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A potential setup for perturbative confinement

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Abstract

A few years ago, 't Hooft suggested a way to discuss confinement in a perturbative fashion. The original idea was put forward in the Coulomb gauge at tree level. In recent years, the concept of a nonperturbative short distance linear potential also attracted phenomenological attention. Motivated by these observations, we discuss how a perturbative framework, leading to a linear piece in the potential, can be developed in a manifestly gauge and Lorentz invariant manner, which moreover enjoys the property of being renormalizable to all orders. We provide an effective action framework to discuss the dynamical realization of the proposed scenario in Yang-Mills gauge theory.

1 Motivation

In [1, 2, 3], 't Hooft launched the idea that confinement can be looked upon as a natural renormalization phenomenon in the infrared region of a Yang-Mills gauge theory. He employed the Coulomb gauge, $\partial_i A_i = 0$, in which case the kinetic (quadratic) part of the gauge field action becomes

$$S_{YM} = -\frac{1}{4}\int \mathrm{d}^4x F_{\mu\nu}^2 \to \int \mathrm{d}^4x \left(-\frac{1}{2}(\partial_i A_j)^2 + \frac{1}{2}(\partial_0 A_j)^2 + \frac{1}{2}(\partial_j A_0)^2\right) . \tag{1.1}$$

The usual (classical) Coulomb potential is recovered as the solution of the equation of motion for A_0 in the presence of static charges with strength α_s (= source terms) separated from each other by a vector $\mathbf{r}$,

$$V_{Q\overline{Q}}(\mathbf{r}) = -\frac{\alpha_s}{r} . \tag{1.2}$$

He then proposed that some (unspecified) infrared quantum effects will alter the kinetic part into

$$\int \mathrm{d}^4x \left(-\frac{1}{2}(\partial_i A_j)^2 + \frac{1}{2}(\partial_0 A_j)^2 + \frac{1}{2}(\partial_j A_0)^2\right) + \int \mathrm{d}^4x \left(-\frac{1}{2}\partial_j A_0 \frac{2\sigma/\alpha_s}{-\partial_j^2 + 2\sigma/\alpha_s}\partial_j A_0\right) . \tag{1.3}$$

As a consequence, the Coulomb potential in momentum space gets modified into

$$V_{Q\overline{Q}}(\mathbf{p}) = -\frac{4\pi\alpha_s}{\mathbf{p}^2} - \frac{8\pi\sigma}{\mathbf{p}^4} , \tag{1.4}$$

which corresponds to

$$V_{Q\overline{Q}}(\mathbf{r}) = -\frac{\alpha_s}{r} + \sigma r , \tag{1.5}$$

which is nothing else than a confining potential of the Cornell type [4]. We made use of the well-known identity $\partial_i^2 \frac{1}{r} = -4\pi\delta(\mathbf{r})$, which also allows one to define a regularized version of the Fourier transform of $\frac{1}{\mathbf{p}^4}$, since $\partial_i^2(r) = \frac{2}{r}$. Indeed, calling $f(\mathbf{p})$ the Fourier transform of r, we can write

$$\partial_i^2\partial_i^2 \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} f(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}} = -8\pi \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} , \tag{1.6}$$

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which leads to

$$f(\mathbf{p}) = -\frac{8\pi}{\mathbf{p}^4} \,. \tag{1.7}$$

Of course, this is an appealing idea, at it might give a way to handle confining theories in a relatively "simple" way, modulo the fact that the origin of the parameter (= string tension) σ is still rather unclear. It was argued that the coefficient $\frac{\sigma}{\alpha_s}$ has to be adjusted in such a way that higher order corrections converge as fast as possible [1, 2].

In this work, we intent to set a modest step forward in this program. First of all, we would like to avoid the use of a non-Lorentz covariant gauge fixing as the Coulomb one, in fact, we should rather avoid using any preferred gauge and produce a Lorentz and gauge invariant version of the 't Hooft mechanism. Secondly, in [1, 2] it was assumed that the infrared effects would not reflect on the ultraviolet sector. Here, we can even explicitly prove the ultraviolet renormalizability of the procedure. We also point out shall how it would be possible to dynamically realize this perturbative confinement scenario, starting from the original Yang-Mills action.

Let us also refer to [5], which gives a second motivation for this work. In the phenomenological paper [5], the issue of physical $\frac{1}{q^2}$ power corrections was discussed. Such $\frac{1}{q^2}$ corrections are in principle forbidden to appear in the usual Operator Product Expansion (OPE) applied to physical correlators, since there is no local dimension 2 gauge invariant condensate to account for the quadratic power correction. This wisdom was however challenged in [5], by including nonperturbative effects beyond the OPE level. Next to the motivation based on ultraviolet renormalons and/or approaches in which the Landau pole is removed from the running coupling, which lead to $\frac{1}{q^2}$ uncertainties when studying the correlators, it was noticed that a linear piece survives in the heavy quark potential up to short distances. This means that a Cornell potential (1.5) could also leave its footprints at distances smaller than might be expected. In the meantime, the notion of a short distance linear potential has also been discussed by means of the gauge/gravity duality approach (AdS/QCD), see e.g. [6, 7]. Notice hereby that the string tension at short distances does not have to concur with the one at larger distances [6, 7].

2 Constructing the starting action and some of its properties

We shall work in Euclidean space. We shall make a small detour before arriving to our actual purpose of the note. We start from the usual Yang-Mills action, and we couple the nonlocal gauge invariant operator

$$O(x) = F^a_{\mu\nu}(x)\left[\frac{1}{D^2_\rho}\right]^{ab}(x)F^b_{\mu\nu}(x) \tag{2.1}$$

to it by means of a global "source" J^2, i.e. we consider

$$S_{YM} + S_O \quad = \quad \frac{1}{4}\int \mathrm{d}^4 y F^a_{\mu\nu}F^a_{\mu\nu} - \frac{J^2}{4}\int \mathrm{d}^4 x O(x) \,. \tag{2.2}$$

This particular operator was first put to use in [8, 9] in the context of a dynamical mass generation for $3D$ gauge theories.

We introduced the formal notation $\frac{1}{D^2}$, which corresponds to the (nonlocal) inverse operator of D^2, i.e.

$$\frac{1}{D^2}(x)f(x) \equiv \int \mathrm{d}^4 y \left[\frac{1}{D^2}\right](x-y)f(y) \tag{2.3}$$

for a generic function $f(x)$, whereby

$$D^2(x)\left[\frac{1}{D^2}\right](x-y) = \delta(x-y) \,. \tag{2.4}$$

Imposing a gauge fixing by adding a gauge fixing term and corresponding ghost part S_{gf} to the action

$$S = S_{YM} + S_O + S_{gf} \,, \tag{2.5}$$

it was shown in [10, 11] that the partition function,

$$\int [\mathrm{d}\Phi] e^{-S} \,, \tag{2.6}$$

can be brought in a localized form by introducing a pair of complex bosonic antisymmetric tensor fields $\left(B^a_{\mu\nu},\overline{B}^a_{\mu\nu}\right)$ and of complex anticommuting antisymmetric tensor fields $\left(\overline{G}^a_{\mu\nu},G^a_{\mu\nu}\right)$, both belonging to the adjoint representation, so that the nonlocal action S gets replaced by its equivalent local counterpart[1]

$$S' = \int \mathrm{d}^4x \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{i}{4}J(B-\overline{B})^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{4}\left(\overline{B}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} - \overline{G}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu}\right)\right] , \tag{2.7}$$

such that

$$\int [\mathrm{d}\Phi] e^{-S} = \int [\mathrm{d}\Phi] e^{-S'} . \tag{2.8}$$

The shorthand notation Φ represents all the fields present in S or S'. The covariant derivative is given by

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - g f^{abc}A^c_\mu . \tag{2.9}$$

From now on, we can forget about the original starting point (2.2), and start our discussion from the local action (2.7), whereby J can now also be considered to be a local source $J(x)$, coupled to the operator $(B-\overline{B})^a_{\mu\nu}F^a_{\mu\nu}$.

This is however not the end of the story. It was proven in [10, 11] that S' must be extended in order to obtain a renormalizable action. More precisely, the complete starting action is given by

$$\begin{aligned}
\Sigma = &\int \mathrm{d}^4x \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{iJ}{4}(B-\overline{B})^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{4}\left(\overline{B}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} - \overline{G}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu}\right)\right.\\
&- \frac{3}{8}J^2\lambda_1\left(\overline{B}^a_{\mu\nu}B^a_{\mu\nu} - \overline{G}^a_{\mu\nu}G^a_{\mu\nu}\right) + J^2\frac{\lambda_2}{32}\left(\overline{B}^a_{\mu\nu} - B^a_{\mu\nu}\right)^2\\
&\left. + \frac{\lambda^{abcd}}{16}\left(\overline{B}^a_{\mu\nu}B^b_{\mu\nu} - \overline{G}^a_{\mu\nu}G^b_{\mu\nu}\right)\left(\overline{B}^c_{\rho\sigma}B^d_{\rho\sigma} - \overline{G}^c_{\rho\sigma}G^d_{\rho\sigma}\right) + \varsigma J^4\right] + S_{gf} ,
\end{aligned} \tag{2.10}$$

We shall clarify the significance of the vacuum term ςJ^4, with ς a dimensionless parameter, after (3.1). λ^{abcd} is an invariant rank 4 tensor coupling, subject to the following symmetry constraints

$$\lambda^{abcd} = \lambda^{cdab} \,, \lambda^{abcd} = \lambda^{bacd} , \tag{2.11}$$

which can be read off from the vertex that λ^{abcd} multiplies [10, 11].

In general, an invariant tensor λ^{abcd} is defined by means of [12]

$$\lambda^{abcd} = \mathrm{Tr}(t^a t^b t^c t^d) , \tag{2.12}$$

with t^a the $SU(N)$ generators in a certain representation r. (2.12) is left invariant under the transformation

$$t^a \to U^+ t^a U , \qquad U = \mathrm{e}^{i\omega^b t^b} , \tag{2.13}$$

which leads for infinitesimal ω^a to the generalized Jacobi identity [12]

$$f^{man}\lambda^{mbcd} + f^{mbn}\lambda^{amcd} + f^{mcn}\lambda^{abmd} + f^{mdn}\lambda^{abcm} = 0 . \tag{2.14}$$

It are the radiative corrections which necessitate the introduction of the extra terms $\propto \lambda_{1,2}J^2$, as well as the quartic interaction $\propto \lambda^{abcd}$ [10, 11]. The quantities λ_1 and λ_2 are two a priori independent scalar "couplings".

It can be easily checked that (2.10) is gauge invariant, $\delta_\omega S = 0$, w.r.t. to the infinitesimal gauge variations

$$\delta_\omega A^a_\mu = -D^{ab}_\mu\omega^b \,, \delta_\omega B^a_{\mu\nu} = g f^{abc}\omega^b B^c_{\mu\nu} \,, \delta_\omega \overline{B}^a_{\mu\nu} = g f^{abc}\omega^b \overline{B}^c_{\mu\nu} \,, \delta_\omega G^a_{\mu\nu} = g f^{abc}\omega^b G^c_{\mu\nu} \,, \delta_\omega \overline{G}^a_{\mu\nu} = g f^{abc}\omega^b \overline{G}^c_{\mu\nu} . \tag{2.15}$$

Using a linear covariant gauge,

$$S_{gf} = \int \mathrm{d}^4x \left(\frac{\alpha}{2}b^a b^a + b^a\partial_\mu A^a_\mu + \overline{c}^a\partial_\mu D^{ab}_\mu c^b\right) , \tag{2.16}$$

1. Performing the Gaussian path integration over $(B,\overline{B},G,\overline{G})$ leads back to (2.2).

it was shown in [10, 11] that the action Σ, (2.10), is renormalizable to all orders of perturbation theory, making use of the algebraic formalism and BRST cohomological techniques [13]. Indeed, the action (2.10) enjoys a nilpotent BRST symmetry, generated by

$$
\begin{aligned}
sA_\mu^a &= -D_\mu^{ab}c^b \,, sc^a = \frac{g}{2} f^{abc}c^b c^c \,, sB_{\mu\nu}^a = g f^{abc}c^b B_{\mu\nu}^c \,, s\overline{B}_{\mu\nu}^a = g f^{abc}c^b \overline{B}_{\mu\nu}^c \,, \\
sG_{\mu\nu}^a &= g f^{abc}c^b G_{\mu\nu}^c \,, s\overline{G}_{\mu\nu}^a = g f^{abc}c^b \overline{G}_{\mu\nu}^c \,, s\overline{c}^a = b^a \,, sb^a = 0 \,, s^2 = 0, s\Sigma = 0 \,.
\end{aligned}
\tag{2.17}
$$

Later on, the renormalizability was also confirmed in the more involved maximal Abelian gauge [14].

If we put the source $J = 0$, we expect to recover the usual Yang-Mills theory we started from, see (2.2). Though, the action (2.10) with $J = 0$,

$$
\begin{aligned}
S'_{YM} &= \int d^4x \left[\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{4}\left(\overline{B}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} B_{\mu\nu}^c - \overline{G}_{\mu\nu}^a D_\sigma^{ab} D_\sigma^{bc} G_{\mu\nu}^c \right) \right. \\
&\left. + \frac{\lambda^{abcd}}{16} \left(\overline{B}_{\mu\nu}^a B_{\mu\nu}^b - \overline{G}_{\mu\nu}^a G_{\mu\nu}^b \right) \left(\overline{B}_{\rho\sigma}^c B_{\rho\sigma}^d - \overline{G}_{\rho\sigma}^c G_{\rho\sigma}^d \right) \right] \,,
\end{aligned}
\tag{2.18}
$$

seems to differ from the ordinary gluodynamics action S_{YM}. This is however only apparent. Following [11, 15], we introduce the nilpotent "supersymmetry" $\delta^{(2)}$,

$$
\delta^{(2)} B_{\mu\nu}^a = G_{\mu\nu}^a \,, \delta^{(2)} G_{\mu\nu}^a = 0 \,, \delta^{(2)} \overline{G}_{\mu\nu}^a = \overline{B}_{\mu\nu}^a \,, \delta^{(2)} \overline{B}_{\mu\nu}^a = 0 \,, \delta^{(2)}\delta^{(2)} = 0 \,, \delta^{(2)} \left(S'_{YM} + S_{gf} \right) = 0 \,. \tag{2.19}
$$

based on which it can be shown that the newly introduced tensor fields $\{B_{\mu\nu}^a, \overline{B}_{\mu\nu}^a, G_{\mu\nu}^a, \overline{G}_{\mu\nu}^a\}$ do not belong to the cohomology of $\delta^{(2)}$, as they constitute pairs of $\delta^{(2)}$-doublets, and as such completely decouple from the physical spectrum [13]. This means that S_{YM} and S'_{YM} share the same physical degrees of freedom, being 2 transverse gluon polarizations, as can be proven using the BRST cohomology [15].

In addition, the tensor coupling λ^{abcd} cannot enter the Yang-Mills correlators constructed from the original Yang-Mills fields $A_\mu^a, b^a, \overline{c}^a, c^a$ as it is coupled to a $\delta^{(2)}$-exact term, $\propto \delta^{(2)} \left[\left(\overline{B}_{\mu\nu}^a B_{\mu\nu}^b - \overline{G}_{\mu\nu}^a G_{\mu\nu}^b \right) \left(\overline{G}_{\rho\sigma}^c B_{\rho\sigma}^d \right) \right]$, hence λ^{abcd} w.r.t. Yang-Mills correlators plays a role akin to that of a gauge parameter w.r.t. gauge invariant correlators.

The gauge invariant action S'_{YM}, (2.18), is thus perturbatively completely equivalent with the usual Yang-Mills action: it is renormalizable to all orders of perturbation theory, and the physical spectrum is the same. The advantage of S'_{YM} is that it allows to couple a gauge invariant local composite operator to it, which is written down in (2.10). This means that we can probe Yang-Mills gauge theories with this particular operator, and investigate the associated effective action, to find out whether a gauge invariant condensate is dynamically favoured.

3 The effective action for the gauge invariant operator $(B_{\mu\nu}^a - \overline{B}_{\mu\nu}^a)F_{\mu\nu}^a$

We consider the functional $W(J)$, given by

$$
e^{-W(J)} = \int [d\Phi] e^{-S'_{YM} - \int d^4x \left(\frac{iJ}{4}(B-\overline{B})_{\mu\nu}^a F_{\mu\nu}^a - \frac{3}{8} J^2 \lambda_1 \left(\overline{B}_{\mu\nu}^a B_{\mu\nu}^a - \overline{G}_{\mu\nu}^a G_{\mu\nu}^a \right) + J^2 \frac{\lambda_2}{32} \left(\overline{B}_{\mu\nu}^a - B_{\mu\nu}^a \right)^2 + \varsigma J^4 \right)} \,. \tag{3.1}
$$

Here, we can appreciate the role of the ςJ^4 term. Upon integrating over the fields, it becomes clear that we need a counterterm $\delta\varsigma J^4$ to remove the divergent J^4-quantum corrections to $W(J)$. Hence, we need a parameter ς to absorb this counterterm $\delta\varsigma J^4$. Although it seems that we are introducing a new free parameter into the action in this manner, ς can be made a unique function of the coupling constant(s) by requiring a homogenous renormalization group equation for the effective action, see [16] for applications to the $\lambda\phi^4$ and Coleman-Weinberg model.

We now define in the usual way

$$
\varphi(x) = \frac{\delta W(J)}{\delta J(x)} \,. \tag{3.2}
$$

The original theory (i.e. Yang-Mills) is recovered in the physical limit $J = 0$, in which case we have

$$
\varphi = \frac{i}{4} \left\langle (B_{\mu\nu}^a - \overline{B}_{\mu\nu}^a) F_{\mu\nu}^a \right\rangle \,. \tag{3.3}
$$

If we construct the effective action $\Gamma(\varphi)$, we can thus study the condensation of the gauge invariant operator $(B^a_{\mu\nu}-\overline{B}^a_{\mu\nu})F^a_{\mu\nu}$. The functionals $\Gamma(\varphi)$ and $W(J)$ are related through a Legendre transformation

$$\Gamma(\varphi) \quad = \quad W(J) - \int \mathrm{d}^4x\, J(x)\varphi(x)\,. \tag{3.4}$$

The vacuum corresponds to the solution of

$$\frac{\partial}{\partial\varphi}\Gamma(\varphi) \quad = \quad 0\ (=-J)\,, \tag{3.5}$$

with minimal energy. From now on, we shall restrict ourselves to space-time independent φ and J.

In the current situation, we shall have to perform the Legendre transformation explicitly [17]. Let us give an illustrative example with a "toy functional" $W(J)$

$$W(J) = \frac{a_0}{4}J^4 + \frac{g^2}{4}J^4\left(a_1 + a_2\ln\frac{J}{\overline{\mu}}\right) + \text{higher order terms}\,, \tag{3.6}$$

where $\overline{\mu}$ is the renormalization scale. Hence

$$\varphi = a_0 J^3 + g^2 J^3\left(a_1 + \frac{a_2}{4} + a_2\ln\frac{J}{\overline{\mu}}\right) + \text{higher order terms}\,, \tag{3.7}$$

which leads to

$$J = \left(\frac{\varphi}{a_0}\right)^{1/3}\left(1 - \frac{g^2}{3a_0}\left(a_1 + \frac{a_2}{4} + a_2\ln\frac{(\varphi/a_0)^{1/3}}{\overline{\mu}}\right)\right) + \text{higher order terms}\,. \tag{3.8}$$

The trivial vacuum with $\varphi = 0$ is of course always recovered, but there is the possibility for an alternative solution $\varphi \neq 0$, when solving the equation $0 = -J = \frac{\partial\Gamma}{\partial\varphi}$.

In practice, one can determine $W(J)$ up to the lowest orders in perturbation theory. $\Gamma(\varphi)$ itself is obtained by substituting (3.8) into (3.4) to reexpress everything in terms of φ.

We are now ready to have a look at the effective action in the *condensed vacuum*. We shall find that the *tree level* action gets modified in the following way

$$\begin{aligned}\Sigma \to \Sigma' \quad \equiv \quad & S'_{YM} + \int \mathrm{d}^4x\left[\frac{im}{4}(B-\overline{B})^a_{\mu\nu}F^a_{\mu\nu} - \frac{3}{8}m^2\lambda_1\left(\overline{B}^a_{\mu\nu}B^a_{\mu\nu} - \overline{G}^a_{\mu\nu}G^a_{\mu\nu}\right) + m^2\frac{\lambda_2}{32}\left(\overline{B}^a_{\mu\nu} - B^a_{\mu\nu}\right)^2\right] \\ & + \text{higher order terms}\,,\end{aligned} \tag{3.9}$$

with

$$m = \left(\frac{\varphi}{a_0}\right)^{1/3}\,, \tag{3.10}$$

since at tree level we only have to take the lowest order term of (3.8) with us.

The actual computation of the effective action for the gauge invariant local composite operator $(B^a_{\mu\nu}-\overline{B}^a_{\mu\nu})F^a_{\mu\nu}$ will be the subject of future work, as this requires a rather large amount of calculations and the knowledge of yet undetermined renormalization group functions to two-loop order [16, 18]. Anyhow, we expect that the theory will experience a gauge invariant dimensional transmutation, leading to $\left\langle(B^a_{\mu\nu}-\overline{B}^a_{\mu\nu})F^a_{\mu\nu}\right\rangle \sim \Lambda^3_{QCD}$. Further steps towards the effective potential calculation were set in the recent work [18].

4 The link with perturbative confinement

We did not substantiate yet the role of the extra parameters λ_1 and λ_2. We consider the case

$$\lambda_1 = \frac{2}{3}\,, \qquad \lambda_2 = 0\,. \tag{4.1}$$

Returning for a moment to the Coulomb gauge in the static case[2], it is easy to verify at lowest (quadratic) order that the (A_0, A_0) sector exactly reduces to that of (1.1), by integrating out the extra fields.

Since we have the freedom to choose the tree level ("classical") values for λ_1 and λ_2 as we want, we can always make the confining scenario work by assigning the values (4.1). The higher order quantum corrections will consequently induce perturbative corrections in the couplings g^2 and λ^{abcd} to the leading order Cornell potential[3]. At the current time we cannot make more definite statements about this, as the corresponding renormalization group functions of λ_1 and λ_3 have not yet been calculated explicitly, see also [18]. The upshot would of course be to keep the expansion under control, i.e. to have a reasonably small expansion parameter. If the dynamically generated mass scale is sufficiently large, one can readily imagine to have an effective coupling constant g^2 which is relatively small due to asymptotic freedom. It is perhaps noteworthy to recall the possible emergence of linear piece of the potential at short distance: restricting to short distance, i.e. high momentum, might be useful in combination with asymptotic freedom.

Anyhow, we envisage that the essential nontrivial dynamics would be buried in the tree level mass parameter (i.e. the nontrivial condensate φ), which characterizes an effective action with confining properties. One can then perform a perturbative weak coupling expansion around this nontrivial vacuum.

5 The static quark potential via the Wilson loop

So far, we have been looking at the Coulomb gauge to get a taste of the inter quark potential. However, there is a cleaner (gauge invariant) way to define the static inter quark potential $V_{Q\overline{Q}}(\mathbf{r})$. As it is well known, $V_{Q\overline{Q}}(\mathbf{r})$ can be related to the expectation value of a Wilson loop, see e.g. [19, 20]. More precisely,

$$V_{Q\overline{Q}}(\mathbf{r}-\mathbf{r}') = \lim_{T\to\infty}\frac{1}{T}\ln\frac{\mathrm{Tr}\langle\mathcal{W}\rangle}{\mathrm{Tr}\langle\mathbf{1}\rangle}\,, \tag{5.1}$$

with the Wilson loop $\mathcal{W}$ defined by

$$\mathcal{W} = \mathcal{P}\,\mathrm{e}^{g\oint_{\mathcal{C}} A_\mu \mathrm{d}x_\mu}\,, \tag{5.2}$$

where the symbol $\mathcal{P}$ denotes path ordering, needed in the non-Abelian case to ensure the gauge invariance of $\mathrm{Tr}\mathcal{W}$. The symbol $\mathbf{1}$ is the unit matrix corresponding to the representation R of the "quarks". Let t^a be the corresponding generators. We shall consider a rectangular loop $\mathcal{C}$ connecting 2 charges at respective positions $\mathbf{r}$ and $\mathbf{r}'$, with temporal extension $T\to\infty$.

To explicitly calculate (5.1), we shall mainly follow [21]. First, we notice that at $T\to\infty$, $F_{\mu\nu}^2\to 0$, i.e. A_μ becomes equivalent to a pure gauge potential[4], $A_\mu = 0$, meaning that we can rewrite the trace of the Wilson loop as

$$\mathrm{Tr}\mathcal{W} = \mathrm{Tr}\mathcal{P}\,\mathrm{e}^{g\int A_0(\mathbf{r},t)\mathrm{d}t - g\int A_0(\mathbf{r}',t)\mathrm{d}t}\,. \tag{5.3}$$

We introduce the current,

$$J_\mu^a(\mathbf{x},t) = g\delta_{\mu 0}t^a\delta^{(3)}(\mathbf{x}-\mathbf{r}) - g\delta_{\mu 0}t^a\delta^{(3)}(\mathbf{x}-\mathbf{r}')\,, \tag{5.4}$$

to reexpress the expectation value of (5.3) as

$$\mathrm{Tr}\langle\mathcal{W}\rangle = \frac{\mathcal{P}}{\mathcal{N}}\int[\mathrm{d}\Phi]\mathrm{e}^{-\Sigma' + \int \mathrm{d}^4x J_\mu^a A_\mu^a)}\,, \tag{5.5}$$

with $\mathcal{N}$ the appropriate normalization factor.

We are now ready to determine the potential explicitly. We limit ourselves to lowest order, in which case the path ordering is irrelevant, and we find

$$V_{Q\overline{Q}}(\mathbf{r}-\mathbf{r}') = \frac{1}{\mathrm{Tr}\mathbf{1}}\lim_{T\to\infty}\frac{1}{T}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{2}J_\mu^a(p)D_{\mu\nu}^{ab}(p)J_\nu^b(-p)\,, \tag{5.6}$$

with

$$J_\mu^a(p) = 2\pi g\delta(p_0)(e^{-i\mathbf{p}\cdot\mathbf{r}} - e^{-i\mathbf{p}\cdot\mathbf{r}'})\delta_{\mu 0}t^a\,, \tag{5.7}$$

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2. Meaning that we formally set "$\partial_0 = 0$".
3. We shall comment on the role of the tensor coupling λ^{abcd} later on in this note.
4. We discard gauge potentials with nontrivial topology.

and with

$$D^{ab}_{\mu\nu}(p) = D_{\mu\nu}(p)\delta^{ab}\,, \qquad D_{\mu\nu}(p) = \frac{p^2+m^2}{p^4}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \frac{\alpha}{p^2}\frac{p_\mu p_\nu}{p^2}\,, \tag{5.8}$$

the gluon propagator. Proceeding with (5.6), we get

$$\begin{aligned} V_{Q\overline{Q}}(\mathbf{r}-\mathbf{r}') &= \lim_{T\to\infty}\frac{1}{2T}C_2(R)\int\frac{\mathrm{d}^4p}{(2\pi)^4}g^2\delta^2(p_0)(2\pi)^2(e^{-i\mathbf{p}\cdot\mathbf{r}}-e^{-i\mathbf{p}\cdot\mathbf{r}'})(e^{i\mathbf{p}\cdot\mathbf{r}}-e^{i\mathbf{p}\cdot\mathbf{r}'})D_{00}(p) \\ &= \lim_{T\to\infty}\frac{1}{2T}C_2(R)g^2 2\pi\delta(0)\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}(e^{-i\mathbf{p}\cdot\mathbf{r}}-e^{-i\mathbf{p}\cdot\mathbf{r}'})(e^{i\mathbf{p}\cdot\mathbf{r}}-e^{i\mathbf{p}\cdot\mathbf{r}'})D_{00}(p)_{p_0=0} \\ &= -g^2C_2(R)\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\frac{\mathbf{p}^2+m^2}{\mathbf{p}^4} - g^2C_2(R)\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\frac{\mathbf{p}^2+m^2}{\mathbf{p}^4}\mathrm{e}^{i\mathbf{p}(\mathbf{r}-\mathbf{r}')}\,. \end{aligned} \tag{5.9}$$

We used that $\lim_{T\to\infty} T = \lim_{T\to\infty}\int_{-T/2}^{T/2}\mathrm{d}t = 2\pi\delta(0)$. The first term of (5.9) corresponds to the (infinite) self energy of the external charges [21], so we can neglect this term to identify the interaction energy, which yields after performing the Fourier integration

$$V_{Q\overline{Q}}(\mathbf{r}-\mathbf{r}') = \frac{g^2C_2(R)}{8\pi}m^2|\mathbf{r}-\mathbf{r}'| - \frac{g^2C_2(R)}{4\pi}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,. \tag{5.10}$$

We nicely obtain a Cornell potential, with the string tension in representation R given by $\sigma(R) = \frac{g^2}{8\pi}C_2(R)m^2$. Notice that the so-called Casimir scaling [22] of $\sigma(R)$ is straightforwardly fulfilled, at least at the considered order.

If we consider our model in a specific gauge, for example the Landau gauge, we see the presence of a $\frac{1}{p^4}$ singularity in the (tree level) gluon propagator (5.8). Actually, it was already argued in [23] that such pole would induce the area law of the Wilson loop, if present in *some* gauge. In the Landau gauge in particular, lattice data have already ruled out since long such a highly singular gluon propagator, see [24] for a recent numerical analysis.

A first observation is that we presented only a lowest order calculation, based on the tree level gluon propagator. We did not consider quantum corrections, on neither the Wilson loop's expectation value nor gluon propagator. A more sophisticated treatment would also have to take into account that our naive string tension σ, related to the condensate $\langle B-\overline{B}\rangle F$, will run with the scale. This would ask for a renormalization group improved treatment. We already mentioned in the introduction that the string tension at short distance (large energy scale) does not have to concur with the one at large distances (small energy scale) [6, 7].

We must also remind that most gauges, in particular, the Landau gauge, are plagued by the Gribov copy problem, which also influence the infrared dynamics of a gauge theory [25, 26]. The latter problem can be overcome as we are not obliged to work in the Landau gauge, since we have set up a gauge invariant framework. In most other gauges, it is not even known how to tackle e.g. the gauge copy problem in a more or less tractable way, or there are no copies at all in certain gauges[5]. As an example of the latter gauges, let us impose the planar gauge [27] via a gauge fixing term $S_{gf} = \int \mathrm{d}^4x\frac{1}{2n^2}n\cdot A\partial^2 n\cdot A$. The gluon propagator becomes a bit complicated

$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(\frac{p^2+m^2}{p^4}\delta_{\mu\nu} + m^2\frac{p^2+m^2}{p^4}\frac{n^2}{(p\cdot n)^2}\frac{p_\mu p_\nu}{p^2} - \frac{p^2+m^2}{p^4}\frac{n_\mu p_\nu}{p\cdot n} - \frac{(p^2+m^2)^2}{p^6}\frac{p_\mu n_\nu}{p\cdot n}\right), \tag{5.11}$$

nevertheless the result (5.10) is recovered, after some algebra.

6 Symmetry breaking pattern

We already mentioned the useful supersymmetry $\delta^{(2)}$, which is however broken if $\left\langle (B^a_{\mu\nu}-\overline{B}^a_{\mu\nu})F^a_{\mu\nu}\right\rangle \neq 0$ (i.e. $m\neq 0$). Hence, we should worry about the emergence of an extra (undesired) massless degree of freedom: the associated Goldstone fermion[6]. The situation is however more complicated than this. The starting action S'_{YM}

5. Some of these gauges then suffer from other problems.
6. Not boson, as δ_2 transforms bosons into fermions and vice versa.

enjoys the following set of (nilpotent) supersymmetries

$$\delta^{(1)} = \int d^4x \left(B^a_{\mu\nu} \frac{\delta}{\delta G^a_{\mu\nu}} - \overline{G}^a_{\mu\nu} \frac{\delta}{\delta \overline{B}^a_{\mu\nu}} \right), \qquad \delta^{(3)} = \int d^4x \left(\overline{B}^a_{\mu\nu} \frac{\delta}{\delta G^a_{\mu\nu}} - \overline{G}^a_{\mu\nu} \frac{\delta}{\delta B^a_{\mu\nu}} \right),$$
$$\delta^{(2)} = \int d^4x \left(\overline{B}^a_{\mu\nu} \frac{\delta}{\delta \overline{G}^a_{\mu\nu}} + G^a_{\mu\nu} \frac{\delta}{\delta B^a_{\mu\nu}} \right), \qquad \delta^{(4)} = \int d^4x \left(B^a_{\mu\nu} \frac{\delta}{\delta \overline{G}^a_{\mu\nu}} + G^a_{\mu\nu} \frac{\delta}{\delta \overline{B}^a_{\mu\nu}} \right), \tag{6.1}$$

in addition to the bosonic symmetries generated by

$$\Delta^{(1)} = \int d^4x \left(B^a_{\mu\nu} \frac{\delta}{\delta B^a_{\mu\nu}} - \overline{B}^a_{\mu\nu} \frac{\delta}{\delta \overline{B}^a_{\mu\nu}} \right), \qquad \Delta^{(2)} = \int d^4x \left(G^a_{\mu\nu} \frac{\delta}{\delta G^a_{\mu\nu}} - \overline{G}^a_{\mu\nu} \frac{\delta}{\delta \overline{G}^a_{\mu\nu}} \right). \tag{6.2}$$

It appears that a nonvanishing $\left\langle (B^a_{\mu\nu} - \overline{B}^a_{\mu\nu}) F^a_{\mu\nu} \right\rangle$ results in the dynamical breakdown of the continuous symmetries $\delta^{(1),(2),(3),(4)}$ and $\Delta^{(1)}$. Though, a little more care is needed. Not all the breakings are independent, as one checks that

$$\delta^{(1)-(3)} \equiv \delta^{(1)} - \delta^{(3)}, \qquad \delta^{(2)-(4)} \equiv \delta^{(2)} - \delta^{(4)}, \qquad \Delta^{(1)}, \tag{6.3}$$

are clearly dynamically broken for $\langle (B - \overline{B}) F \rangle \neq 0$, since can write

$$\left\langle (B^a_{\mu\nu} - \overline{B}^a_{\mu\nu}) F^a_{\mu\nu} \right\rangle = \left\langle \delta^{(1)-(3)} \left[G^a_{\mu\nu} F^a_{\mu\nu} \right] \right\rangle = -\left\langle \delta^{(2)-(4)} \left[\overline{G}^a_{\mu\nu} F^a_{\mu\nu} \right] \right\rangle = \left\langle \Delta^{(1)} \left[(B^a_{\mu\nu} + \overline{B}^a_{\mu\nu}) F^a_{\mu\nu} \right] \right\rangle, \tag{6.4}$$

while

$$\delta^{(1)+(3)} \equiv \delta^{(1)} + \delta^{(3)}, \qquad \delta^{(2)+(4)} \equiv \delta^{(2)} + \delta^{(4)}, \qquad \Delta^{(2)}, \tag{6.5}$$

are still conserved.

If a nonzero value of $\left\langle (B^a_{\mu\nu} - \overline{B}^a_{\mu\nu}) F^a_{\mu\nu} \right\rangle$ is dynamically favoured, 2 Goldstone fermions and 1 Goldstone boson seem to enter the physical spectrum. As this would be a serious problem[7], we need to find a way to remove these from the spectrum. A typical way to kill unwanted degrees of freedom is by imposing constraints on the allowed excitations. Consistency is assured when this is done by using symmetry generators to restrict the physical subspace. First, we have to identify the suitable operators to create/annihilate the Goldstone particles. As it is well known, these are provided by the Noether currents corresponding to (6.3), which can be derived from the action S'_{YM}. We obtain

$$j^{(1)-(3)}_\mu = -B^a_{\alpha\beta} D^{ab}_\mu \overline{G}^b_{\alpha\beta} + \overline{G}^a_{\alpha\beta} D^{ab}_\mu B^b_{\alpha\beta} + \overline{B}^a_{\alpha\beta} D^{ab}_\mu \overline{G}^b_{\alpha\beta} - \overline{G}^a_{\alpha\beta} D^{ab}_\mu \overline{B}^b_{\alpha\beta},$$
$$j^{(2)-(4)}_\mu = \overline{B}^a_{\alpha\beta} D^{ab}_\mu G^b_{\alpha\beta} - G^a_{\alpha\beta} D^{ab}_\mu \overline{B}^b_{\alpha\beta} - B^a_{\alpha\beta} D^{ab}_\mu G^b_{\alpha\beta} + G^a_{\alpha\beta} D^{ab}_\mu B^b_{\alpha\beta}, \tag{6.6}$$

after a little algebra. Let us now define what physical operators are. First of all, they are expected to be gauge invariant[8]. Secondly, based on $\Delta^{(2)}$ we can also introduce a $\mathcal{G}$-ghost charge, with $\mathcal{G}(G^a_{\mu\nu}) = +1$, $\mathcal{G}(\overline{G}^a_{\mu\nu}) = -1$, and demand that physical operators are $\mathcal{G}$-neutral. In addition, we also can request invariance w.r.t. $\delta^{(1)+(3)}$ and $\delta^{(2)+(4)}$.

Let us mention the following useful relations

$$\delta^{(1)+(3)} j^{(2)-(4)}_\mu = \delta^{(2)+(4)} j^{(1)-(3)}_\mu = 2(\overline{B}^a_{\alpha\beta} D^{ab}_\mu B^b_{\alpha\beta} - B^a_{\alpha\beta} D^{ab}_\mu \overline{B}^b_{\alpha\beta}) \neq 0,$$
$$\delta^{(1)+(3)} j^{(1)-(3)} = \delta^{(2)+(4)} j^{(2)-(4)} = 0. \tag{6.7}$$

The currents $j^{(2)-(4)}_\mu$ or $j^{(1)-(3)}_\mu$ are thus not physical operators. Although gauge invariant, (6.7) tells us these are not $\delta^{(1)+(3)}$ or $\delta^{(2)+(4)}$ invariant. Moreover, since $\mathcal{G}(j^{(2)-(4)}_\mu) = +1$, and $\mathcal{G}(j^{(1)-(3)}_\mu) = -1$, also the $\mathcal{G}$-neutrality is not met.

We can assure $\mathcal{G}$-neutrality by e.g. taking a product $j^{(2)-(4)} j^{(1)-(3)}$, but this does not ensure $\delta^{(1)+(3)}$ or $\delta^{(2)+(4)}$ invariance, which can be easily checked using (6.7).

7. These extra particles carry no color, so there is no reason to expect that these would be confined or so, thereby removing themselves from the physical spectrum.
8. Or more precisely, BRST closed but not exact, after fixing the gauge.

Concerning the current k_μ associated with $\Delta^{(1)}$, we find

$$k_\mu \;=\; -B^a_{\alpha\beta}D^{ab}_\mu\overline{B}^a_{\alpha\beta}+\overline{B}^a_{\alpha\beta}D^{ab}_\mu B^a_{\alpha\beta}\,, \tag{6.8}$$

hence

$$\begin{aligned}\delta^{(1)+(3)}k_\mu &= B^a_{\alpha\beta}D^{ab}_\mu\overline{G}^a_{\alpha\beta}-\overline{G}^a_{\alpha\beta}D^{ab}_\mu B^a_{\alpha\beta}+\overline{G}^a_{\alpha\beta}D^{ab}_\mu\overline{B}^a_{\alpha\beta}-\overline{B}^a_{\alpha\beta}D^{ab}_\mu\overline{G}^a_{\alpha\beta}\neq 0\,,\\ \delta^{(2)+(4)}k_\mu &= -G^a_{\alpha\beta}D^{ab}_\mu\overline{B}^a_{\alpha\beta}+\overline{B}^a_{\alpha\beta}D^{ab}_\mu G^a_{\alpha\beta}-B^a_{\alpha\beta}D^{ab}_\mu G^a_{\alpha\beta}+G^a_{\alpha\beta}D^{ab}_\mu B^a_{\alpha\beta}\neq 0\,.\end{aligned} \tag{6.9}$$

Since the symmetries we are using are not unrelated, it is evidently no surprise that k_μ, $j_\mu^{(2)-(4)}$ and $j_\mu^{(1)-(3)}$ are transformed into each other. The question remains however whether we can build combinations[9] of these which enjoy all the necessary invariances? Let us try to construct one, starting from $j^{(2)-(4)}$. We shall use a more symbolic notation. It can be checked that e.g.

$$\delta^{(2)+(4)}\left(\overline{G}j^{(2)-(4)}+(B+\overline{B})K-Gj^{(1)-(3)}\right)\;=\;0\,, \tag{6.10}$$

but

$$\delta^{(1)+(3)}\left(\overline{G}j^{(2)-(4)}+(B+\overline{B})K-Gj^{(1)-(3)}\right)=-4\overline{G}k-2(B+\overline{B})j^{(1)-(3)}\,. \tag{6.11}$$

So far, we have been unable to construct suitable invariant operators. We are lead to believe that this is generally true, in return we could state that the Goldstone modes can be expelled from the spectrum. An explicit proof is however lacking hitherto.

7 A few words on the tensor coupling λ^{abcd}

In the massless case, the precise value of the tensor coupling λ^{abcd} is irrelevant, as it cannot influence the dynamics of the (physical) Yang-Mills sector of the theory as explained above. However, when studying the effective action for $\varphi=\langle(B-\overline{B})F\rangle$, λ^{abcd} plays a role. We might see this as a drawback, as then a new independent coupling would enter the game. As our setup was to deal with confinement in usual gauge theories with a single gauge coupling g^2, we would like to retain solely g^2 as the relevant parameter. This can be nicely accommodated for by invoking the renormalization group equations to reduce the number of couplings. In the presence of multiple couplings, one can always opt to choose a primary coupling and express the others in term of this one. For consistency, no sacrifices should be made w.r.t. the renormalization group equations, therefore we shall search for a fix point $\lambda_*^{abcd}(g^2)$, such that $\mu\frac{\partial}{\partial\mu}\lambda_*^{abcd}=0$.

We recall the result of [11], where it was calculated, using dimensional regularization ($d=4-2\varepsilon$) and using the $\overline{\text{MS}}$ scheme, that

$$\begin{aligned}\mu\frac{\partial}{\partial\mu}\lambda^{abcd} &= -2\varepsilon\lambda^{abcd}+\left[\frac{1}{4}\left(\lambda^{abpq}\lambda^{cpdq}+\lambda^{apbq}\lambda^{cdpq}+\lambda^{apcq}\lambda^{bpdq}+\lambda^{apdq}\lambda^{bpcq}\right)\right.\\ &\quad\left.-\,12C_A\lambda^{abcd}a\,+\,8C_Af^{abp}f^{cdp}a^2\,+\,16C_Af^{adp}f^{bcp}a^2\,+\,96d_A^{abcd}a^2\right]+\ldots\,,\end{aligned} \tag{7.1}$$

with $a=\frac{g^2}{16\pi^2}$, and we also rescaled $\lambda^{abcd}\to\frac{1}{16\pi^2}\lambda^{abcd}$. We clearly notice that $\lambda^{abcd}=0$ is not a fixed point of this renormalization group equation. We must thus look out for an alternative fixed point $\lambda_*^{abcd}\neq 0$.

We shall restrict ourselves to the simplest case: we take $SU(2)$ as gauge group, and only consider gauge fields in the adjoint representation. Doing so, we can simplify (7.1) a bit by explicitly computing the completely symmetric rank 4 tensor d_A^{abcd} [12], and by looking for tensor structures that can be used to construct a rank 4 tensor consistent with the constraints (2.14) and (2.11).

The generators of the adjoint representation of $SU(2)$, are given by $(t^a)_{bc}=i\varepsilon^{abc}$. We can compute d_A^{abcd}, which is defined by means of a symmetrized trace STr as

$$d_A^{abcd}\;=\;\mathrm{STr}\left(t^at^bt^ct^d\right)\;=\;\left[\delta^{ab}\delta^{cd}+\delta^{ad}\delta^{bc}\right]_{\text{symmetrized w.r.t. }\{a,b,c,d\}}\;=\;\frac{2}{3}\left(\delta^{ab}\delta^{cd}+\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc}\right)\,. \tag{7.2}$$

9. These combinations may of course contain other operators too.

Moreover, we can also simplify the other tensor appearing in (7.1), namely ($C_A=2$)

$$8C_A f^{abp} f^{cdp} a^2 + 16 C_A f^{adp} f^{bcp} a^2 \quad = \quad -16\delta^{ac}\delta^{bd} - 16\delta^{ad}\delta^{bc} + 32\delta^{ab}\delta^{cd} \,. \tag{7.3}$$

Using the constraints (2.14) as definition of any building block of our tensor λ_*^{abcd}, one can check that the following rank 4 color tensors are suitable (linearly independent) candidates

$$\mathcal{O}_1^{abcd} \quad = \quad \delta^{ab}\delta^{cd} \,, \qquad \mathcal{O}_2^{abcd} \;=\; \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc} \,. \tag{7.4}$$

Clearly, d_A^{abcd} and the tensor (7.3) are particular linear combinations of the tensors in (7.4). We now propose

$$\lambda_f^{abcd}(a) = y_1 \mathcal{O}_1^{abcd} a + y_2 \mathcal{O}_2^{abcd} a \qquad\qquad y_i \in \mathbb{R} \,, \tag{7.5}$$

and we demand that the l.h.s. of (7.1) vanishes when (7.5) is substituted into it, with $\varepsilon = 0$. This leads to

$$\left\{ \begin{array}{lcr} y_1 & \approx & 67.6 \\ y_2 & \approx & -43.6 \end{array} \right. \,, \qquad\qquad \left\{ \begin{array}{lcr} y_1 & \approx & 28.4 \\ y_2 & \approx & -4.4 \end{array} \right. \,. \tag{7.6}$$

We conclude that the renormalization group equation $\mu\frac{\partial}{\partial\mu}\lambda^{abcd} = \beta^{abcd} = 0$ possesses a fixed point in $d=4$, at least at 1-loop for the gauge group $SU(2)$ in the presence of only gauge fields.

We end this note by briefly returning to the issue of $\frac{1}{q^2}$ power corrections. In [28, 29], these were related to (part of) the dimension two condensate $\langle A^2_{\min}\rangle = (VT)^{-1}\left\langle \min_{g\in SU(N)} \int \mathrm{d}^4x (A_\mu^g)^2 \right\rangle$. The nonlocal operator $A^2_{\min}$ reduces to A^2 in the Landau gauge, hence the interest in this gauge [28, 29]. Although the mechanism discussed in this Letter might seem to be completely different, this is however not the case. The nonperturbative mass scale, set by the condensation of the gauge invariant operator (3.3), will also fuel a nonvanishing A^2 condensate in the Landau gauge, i.e. $\langle A^2\rangle \propto m^2$, already in a perturbative loop expansion. As such, at least part of the nonperturbative information stored in $\langle A^2\rangle$ could be attributed to the gauge invariant condensate introduced in this work.

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