and 106 last-year trainees), enrolled in two teacher education institutes in Flanders (academic year 2009-2010). In Flanders, elementary school teachers follow a three-year professional bachelor degree. Flemish elementary school teachers are all-round teachers, and therefore preservice teachers are trained in all school subjects, including mathematics. The total group consisted of 43 male and 247 female students, which is representative for the Flemish teacher population. Participants' average age was 19.63 (SD = 1.77) years.

Prior to entering teacher education, 197 participants attained a general secondary education diploma preparing for higher education (academic track), 93 participants completed a technical or vocational track, not necessarily geared to enter higher education. Both teacher education programs equally focus on fractions (See Appendix A). A first block is devoted to repetition of basic fraction knowledge, while a second block focuses on how to teach fractions. Total teaching time spent to fractions during

teacher education varies between five and seven hours. The focus 'How to teach fractions' in the first teacher education institute is programmed in the first half of the second year of teacher education. In the second teacher education institute it is programmed in the second half of the first year, but after the current study was carried out.

5.2. Instrument

Based on the review of elementary school students' understanding of fractions (cfr. supra), a test was developed and administered to measure preservice teachers' understanding of fractions. A detailed description of all test items is provided in Appendix B. The first part of the test includes 39 items addressing respondents' *conceptual knowledge of fractions*. These 39 test items were used in previous studies measuring students' conceptual knowledge of fractions (Baturo, 2004; Boulet, 1998; Charalambous & Pitta-Pantazi, 2007; Clarke, et al., 2007; Cramer, Behr, Post, & Lesh, 1997; Davis, Hunting, & Pearn, 1993; Hannula, 2003; Kieren, 1993; Lamon, 1999; Marshall, 1993; Ni, 2001; Noelting, 1980; Philippou & Christou, 1994).

The second part of the test consists of 13 test items addressing respondents' *procedural knowledge of fractions*; these items were sampled from mathematics textbooks. In addition, for respectively two and five items of the first and second part of the test respondents were required to indicate how they would explain the underlying rationale to students. These items aimed at measuring preservice teachers' *specialized content knowledge of fractions*.

All test items corresponded to the elementary school mathematics curriculum. Items measuring procedural or conceptual knowledge were scored dichotomously: correct/incorrect. Items measuring specialized content knowledge, were scored a second time leading to a 0, 1, or 2 point score. Scoring for the specialized content knowledge depended on the nature of the justification or clarification. If respondents could not explain the rationale, presented a wrong explanation, or simply articulated the rule, a 0 score was awarded (e.g., 2/5 x 3/5: 'I would formulate the rule: nominator times nominator; denominator times denominator'). A partially correct justification/explanation, resulted in a score 1. The latter included responses that were too abstract for elementary school students, or partially correct (e.g., 2/5 x 3/5: 'I would start with an example of multiplication of natural numbers: 2 times 3.

Students know it equals 6. Next I would rewrite the natural numbers as fractions: 2/1 x3/1; this equals 6/1. Then I would show that in order to multiply two fractions, one has to multiply both the nominators and both the denominators.'). Completely correct explanations/justifications resulted in a score 2 (e.g., 2/5 x 3/5: 'I would draw a square on the blackboard and ask students to divide the square in five equal pieces and let them color three of the five equal parts: this is 3/5 of the original square. Next I would ask to divide the colored part again in five equal pieces and let them mark in another color two of the five equal parts. This is 2/5 of 3/5. The original square is now divided in 25 equal pieces and the result of the multiplication comprises 6 of the 25 pieces. So, actually, we divided the original square in 25 equal-sized pieces and we took 6 such pieces. And thus, the result of the multiplication is 6/25.').

Mean scores for the conceptual and procedural subtests and for specialized content knowledge were calculated, resulting in an average score ranging from 0 to 2 for the specialized content knowledge subtest and from 0 to 1 for the conceptual and procedural subtest.

A trial version of the test was screened by two teacher trainers and by two experienced inservice elementary school teachers. They were asked (1) whether the test items correspond to the elementary school mathematics curriculum and (2) whether they had suggestions for improving the wording of the items. All items were judged to correspond to the curriculum; the wording of some items was improved on the base of concrete suggestions.

5.3. Procedure

All tests were delivered to the participating teacher education institutes; completed tests were returned to the researchers. At the time of test administration, all first year student teachers had already been taught basic fraction knowledge; but none was trained to teach fractions. All third year students were both taught basic fraction knowledge and trained to teach fractions. Informed consent was obtained from participating student teachers. Student teachers were informed that test scores would not affect their evaluation. Confidentiality of personal data was stressed. Respondents could refuse to provide

personal background details. All student teachers participated in the study, none refused and no missing data were found in the data set.

Teacher educators were given a protocol in view of the test administration containing guidelines with regard to the maximum time-frame, the introduction of the test to the student teachers, and the test administration. A time-frame of 100 minutes was set. All participants handed in the completed test within this time-frame. At the beginning of the test administration, the teacher educator introduced the test to the preservice teachers. The test started with background questions on the first page, requesting student background data: name, gender, and prior secondary education diploma. All returned test forms were scored by one member of the research team.

5.4. Research design and analysis approach

The first research question was approached in two ways. First, the difference between student teachers' conceptual and procedural knowledge of fractions was analyzed. Second, we focused specifically on student teachers' conceptual knowledge of fractions and analyzed scores in relation to the five sub-constructs.

With regard to the difference between student teachers' conceptual and procedural knowledge of fractions, the design reflects a 2*2*2*2 mixed ANOVA design. The first factor was the between-subjects factor of gender. The second factor was the between-subjects factor of track in secondary education (general oriented secondary education versus practical oriented secondary education). The third factor was the between-subjects factor of year of teacher education (first-year versus third-year teacher trainees). A fourth factor was based on the within-subjects factor of type of knowledge (procedural knowledge versus conceptual knowledge). The dependent variable was the participants' average score. Whereas the first two factors were included in the research design as background variables, the third and fourth factor were included as variables of interest.

Considering student teachers' conceptual knowledge of fractions, the design employed was also a 2*2*2*2 mixed ANOVA design. Between-subjects factors were the same as in the previous section: gender, track in secondary education, and year of teacher education. A fourth factor was a within-

subjects factor of conceptual knowledge sub-construct of which the five levels were defined by the five sub-constructs: part-whole, ratio, operator, quotient, and measure. The dependent variable was the participants' average score. Again the first two factors were included as background variables; the third and fourth factor were included as variables of interest.

With regard to the second research question, the design employed was a 2*2*2 ANOVA design. The three between-subjects factors were based on gender, track in secondary education, and year of teacher education. The dependent variable was the participants' average score for the specialized content knowledge subtest. Once more, the first two factors were considered as background variables; the third factor as a variable of interest.

6. Results

6.1. Procedural and conceptual knowledge

The average score for the complete fractions test was .81 (*SD* = .11), .86 (*SD* = .15) for the procedural knowledge subtest, and .80 (*SD* = .12) for the conceptual knowledge subtest (see Table 1).

Table 1. Average score (and standard deviation) on the fractions test

	Procedural knowledge			Conceptual knowledge			Total		
	Male	Female	Total	Male	Female	Total	Male	Female	Total
AT	.89 (.17)	.86 (.14)	.87 (.15)	.86 (10)	.82 (.10)	.82 (.11)	.87 (.10)	.83 (.10)	.83 (.10)
TT	.85 (.16)	.84 (.17)	.84 (.17)	.84 (.09)	.71 (.14)	.74 (.14)	.84 (.10)	.74 (.12)	.76 (.13)
Total	.88 (.17)	.85 (.15)	.86 (.15)	.85 (.10)	.79 (.13)	.80 (.12)	.86 (.10)	.80 (.11)	.81 (.11)

Note. AT = academic track; TT = technical or vocational track.

There was a significant main effect of gender ($F(1, 282) = 5.27$, $p < .05$, partial $\eta^2 = .02$), track in secondary education ($F(1, 282) = 6.88$, $p < .01$, partial $\eta^2 = .02$), and type of knowledge ($F(1, 282) = 15.78$, $p < .0001$, partial $\eta^2 = .05$). There was no significant main effect of year of teacher education ($F(1,282) = 1.75$, $p = .187$). The gender by type of knowledge interaction ($F(1, 282) = 4.01$, $p < .05$, partial $\eta^2 = .01$), and the gender by type of knowledge by secondary education interaction ($F(1, 282) = 4.47$, $p < .05$, partial $\eta^2 = .02$) were also significant. The significant main effects show that male student teachers scored higher than female students teachers on the whole fractions test, that student teachers from an academic track scored higher on the whole fractions test than those from a technical

or vocational track in secondary education, and that scores for procedural knowledge of fractions were higher than scores for conceptual knowledge of fractions (see Table 1). Related effect sizes were small (cfr. supra). The absence of a significant main effect of year of teacher education indicates that third-year trainees did not perform significantly different as compared to first-year trainees on the whole fractions test.

The gender by type of knowledge interaction implies that the difference between male and female student teachers was significantly smaller for procedural knowledge as compared to the gender difference on the whole fractions test score (See Table 1). Moreover, male student teachers scored higher than female students teachers on conceptual knowledge (t =3.41, df = 288, p < .005, one-tailed), but not on procedural knowledge (t =.86 , df = 288, p = .19, one-tailed). The gender by type of knowledge interaction also indicates that the difference between the scores for procedural and conceptual knowledge for male students was significantly smaller as compared to the difference for the entire group of respondents. Moreover, the scores for procedural knowledge were significantly higher than scores for conceptual knowledge for female student teachers (t =6.90, df = 246, p < .0001, one-tailed), but not for male student teachers (t =1.04, df = 42, p = .15, one-tailed).

The gender by type of knowledge by secondary education interaction reflects that female student teachers from an academic track in secondary education scored significantly higher for conceptual knowledge than female student teachers from a technical or vocational track (t = 5.89 , df = 114, p < .0001, one-tailed), while this did not hold for procedural knowledge (t = 1.23, df = 120.71, p = .11, one-tailed). Male student teachers from an academic track did not score significantly higher than male student teachers from a technical or vocational track (conceptual knowledge: t = .83, df = 41, p = .21, one-tailed; procedural knowledge: t = .97, df = 41, p = .17, one-tailed).

6.2. Conceptual knowledge: sub-constructs

There was a significant main effect of gender (F(1, 282) = 12.56, p < .0005, partial η^2 = .04), track in secondary education (F(1, 282) = 9.26, p < .005, partial η^2 = .03), and sub-construct (F(3.38, 953.24) =

56.15, $p < .0001$, partial $\eta^2 = .17$). There was no significant main effect of year of teacher education ($F(1,282) = 0.501$, $p = .480$).

The significant main effects indicate that male student teachers scored higher than female student teachers; that student teachers from an academic track scored higher on the subtest measuring conceptual knowledge than those from a technical or vocational track (see Table 2).

The absence of a significant main effect of year of teacher education indicates that third-year trainees did not perform significantly different on the subtest measuring conceptual knowledge as compared to first-year trainees.

Table 2. Average score (and standard deviation) for the sub-constructs

		Secondary education		
		AT	TT	Total
Part-whole	Male	.92 (.10)	.92 (.10)	.92 (.10)
	Female	.90 (.11)	.80 (.18)	.87 (.14)
	Total	.90 (.11)	.82 (.17)	.88 (.14)
Ratio	Male	.97 (.05)	.95 (.10)	.96 (.07)
	Female	.94 (.10)	.91 (.12)	.93 (.10)
	Total	.94 (.09)	.92 (.11)	.93 (.10)
Operator	Male	.79 (.18)	.78 (.19)	.79 (.18)
	Female	.77 (.18)	.62 (.23)	.73 (.21)
	Total	.78 (.18)	.65 (.23)	.74 (.21)
Quotient	Male	.82 (.24)	.79 (.21)	.81 (.22)
	Female	.80 (.22)	.65 (.26)	.76 (.24)
	Total	.81 (.22)	.67 (.25)	.76 (.24)
Measure	Male	.77 (.20)	.70 (.20)	.74 (.20)
	Female	.64 (.22)	.51 (.24)	.60 (.24)
	Total	.66 (.22)	.55 (.24)	.62 (.24)
Total	Male	.87 (.08)	.85 (.08)	.87 (.08)
	Female	.82 (.10)	.72 (.13)	.79 (.12)
	Total	.82 (.10)	.74 (.13)	.80 (.12)

Note. AT = academic track; TT = technical or vocational track.

Paired t-tests were performed to further analyze the significant main effect of sub-construct (see Table 3). As can be derived from Table 3, the results reveal a hierarchy in the mastery level of the sub-constructs. The score for the ratio sub-construct was significantly higher than the scores for all other sub-constructs. The score for the part-whole sub-construct was significantly higher than the scores for the quotient, operator, and measure sub-construct. The score for the quotient sub-construct was significantly higher than the scores for the operator and measure sub-construct. The score for the operator sub-construct was significantly higher than the score for the measure sub-construct, and

consequently, the score for the measure sub-construct was significantly lower than the scores for all other sub-constructs.

Table 3. T-values for differences between sub-constructs (row minus column)

	Part-whole	Ratio	Operator	Quotient	Measure
Part-whole	/	-6.58**	12.36**	8.75**	20.45**
Ratio		/	16.70**	12.50**	23.00**
Operator			/	-1.82*	7.79**
Quotient				/	8.97**
Measure					/

df = 289; * $p < .05$; ** $p < .0001$

A more detailed inspection of responses at item level revealed some remarkable results. First, in total 63.10% of the respondents was not able to give a number located between $\frac{1}{8}$ and $\frac{1}{9}$ (item 19) and 43.44% could not solve item 18: 'By how many times should we increase 9 to get 15?'. Furthermore, 35.86% did not answer item 29 correctly: 'Which of the following are numbers? Circle the numbers: A, 4, *, 1.7, 16, 0.006, $\frac{2}{5}$, 47.5, $\frac{1}{2}$, $, $1\frac{4}{5}$'; and 35.52% could not locate the number one on a number line when the origin and a given number ($2\frac{1}{4}$) was given (item 21.2). In addition, also 35.52% was not able to solve item 24: 'Peter prepares 14 cakes. He divides these cakes equally between his 6 friends. How much cake does each of them get?'.

Since the nature of the responses to items 19 and 29 reflected patterns, an error analysis was performed. Item 19 asks respondents whether there is a fraction located between $\frac{1}{8}$ and $\frac{1}{9}$. If they thought so, respondents were asked to write down a fraction located between the two given fractions. Only 36.90% (n = 107) answered this question correctly. Errors: 75 students were not able to answer the question, 55 wrote down a fraction that was not located between the two given numbers, and 53 indicated explicitly that no fraction exists between $\frac{1}{8}$ and $\frac{1}{9}$. As such, 18.28% of all the respondents came up with a wrong answer because they explicitly thought that there are no fractions located between $\frac{1}{8}$ and $\frac{1}{9}$. Item 29 asks respondents to circle the numbers in a given row of representations. In total, 186 (64.14%) did well. Errors: 85 students neglected the fractions; 5 respondents did only encircle the natural numbers, 2 respondents did encircle both numerators and denominators, and 12

made another type of error. As such, 92 respondents (31.72%) made an error that states that a fraction is not a number.

6.3. Specialized content knowledge

The average score for the specialized content knowledge subtest was 0.42 (*SD* = 0.20) out of a maximum of 2. There was a significant main effect of track in secondary education ($F(1, 282) = 4.05$, $p < .05$, partial $\eta^2 = .01$) and a significant interaction effect of gender by year of teacher education ($F(1, 282) = 3.97$, $p < .05$, partial $\eta^2 = .01$). Though these differences were significant, the effect sizes indicate that we observed rather small variations. There was no significant main effect of gender ($F(1,282) = 0.002$, $p = .960$) and no significant main effect of year of teacher education ($F(1,282) = 0.328$, $p = .568$).

According to the significant main effects, student teachers from an academic track scored significantly higher on the specialized content knowledge subtest than those who followed a technical or vocational track in secondary education (see Table 4).

Table 4. Average score (and standard deviation) for specialized content knowledge

	First year teacher training			Third year teacher training			Total		
	Male	Female	Total	Male	Female	Total	Male	Female	Total
AT	.44 (.13)	.43 (.18)	.44 (.17)	.43 (.23)	.46 (.21)	.46 (.21)	.44 (.16)	.45 (.19)	.44 (.19)
TT	.46 (.19)	.32 (.19)	.35 (.20)	.29 (.00)	.40 (.23)	.38 (.21)	.42 (.19)	.34 (.21)	.35 (.20)
Total	.45 (.16)	.39 (.19)	.40 (.19)	.38 (.20)	.45 (.22)	.44 (.22)	.43 (.17)	.41 (.20)	.42 (.20)

Note. AT = academic track; TT = technical or vocational track.

The absence of the significant main effect 'year of teacher education' implies that across all respondents, teacher education year did not had a significant impact on the student teachers' score for specialized content knowledge of fractions. The gender by year of teacher education interaction implies that the difference between first and third year male students was significantly different as compared to the difference between the entire group of first and third year students. The gender by year of teacher education interaction also means that the difference between male and female third

year students was significantly different as compared to the difference between entire group of male and female students (see Table 4).

7. Discussion and conclusion

A major concern regarding increasing mathematics standards expected of students should be teachers' preparation to address these standards (Jacobbe, 2012; Kilpatrick, et al., 2001; Stigler & Hiebert, 1999; Zhou, et al., 2006). Fractions is known to be an important yet difficult subject in the mathematics curriculum (Newton, 2008; Siegler, et al., 2010; Van Steenbrugge, et al., 2010). Compared to the large amount of research that focuses on students' knowledge of fractions, little is known, however, about both inservice and preservice teachers' knowledge of fractions (Moseley, et al., 2007; Newton, 2008). This is a critical observation since particularly in elementary education, it is a common misconception that school mathematics is fully understood by the teachers and that it is easy to teach (Ball, 1990; Jacobbe, 2012; NCTM, 1991; Verschaffel, et al., 2005). Therefore, and given that teacher education is considered to be a crucial period in order to obtain a profound understanding of fractions (Borko, et al., 1992; Ma, 1999; Newton, 2008; Toluk-Ucar, 2009; Zhou, et al., 2006), this study focused on preservice teachers' knowledge of fractions.

A common approach to analyze the required content knowledge to teach effectively is by means of a review of students' understanding to determine the difficulties students encounter with mathematics (Ball, et al., 2008; Stylianides & Ball, 2004). Following this methodology, we reviewed studies related to students' understanding of fractions, revealing a gap between students' procedural and conceptual knowledge of fractions (Aksu, 1997; Bulgar, 2003; Post, et al., 1993; Prediger, 2008). Analysis of students' conceptual understanding of fractions illustrated that students are most successful in tasks about the part-whole sub-construct, whereas students' knowledge of the sub-construct measure is disappointing (Charalambous & Pitta-Pantazi, 2007; Clarke, et al., 2007; Hannula, 2003; Martinie, 2007).

Research suggests that preservice teachers' knowledge of fractions mirrors similar misconceptions as revealed by research of elementary school students' knowledge of fractions (Newton, 2008; Silver,

1986; Tirosh, 2000). Previous studies were however too narrow in scope to analyze the difficulties preservice teachers encounter when learning fractions as revealed in our overview of students' understanding of fractions. Therefore, we decided to use a more comprehensive test measuring both preservice teachers' conceptual and procedural knowledge of fractions and their competence in explaining the underlying rationale.

Regarding the first research question, preservice teachers' procedural and conceptual knowledge about fractions were analyzed. Since test items corresponded to the elementary school mathematics curriculum, and since the Flemish Government stresses that preservice teachers, regarding content knowledge, should master at least the attainment targets of elementary education (Ministry of the Flemish Community Department of Education and Training, 2007), it can be concluded that an average score of .81 is not completely sufficient to teach these contents. Moreover, detailed results revealed that even third-year student teachers made many errors. Across all respondents, scores for procedural knowledge were significantly higher than scores for conceptual knowledge. Though the related effect size indicated that the difference was small, the latter reflects the finding a gap between elementary school students' procedural and conceptual knowledge of fractions (Aksu, 1997; Bulgar, 2003; Post, et al., 1993; Prediger, 2008). In addition, sub-scores for the five fraction sub-constructs were studied in detail. Large and significant differences in the mastery of the various sub-constructs were found. The findings again mirror the results from studies involving elementary school students who seem to master especially the part-whole sub-construct while scores for the measure sub-construct are disappointing (Charalambous & Pitta-Pantazi, 2007; Clarke, et al., 2007; Hannula, 2003; Martinie, 2007). Moreover, more than one third of the preservice elementary school teachers did not encircle the fractions out of a set of characters when asked to circle the numbers. This also reflects the finding that elementary school students often did not internalize that a fraction represents a single number (Post, et al., 1993). These results raise questions considering preservice teachers' common content knowledge of fractions. This is a critical finding since this kind of knowledge is found to play a crucial role when teachers plan and carry out instruction in teaching mathematics (Ball, et al., 2008) and consequently is considered as a cornerstone of teaching for proficiency (Kilpatrick, et al., 2001).

With regard to the second research question of the study, we addressed preservice teachers' skill in explaining the underlying rationale (i.e. explaining why a procedure works or justifying their answer on a conceptual question). This kind of knowledge, specialized content knowledge, refers to the mathematical knowledge and skill unique to teaching (Ball, et al., 2008). The average score for preservice teachers' specialized content knowledge was only .42 (maximum = 2.00), which can be considered as a low score, that – although there is no bench mark available – questions preservice teachers' specialized content knowledge level. This is an important finding since research clearly points at the differential impact of teachers who have a deeper understanding of their subject (Hattie, 2009). The present results question the nature and impact of teacher education. The latter is even more important, since we observe that the year of teacher education students are in did not had a significant impact on preservice teachers' common content knowledge, nor on their specialized content knowledge of fractions, implying that third year students did not perform better than first year students. Analysis of the fractions-related curriculum in teacher education learns that this is hardly surprising, since only a limited proportion of teaching time in teacher education is spent on fractions. Given that fractions are considered an essential foundational skill for future mathematics success and as a difficult subject to learn and to teach (Hecht, et al., 2003; Newton, 2008; Van Steenbrugge, et al., 2010), questions can be raised about the fact that fractions represent only a very small proportion of the curriculum related time in teacher education. Along the same line, one can doubt whether it is feasible to prepare preservice teachers to teach every subject in elementary education. A practical alternative, as suggested by the National Mathematics Advisory Panel (2008), could be to focus on fewer teachers who are specialized in teaching elementary mathematics. Also, simply increasing the number of lessons in teacher education that focus on fractions would be insufficient; preservice teachers should be provided with mathematical knowledge useful to teaching well (Kilpatrick, et al., 2001). Therefore, teacher education programs could familiarize preservice teachers with common, sometimes erroneous processes used by students (Tirosh, 2000) and include explicit attempts to encourage their flexibility (a tendency to use alternate methods when appropriate) in working with fractions (Newton, 2008).

The implication of our finding that only a limited proportion of teaching time in teacher education is spent on fractions and on how to teach fractions relates not only to mathematics education of preservice teachers, but to teacher education in general. It suggests that the move from teacher "training" to teacher "education", initiated in the 1980s (Verloop, Van Driel, & Meijer, 2001), is yet not fully implemented. Preservice teachers can replicate most of the procedures they have been taught, but they are not 'empowered' with a deeper understanding (Darling-Hammond, 2000).

Concluding, the present study indicates that Flemish preservice teachers' knowledge of fractions mirrors students' inadequate knowledge of fractions. Their level of common content knowledge and in particular their level of specialized content knowledge of fractions is below a required level. Moreover, teacher education seems to have no impact on its development. These findings might give impetus to teacher education institutes to reflect on on how to familiarize preservice teachers with teaching fractions. Future research focusing on approaches to improve teacher education's impact on preservice teachers' level of common content knowledge and specialized content knowledge of fractions in particular and of mathematics more broadly, can have a significant impact on improving the content preparation of preservice teachers.

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APPENDIX. Description of the 52 test items measuring conceptual, procedural, and specialized content knowledge of fractions

Conceptual knowledge

- Sub-construct: part-whole

1. [Three drawings of rectangles divided in parts of which some are shaded are given.] Which of the following corresponds to the fraction 2/3? Circle the correct answer.

7. [A triangle, divided in 2 rectangles and 4 triangles of which 1 triangle is shaded is given. The two rectangles are equally sized; the 4 triangles are all exactly half of the size of the rectangles.] Which part of the triangle is represented by the grey part? Answer by means of a fraction.

8. [A drawing of a rectangle is given.] The rectangle below represents 2/3 of a figure. Complete the whole figure.

13. [A picture of 4 marbles is given]. If this represents 2/5 of a set of marbles, draw the whole set of marbles below.

14.1. [A drawing of 4 triangles and 5 circles is given.] What part of the total number of the objects shown in the picture are the triangles included in this picture?

14.2. [The same drawing of 4 triangles and 5 circles is given.] What part of the triangles shown in the picture above, do two triangles represent?

16. [18 dots are given, of which 12 are black-colored.] Which part of the dots is black-colored?

25. [A drawing of a triangle, divided into3 equally-sized parts is given.] Color ¾ of the rectangle below.

26.1. [A circle divided in some parts is given. Each part is allocated to a corresponding character.] Which part of the circle is represented by B?

26.2. [The same circle divided in some parts is given. Each part is allocated to a corresponding character.] Which part of the circle is represented by D?

- Sub-construct: ratio

2. [a drawing of 3 pizzas allocated to 7 girls, and 1 pizza allocated to 3 boys is given.] Who gets more pizza: the boys or the girls?

3.1. [A square divided in 6 equally-sized rectangles of which 1 is shaded is given on the left. On the right, 24 diamonds are given.] Use the diagram on the right to represent an equivalent fraction to the one presented on the left.

3.2. [On the left, 4 diamonds are given of which 1 is encircled. On the right, 1 rectangle is divided into 16 equally-sized squares.] Use the diagram on the right to represent an equivalent fraction to the one presented on the left.

3.3. [A rectangle divided in 6 equally-sized squares of which 4 are shaded is given on the left. On the right, 24 diamonds are given.] Use the diagram on the right to represent an equivalent fraction to the one presented on the left.

9*[5]. [Two equal-sized squares, one divided in 7 equal parts, the other divided in 4 equal parts are given. By means of balloons, Hannah states that '7/7 is larger than 4/4 because it has more pieces' and Jonas states that '4/4 is larger because its pieces are larger'.] What do you think? Who is right? Please justify your answer.

10. [A rectangle divided into 18 parts of equal size of which 10 are shaded is given. Also 5 circles of which some part is shaded are given.] The proportion of the area shaded in the following rectangle is approximately the same with the proportion of the area shaded in which circle? (Circle ONE answer only.)

17.1. Bram and Olivier are making lemonade. Whose lemonade is going to be sweater if the kids use the following recipes? Bram: 2 spoons of sugar for every 5 glasses of lemonade; Olivier: 1 spoon of sugar for every 7 glasses of lemonade.

17.2. Bram and Olivier are making lemonade. Whose lemonade is going to be sweater if the kids use the following recipes? Bram: 2 spoons of sugar for every 5 glasses of lemonade; Olivier: 4 spoons of sugar for every 8 glasses of lemonade.

27. Piet and Marie are preparing an orange juice for their party. Below you see the two recipes. Which recipe will taste the most 'orange'? Recipe 1: 1 cup of concentrated orange juice and 5 cups water. Recipe 2: 4 cups of concentrated orange juice and 8 cups of water.

- Sub-construct: operator

15.1. Without carrying out any operations, decide whether the following statement is correct or wrong. If we divide a number by 4 and then multiply the result by 3, we are going to get the same result we would get if we multiplied this number by ¾.

15.2. Without carrying out any operations, decide whether the following statement is correct or wrong. If we divide a number by 7 and then multiply the result by 28 we are going to get the same result we would get if we multiplied this number by ¼.

15.3. Without carrying out any operations, decide whether the following statement is correct or wrong. If we divide a number by 4 and then multiply the result by 2 we are going to get the same result we would get if we divided this number by 2/4.

[5] An asterix (*) indicates that the item in addition was used to measure respondents' specialized content knowledge.

18*. Please answer the following question. Then explain how you got your answer. **‘**By how many times should we increase 9 to get 15?’

28.1. [A drawing of a machine that outputs ¼ of the input number is given.] If the input number is equal to 48, the output number will be …?

28.2. [A drawing of a machine that outputs 2/3 of the input number is given.] If the input number is equal to 12, the output number will be …?

- Sub-construct: quotient

4. Decide whether the following statement is correct or wrong: ‘2/3 is equal to the quotient of the division 2 divided by 3.’

5. Three pizzas were evenly shared among some friends. If each of them gets 3/5 of the pizza, how many friends are there altogether?

11. [A drawing of 5 girls and 3 pizzas is given.] Three pizza’s are equally divided among five girls. How much pizza will each of them get?

12. Which of the following correspond to a division? 137 + 45 = ; 350 : 30 = ; 234 – 124 = ; 12/124 = ; 45*123 = ; $\frac{2}{3}$

24. Peter prepares 14 cakes. He divides these cakes equally between his 6 friends. How much cake does each of them get?

- Sub-construct: measure

6.1. [A number line is given, with a range from 0 to 6.] Locate 9/3 on the number line.

6.2. [A number line is given, with a range from 0 to 6.] Locate 11/6 on the number line.

19. Is there a fraction that appears between 1/8 and 1/9? If yes, give an example.

20. Draw below a number line and locate 2/3 on it.

21.1. [A number line with the origin and 5/9 located on is given.] Locate number 1 on the number line.

21.2. [A number line with the origin and $2\frac{1}{4}$ located on is given.] Locate number 1 on the number line.

23.1. Use two of the following numbers to construct a fraction as close as possible to 1. [The numbers 1,3,4,5,6,7 are given.]

23.2. Use two of the following numbers to construct a fraction as close as possible to 0. [The numbers 1,3,4,5,6,7 are given.]

29. Which of the following are numbers? Put a circle around them. [Next is given: A, 4, *, 1.7, 16, 0.006, $\frac{2}{5}$, 47.5, $\frac{1}{2}$, $, $1\frac{4}{5}$]

Procedural knowledge

- Find the answer to the following

40. 3/5 + 4/5 = …

41. 5/8 – 1/4 = …

42. 3/5 * 3/4 = …

43. 1/3 : 4 = …

44. 8/3 * 4/5 =

45. 3/4 + 1/3 = …

46. 5/6 – 1/4 = …

47. 6/7 : 2/3 = …

- Find the answer to the following. Illustrate each time how you would explain this to your pupils. You can use the following pages to write down the illustrations.

48*. 5/6 – 1/4 = …

49*. 2/6 + 1/3 = …

50*. 5: 1/2 = …

51*. 2/5 * 3/5 = …

52*. 3/4 : 5/8 = …

Chapter 5

Teaching fractions for conceptual understanding: An observational study in elementary school

Chapter 5

Teaching fractions for conceptual understanding: An observational study in elementary school[6]

Abstract

This study analyzed how fractions are taught in the fourth grade of elementary education in Flanders, the Dutch speaking part of Belgium. Analysis centered on features that facilitated students' conceptual understanding. The findings suggested that the teaching of fractions in Flanders supported students' procedural understanding rather than their conceptual understanding of fractions. The study further revealed that the orientation toward conceptual understanding differed according to the mathematical idea that was stressed. Finally, the results revealed a consistency in the transition from the task as presented in the teacher's guide to the task as set up by the teacher, and an inconsistency in the transition from the task as set up by the teacher to the task as enacted through individual guidance by the teacher. Implications are discussed.

[6] Based on:
Van Steenbrugge, H., Remillard, J., Verschaffel, L., Valcke, M., & Desoete, A. Teaching fractions for conceptual understanding: An observational study in elementary school. Manuscript submitted for publication in *The Elementary School Journal*

1. Teaching fractions

In the chapter 'Rational Number, Rate, and Proportion' in the *Handbook of Research on Mathematics Teaching and Learning*, Behr, Harel, Post, and Lesh (1992) concluded that they were unable to find a significant body of research that focused explicitly on teaching rational number concepts. By making this statement, Behr and colleagues highlighted the dearth of findings that could offer guidance for teaching the domain that includes fractions (Lamon, 2007). A notable exception on this point is the work of Streefland, who developed, implemented, and evaluated a curriculum for fractions in elementary school in The Netherlands that was built according to a constructivist approach (Streefland, 1991). In her chapter 'Rational Numbers and Proportional Reasoning' in the *Second Handbook of Research on Mathematics Teaching and Learning*, Lamon (2007) does report on research that has taken rational number concepts into the classroom and as such offers empirically grounded suggestions for teaching. Illustrating the growing interest and body of research in the field of teaching fractions is the practice guide 'Developing effective fractions instruction for kindergarten through 8th grade' (Siegler et al., 2010), published by the *Institute of Educational Sciences* [IES], the research arm of the U.S. Department of Education. The five presented recommendations in this practice guide range from proposals related to the development of basic understanding of fractions in young children to more advanced understanding of older students as they progress through elementary and middle school; one recommendation addresses teachers' own understanding and teaching fractions. Whereas the recommendations vary in their particulars, they all reflect the importance of conceptual understanding of fractions (Siegler et al., 2010, p. 8). Siegler and colleagues state however that to date, still less research is available on fractions than on whole numbers, and that a greater number of studies related to the effectiveness of alternative ways of teaching fractions is needed.

This study is a response to calls for greater focus on the teaching of fractions, and within that, a response to the call for more attention to the development of conceptual understanding of fractions. The aim of the study is to take stock of how Flanders, the Dutch speaking part of Belgium, is doing in response to this call. To do so, we examined how fractions are represented in the most commonly used curriculum programs in Flanders and at how fractions lessons from these curriculum programs are

implemented, Our rationale for including analysis of how the written curriculum is implemented is informed by research on curriculum enactment that illustrates that teachers use curriculum resources in different ways and that written plans are transformed when teachers enact them in the classroom (Stein, Remillard, & Smith, 2007). By providing a picture of how fractions are currently taught in 20 classrooms, this study informs the research field about the current ways of teaching fractions which can stimulate discussion and result in a more precisely oriented focus on alternative ways of teaching fractions.

2. Conceptual framework

The conceptual framework applied to analyze how teachers teach fractions was based on the mathematics task framework as adopted in a study that analyzed enhanced instruction as a means to build students' capacity for mathematical thinking and reasoning (Stein, Grover, & Henningsen, 1996). Taking the mathematical task as the unit of analysis, Stein et al. (1996) demonstrated changes in cognitive demand of mathematical tasks as they are implanted during instruction. They frequently found differences in the demand of the tasks as they appeared in instructional materials, as they were set up by the teacher, and as they were implemented by students in the classroom. This framework was later adapted by Stein et al. (2007) to elaborate the role that teachers play in these curricular shifts. Their review of the literature identified three phases in the curriculum implementation chain: curriculum as written, as intended by the teacher, and as enacted in the classroom. Figure 1 combines these two frameworks and shows through shading those components that were the focus of this study.

2.1. Mathematical tasks

Examination of the teaching of fractions was framed by the concept of mathematical tasks. This concept builds on what Doyle (1983) described as academic tasks. Doyle underlined the centrality of academic tasks in creating learning opportunities for students (Silver & Herbst, 2007). In this study, we used the Stein et al.'s (1996) definition of a mathematical task as a classroom activity that aims to focus students' attention on a specific mathematical idea. The conception of Stein and colleagues of

mathematical tasks is similar to Doyle's notion of academic tasks in that it determines the content that students learn, how students learn this content, and by means of which resources that students learn this content. It is different from Doyle's notion of academic task regarding the duration: an activity is not classified as another mathematical task, until the underlying mathematical idea changes In the current study, instructional time of the analyzed lessons was typically divided in one or two mathematical tasks and as such, the mathematical tasks can be considered as broad units of analysis, which was in accordance with the plea for broad units of analysis to describe the complex nature of teaching (Hiebert et al., 2003; Hiebert & Grouws, 2007; Stigler & Hiebert, 1999).

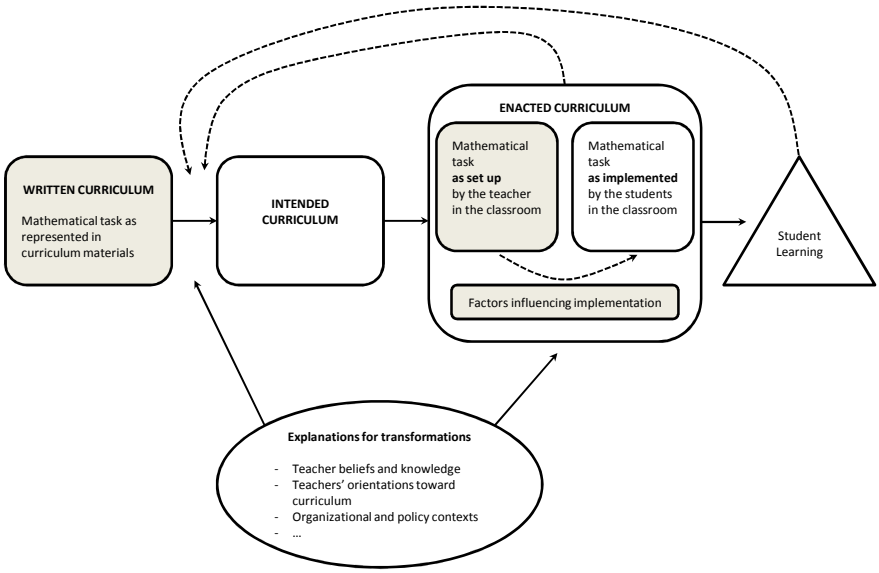

Figure 3. Conceptual framework based on Stein, Grover, and Henningsen, 1996; Stein, Remillard, and Smith, 2007

A central theme in research related to academic tasks is the extent to which tasks can change their character as they pass through the curriculum chain as depicted in the conceptual framework (Stein et al., 1996, p. 460). For example, Stein et al. (1996) found that teachers often lowered the nature of tasks because of their focus on correctness of the answer, or because the teachers did too much for the students.

This study focused on three aspects of the conceptual framework. Given that curriculum programs are considered to be a main source of the mathematical tasks as presented by the teacher (Stein et al., 2007), a first focus of the study related to the *task as represented in the teacher's guide*. The task as represented in the teacher's guide refers to the way in which the task set up during instruction is described in the teacher's guide to inform the teacher on how to 'optimally' set up and implement the specific mathematical idea. Second, we analyzed the *task as set up by the teacher*, given the importance of the enacted curriculum to shape students' learning experiences (Carpenter & Fennema, 1988; Stein et al., 2007; Wittrock, 1986). The task as set up refers to the task as introduced by the teacher. The kinds of assistance provided by the teacher to students that are having difficulties, is considered to be a factor that influences how tasks are implemented by the students in the classroom (Stein et al., 1996). This was also part of the current study's focus. We defined this as the *task as enacted through individual guidance provided by the teacher to students that are having difficulties*. The mathematical task as represented in the teacher's guide, as set up by the teacher, and as enacted through individual guidance provided by the teacher to students that are having difficulties were examined on task features that are considered to facilitate students' conceptual understanding. We describe this below.

2.2. Task features that relate to students' conceptual understanding

Teaching that primarily facilitates students' skill efficiency is often described as rapidly paced, teacher-directed instruction in which the teacher plays a central role in the organization and presentation of a mathematical problem to students that is followed by a substantial amount of error free practice of a similar set of problems completed by students individually (Hiebert & Grouws, 2007; Stein et al., 1996). Students' work, then, can be described as memorization of facts and applying procedures without understanding of when and why to apply these procedures (Stein et al., 1996).

A key feature of teaching for conceptual understanding can be described as *students struggling with important mathematics*: "By struggling with important mathematics we mean the opposite of simply being presented with information to be memorized or being asked only to practice what has been

demonstrated" (Hiebert & Grouws, 2007, pp. 387-388). Along this line, research points at maintenance of a high level of cognitive demand during lesson enactment as an important factor in students' learning gains (Boaler & Staples, 2008; Stein & Lane, 1996; Stigler & Hiebert, 2004). Furthermore, students struggling with important mathematics also implies that students must be given opportunities to make themselves sense of mathematics. Therefore, students should be encouraged to discuss ideas with each other and must be given meaningful and worthwhile tasks; such tasks use contextualized problems, contain multiple solution strategies, encourage the use of different representations, and ask students to communicate and justify their solution methods (Hiebert & Wearne, 1993; Stein et al., 1996). This is also the kind of teaching mathematics that is plead for in several countries with the adoption of new standards (Bergqvist & Bergqvist, 2011; Lloyd, Remillard, & Herbel-Eisenman, 2009; NCTM, 2000; Verschaffel, 2004).

Underlining the importance of teaching for conceptual understanding, several studies have revealed that lessons that focus on students' conceptual understanding also promote students' skills (Hiebert & Grouws, 2007). However, a major difference lies in the finding that students who developed skill by means of conceptual understanding more fluently applied that skill: they were better able to adjust their skill to changing circumstances (Bjork, 1994; Hiebert & Grouws, 2007).

Given the importance of teaching for conceptual understanding, in the current study, the mathematical tasks were analyzed on the following task features: the extent to which the task makes use of contextualized problems, the extent to which the task stimulates collaboration between students, the extent to which the task lends itself to be solved by means of multiple solution strategies, the extent to which the task can be depicted by several representations, and the extent to which the task encourages to predict and/or justify the solution methods. Features of selected tasks in the teacher's guide relate to the extent to which the teacher's guide encourages the teacher to incorporate these features. During task set up, task features refer to the extent to which the task as announced by the teacher incorporates or encourages these different features. Task features during the assistance provided by the teacher refers to the extent to which the teacher incorporates or encourages these features while helping students with difficulties.

3. Research questions

The overall aim of the study is to analyze how teachers teach fractions. Guided by the conceptual framework, the following research questions were put forward:

- To what extent does the teaching of fractions in Flanders (task as presented in the teacher guide, task as set up by the teacher, and task as enacted through individual guidance provided by the teacher to students who experience difficulties) reflect features that foster students' conceptual understanding of fractions? Is there a relationship with the particular curriculum program used or the specific mathematical idea being stressed?
- To what extent do the instructional features change as instruction moves from tasks as written in the curriculum, to how they are set up in the classroom, to how they are enacted through individual guidance provided by the teacher?

4. Methodology

In order to pursue these questions, we analyzed 24 video recorded lessons on fractions of 20 teachers. Teachers were using one of the three most predominately used curriculum programs in Flanders. Using the task features listed above, we analyzed the tasks as they appeared in the curriculum guides, as they were set up by the teacher during the lesson, and how they were represented to students during individualized assistance by the teacher.

4.1. Data sources

Transcriptions of videotaped classroom lessons formed the basis of the data used for analysis. Classroom observations took place during Spring 2010 and were video recorded by trained observers. Each observation covered one complete mathematics lesson.

The observers were students in educational sciences enrolled in the course 'mathematics education'. During two consecutive sessions, students were given information of the background and aim of the study, and of the practical aspects of the study (i.e., the necessity to record one complete lesson and to stay focused on the teacher, how to complete the informed consent, and how to introduce themselves

to the school principals and the teachers). Students were also presented fragments of a recorded lesson that was discussed afterwards. Students were asked to videotape two lessons of fractions in fourth grade of elementary education. Between the first and second observation, and after the second observation, students met each other in groups of ten, supervised by the first author to share findings, obstacles and other experiences with each other.

4.2. Sampling procedure

In total, based on an at random selection, 20 Flemish schools participated in the study. Of every school one fourth grade teacher participated in the study. As a selection criterion, schools had to use one of the most frequently used curriculum programs in Flanders: Kompas (KP), Nieuwe Tal-rijk (NT), and Zo gezegd, zo gerekend! (ZG) [i]. (Abbreviations are used going forward). This resulted in a total number of 29 lessons considered for analysis. Table 1 gives an overview of the number of lessons, schools and teachers that were considered for analysis, and the total number of lessons, schools and teachers that finally were included in the analysis. From our initial pool of 29 lessons, we selected 24 lessons. Selection secured an equal amount of lessons of each curriculum program (n = 8), and a maximum overlap related to the mathematical ideas covered across the three curriculum programs. As such, 24 observed lessons were included in the analysis.

Table 3. Overview of data pool

	Considered for analysis			Included in the analysis		
	Lessons	Schools	Teachers	Lessons	Schools	Teachers
Kompas	10	8	8	8	8	8
Nieuwe Tal-rijk	9	6	6	8	6	6
Zo gezegd, zo gerekend!	10	6	6	8	6	6
Total	29	20	20	24	20	20

Note. KP = Kompas; NT = Nieuwe Tal-rijk; ZG = Zo gezegd, zo gerekend!

Fourteen lessons included only one mathematical task set up by the teacher; 5 lessons included two mathematical tasks set up by the teacher, and another 5 lessons, included three mathematical tasks set up by the teacher. For lessons with two or three mathematical tasks set up by the teacher, the mathematical task that occupied the largest percentage of time was selected.

Four lessons of KP, three lessons of NT and one lesson of ZG mainly focused on fractions and decimals. Four lessons of KP, two lessons of NT and two lessons of ZG mainly centered on comparing and ordering fractions; and three lessons of NT and five lessons of ZG primarily focused on equivalent fractions. As such, 24 lessons were included in the analysis. Related to every observed lesson, for each selected task as set up by the teacher, we selected the task as represented in the teacher's guide that addressed the same mathematical idea. In addition, we selected two tasks as enacted through individual guidance provided by the teacher that also addressed the same mathematical idea as in the task as set up by the teacher. As such, for each observed lesson, the underlying mathematical idea was the same for the task as represented in the teacher guide, as set up by the teacher, and as enacted through individual guidance provided by the teacher. This resulted in a total number of 88 task to be analyzed[ii].

4.3. Coding

QSR NVivo 9 was used to code the selected mathematical tasks. All video recorded lessons were transcribed in detail to cover the conversations between the teacher and students. Coding was based on these transcriptions, and the corresponding video fragment was looked at again only when the transcription did not provide sufficient information to make a decision. In a first phase, the mathematical tasks as presented in the teacher guide, as set up by the teacher, and as enacted through individual guidance provided by the teacher were selected. In a second phase, we coded the selected tasks. The coding scheme was based on the conceptual framework presented earlier and was tested and revised until we ended up with the actual coding scheme. We used one unique coding scheme for coding the mathematical tasks as presented in the teacher guide, as set up by the teacher, and as enacted through individual guidance provided by the teacher, which is in correspondence with Stein et al. (2007) who state that the research field would benefit from establishing common structures for examining both the written curriculum and the enacted curriculum.

As a first step, the coding scheme required to describe the mathematical idea that was stressed in the mathematical task. Three kinds of mathematical ideas were stressed throughout all analyzed

mathematical tasks: the relationship between fractions and decimals, the ordering and comparing of fractions, and equivalent fractions. The first kind of tasks included parts of lessons in which fractions were converted into decimals and decimals into fractions by means of Cuisenaire rods or an external number line, positioning fractions and decimals on a number line, and comparing fractions and decimals by means of area models. The second kind of tasks included lessons that focused on comparing and ordering fractions, either by means of a number line or by means of other representations. The last category of tasks included lessons that centered on finding equivalent fractions for a given fraction and on finding the most reduced form of a given fraction.

After description of the mathematical idea that was stressed in the task, the coding scheme required to make six decisions related to features of the mathematical task. Decisions had to be made regarding the inclusion of real-life objects, the collaborative venture of the task (did students need to cooperate?), the number of solution strategies[iii], the number and kind of representations, whether representations were linked to each other or not, and the requirement for students to produce mathematical explanations or justifications. All fragments were coded by first author. To ensure coding validity, a second researcher was trained and asked to code 3 randomly selected lessons. To measure inter-rater reliability, Krippendorff's alpha was calculated for each decision to be made in the coding scheme and ranged from .80 to 1.00 and was as such above the customary border of $\alpha \geq .80$ (Krippendorff, 2009). This means that at least 80% of the codings were perfectly reliable whereas 20% at most were due to chance.

5. Results

We start this section with a description of and a reflection on one sample lesson. This will, as we explain in the first reflection, set out the structure and the specific approach of the analysis.

5.1. A lesson on equivalent fractions

Below, we describe a lesson in which a teacher helps her students to understand the meaning of equivalent fractions and to find equivalent fractions. At the moment of the lesson, students are familiar with the part-whole notion of fractions.

Starting the lesson, the teacher asks her students to take their textbook, a stencil, fractions box, and crayons. The students are asked to put the fractions box in front of them and the rest of their materials aside of the desk.

An illustration of fractions box is shown in Figure 2. The fractions box consists of a template which gives place to 9 units. The teacher consistently refers to each unit on the template as one cake. The box further consists of units and pieces of 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/9, and 1/10.

Figure 4. A student uses the fractions box to find equivalent fractions for 1/2

This is how the conversation between the teacher and the students continues after the students opened their fractions boxes:

> T: I would like everyone to fill one cake with two halves. [The students fill one whole on their template with two pieces of 1/2].
>
> T: Everyone now takes one half away. No we have a hole in the cake. How big is that hole?
>
> S: It is the fraction 1/2. [The teacher now writes 1/2 on the blackboard].
>
> T: Now I would like you to fill that whole with other pieces that are all equally-sized. Once you've found one solution, you can search for other solutions because there is more than one solution. [The students fill the half with equally-sized pieces].

T: Okay, everyone now has to look at the blackboard. What did we found? [Teacher wrote '1/2 = ' on the blackboard] T: I wrote '1/2 equals' on the blackboard because, as we mentioned earlier, the piece we filled in equals 1/2.

S: 1/2 equals two pieces of 1/4.

T: How do we write this in one fraction?

S: 2/4 [The teacher writes this down on the blackboard: 1/2 = 2/4]

T: Who found something else?

S: 1/2 equals 3/6 [Below 1/2 = 2/4, the teacher writes this down on the blackboard: 1/2 = 3/6]

T: Who found something else?

S: 1/2 equals 5/10 [Below 1/2 = 3/6, the teacher writes this down on the blackboard: 1/2 = 5/10]

[The teacher points at the blackboard] T: What can we say of those fractions?

S: All those fractions represent the same size.

T: Yes, it doesn't matter if I eat 1/2 or 3/6 or 5/10 of the cake: it all represents the same size of the cake. All those fractions represent the same size, the same piece. We call them equivalent fractions. [Teacher writes the title 'Equivalent fractions' on the blackboard].

The lesson continues with a similar exercise in which students search for equivalent fractions for 1/3 by means of their fractions box. Afterwards, the lesson continues as follows:

T: Unfortunately, we are not always able to use our fractions box to find equivalent fractions. Imagine for a moment that we don't have our fraction boxes and look at the equivalent fractions that are written on the blackboard. How can we find then equivalent fractions? [The students are given some time to think about it].

S: We have to multiply both numbers with a same number.

T: Try to say it in a more scholarly way.

S: We have to multiply both the numerator and the denominator with a same number.

[Teacher checks if this holds for all equivalent fractions on the blackboard]. T: Actually, it is quite easy to find equivalent fractions!

T: Please take all your stencil (see Figure 3).

T: You can see several fraction strips on the stencil. Look at the first picture and tell me in how many pieces we the first fraction strip is divided.

S: 9.

T: OK, next to the fraction strip, you see the fraction 6/9. I want you all to color 6/9 of the fraction strip. [Students color 6 of the 9 pieces of the first fraction strip; the teacher writes the fraction 6/9 on the blackboard].

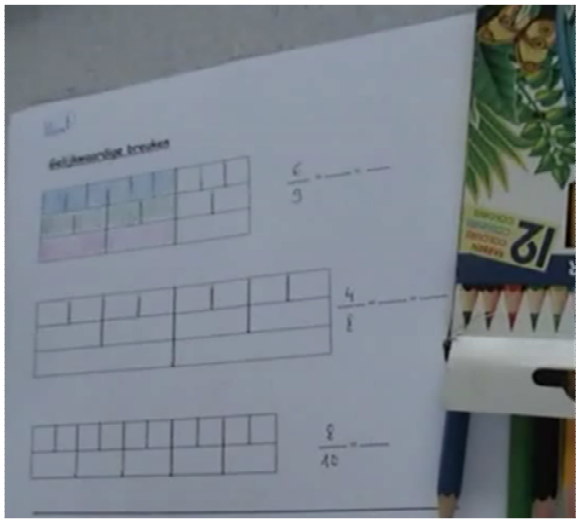

Figure 5. Student uses fraction strips to find equivalent fractions for 6/9

T: Now, take another color, and I would like you to color in the second fraction strip a piece that is equally-sized as the one you colored in the fraction strip above. [Students color 4 of the 6 pieces in the second fraction strip].

T: You can see several fraction strips on the stencil. Look at the first picture and tell me in how many pieces we the first fraction strip is divided.

S: 9.

T: OK, next to the fraction strip, you see the fraction 6/9. I want you all to color 6/9 of the fraction strip. [Students color 6 of the 9 pieces of the first fraction strip; the teacher writes the fraction 6/9 on the blackboard].

T: Now, take another color, and I would like you to color in the second fraction strip a piece that is equally-sized as the one you colored in the fraction strip above. [Students color 4 of the 6 pieces in the second fraction strip].

T: Now, take yet another color and color in the third fraction strip a piece that is equally-sized as the one you colored in the two fraction strips above. [Students color 2 of the 3 pieces in the third fraction strip].

T: What do we know of all our colored pieces?

S: They are equal in size.

T: OK, we still know that in the first fraction strip, we colored 6/9. Now I want you to tell me what piece we colored in the second fraction strip.

T: The second fraction strip consists of how many pieces?

S: 6 [The teacher writes the denominator 6 on the blackboard]

T: How many pieces did we colored?

S: 4 [The teacher writes the nominator 4 on the blackboard]

T: And what did we color in the third fraction strip?

S: 2/3 [The teacher writes the fraction 2/3 on the blackboard].

T: Right. And what can we say about those three fractions?

S: They are equivalent fractions.

T: Right. They have the same value. Look for a moment at the fractions 6/9 and 2/3; fraction 2/3 is the same as 2/9 but in a reduced form. We might reduce fractions; that can make it easier for us.

T: How can we go from the fraction 6/9 to 2/3

S: By dividing both the numerator and the denominator by 3.

T: Yes, again, we see that it is important to divide both the numerator and the denominator by a same number.

The lesson continues with a similar exercise. After finishing that exercise, students are asked to put the stencil and the fractions box aside and to take their textbook. All the fractions boxes are then collected.

Students now have to complete exercises in which they must find equivalent fractions. Students work individually and in case they have problems, they raise their finger and the teacher then comes to help them. Below are two conversations between the teacher and students who are having difficulties.

Conversation 1: a student isn't able to find an equivalent fraction for 1/2

[The teacher points at the board].

T: In order to find an equivalent fraction, we have to multiply both the numerator and the denominator with a same number. Let's multiply them with 2; what do we get?

S: 1/4.

T: No, you multiplied only the denominator with 2; you must also multiply the numerator with 2.

S: 2/4.

T: Okay. And now an equivalent fraction for 2/5…

Conversation 2: A student isn't able to reduce the fractions 2/4 and 3/6.

T: Okay, you [the neighbor of the student] also listen to what I am saying.

[The teacher points at the blackboard]. If we want to reduce a fraction, we must always divide both the numerator and the denominator with a same number. Let's divide them by 2. What do we get?

S: 1/2.

T: Okay.

S: But 3/6 …?

T: Yes: we always start with trying to divide them by 2. If that doesn't work, you try to divide them by 3, or by 4…

At the end of the lesson, students put their textbook aside of their desks and the textbooks are collected.

5.2. Lesson Analysis

After observing the lesson as outlined in the vignette, several aspects triggered our attention. There seemed to be two major sections in the lesson. A first section, that we described as instructional time comprised the learning of new content (in this case: equivalent fractions). Strongly guided by the teacher, during this mainly whole-class moment, students learned to use multiple representations and strategies to find equivalent fractions. Notably, both representations were not linked to each other: the students learned to find equivalent fractions by means of their fractions boxes and afterwards, they learned to do so by means of fraction strips, but it was not explicitly made clear that, for example, 1/2 and 2/4 are equivalent fractions and that students might come to this solution by means of their fractions box and by means of the use of fraction strips. After they learned to find equivalent fractions by means of the fractions box and by means of fraction strips, students inductively retrieved the rule to find equivalent fractions. During this instructional phase, the teacher links, though very briefly, the exercises with real-life situations ("Think of a half a cake, and try to fill in the other half of the cake with equally-sized pieces").

During the second section of the lesson that we described as practice time, students practiced the learned content on their own and were – if they encountered problems – helped individually by the teacher. When the teacher helped students with problems in finding equivalent fractions, the teacher immediately pointed at applying the rule to find equivalent fractions without referring to helpful representations and other solution strategies, nor to real-life objects. Moreover, at the start of the practice time, all the fractions boxes were collected and removed from the desks, not allowing students to use these in case they might want to.

As such, the structure of the sample lesson did not reflect a way of teaching that is considered to support students' conceptual understanding: teacher-directed instruction followed by a substantial amount of practice of a similar set of problems completed by students on their own (Hiebert & Grouws, 2007; Stein et al., 1996). We also noticed a sharp decline in features that might facilitate students' conceptual understanding as we move from instructional time to practice time. This observation suggests a differentiation in instruction. Students who remembered from instructional time

the conceptual meaning of finding equivalent fractions might not experience problems in finding equivalent fractions during practice time, and might know what they were doing. Students with difficulties during practice time might get the impression, when the teacher helped them by immediately refreshing the rule and only referring to that rule in order to find equivalent fractions, that mathematics is about learning and applying rules rather than understanding what they are doing.

We are interested if the picture provided by a sample lesson is can be considered as a general pattern when teachers in Flanders teach – and students learn fractions. This is the focus of our subsequent analyses. We first zoomed in on the structure of all 24 observed lessons and afterwards on the features of all 84 analyzed mathematical tasks.

5.3. Structure of the lessons: facilitating skill efficiency rather than conceptual understanding

All the lessons started with a short introduction that mostly included the subject of the lesson and in which students were asked to take their materials (textbooks, pencils, …) and sometimes previous content was briefly refreshed. The introduction was then followed by a whole-class instruction moment that was strongly guided by the teacher (hereafter referred to as 'instructional time'). Typically, instructional time addressed teaching of new content, or teaching of previously learned content. After instruction, students usually practiced the learned content on their own and were – if they encountered problems – helped individually by the teacher (hereafter referred to as 'practice time'). Thus, the overall picture is that pairwise or group learning during instructional and practice time was marginal. Lessons were closed by collecting textbooks; during two observations, closing of the lesson also comprised a summary of the learned content.

Introduction ranged from 20 seconds to 11 minutes and covered on average 4% of the lesson. Instructional time covered all the tasks as set up by the teacher, ranged from 8 to 40 minutes and covered on average 49% of the total lesson duration. Coded mathematical tasks as set up by the teacher ranged from 6 to 40 minutes, with an average length of 20 minutes. On average the coded mathematical task as set up by the teacher covered 85% of the total instructional time. Practice time ranged from 3 to 40 minutes and covered on average 44% of the lesson. Closing ranged from 0 to 5

minutes and covered on average 2% of the lesson. Two percent of total lesson duration was coded as not related to mathematics. This included moments in which a colleague of the teacher entered the class and had a conversation with the teacher and moments in which the teacher left the classroom.

In two of the 24 observed lessons, there were no moments in which the teacher helped students struggling with mathematics; only students who knew the answer of the problems were given the opportunity to answer in these two whole-class lessons. Whereas this finding does not allow to state that students weren't struggling with mathematics, it does suggest that mathematics was conceived as something you know or not, and in case you aren't able to come up with a straightforward answer, you shouldn't struggle to find one. This is important since students' struggling with mathematics is considered as an important feature that facilitates students' conceptual understanding (Hiebert & Grouws, 2007).

The description of the lessons as presented above mirrored the structure of the sample lesson as described in the vignette and reflected a structure that did not facilitate students' conceptual understanding: teacher-directed instruction with a central role for the teacher, followed by a substantial amount of practice of a similar set of problems completed by students individually (Hiebert & Grouws, 2007; Stein et al., 1996).

Below, we analyzed to which extent the features of the tasks as represented in the teacher guide, as set up by the teacher, and as enacted through individual guidance by the teacher to students who experience difficulties, facilitated students' conceptual understanding of fractions. We reflected on these findings in the second reflection (see below).

5.4. Task as represented in the teacher's guide

Table 2 gives an overview of the features of all 24 coded tasks as presented in the teacher's guide. We first looked for a general pattern based on all 24 coded tasks (see column 'Total' in Table 2). This overall picture revealed mixed findings related to the presence of features that might facilitate students' conceptual understanding. The majority of the tasks addressed students' conceptual understanding by stressing the use of multiple solution strategies and multiple representations.

However, the majority of tasks also stressed features that did not address students' conceptual understanding: remaining in the abstract world of mathematics, the absence of a strong collaboration between students, and the absence of the need to justify the solution method. Almost half of the 24 tasks suggested to link the multiple representations to each other.

Table 4. Presence of features (in percentages) of tasks as represented in the teacher's guide

	Curriculum program			**Mathematical idea**			
	KP (n = 8)	NT (n = 8)	ZG (n = 8)	F & D (n = 8)	C & O (n = 8)	E. F. (n = 8)	**Total** (n = 24)
Context							
Abstract world of math	100	100	/	88	75	38	67
Real-life objects	/	/	100	12	25	62	33
Collaborative venture							
Alone	/	/	/	/	/	/	/
Duo or small groups	/	/	12	/	12	/	4
Teacher to students	100	100	88	100	88	100	96
Solution strategies							
Single	50	25	/	75	/	/	25
Multiple	50	75	100	25	100	100	75
Representations							
Single	50	25	12	75	12	/	29
Multiple	50	75	88	25	88	100	71
Representations - links							
Not linked	50	100	22	100	25	38	54
Linked	50	/	88	/	75	62	46
Justification							
Not required	100	88	50	75	100	62	79
Required	/	12	50	25	/	38	21

Note. KP = Kompas; NT = Nieuwe Tal-rijk; ZG = Zo gezegd, zo gerekend!; 'F & D' = Fractions and decimals; 'C & O' = Comparing and ordering fractions; 'E. F.' = Equal fractions.

Given that curriculum programs are considered as a main source for mathematical tasks to be used by the teacher (Stein et al., 2007), we also made a comparison of task features based on the curriculum programs (see the columns 'KP', 'NT', 'ZG' in Table 2). Tasks represented in the teacher's guide of ZG encouraged most the development of conceptual understanding of fractions: all tasks referred to real-life objects whereas none of the tasks of KP and NT did, some tasks encouraged teachers to let

students work together in pairs or in small groups whereas none of the tasks of KP and NT did, and all of the tasks included multiple strategies. Tasks of ZG also included more often multiple representations and linked representations more often to each other as compared to tasks as represented in NT and KP, and tasks of ZG also required more often justification of the solution strategies. Tasks of NT added more to students' conceptual understanding of fractions than tasks of KP did: they included more often multiple strategies and multiple representations, and required more often justification of the solution strategies. Tasks of NT did not include links between the representations whereas KP did in 50% of the tasks.

When we made a comparison based on the underlying mathematical idea of the coded task (see the columns 'F & D', 'C & O', and 'E. F.' in Table 2), the following picture emerges. Mathematical tasks that related to fractions and decimals contrasted with tasks that related to comparing and ordering fractions and equivalent fractions in a way that did not support the development of students' conceptual understanding of fractions. All or most of the tasks that related to fractions and decimals, did not refer to real-life objects, did not require strong collaboration between the students, focused attention on one solution strategy, presented one representation, did not link representations to each other and did not require justification of the solution method. There were no remarkable differences related to comparing and ordering fractions and equivalent fractions: three features that related to equivalent fractions (inclusion of real-life objects, multiple representations and requirement of justification) and two features that related to comparing and ordering fractions (collaboration between students, presentation of links between the representation) were scored more in favor of supporting the development of students' conceptual understanding of fractions; on one feature (inclusion of multiple solution strategies) they both scored the same. We now turn to features of tasks as set up by the teacher.

5.5. Task as set up by the teacher

Table 3 gives an overview of the features of all 24 coded tasks as set up by the teacher. Again, we first looked for a general pattern based on all 24 coded tasks (see column 'Total' in Table 3). The overall

picture revealed a same pattern as observed in the sample lesson (see 'A vignette: a lesson on equivalent fractions'). The majority of the tasks addressed students' conceptual understanding by stressing the use of multiple solution strategies and multiple representations. However, the majority of tasks also stressed features that did not address students' conceptual understanding: all the tasks were set up in a way in which the teacher guides, directs and instructs the students, and as such, did not reflect strong collaboration between students. Most tasks did link the representations to each other and did not require students to justify their solution. Half of the tasks remained in the abstract world of mathematics.

A comparison based on the three curriculum programs (see the columns 'KP', 'NT', 'ZG' in Table 3) again, revealed that KP added least to the development of students' conceptual understanding of fractions whereas there were no straightforward differences between teachers teaching with ZG and NT. Tasks as set up by teachers working with KP included seldom real-life objects, seldom linked representations to each other, and required in most of the tasks no justification for solution strategies. Half of the tasks as set up by teachers working with KP focused on only one solution strategy and a single representation. Some features of tasks set up by teachers teaching with ZG supported more students' conceptual understanding of fractions (referring to real-life objects, requirement of justification) as compared to NT, sometimes it was vice versa (inclusion of multiple representations and linking the representations to each other), and sometimes task set up by teachers teaching with ZG or NT they were coded equally (attention to multiple solution strategies)

When we made a comparison based on the underlying mathematical idea of the coded task (see the columns 'F & D', 'C & O', and 'E. F.' in Table 3), a similar picture as in the previous section (Task as represented in the teacher's guide) emerged. Mathematical tasks that related to fractions and decimals contrasted with tasks that related to comparing and ordering fractions and equivalent fractions in a way that did not support the development of students' conceptual understanding of fractions. In most of the tasks that related to fractions and decimals, there was no link to real-life objects, only one solution strategy was stressed, no multiple representations were included, tasks were not linked, and tasks did not require students to justify their solution method. Again, there were no straightforward differences

related to comparing and ordering fractions and equivalent fractions: two features of tasks that related to comparing fractions (inclusion of multiple solution strategies and linking representations to each other) and one feature of tasks that related to equivalent fractions (inclusion of real-life objects) were scored more in favor of supporting the development of students' conceptual understanding of fractions, for two features (inclusion of multiple representations, requirement of justification) tasks that related to comparing and ordering fractions and equivalent fractions both scored the same.

Table 5. Presence of features (in percentages) of tasks as set up by the teacher

	Curriculum program			**Subject**			
	KP (n = 8)	NT (n = 8)	ZG (n = 8)	F & D (n = 8)	C & O (n = 8)	E. F. (n = 8)	**Total** (n = 24)
Context							
Abstract world of math	62	75	12	88	38	25	50
Real-life objects	38	25	88	12	62	75	50
Collaborative venture							
Alone	/	/	/	/	/	/	/
Duo or small groups	/	/	/	/	/	/	/
Teacher to students	100	100	100	100	100	100	100
Solution strategies							
Single	50	25	25	88	/	12	33
Multiple	50	75	75	23	100	88	67
Representations							
Single	50	/	12	62	/	/	21
Multiple	50	100	88	38	100	100	79
Representations - links							
Not linked	88	50	88	88	50	88	75
Linked	12	50	12	12	50	12	25
Justification							
Not required	62	62	50	75	50	50	58
Required	38	38	50	25	50	50	42

5.6. Task as enacted through individual guidance provided by the teacher to students with difficulties

Table 4 gives an overview of the features of all 40 coded tasks as enacted through individual guidance provided by the teacher to students with difficulties. Again, we first looked for a general pattern based on all 40 coded tasks (see column 'Total' in Table 4).

The overall picture revealed a same pattern as observed in the sample lesson (see 'A vignette: a lesson on equivalent fractions'). The results revealed that a majority of tasks required students to work on their own, remained in the abstract world of mathematics, focused on a single solution strategy and a single representation, did not link representations to each other, and did not require students to justify their answer.

A comparison based on the three curriculum programs (see the columns 'KP', 'NT', 'ZG' in Table 4) revealed an absence of straightforward differences. Tasks as enacted through individual guidance by teachers working with KP, NT, or ZG reflected to an equally high extent features that did not facilitate students' conceptual understanding of fractions: most tasks from either KP, NT, or ZG did not refer to real-life objects, required students to work on their own, focused attention on one solution strategy and one representation, did not link representations to each other, and did not require students to justify their solution method.

When we made a comparison based on the underlying mathematical idea of the coded task (see the columns 'F & D', 'C & O', and 'E. F.' in Table 4), a similar picture as in the previous sections ('Task as represented in the teacher's guide' and 'Task as set up by the teacher') emerged. Once again, mathematical tasks that relate to fractions and decimals contrasted with tasks that relate to comparing and ordering fractions and equivalent fractions in a way that support to a lesser extent the development of students' conceptual understanding of fractions. It should be stated however, that also for mathematical tasks that relate to comparing and ordering fractions and equivalent fractions, presence of features that might facilitate students' conceptual understanding was low. In most or all of the tasks that related to fractions and decimals, there was no link to real-life objects, one solution strategy and one representation was stressed, representations were not linked to each other, and justification of solution method was not required. Again, there were no straightforward differences for tasks related to comparing fractions and equivalent fractions. Whereas tasks that relate to comparing and ordering fractions did include real-life objects to a slightly higher degree, in general, tasks that relate to comparing fractions and equivalent fractions did score similar for inclusion of multiple strategies, multiple representations, linking the representations to each other, and requirement of justification.

Table 6. Presence of features (in percentages) of tasks as enacted through individual guidance by the teacher

	Curriculum program			Subject			
	KP (n = 11)	NT (n = 12)	ZG (n = 17)	F & D (n = 12)	C & O (n = 11)	E. F. (n = 17)	**Total** (n = 40)
Context							
Abstract world of math	82	100	82	100	73	88	87
Real-life objects	18	/	18	/	27	12	13
Collaborative venture							
Alone	91	100	53	75	82	76	78
Duo or small groups	/	/	6	8	/	/	2
Teacher to students	9	/	41	17	18	24	20
Sollution strategies							
Single	82	75	59	92	64	59	70
Multiple	18	25	41	8	36	41	30
Representations							
Single	82	83	88	92	88	82	85
Multiple	18	17	12	8	18	18	15
Representations - links							
Not linked	91	83	100	100	91	88	92
Linked	9	17	/	/	9	12	8
Justification							
Not required	91	100	94	100	91	94	95
Required	9	/	6	/	9	6	5

Note. KP = Kompas; NT = Nieuwe Tal-rijk; ZG = Zo gezegd, zo gerekend!; 'F & D' = Fractions and decimals; 'C & O' = Comparing and ordering fractions; 'E. F.' = Equal fractions.

5.7. Second reflection

This second reflection, a reflection based on all observed lessons, confirmed the findings of a first reflection based on a sample lesson. The structure of the lessons that we observed, mirrored the lesson structure that scholars describe as focusing on students' skill efficiency (Hiebert & Grouws, 2007; Stein et al., 1996). Instruction was set up in a way in which the teacher guides, directs and instructs the students. This was then followed by a substantial amount of practice of a similar set of problems that students completed on their own. An analysis of the features of the 84 tasks that were included in the study also confirmed the outcomes of the first reflection: some features of the tasks as set up by the

teacher supported students' conceptual understanding of fractions (focus on multiple solution strategies and multiple representations), others (remaining in the abstract world of mathematics, absence of strong collaboration between students, not linking representations to each other, mostly not requiring justification of the solution method) did not. This finding suggests that only part of the features that are considered to facilitate students' conceptual understanding are present in lessons related to teaching fractions.

A major distinguishing aspect regarding the task features of mathematical tasks, was the mathematical idea that was stressed in the task: tasks that related to fractions and decimals were consistently coded as less supporting students' conceptual understanding as compared to tasks that related to comparing and ordering fractions, and equivalent fractions. We observed this throughout the observations for tasks as presented in the teacher's guide, tasks as set up by the teacher, and tasks as enacted through guidance provided by the teacher to students with difficulties. This finding suggests a differentiation of instruction based on the mathematical idea that is the focus of the task.

Furthermore, the results revealed differences in task features related to the three curriculum programs (KP, NT, ZG) and the mathematical ideas that were stressed in the mathematical tasks (fractions and decimals, comparing and ordering fractions, equivalent fractions). Although there was to some extent an overlap between the curriculum programs and the mathematical ideas that were stressed (see 'Sampling procedure'), we did notice trends that we want to report on. KP contrasted with ZG and NT in a way that did not favor students' conceptual understanding for tasks as presented in the teacher's guide and tasks as set up by the teacher, but this difference melted away when instruction moved to task as enacted by through individual guidance provided by the teacher. This finding points at two points of attention. First, it confirmed the suggestion that curriculum programs are a main source of the mathematical tasks as set up by the teacher (Stein et al., 2007). Second, it revealed that this did not hold when the teacher helps struggling students individually.

The analysis of tasks as presented in the teacher's guide, as set up by the teacher, and as enacted through individual guidance by the teacher revealed that the features of tasks as set up by the teacher resembled the features of tasks as presented in the teacher's guide. This was not the case regarding the

features of tasks set up by the teacher and tasks as enacted through individual guidance by the teacher. To study this more deeply, we analyzed the specific transition of a task moving from presented in the teacher guide to set up by the teacher to enacted through individual guidance provided by the teacher. We did so by focusing on features related to the task's context, solution strategies an representations. This is the focus in the next section.

5.8. Change of features as instruction moves from tasks as represented in the teacher's guide to how they are set up in the classroom, to how they are enacted through individual guidance provided by the teacher

In order to analyze the extent to which task features change as instruction unfolds from tasks as represented in the teacher's guide to how they are set up by the teacher, to how they are enacted through the individual assistance provided by the teacher to students who experienced difficulties, two matrices were generated. A first matrix captured consistency in transition from tasks as presented in the teacher's guide to the tasks as set up by the teacher. The row headings listed the codes assigned to the tasks as represented in the teacher's guide and the column headings listed the codes for the corresponding tasks as set up by the teacher. The second matrix captured consistency in transition from tasks as set up by the teacher to the tasks as enacted through individual guidance provided by the teacher. The row headings listed the codes assigned to the tasks as set up by the teacher and the column headings listed the codes for the corresponding tasks as enacted through individual guidance by the teacher. Each cell contained the corresponding percentage and frequency. Percentages on the diagonals of the matrices represented consistency between a) the tasks as presented in the teacher's guide and corresponding tasks as set up by the teacher (matrix 1) and b) tasks as set up by the teacher and corresponding tasks as enacted through individual guidance by the teacher (matrix 2). Off-diagonal cells represented inconsistencies.

Matrix 1 revealed a high level of consistency between the tasks as presented in the teacher's guide and the corresponding tasks as set up by the teacher: percentages on the diagonal ranged from 69% to 100%. For example, 83% of all the tasks as presented in the teacher's guide that were coded as

stressing multiple solution strategies were also set up by the teacher in a way that made appeal to multiple solution strategies.

Table 7. Matrix 1: transition from tasks as presented in the teacher's guide to the tasks as set up by the teacher

	Task as set up during instruction					
	Solution strategies		**Representations**		**Context**	
Task as represented in teacher guide	Single	Multiple	Single	Multiple	Abstract	Real-life objects
Solution strategies						
Single (n = 6)	***83%*** *(5)*	17% *(1)*				
Multiple (n = 18)	17% *(3)*	***83%*** *(15)*				
Representations						
Single (n = 7)			***71%*** *(5)*	29 *(2)*		
Multiple (n = 17)			0	***100%*** *(17)*		
Context						
Abstract (n = 16)					***69%*** *(11)*	31% *(5)*
Real-life objects (n = 8)					13% *(1)*	***87%*** *(7)*

Matrix 2 revealed a different pattern as compared to the pattern observed in matrix 1. Percentages on the diagonal were high for task features that did not support students' conceptual understanding of fractions: remaining in the abstract world of mathematics, focus on one solution strategy and one representation. For example, 90% of all the tasks that were set up by the teacher in a way that focused on a single representation, were also enacted through individual guidance by the teacher in a way that focused on a single representation. This reveals a consistency between tasks as set up by the teacher and the corresponding tasks as enacted through individual guidance by the teacher regarding features that did not support students' conceptual understanding of fractions. Percentages on the off-diagonal cells were high for features that might facilitate students' conceptual understanding of fractions. For example, 83% of all the tasks that were set up by the teacher in a way that focused on multiple representations, were however enacted through individual guidance by the teacher in a way that

focused on a single representation. This reveals an inconsistency between tasks as set up by the teacher and the corresponding tasks as enacted through individual guidance by the teacher regarding features that support students' conceptual understanding of fractions.

These findings, related to the teaching of fractions, confirmed that lower demanding tasks are more likely to retain their character whereas higher demanding tasks are more likely not to retain their character (Hiebert et al., 2003; Stein et al., 1996).

Table 8. Matrix 2: transition from tasks as set up by the teacher to the tasks as enacted through individual guidance provided by the teacher

	Task as enacted through individual guidance provided by the teacher					
	Solution strategies		**Representations**		**Context**	
Task as set up during instruction	Single	Multiple	Single	Multiple	Abstract	Real-life objects
Solution strategies						
Single (n = 14)	***86%*** *(12)*	14% *(2)*				
Multiple (n = 26)	62% *(16)*	***38%*** *(10)*				
Representations						
Single (n = 10)			***90%*** *(9)*	10% *(1)*		
Multiple (n = 30)			83% *(25)*	***17%*** *(5)*		
Context						
Abstract (n = 18)					***100%*** *(18)*	0
Real-life objects (n = 22)					77% *(17)*	***23%*** *(5)*

6. Discussion

6.1. Implications for practice

Despite the worldwide adoption of standards that stress the importance of teaching mathematics for conceptual understanding (Bergqvist & Bergqvist, 2011; Lloyd et al., 2009; NCTM, 2000; Verschaffel, 2004), the present study's findings suggested that teachers in Flanders teach fractions in a way that does support students' procedural understanding rather than their conceptual understanding of fractions. The structure of the lessons still mirrored the structure of lessons from typical

mathematics classrooms before the adoption of the standards (Porter, 1989; Stodolsky, 1988) and the majority of mathematical tasks possessed both features that might facilitate (focus on multiple solution strategies and multiple representations) and features that might not facilitate students' conceptual knowledge of fractions (remaining in the abstract world of mathematics, absence of strong collaboration between students, not linking representations to each other, absence of requirement of justification of the solution method). Furthermore, we noticed a sharp decline in features that related to students' conceptual understanding as instruction moves to individual guidance provided by the teacher. In this respect, our findings corroborate prior research that maintenance of demanding features is difficult (Hiebert et al., 2003; Stein et al., 1996) also in the teaching of fractions. It also illustrates that the problem of maintenance of demanding features remains a persistent problem. This finding underlines the quest of Stein et al. (1996) for staff development efforts that aim to help teachers to implement tasks in a way that fosters students' conceptual understanding of mathematics in general and fractions in particular. In addition, since the findings also revealed that mathematical tasks that related to fractions and decimals were consistently coded as less supporting students' conceptual understanding as compared to tasks that related to comparing and ordering fractions and equivalent fractions, curriculum developers, teachers, and staff development efforts might, within their focus on teaching fractions for conceptual understanding, target especially the link between fractions and decimals.

6.2. Implications for research

The coding scheme and the conceptual framework on which the coding scheme was based, proved to be useful to cope with the complex nature of teaching. Moreover, the definition of mathematical tasks as broad units of analysis also helped to gain insight in the teaching of fractions. The distinction between tasks as presented in the teacher's guide, tasks as set up during instruction, and tasks as enacted through individual guidance by the teacher was also useful since it helped to describe the process of instruction as it unfolds in the class.

The findings of the current study have implications for studies that aim to respond to the quest for more studies related to alternative ways of teaching fractions (Lamon, 2007; Siegler et al., 2010).

Related studies might target the current prevailing structure of the lessons in which students during practice applied the rules as presented by the teacher during instruction. Given the many links of fractions with everyday life, students could instead learn fractions while doing activities that require them to conjecture, justify, interpret, work together, link representations to each other, etc.

Since the results clearly pointed that the orientation toward conceptual understanding differed based on whether the mathematical task was related to fractions and decimals, comparing and ordering fractions, and equivalent fractions, studies that aim to target alternative ways of teaching fractions might also pay considerable attention to teaching that aims to help students to understand the links between fractions and decimals.

Whereas Stein et al. (2007) asked for studies that addressed the whole curriculum chain (written, intended, enacted curriculum, and student learning), the current study addressed the written and enacted curriculum since the written, and especially the enacted curriculum is found to impact students' learning (Carpenter & Fennema, 1988; Stein et al., 2007; Wittrock, 1986). The assistance provided by the teacher to students who are struggling is considered to be a mediating variable between the task as set up by the teacher and the task as implemented by the students (Stein et al., 1996) and was also addressed in this study. However, we did not control for other mediating variables between the different phases of the curriculum chain as depicted in the conceptual framework. Other studies might include these mediating variables, the intended curriculum and students' performance in the analysis.

[i] In line with the in 1997 adopted attainment targets, the three curriculum programs (KP, NT, ZG) embody an approach to mathematics teaching and learning that was previously uncommon (focusing on mathematical thinking and reasoning, inclusion of problem solving activities, making use of realistic contexts, the use of calculators, collaboration, communication, and the development of a critical attitude related to numerical information). All curriculum programs cluster lessons in a week, a block or a theme addressing the main content domains of mathematics education (numbers and calculations, measurement, geometry).

[ii] For 18 of the 24 selected tasks as set up by the teacher, two tasks as enacted through individual guidance provided by the teacher to students experiencing difficulties were selected. For five lessons, we could not select two tasks as enacted through individual guidance provided by the teacher, because instruction took the major time of the lesson and practice was too short to allow for selecting two tasks. In one lesson, we selected three tasks as enacted through individual guidance provided by the teacher in order to cover to whole range of tasks as enacted through individual guidance provided by the teacher to students experiencing difficulties.

[iii] Single representation refer to either single symbol representations or single nonsymbolic representations. Single symbol representations refer to representations that are entirely composed of numerals, mathematical symbols, mathematical notation. Single nonsymbolic representations refer to representations that incorporate both a symbol and a nonsymbol (e.g., manipulative, picture).

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Chapter 6

General discussion and conclusion

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1. Problem statement

In the introduction of this dissertation we described gaps related to the research field of fractions. We stated that fractions are considered a critical (Kilpatrick, Swafford, & Findell, 2001; Kloosterman, 2010; NCTM, 2007; Siegler et al., 2010; Van de Walle, 2010), but difficult subject for students to learn (Akpinar & Hartley, 1996; Behr, Wachsmuth, Post, & Lesh, 1984; Bulgar, 2003; Hecht, Close, & Santisi, 2003; Lamon, 2007; Newton, 2008; Siegler et al., 2010). Worldwide, students experience difficulties when learning fractions. The range of studies over the past years revealed that this problem is persistent. This also appears to be the case in Flanders, as two sample surveys, administered respectively in 2002 and 2009, revealed that on both measurement occasions, only 64% of the last-year Flemish elementary school students mastered the attainment targets – minimum goals that all students should master at the end of elementary school, approved by the Flemish Government – related to fractions and decimals. This finding, in addition to the outcomes of the study that we reported on in Chapter 2, constitutes the basis for the focus on fractions in the present dissertation.

We further pointed at the need for more studies focusing on preservice and inservice teachers' knowledge of fractions (Moseley, Okamoto, & Ishida, 2007; Newton, 2008). Given that teacher education is considered to be crucial for teachers to develop a deep understanding of fractions (Borko et al., 1992; Ma, 1999; Newton, 2008; Toluk-Ucar, 2009; Zhou, Peverly, & Xin, 2006), and that a major concern related to increasing the mathematics standards expected of students should be teachers' preparation to address these standards (Jacobbe, 2012; Kilpatrick et al., 2001; Siegler et al., 2010; Stigler & Hiebert, 1999; Zhou et al., 2006), we analyzed Flemish preservice teachers' knowledge of fractions in Chapter 4 of this dissertation.

Finally, we discussed a growing body of research related to fractions that explicitly focuses on the teaching of fractions (Lamon, 2007; Siegler et al., 2010). The importance of studying actual teaching

is also stressed in research related to teachers' use of curriculum materials, placing the teacher as a central actor in the process of transforming the written curriculum (Lloyd, Remillard, & Herbel-Eisenman, 2009; Stein, Remillard, & Smith, 2007). Related research describes a curriculum chain that comprises a written, an intended, an enacted curriculum, and mediating factors between these phases (Stein et al., 2007). In Chapter 3 we focus on one such mediating variable, namely teachers' views of curriculum programs. In Chapter 5, we zoom in on how teachers in Flanders teach fractions.

2. Research objectives

The initial aim of the dissertation was to set up research on mathematical difficulties. Based on the outcomes of Chapter 2, where we explored mathematical difficulties as reported by the teachers, we decided to focus on fractions and to analyze teachers' views of curriculum programs more in-depth. In this respect, the general aim of the dissertation – that resulted from our decision to focus on fractions – was to analyze preservice teachers' knowledge of fractions and to analyze how fractions are taught in Flanders. In Chapter 1, four research objectives were introduced related to the aims of the dissertation. These research objectives were addressed in the empirical studies reported in Chapter 2 to 5.

RO 1. Analysis of the prevalence of mathematical difficulties in elementary education as reflected in teacher ratings

RO 2. Analysis of teachers' views of curriculum programs

RO 3. Analysis of preservice teachers' knowledge of fractions

RO 4. Analysis of the teaching of fractions

In Chapter 2 we reported on an exploratory study set up to gain insight in mathematical difficulties as reported by the teacher. The main focus of this study was related to difficulties inherent to mathematics and enabled us to present a grade-specific overview of difficult subjects in the mathematics curriculum (RO 1). In addition, we also focused on difficulties that stemmed from the curriculum programs. We elaborated on this in Chapter 3, where we studied teachers' views of curriculum programs (RO 2). In Chapter 4 we studied Flemish preservice teachers' common content

and specialized content knowledge of fractions (RO 3). Finally, in Chapter 5 we analyzed to which extent elementary school teachers in Flanders were teaching fractions for conceptual understanding (RO 4) by means of an observational study.

2.1. RO 1. Analysis of the prevalence of mathematical difficulties in primary education as reflected in teacher ratings

This exploratory study aimed to provide insight in mathematical difficulties (a) inherent to mathematics and as such, difficult for students to learn and (b) related to the curriculum program, as reported by the teachers on a 5-point Likert scale. Data were collected by means of three grade-specific questionnaires. We developed these questionnaires based on the three predominant curricula in Flemish elementary education. In total, 918 teachers of 243 schools completed the questionnaires. We used quantitative research techniques to analyze the data.

Main findings

The findings revealed that some subjects were reported by the teachers to be difficult in every grade in which the subject was listed in the curriculum, namely fractions (1^{st} to 6^{th} grade), divisions (1^{st} to 6^{th} grade), numerical proportions (3^{rd} to 6^{th} grade), scale (5^{th} to 6^{th} grade) and almost every problem solving item (1^{st} to 6^{th} grade). Items that were considered to be difficult in at least half of the grades in which the subject was listed in the curriculum were estimation (4^{th} – 6^{th} grade), long divisions (5^{th} and 6^{th} grade), length (2^{nd} to 4^{th} grade), content (1^{st}, 2^{nd}, 3^{rd}, 5^{th}, 6^{th} grade), area (4^{th} and 5^{th} grade), time (1^{st} to 5^{th} grade), and the metric system (5^{th} grade).

Furthermore, it was established that the proportion difficult subjects was the highest in the second grade, followed by the first, fifth, fourth, third, and sixth grade. The proportion difficult subjects ranged from 23% to 49%, which let us to conclude that, in general, mathematics is a difficult area to learn for elementary school students.

Thirdly, as we asked the teachers to report on the applied curriculum program, we were able to present an overview of the frequently used curriculum programs in Flanders. Five curriculum

programs[7] were used by 89% of the respondents: 'Eurobasis' (27%), 'Zo gezegd, zo gerekend!' (25%), 'Kompas' (15%), 'Nieuwe Tal-rijk' (12%), and 'Pluspunt' (10%).

Finally, with regard to the reported difficulties related to the curriculum program, the findings suggested differences between the curriculum programs. This is more deeply analyzed in Chapter 3.

Strengths, limitations, implications

A major strength of the study is the strong inclusion of teachers' perspective which is – notwithstanding the prevailing extended view on teacher professionalism – exceptional rather than standard (Bryant et al., 2008). However, within the strong focus on the teachers' perspective, we did not analyze important aspects such as teachers' practices and students' outcomes (Correa, Perry, Sims, Miller, & Fang, 2008; Pajares, 1992; Phillipp, 2007; Staub & Stern, 2002). Future research could therefore apply a more integrated approach and combine teacher knowledge, teacher practices, and student outcomes in one single study.

As reported, we used quantitative techniques to analyze the data. Given the large sample size, this was helpful to provide a general picture. A qualitative research approach, however, could complement this study by going more deeply into it. Instead of merely collecting teacher ratings of difficulties for students, teachers in can also be asked to make this explicit and to illustrate what exactly causes the difficulties.

This study was exploratory in nature and its implications related primarily to the upset of the dissertation. A first implication was related to the subject of this dissertation. As fractions were consistently reported by the teachers as being difficult for their students, and as students' performance results reveal the same pattern (Ministry of the Flemish Community Department of Education and Training, 2004, 2010), we decided to focus further on fractions in Chapter 3 and Chapter 4. Secondly, we also decided to go more deeply into the difficulties related to the curriculum program; this is done

[7] Kompas is an updated version of Eurobasis. At the moment this study was set up, no version was yet available of Kompas for 4th, 5th and 6th grade.

in Chapter 3 where we used the related teacher ratings as an indicator of their views toward curriculum programs.

2.2. RO 2. Analysis of teachers' views of curriculum programs

Based on the outcomes in Chapter 2, we decided to analyze the teacher ratings of their curriculum programs more deeply in Chapter 3. In this study, we used teacher ratings as a measure for their views toward curriculum programs. A subsample of Chapter 2 was included in this study (n = 814): only teachers working with one of the five most frequently used curriculum programs were included in the study.

Research stresses the importance of variables mediating between the written, the intended, and the enacted curriculum (Atkin, 1998; Christou, Eliophotou-Menon, & Philippou, 2004; Macnab, 2003; Stein et al., 2007). Teachers' orientations toward curriculum are regarded as such a mediating variable. These orientations influence how teachers engage with the materials and use them in teaching (Remillard & Bryans, 2004). Teachers' orientations toward curriculum reflect teachers' ideas about mathematics teaching and learning, teachers' views of curriculum materials in general, and teachers' views of the particular curriculum they are working with. Whereas research pointed out that the unique combination of these ideas and views of teachers (i.e., their orientations toward curriculum) influences the way they use the curriculum, the study also revealed that the ideas about mathematics teaching and learning and views of curriculum materials in general and of the particular curriculum they are working with on their own also proved to be a mediating variable (Remillard & Bryans, 2004). In addition to the study of teachers' views (n = 814), we also studied in a subsample of the teachers (n = 89) whether or not the performance results of their students (n = 1579) differed significantly based on the curriculum programs used in the classroom. This enabled us to analyze whether differences in teachers' views of curriculum programs are related to differences in students' performance results or not.

Main findings

The results revealed significant differences in teachers' views of curriculum programs, based on the curriculum program used in class. We observed clear patterns in teachers' views of curriculum programs. Teachers' views of curriculum programs were more positive in case the curriculum programs address one content domain of mathematics (numbers and calculations, measurement, geometry) per lesson and provide more support for the teachers, such as providing additional materials, a more detailed description of the course, additional didactical suggestions, and theoretical background knowledge about mathematics. Whereas we were not able to control for other variables, the results suggested that curriculum programs matter with regard to teachers' views of curriculum programs.

The study further revealed that students' performance results did not vary significantly based on the curriculum program used in class. This underlines the fact that teachers' views of curriculum programs is but one mediating variable and that in addition, it would be useful to include other mediating variables in the analysis, such as teachers' beliefs about mathematics teaching and learning, teachers' views of curriculum materials in general, teachers' knowledge, teachers' professional identity, teacher professional communities, organizational and policy contexts, and classroom structures and norms (Remillard & Bryans, 2004; Stein et al., 2007).

Strengths, limitations, implications

To our knowledge no previous studies combined an analysis of teachers' views of curriculum programs and related these to students' performance results on such a large scale. Whereas this approach enabled us to look for differences in teachers' views that are most likely not based on coincidence, the large-scale study also limited the grain size to study teachers' views. Further, though the sampling approach helped to involve a large set of respondents, it was not based on random selection (the project was announced through different media and if teachers showed interest, they were contacted by the researcher). As such, we were not able to counter a potential sampling bias in the study, including teachers who already developed clear and explicit views of curriculum programs.

Thirdly, given that this study was part of a larger research project that centered on mathematical difficulties, we analyzed teachers' views of curriculum programs by building on their experiences with the curriculum programs and by focusing on learning difficulties related to the curriculum programs. Future studies might shift the focus on the strengths of curriculum programs instead of focusing on the weaknesses.

The observation of a discrepancy between teachers' views and students' performance results stressed the need for observational studies about the way teachers actually implement curriculum programs. Observational studies could reveal if teachers are compensating for anticipated difficulties related to curriculum programs. In line with this implication, we included an observational study related to the teaching of fractions in Chapter 5.

2.3. RO 3. Analysis of preservice teachers' knowledge of fractions

Building on the work of Shulman and colleagues (Shulman, 1986a, 1987; Wilson, Shulman, & Richert, 1987), Ball, Hill and colleagues (Ball, Thames, & Phelps, 2008; Hill & Ball, 2009; Hill, Ball, & Schilling, 2008) analyzed the mathematical knowledge needed to teach mathematics. Their findings pointed at two domains of content knowledge: *common content knowledge* and *specialized content knowledge*. *Common content knowledge* refers to knowledge that is not unique to teaching. Teachers need to be able to multiply two fractions, but also in other professions this kind of knowledge is needed. This kind of knowledge plays a crucial role in the planning and carrying out of instruction (Ball et al., 2008) and is still considered to be a cornerstone of teaching for proficiency (Kilpatrick et al., 2001). *Specialized content knowledge* refers to the mathematical knowledge and skill unique to teaching (Ball et al., 2008). For instance, teachers must be able to explain why you multiply both the numerators and denominators when multiplying fractions, whereas for others it is sufficient to be able to perform the multiplication without being able to explain the rationale behind the rule. In their study, Ball et al. (2008) were surprised about the important presence of teachers' specialized content knowledge. In this study, we analyzed preservice teachers' content knowledge of fractions.

One approach to investigate what effective teaching requires in terms of content knowledge, is reviewing studies related to students' understanding to determine the mathematics difficulties encountered by students (Ball et al., 2008; Stylianides & Ball, 2004). Therefore, in this study we began by reviewing literature related to students' knowledge of fractions. The review revealed a gap between students' procedural and conceptual knowledge of fractions (Aksu, 1997; Bulgar, 2003; Post, Cramer, Behr, Lesh, & Harel, 1993; Prediger, 2008), resulting in a rather instrumental understanding of the procedures (Aksu, 1997; Hecht et al., 2003; Prediger, 2008). Regarding the conceptual understanding of fractions, research pointed at a multifaceted nature of fractions (Baroody & Hume, 1991; Cramer, Post, & delMas, 2002; English & Halford, 1995; Grégoire & Meert, 2005; Kilpatrick et al., 2001) and distinguished five sub-constructs to be mastered by students in order to develop a full understanding of fractions (Charalambous & Pitta-Pantazi, 2007; Hackenberg, 2010; Kieren, 1993; Kilpatrick et al., 2001; Lamon, 1999; Moseley et al., 2007). Related sudies revealed that students were most successful in assignments regarding the part-whole sub-construct, and that in general, they had too less knowledge of the other sub-constructs; especially knowledge regarding the measure sub-construct seemed to be lacking (Charalambous & Pitta-Pantazi, 2007; Clarke, Roche, & Mitchell, 2007; Hannula, 2003; Martinie, 2007).

In the present study, we centered on 184 first-year and 106 last-year preservice teachers' common content knowledge as measured by their conceptual and procedural knowledge of fractions on the one hand and on preservice teachers specialized content knowledge as measured by their skill in explaining the underlying rationale on the other hand.

Main findings

Preservice teachers' average score for the fractions test was .81 (maximum = 1.00). As the test items were retrieved either from previous tests to measure students' knowledge of fractions or from exercises in mathematics textbooks for students, we concluded that this is not sufficient to teach these contents. This is an important finding given that the Flemish Government stresses that preservice teachers should master at least the attainment targets of elementary education (Ministry of the Flemish

Community Department of Education and Training, 2007). This is also an interesting finding given that research found that this kind of knowledge (i.e., common content knowledge) is important for the planning and carrying out of instruction (Ball et al., 2008). The findings further revealed that preservice teachers' knowledge of fractions mirrored largely students' knowledge of fractions.

The average score of preservice teachers' specialized content knowledge was only .42 (maximum = 2.00). This can be considered to be a low score, that questions preservice teachers' specialized content knowledge level. This is an interesting finding because research points at the differential impact of teachers who have this kind of deeper understanding of the subject (Hattie, 2009). Furthermore, we did not observe significant differences regarding first-year and last-year preservice teachers' common content and specialized content knowledge. Analysis of the fractions-related curriculum in teacher education learned that this is hardly surprising, because only a limited proportion of teaching time in teacher education was spent on fractions.

Strengths, limitations, implications

Research suggests that preservice teachers' knowledge of fractions mirrors similar misconceptions as revealed by research of elementary school students' knowledge of fractions (Newton, 2008; Silver, 1986; Tirosh, 2000). However, previous studies (e.g. Cai & Wang, 2006; Isiksal & Cakiroglu, 2011; Izsak, 2008; Moseley et al., 2007; Newton, 2008) were too narrow in scope to analyze the difficulties that were revealed in our overview of students' understanding of fractions. Therefore, in the current study, we addressed both preservice teachers' procedural and conceptual knowledge (i.e. their common content knowledge). Conceptual knowledge comprised knowledge of the five sub-constructs: part-whole, ratio, division, operator, and number. As research also stressed the importance of teachers' specialized content knowledge (Ball et al., 2008), we also included this aspect in the current study. Furthermore, inclusion of both first-year and last-year preservice teachers made it possible to analyze to some extent the role of teacher education in this respect.

The study applied a cross-sectional design, which was useful regarding the data collection. A major drawback is that we were not able to control for differences between both groups of respondents (first-year and third-year preservice teachers). A longitudinal study could tackle this limitation.

As to the implications of the study, the finding that preservice teachers' common and specialized content knowledge were limited and that preservice teachers' common content knowledge mirrored students' knowledge of fractions suggested that, indeed, attempts to augment (preservice) teachers' knowledge might be a fruitful way to increase the mathematics standards expected of students (Jacobbe, 2012; Kilpatrick et al., 2001; Stigler & Hiebert, 1999; Zhou et al., 2006). A second implication relates to the fact that fractions, known to be an important yet difficult subject for students (Akpinar & Hartley, 1996; Behr et al., 1984; Bulgar, 2003; Hecht et al., 2003; Kilpatrick et al., 2001; Kloosterman, 2010; Lamon, 2007; NCTM, 2007; Newton, 2008; Siegler et al., 2010; Van de Walle, 2010), represented only a very small proportion of the curriculum in teacher education. Given that fractions are only one of the many subjects, one can doubt whether it is feasible to prepare preservice teachers to teach every subject in elementary education. A practical alternative, as suggested by the National Mathematics Advisory Panel (2008), might be to focus on fewer teachers who are specialized in teaching elementary mathematics. Another option is to extend teacher education, but, simply increasing the number of lessons in teacher education that focus on fractions would be insufficient; preservice teachers should be provided with mathematical knowledge useful to teaching well (Kilpatrick et al., 2001). Teacher education programs could then pay considerable attention to the aspects that constitute teachers' mathematical knowledge for teaching (Ball et al., 2008; Hill & Ball, 2009; Hill, Ball, et al., 2008). Finally, the outcomes of the study relate to teacher education in general. It suggests that the move from teacher "training" to teacher "education", initiated in the 1980s (Verloop, Van Driel, & Meijer, 2001), has not yet been implemented. Preservice teachers seemed to be able to replicate most of the procedures they have been taught, but they are not 'empowered' with a deeper understanding (Darling-Hammond, 2000).

2.4. RO 4. Analysis of the teaching of fractions

This study built on Chapter 2 in its focus on fractions, and on Chapter 3 in its focus on the enacted curriculum. By analyzing how fractions were taught in Flanders, this study addressed the call for a greater focus on the teaching of fractions (Lamon, 2007), and within that, a response to the call for more attention to the development of conceptual understanding of fractions (Siegler et al., 2010). We built on curriculum research that identifies the teacher as a central actor in the process of transforming curriculum ideals (Lloyd et al., 2009; Stein et al., 2007). This implies acceptance of a difference between the curriculum as represented in instructional materials and the curriculum as enacted during lessons. Therefore, we analyzed both the teacher's guide and the enacted curriculum. We did so by analyzing mathematical tasks, broad units of a classroom activity that aim to focus students' attention on a specific mathematical idea. In total, 88 mathematical tasks were analyzed: 24 mathematical tasks as represented in the teacher's guide, 24 mathematical tasks as set up by the teacher, and 40 tasks as enacted through individual guidance by the teacher.

Main findings

The findings of the study suggested that teachers in Flanders teach fractions in a way that supports students' procedural understanding rather than their conceptual understanding of fractions. This was evident in the structure of the lessons and in the features of the analyzed tasks.

The structure of the lessons can be characterized as teacher-directed instruction followed by a substantial amount of practice of a similar set of problems completed by students on their own, and as such, did not reflect a way of teaching that is considered to support students' conceptual understanding (Hiebert & Grouws, 2007; Stein, Grover, & Henningsen, 1996). The majority of mathematical tasks possessed both features that facilitated (focus on multiple solution strategies and multiple representations) and features that did not facilitate students' conceptual understanding of fractions (remaining in the abstract world of mathematics, absence of strong collaboration between students, not linking representations to each other, absence of requirement of justification of the solution method).

Moreover, whereas the results revealed a consistency in task features as the task moved from presented in the teacher's guide to set up by the teacher, the results also revealed a sharp decline in task features that related to students' conceptual understanding as instruction moved from tasks as set up by the teacher to enactment through individual guidance provided by the teacher. In this respect, our findings corroborate prior research that maintenance of demanding features is difficult (Hiebert et al., 2003; Stein et al., 1996) also in the teaching of fractions. It also illustrates that the problem of maintenance of demanding features remains a persistent problem.

Finally, the study revealed that the orientation toward conceptual understanding differed to some extent according to the curriculum program used by the teacher, but mainly to the mathematical idea that was stressed. Mathematical tasks related to fractions and decimals were consistently coded as less supporting students' conceptual understanding as compared to tasks that related to comparing and ordering fractions and equivalent fractions.

Strengths, limitations, implications

Following the recommendations of Hiebert and colleagues regarding the analysis of teaching (Hiebert & Grouws, 2007; Stigler, Gallimore, & Hiebert, 2000), we opted for the analysis of video data instead of for survey questionnaires or non-registered classroom observations. This enabled us to go back to the data whenever needed. Further, it facilitated reaching an acceptable level of inter-rater reliability, and as such, the use of video data had advantages in terms of validity and reliability. Guided by previous research, we analyzed mathematical tasks (Stein et al., 1996; Stein et al., 2007; Stein, Smith, Henningsen, & Silver, 2000). These were broad units of a classroom activity that aim to focus students' attention on a specific mathematical idea. In doing so, we met the quest of Hiebert and colleagues (Hiebert et al., 2003; Hiebert & Grouws, 2007; Stigler & Hiebert, 1999), who argue that broad units of analysis are preferred, given the complex nature of teaching.

Furthermore, we applied one unique coding scheme to analyze both the written and the enacted curriculum, and as such, addressed several aspects of the curriculum chain. This is in correspondence

with Stein et al. (2007) who stated that the research field would benefit from establishing common structures for examining both the written and the enacted curriculum.

Some limitations regarding the study need to be acknowledged as well. Although this study addressed both the written and the enacted curriculum, we did not examine the entire curriculum chain, from written curriculum over intended curriculum and enacted curriculum to student learning, as recommended by Stein et al. (2007). Moreover, in addition to the video data, interviews and stimulated recall interviews with the teachers, and the inclusion of information about students' background might have strengthened the study.

In our response to the call for more attention to the development of conceptual understanding of fractions (Siegler et al., 2010), we analyzed the data by focusing on features that were considered to facilitate students' conceptual understanding of mathematics in general (Hiebert & Grouws, 2007; Stein et al., 1996). We did not, however,analyze the lessons from a fractions-specific didactical point of view. It might be useful to include this in future research.

As to the implications of the study, the finding that teachers in Flanders taught fractions in a way that did support students' procedural understanding rather than their conceptual understanding indicates that despite a worldwide adoption of standards that stress the importance of teaching mathematics for conceptual understanding (Bergqvist & Bergqvist, 2011; Lloyd et al., 2009; NCTM, 2000; Verschaffel, 2004), at least with regard to the teaching of fractions, there seems to be a wide gap between theory and practice. Related staff development efforts, as recommended by Stein et al. (1996), might be a means to close this gap.

The observation of a decline in features that might facilitate students' conceptual understanding as the instruction moved from task as set up by the teacher to the task as enacted through individual guidance by the teacher, suggested a differentiation in instruction. Some students forgot or did not understood the conceptual meaning of the task as set up during instruction. Since teachers generally focussed on immediately refreshing the rule and only referring to that rule during individual guidance, these students might, experience fractions as learning and applying rules rather than understanding

what they are doing. Consequently, there appears to be a differentiation in instruction as compared to students who did understood the conceptual meaning during task set up.

Finally, the finding that the orientation toward conceptual understanding differed according to the mathematical idea that was stressed, suggests that research into alternative approaches for teaching fractions as recommended by Siegler et al. (2010) might target explicitly the relationship between fractions and decimals.

3. General discussion

We started the dissertation with the statement that fractions are an essential but difficult subject to learn (e.g. Behr, Harel, Post, & Lesh, 1992; Lamon, 2007; Newton, 2008; Siegler et al., 2010). This, in addition to the outcomes of the exploratory study reported in Chapter 2, formed the fundament to focus on fractions in this dissertation. In addition, also based on the outcomes of Chapter 2, we analyzed teacher ratings of the curriculum programs more deeply in Chapter 3. We used these ratings as an indicator of teachers' views of curriculum programs. Regarding the research on fractions, we pointed at the relevance of research focusing on preservice teachers' knowledge of fractions (Borko et al., 1992; Jacobbe, 2012; Ma, 1999; Moseley et al., 2007; Newton, 2008; Siegler et al., 2010; Stigler & Hiebert, 1999; Toluk-Ucar, 2009; Zhou et al., 2006). We also underlined the importance and relevance of research addressing the teaching of fractions explicitly (Lamon, 2007; Lloyd et al., 2009; Siegler et al., 2010; Stein et al., 2007). Therefore, we addressed Flemish preservice elementary school teachers' common and specialized content knowledge of fractions (Chapter 4) and their approach in teaching fractions in Flanders (Chapter 5).

The exploratory study in Chapter 2 enabled us to provide a grade-specific overview of subjects in the mathematics curriculum that teachers perceive as difficult for the students to learn. The teachers further also rated the extent to which the curriculum programs caused difficulties in learning the subjects. The results suggested differences that could be related to the curriculum programs.

In Chapter 3, we elaborated more on teacher ratings of difficulties in student learning produced by the curriculum programs. This was used as an indicator for teachers' views of curriculum programs,

which can be considered as a mediating variable influencing how the curriculum changes as it moves from written, over intended, to enacted curriculum and to student learning (Remillard & Bryans, 2004; Stein et al., 2007). The results revealed patterns in teachers' views of curriculum programs. In general, teachers' views were more positive in case the curriculum programs addressed one content domain of mathematics per lesson and provided more support (additional materials, a more detailed description of the lesson, additional didactical suggestions, mathematical background knowledge). Differences in teachers' views of curriculum programs were not related to differences in students' performance results, which stresses the need to include other mediating variables and to carry out observational studies about the way teachers implement curriculum programs.

In Chapter 4, we analyzed preservice teachers' common and specialized content knowledge. Both forms of knowledge are considered to be an important aspect of knowledge needed to teach mathematics (Ball et al., 2008; Hill & Ball, 2009; Hill, Ball, et al., 2008). Common content knowledge refers to a kind of knowledge not unique to teaching. Specialized content knowledge refers to knowledge and skill unique to teaching. The findings revealed that preservice teachers' common content knowledge of fractions was not sufficient and largely mirrored students' knowledge of fractions. The findings also revealed that preservice teachers' specialized content knowledge of fractions was below an acceptable level. Furthermore, nor for common content knowledge, nor for specialized content knowledge, we observed significant differences between first-year and last-year preservice teachers. An observation of the mathematics curriculum learned that over the three years, only a very limited proportion of teaching time was spent on fractions. The results questioned the role of teacher education.

In Chapter 5, we analyzed how teachers taught fractions in elementary school. We found that, more than ten years after Flanders adopted standards in mathematics that stress the importance of conceptual understanding (Ministry of the Flemish Community Department of Education and Training, 1999; Verschaffel, 2004), teachers were teaching fractions in a way that supported students' procedural understanding rather than their conceptual understanding. The results further suggested a differentiation in instruction for students who did not understand or could not remember the learned

content during instruction and needed help from the teacher during practice. Finally, the study also revealed a differentiation in orientation toward conceptual understanding, based on the mathematical idea that was stressed in the task.

4. General limitations and directions for future research

As also referred to in the acknowledgement, and as sung by the famous Canadian poet Leonard Cohen "There is a crack in everything, that's how the light gets in." (Cohen, 1992). Applying this metaphor to the current dissertation sheds lights on the limitations (the cracks) and on the directions for future research (the light that gets in). As such, the results of this dissertation must be considered in the light of a number of limitations to be addressed in future research. Some limitations were already addressed in relation to the main findings as discussed above. In this part of the dissertation, we will discuss the overarching limitations regarding the study variables and the research design.

4.1. Study variables

In this dissertation, two major groups of variables were addressed. On the one hand we focused on variables related to the use of curriculum programs; on the other hand we addressed variables related to teacher knowledge. For both groups of variables, we have to acknowledge some limitations, which we outline below.

In this doctoral dissertation, we addressed several of the temporal phases of curriculum use as depicted in Figure 1. In Chapter 5, we analyzed the teacher's guide of the curriculum programs regarding the conceptual nature of the mathematical tasks (i.e. the written curriculum) on the one hand and the mathematical tasks as set up during whole-class instruction by the teacher and enacted through individual teacher guidance (i.e. the enacted curriculum). Further, we addressed teachers' views of curriculum programs (Chapter 3), a mediating variable with regard to the transformations in the phases of curriculum use. Correspondingly, we studied whether differences in teachers' views are related with students' performance results.

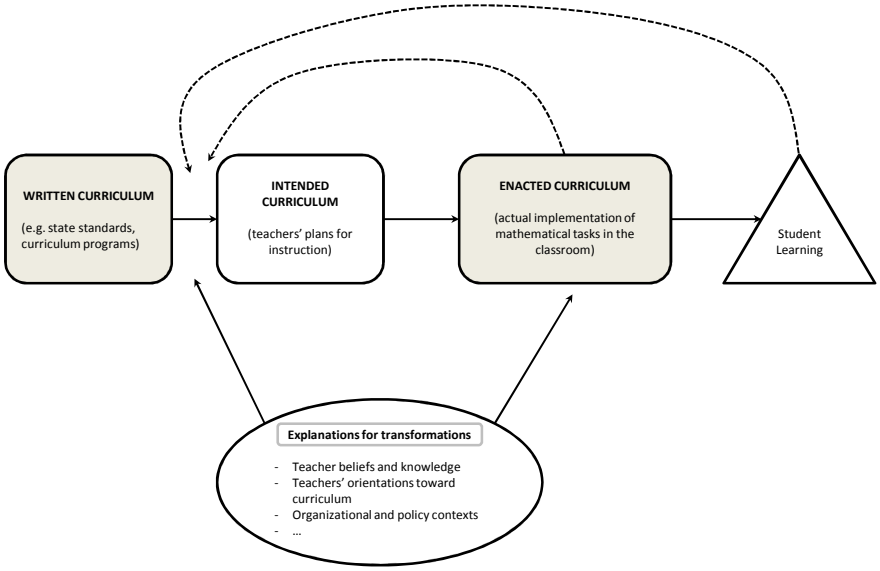

Figure 6. Temporal phases of curriculum use (Stein et al., 2007, p. 322)

In this respect, we addressed several parts of the temporal phases of curriculum use. More particularly, the present dissertation shed light on consistencies and inconsistencies between the written and enacted curriculum, and revealed differences in teachers' views of curriculum programs based on the curriculum program used in class. However, we failed to address all of the temporal phases of curriculum use, as recommended by Stein et al. (2007). The intended curriculum was not included in the studies and only one mediating variable with regard to the transformations in the phases of curriculum use was addressed. Also the impact of the enacted curriculum on student learning was not studied. Therefore, future research might elaborate on this more deeply by analyzing the written, intended, enacted curriculum, and mediating variables, and its impact on student learning for one given set of participants.

Another focus of the dissertation comprised teachers' knowledge for teaching mathematics. In this respect, we built on the work of Ball, Hill, and colleagues (Ball et al., 2008; Hill & Ball, 2009; Hill, Ball, et al., 2008), who in turn built on Shulman's attention to the content specific nature of knowledge for teaching (Shulman, 1986a, 1987; Wilson et al., 1987). Arguing that there is a need for a greater precision about what is meant with content knowledge and pedagogical content knowledge, Ball, Hill, and colleagues are developing a practice-based theory of content knowledge for teaching

mathematics. By using the term ‘mathematical knowledge for teaching’, they focus on the mathematical knowledge needed to carry out the work of teaching mathematics. Figure 2 presents the different domains in mathematical knowledge for teaching. Ball, Hill, and colleagues more particularly point at two major domains: subject matter knowledge and pedagogical content knowledge. They further divide subject matter knowledge in common content knowledge (mathematical knowledge needed by individuals in diverse professions), specialized content knowledge (mathematical knowledge not needed in settings other than teaching), and knowledge at the mathematical horizon (knowledge of how mathematical topics are related over time). They further divide pedagogical content knowledge in content knowledge intertwined with knowledge of how students learn a specific content (e.g. “Teachers must anticipate what students are likely to think and what they will find confusing”; Ball et al., 2008, p. 401), content knowledge intertwined with knowing about teaching (e.g. “Teachers evaluate the instructional advantages and disadvantages of representations used to teach a specific idea and identify what different methods and procedures afford instructionally”; Ball et al., 2008, p. 401), and knowledge of content and curriculum (e.g. familiarity with the curriculum, knowledge of alternative curricula; Shulman, 1986b).

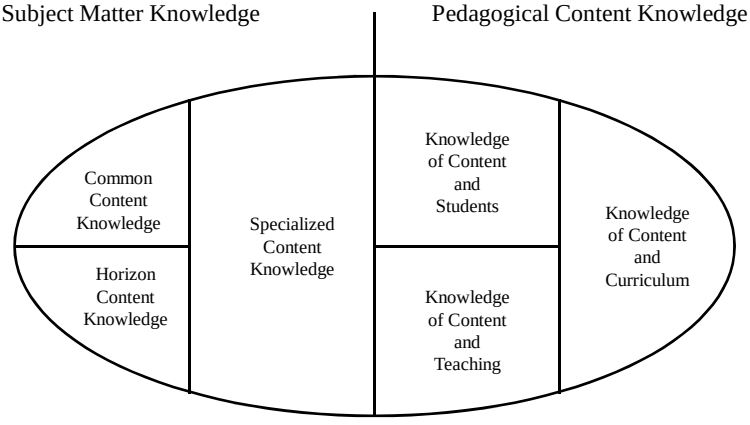

Figure 7. Domains of Mathematical Knowledge for Teaching (Ball et al., 2008, p. 403)

Whereas the conceptualization of mathematical knowledge for teaching (Ball et al., 2008; Hill & Ball, 2009; Hill, Ball, et al., 2008) helped us to get a grasp on the multidimensional character of knowledge for teaching, some warrants need to be taken into account. First, the research on mathematical knowledge for teaching is work in progress and without any doubt revealed that knowledge for

teaching mathematics is multidimensional. Further research is, however, needed to confirm the current findings (Ball et al., 2008). Further, we have to acknowledge that some situations might be managed using different kinds of knowledge (Ball et al., 2008). Whereas we hypothesized that we addressed teachers' knowledge of content and students (pedagogical content knowledge) to provide a grade-specific overview of difficult subjects of the mathematics curriculum (Chapter 2), it might be possible that some teachers leaned solely on their content knowledge of mathematics to decide upon the intrinsic difficulties of mathematical content. Moreover, research findings suggest that even knowledge of content and students is multidimensional (Hill, Blunk, et al., 2008).

Another remark relates to both research on teachers' use of curriculum programs and research on knowledge needed to teach mathematics. As reflected and applied in this dissertation, both fields of research largely developed in parallel, whereas in the practice of teaching, both are related to each other and impact the quality of instruction (Charalambous & Hill, 2012). Whereas both research fields acknowledged the added value of each other, research that addressed both curriculum programs and teacher knowledge and its impact on quality of instruction was virtually nonexistent (Charalambous & Hill, 2012). A special issue of Journal of Curriculum studies, published recently (August 23rd, 2012), addressed this shortcoming, and set up initial steps in combining both fields of research. The findings of these studies suggested that teacher knowledge and curriculum programs have a unique and a joint contribution to the quality for teaching, and that other factors like teachers orientations toward mathematics and mathematics teaching mediated the contribution of teacher knowledge and curriculum programs on the quality of instruction (Charalambous & Hill, 2012; Charalambous, Hill, & Mitchell, 2012; Hill & Charalambous, 2012a, 2012b; Lewis & Blunk, 2012; Sleep & Eskelson, 2012). As such, these findings underline the complex nature of teaching (e.g. Hiebert & Grouws, 2007; Stein et al., 2007) and add to the suggestion of Stein et al. (2007) to address all phases of curriculum use, to do so including teacher knowledge. Also in the current dissertation, this might have been useful.

Finally, in our aim to provide a general picture of teachers' views of curriculum programs, of preservice teachers' knowledge of fractions, and of teaching fractions in Flanders, contextual

variables were not explicitly addressed in the studies. In this respect, research (Cobb, McClain, Lamberg, & Dean, 2003) pointed at the potential impact of professional communities on supporting teachers to teach with curriculum programs that address the kind of mathematics as entailed by the mathematical standards currently applied in many countries (Bergqvist & Bergqvist, 2011; Lloyd et al., 2009; NCTM, 2000; Verschaffel, 2004). Further, the literature also point at the impact of the school context on beginning teachers' motives for applying innovative instructional strategies in class (Ruys, 2012). Consequently, it is thus advisable to include variables related to the school context in longitudinal studies that span both preservice and inservice teachers.

4.2. Research design

We already referred to the fact that a longitudinal study of preservice teachers' knowledge of fractions has advantages as compared to the cross-sectional design we applied in Chapter 4. We can elaborate further on that by arguing that it might have been useful to follow up the development of these preservice teachers' knowledge during their first years after entering the teaching profession. This analysis of the development of their mathematical knowledge for teaching in combination to their use of curriculum programs and its impact on instruction, has the potential to add significantly to the research as plead for by Hill and Charalambous (Charalambous & Hill, 2012; Hill & Charalambous, 2012a).

Second, whereas the sample sizes in Chapters 2, 3, and 4 were reasonably large, the sample size in Chapter 5 comprised 24 lessons on fractions taught by 20 teachers. The number of observed lessons enabled us to construct a picture of how fourth-grade teachers in elementary school were teaching fractions, but inclusion of the whole range of years (grade 1 – grade 6) in future research might result in a richer picture of teaching fractions throughout elementary school.

Finally, the present dissertation was especially designed from a quantitative research paradigm. Whereas this helped us to provide a general picture of teachers' views of curriculum programs, of preservice teachers' knowledge of fractions, and of teaching fractions in Flanders, this inevitably also

resulted in a loss of information. Future research could apply a mixed-method design, and combine quantitative with qualitative studies.

5. Implications of the findings

5.1. Implications for empirical research

Building on the main research findings, the following implications for empirical research can be formulated.

On the basis of the outcomes of the study reported in Chapter 2, we decided to focus on fractions in the following chapters of the present dissertation. However, Chapter 2 revealed that other subjects (i.e. divisions, time, estimation, content, and length) were consistently rated by teachers as being difficult for their students as well. Therefore, future research might also target these subjects and apply both research lines addressed in the current dissertation (i.e. mathematical knowledge for teaching and teachers' use of curriculum materials) in the study of these subjects.

Further, the study reported in Chapter 3 revealed differences in teachers' views depending on the curriculum program used in class. These differences were however not related to differences in students' performance. These results stress the importance for future research to include a combination of variables that might mediate between the phases of curriculum use. In this respect, in a case study of 8 teachers using the same curriculum program, Remillard and Bryans (2004) already pointed at the added value of combining several mediating variables. The findings of Chapter 3 suggest that it might be a fruitful way for future research also to include a combination of mediating variables and to analyze their impact by studying different groups of teachers and curriculum materials.

In accordance to claims that stress the important role of teacher education in the development of teachers' knowledge of fractions (Borko et al., 1992; Ma, 1999; Newton, 2008; Toluk-Ucar, 2009; Zhou et al., 2006), the study in Chapter 4 addressed first-year and last-year preservice teachers' content knowledge of fractions. The study revealed that preservice teachers' common content and specialized content knowledge of fractions was limited, and thus, underlined the finding that it is a

common misconception that school mathematics is fully understood by the teachers and that it is easy to teach (Ball, 1990; Jacobbe, 2012; NCTM, 1991; Verschaffel, Janssens, & Janssen, 2005). As such, future research might address preservice teachers' development of mathematical knowledge for teaching fractions as well as other mathematics subjects (See Chapter 2) more deeply.

The finding in Chapter 5 that more than 10 years after the adoption of standards stressing the importance of teaching mathematics for conceptual understanding (Verschaffel, 2004), the teaching of fractions in Flanders still mainly focuses on students' procedural understanding, stress the need to carry out more research to better understand how the curriculum unfolds from the written text to the enactment in class. The study in Chapter 5 further suggests that studies related to the effectiveness of alternative ways of teaching fractions as recommended by Siegler et al. (2010), might select carefully which aspect of fractions they want to study, since the results illustrated that the orientation toward conceptual understanding differed based on the mathematical idea that was stressed. Finally, the findings corroborate prior research that maintenance of demanding features is difficult (Hiebert et al., 2003; Stein et al., 1996) also in the teaching of fractions.

5.2. Implications for practice and policy

The findings in Chapter 3 revealed that teachers' views of curriculum programs were more positive in case the programs were provided with teacher support, such as additional materials, detailed descriptions of each 'course', additional didactical suggestions and theoretical and mathematical background knowledge, and in case the lessons addressed one content domain. This finding might inform school teams in their choice for a specific curriculum program. This might also inspire curriculum program designers and publishers.

It is often heard that the knowledge level of the entrants in teacher education is decreasing. Surveys related to teacher education preparing future elementary school teachers showed that, prior to entering teacher education, about half of the candidates followed an academic track in secondary education and the other half followed a technical track, not necessarily geared to enter higher education (Ministry of the Flemish Community Department of Education and Training, 2009). The surveys also revealed that

the success rate is higher for the candidates who followed an academic track in secondary education. These findings are in line with the outcomes of the study in Chapter 4, where track in secondary education differentiates between preservice teachers' knowledge level of fractions. The finding that preservice teachers' common content knowledge of fractions was limited also suggest that the knowledge level of entrants, but also of last-year preservice teachers, is insufficient. This inevitably has its impact on the proportion of teaching time in teacher education that is spent on *teaching* fractions. Teacher education programs in our study spent half of their teaching time of fractions on refreshment of knowledge that elementary school students are expected to master at the end of elementary school. This limits the attention that can be paid on didactics regarding how to teach these contents. Also, over the three years of teacher education, and not taking into account the internships at schools, both teacher education programs involved in the study spent respectively only 5 and 7 hours of their teaching time on fractions (of which, as mentioned above, half of the time focused on refreshing common content knowledge). One can whether this is sufficient to learn to teach fractions in all grades of elementary school. These findings might give impetus to teacher education institutes to reflect on the teaching time devoted to fractions and on how to familiarize preservice teachers with teaching fractions.

Finally, the findings in Chapter 5 shed light on the quest of Stein et al. (1996) for staff development efforts that aim to help teachers to implement tasks in a way that fosters students' conceptual understanding of mathematics (and fractions in particular). Since the findings also revealed that the orientation toward conceptual understanding differed based on the mathematical idea that was stressed, these staff development efforts might target specific aspects of fractions. As such, also these findings might initiate teachers and by extension teacher education to reflect on the prevailing focus on rule learning, which seems to be triggered depending on the mathematical idea that is stressed and on the phase in instruction.

6. Final conclusion

Guided by the outcomes of Chapter 2, this dissertation focused on preservice teachers' knowledge of fractions and on the actual teaching of fractions in Flanders. As an extension of Chapter 2, teachers' views of curriculum programs were studied as well. The main findings, based on the four reported studies, indicate that:

- Fractions is but one subject of the mathematics curriculum that merits further investigation.
- Curriculum programs might influence teaching indirectly.
- Common content knowledge of fractions of beginning and last-year preservice teachers is limited.
- Specialized content knowledge of fractions of beginning and last-year preservice teachers is limited.
- The teaching of fractions in Flanders encourages students' procedural understanding, rather than their conceptual understanding.
- The focus on conceptual understanding of fractions differs according to the mathematical idea that is stressed and according to the phase in instruction.

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Nederlandstalige samenvatting

Summary in Dutch

Nederlandstalige samenvatting

Academic output

Academic output

Journals

(a1)

Van Steenbrugge, H., Valcke, M., & Desoete, A. (2010). Mathematics learning difficulties in primary education: teachers' professional knowledge and the use of commercially available learning packages. *Educational studies, 36,* 1, 59-71, DOI: 10.1080/03055690903148639

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Van Steenbrugge, H., Valcke, M., Lesage, E., Desoete, A. & Burny, E. Preservice elementary school teachers' knowledge of fractions: A mirror of students' knowledge? Manuscript submitted for publication in *Journal of curriculum studies*

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Van Steenbrugge, H. (2011). *Een bevraging bij leerkrachten: wat zijn moeilijke onderwerpen uit het wiskundecurriculum, en zijn er antwoordtendensen vast te stellen tussen de wiskundemethoden? [These are the words of teachers: mathematical difficulties, and curriculum programs]*. Presentation at 'Dag van het remediërend onderwijs' [Day of remedial teaching], Heusden-Zolder, Belgium, March, 1.

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