Subitizing and counting as possible screening variables for learning disabilities in mathematics education or learning?

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#### Abstract

This paper aims to highlight the significance of a particular aspect of magnitude processing, namely counting and subitizing or the rapid enumeration of small sets of items, for learning. Emphasis is laid on the historical roots and the conceptual framework as well as on studies on pre-verbal and school-age children. Evidence of the potential value of this research for the assessment of children at risk of mathematical learning disabilities, is presented. Inherent to its nature, subitizing relies on rapid, preverbal analogue magnitude comparisons being triggered. We will highlight the differences with counting, and the implications of shortcomings in counting and subitizing in children with mathematical learning disabilities for the automaticity of number magnitude processing. Furthermore we especially look in this paper at the varying assessment paradigms experimentally controlling for continuous variables which are used in research with different age groups, something which has received insufficient attention in the past. Finally we outline the challenges for future research on mathematical learning disabilities.


Keywords : Magnitude comparison; Paradigms; Assessment; Subitizing; Counting; Mathematical learning disabilities

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Magnitude comparison: a possible screening variable for learning disabilities in mathematics?

Mathematical learning disabilities (MLD) have received far less attention in research than any other academic problem. MLD has been studied less than it deserves, contrary to i.e. reading disabilities (Dowker, 2005; Tymms, 1999), even though MLD is relatively frequent, affecting 3 to14\% of children (Barbaresi, Katusic, Colligan, Weaver, \& Jacobsen, 2005; Lewis, Hitch, \& Walker, 1994; Shalev, Manor, \& Gross-Tsur, 2005). It is becoming more and more common for children with mathematical disabilities, who struggle with aspects of counting linked to dyscalculia, having to be dealt with in classrooms on a daily basis.

There are several models trying to describe or explain the mechanisms underlying quantity processing deficits in children with MLD. Some models focus on immature counting and calculation strategies (Geary, 2004; Jordan, Hanich \& Kaplan, 2003), slow serial elaboration (Geary \& Brown, 1991), deficits in the working memory or speed of processing (Geary, Hoard, Byrd-Craven, Nugent, \& Numtee, 2007), problems retrieving from semantic long term memory (Geary, 2004), problems with visual spatial elaboration (Geary, 2004; Shalev, 2004), and executive deficits (Passolunghi \& Siegel, 2004).

However some researchers consider the above mentioned deficits as 'higher' order problems of children with MLD, with an inborn core deficit in 'the number module' (Butterworth, 1999) or in 'number sense' a term denoting the ability to picture and manipulate numerical magnitude on an internal number line (e.g., Dehaene, 2001; Dehaene, Piazza, Pinel, \& Cohen, 2003; Lander, Bevan, \& Butterworth, 2004; Nuerk, Kaufmann, Zoppoth, \& Willmes, 2004;Wilson \& Dehaene, 2007; Wilson, Revkin, Cohen, Cohen, \& Dehaene, 2006). Von Aster and Shalev (2007) showed that this internal number line, a virtual spatial depiction of numbers arranged by their proximity, have a particular plasticity and continues to develop

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during elementary school. Impairments such as problems with subitizing (Fisher, Gebhardt \& Hartnegg, 2008; Koontz \& Berch, 1996; Landerl, Bevan, \& Butterworth, 2004) and deficiencies in depicting or accessing magnitude information on the mental number line (Dehaene, Bossini, \& Giraux, 1993; Jordan, Hanich \& Kaplan, 2003; Xu \& Spelke, 2000) are considered by these researchers as 'low-level' symptoms consistent with the core deficit theory. Nevertheless the same researchers acknowledge that higher level symptoms (impairments in acquisition of counting and addition procedures and in fact retrieval) may be a derivative of an initial dysfunction of the core number sense system even though there may be other possible causes of these high-level symptoms. Endorsing them, there are researchers who support other models to explain these so called higher order problems with elementary and more advanced arithmetical skills of children with MLD (Geary et al., 2007; Geary, Hoard, \& Hamson, 1999).

The past decade, early numeracy has been receiving growing attention (e.g., Aubrey \& Godfrey, 2003; Stock, Desoete, \& Roeyers, 2007). Empirical evidence suggests that, the earlier we can detect children at risk, the more able we will be to prevent/avert learning difficulties later on (Coleman, Buysse, \& Neitzel, 2006). The current interest in early predictors (e.g., DiPerna, Lei, \& Reid, 2007; Mazzocco \& Thompson, 2005) is encouraged by the hope that, if predictors and core deficits can be addressed as key components in remediation programs, children may not fall further behind (e.g., Gersten, Jordan, \& Flojo, 2005). Nevertheless, the need to measure individual differences on these predictors has led to the development of different instruments to help assess subitizing and counting in young children. At present, the use of different paradigms in order to assess magnitude processing makes it difficult to compare the results of studies. We want to eliminate some of the confusion arising from the lack of consistency in the use of terminology and/or tests in this

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field by means of giving an overview of terms and their meaning. The aim of this contribution is to help clarify some of the issues in the conceptualization and the assessment of subitizing as a magnitude processing mechanism, to review the relationships between counting and subitizing and to analyse the research about possible relationship between magnitude comparison deficits and MLD.

In what follows we shall refer, firstly, to the conceptualisation and historical roots of subitizing; secondly, to the relationship with counting in addition to research paradigms; and finally, to the implications for learning and potential research on early markers for mathematical learning disabilities.

## 1. Conceptualisation and historical roots of counting and subitizing

It is obvious that early mathematics involves "counting" (Wynn, 1990) in the 'count all' or 'sum' strategy in which the child first counts each collection and then counts the combination of two collections starting from one (i.e., $2+5={ }_{-} 1,2 \ldots 1,2,3,4,5 \ldots 1,2,3,4,5,6,7$ ). As practice increases, older children use more effective back-up strategies, such as the 'counton' strategy where they count up from the first addend the number of times indicated by the second addend (i.e., $2+5=\_(2), 3,4,5,6,7$ ) or the 'min' strategy where they count up from the larger addend the number of times indicated by the smaller addend (i.e. " $2+5=5+2=(5), 6,7$ "). It is assumed that the retrieval strategy (" $2+5=7$, I know this by heart") is made possible by the learning and progressive strengthening of memory associations between problems and answers as a result of the repeated use of algorithms (Barrouillet \& Lepine, 2005). Moreover, it has been suggested that children's basic conceptual understanding of how to count objects and their knowledge of the order of numbers play an important role in arithmetic performance because they promote the automatic use of arithmetic related information, allowing attention

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resources to be devoted to more complex arithmetic problem solving (Aunola, Leskinen, Lerkkanen, \& Nurmi, 2004).

Although research looked into counting as a unitary ability, Dowker (2005) suggested that counting knowledge consists of procedural and conceptual aspects.

Procedural knowledge' can be defined as children's ability to perform an arithmetic task, for example, when a child can successfully determine that there are five objects in an array (LeFevre et al., 2006). 'Conceptual knowledge' reflects a child's understanding of why a procedure works or whether a procedure is legitimate (LeFevre et al., 2006). For counting, conceptual knowledge includes understanding five principles (Gallistel \& Gelman,1992):
(a) 'stable-order principle' according to which the order of number words must be invariant across counted sets;
(b) 'one-one principle' according to which every number word can only be attributed to one counted object;
(c) 'cardinality principle' according to which the final number word pronounced in a count represents the numerosity of the set;
(d) 'abstraction principle' according to which any kind of object can be counted;
(e) 'order-irrelevance principle' according to which objects can be counted in any order.

Moderate correlations were found between procedural and conceptual counting knowledge (Le Fevre et al., 2006). Some advocates of the 'continuity hypothesis' (e.g., Gallistel \& Gelman, 1992) claimed that children have conceptual knowledge before their procedural counting skills are well developed. Other researchers reported the opposite (e.g., Frye, Braisby, Lowe, Maroudas, \& Nicholls, 1989). However, the timing of the two types of knowledge may largely depend on the particular task or the development may be iterative (Rittle-Johnson, Siegler, \& Wagner, 2001).

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The term "subitizing" is introduced by Kaufman and his colleagues, in the context of the discrimination of visual numbers, to denote confident judgments of small quantities (Kaufman, Lord, Reese, \& Volkmann, 1949). Since 1949 there is growing research evidence on this preverbal number magnitude processing in animals and infants, although the nature of subitizing remains controversial (Nan, Knösche, \& Luo, 2006). Kaufman and his colleagues defined subitizing as the rapid (40-100 ms/item), accurate, and confident assessments of numbers made for small quantities of items.

Since 1949 research has focused on this ability to determine without any problem the amount of items in small sets of four or less (Benoit, Lehalle \& Jouen, 2004; Dehaene \& Cohen, 1994; Piazza, Mechelli, Prince, \& Butterworth, 2006; Revkin, Piazza, Izard, Cohen, \& Dehaene, 2008; Wender \& Rothkegel, 2000). Subitizing has been established in preverbal human infants (Antell \& Kating, 1983; Starkey \& Cooper, 1980; Xu, 2003) and in a variety of non-human species (Brannon, 2005) including monkeys (Boyson \& Berntson, 1989; Brannon \& Terrace, 2000; Matsuzawa, 2004; Murofushi, 1997; Nieder \& Miller, 2004; Phillips \& Santos, 2007), rats (Davids \& Memmott, 1983), parrots (Pepperberg, 1987) and cats (Thompson et al., 1970). A special group of cells in the cortex of cats (Thompson, Mayers, Robertson, \& Patterson, 1970) and of monkeys (Sawamura, Shima, \& Tanji, 2002) was found to respond selectively to small quantities, regardless of the modality - visual, auditory, or tactile - they were presented in.

Children manifest a sense for numbers very early on in their development (HuntleyFenner \& Cannon, 2000; Schneider et al., 2008). Infants showed the ability to differentiate sets based on the number of items they were containing. Differentiation of visual quantities was established in infants who were watching as slides with a fixed number of two dots were presented repeatedly to them until the length of time they kept watching them started to

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decrease, indicating habituation. At that point, slides with three dots were presented, which in turn were yielding significantly longer viewing times again, indicating dishabituation and therefore differentiation of two versus three (Starkey \& Cooper, 1980). The ability of infants to differentiate quantities is not limited to visual sets of objects. Newborns have been found to differentiate two- and three-syllable words (Bijeljac-Babic, Bertoncini, \& Mehler, 1991) and it was established that six-month-olds were able to differentiate quantities in visual events, such as a puppet making two or three jumps (Wynn, 1996).

Children were found capable of subitizing three items, whereas most adults could subitize four items (Nan, Knösche \& Luo, 2006; Piazza et al., 2006; Simon \& Vaishnavi, 1996; Starkey \& Cooper, 1995; Watson, Maylor, \& Bruce, 2007). However, early findings on number representation been criticized for failing to eliminate alternative stimulus cues such as cumulative surface area, density, or contour length as continuous dimensions (e.g., Brannon, Abbott, \& Lutz; Feigenson, Spelke, \& Carey, 2002). These studies reveal that the ability to differentiate between item sets of different numerosities can not only be observed on small number sets but also on larger sets if the ratio between the two numerosities is $2: 1$ (e.g., comparing 8 squares with 4 squares). In addition some of the previous effects were no longer found comparing small item sets (e.g., 1 vs. 2) experimentally controlled for continuous dimensions such as surface (e.g., Lipton \& Spelke, 2003; Xu, 2003; Xu \& Spelke, 2000; Xu, Spelke, \& Goddard, 2005). The skill to differentiate between larger sets was found to improve rapidly and by the age of 9 months, the ratio of numerosities that can be differentiated was 3:2 (e.g., comparing 6 squares with 4 squares; Lipton \& Spelke, 2003). Moreover, Fischer and Hartnegg (s.d.) postulated a long term development with regard to accuracy and speed in subitizing and counting up to the adult age of 18 years. Green and Bavelier (2003) established training potential even in the subitizing performance of young adult video game players.

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## 2. Magnitude comparison by counting or subitizing

A central role in arithmetic development has been awarded to counting abilities. Counting has been described as the key ability which makes the bridge between the innate sense of numbers and the more advanced arithmetic abilities that are culturally expected (Butterworth, 2004; Sarnecka \& Carey, 2008). Counting can be seen as a prerequisite for a proper development of mathematical abilities (Van de Rijt \& Van Luit, 1999). It is the foundation for future mathematical strategies such as addition and subtraction (Le Fevre et al., 2006) and multiplication (Blöte, Lieffering, \& Ouwehand, 2006).

However, the human brain has also the capacity of correctly recognizing the quantity of a certain amount of items, when they have only been presented for $40-100 \mathrm{~ms}$. This presentation time is too short to count the items by scanning saccades (Watson, Maylor, \& Bruce, 2007). The item number is recognized immediately without counting. This is called subitizing (from the Latin word for "sudden"). The time for subitizing one to four items is about the same. For higher quantities (e.g., comparing 5 squares with 7 squares) the subjects start a process of counting. Response time for counting increases gradually as the numbers grow, indicating that each additional item needs a constant extra time to be counted.

Results of neuro-imaging, neuropsychological and brain stimulation techniques have led to believe that there is a dichotomy between subitizing and counting (Nan, Knösche \& Luo, 2006). Researchers suggest two qualitatively different number magnitude processing mechanisms: a fast parallel mechanism for subitizing sets of one, two or three/four objects and a slow serial mechanism for counting larger sets (Lipton \& Spelke, 2004; Piazza et al., 2003). The question in which part of the brain subitizing and counting takes place is still under discussion. There is evidence suggesting that subitizing is bilaterally represented

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(Colvin, Funnell \& Gazzaniga, 2005; Dehaene \& Cohen, 1994; Gallistel \& Gelmann, 2000; Piazza et al., 2002). There are also studies which show that in normal adults the right hemisphere may have an advantage with regard to subitizing (Pasini \& Tessari, 2001). Sathian et al. (1999) revealed that subitizing activated the occipital cortex whereas counting activated the parietal and the right frontal cortex. However Piazza et al. (2002) showed a common network for subitizing and counting that comprises occipital and parietal areas. There are several different theories attempting to explain number processing and subitizing (Alston \& Humprheys, 2004; Cowan, 2001; Dehaene \& Cohen, 1994; Gallister \& Gelman, 1992; 2000; Logan \& Zbrodoff, 2003; Piazza et al., 2006). Although the concept of subitizing has been present in literature for almost 60 years, there is still no consensus on the clarification of this phenomenon.

## 3. Paradigms

Procedural and conceptual counting knowledge have to be studied with different instruments and by means of different tasks. Procedural counting tasks are designed to measure children's ability to perform an arithmetic task, i.e. to test whether a child can successfully determine that there are five objects in an array (LeFevre et al., 2006). The knowledge of the sequence of counting words (the number row) is one of the most important procedural aspects of counting. Testing procedural knowledge includes the measuring of the ability to count forward and backwards. In general, procedural counting tasks measure the accuracy in counting objects (see subtest 1 Appendix A). Conceptual counting knowledge reflects a child's understanding of why a procedure works or whether a procedure is legitimate (LeFevre et al., 2006). This conceptual counting knowledge can be tested by asking

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children to assess their own counting performances (see subtest 2 Appendix A) or those performed by an animated frog (LeFevre et al., 2006).

Subitizing is studied using habituation and familiarization, visual expectation and enumeration paradigms. In the habituation paradigm changes in viewing time are considered a measure of habituation to repeated presentations and dishabituation to novel stimuli. Initially a certain stimulus is being presented until the infant's attention wanes, recording the total viewing time. Then the same stimulus is presented over and over again until the viewing time reaches a certain criterion, e.g. half of the initial viewing time ("habituation"). Finally a new different stimulus is presented and the increased viewing time is compared to the one of the last habituation trial ("dishabituation"). This paradigm was used on one-week-old infants habituating to 3-dot displays, dishabituating to 2-dot displays (Antell \& Keating, 1983). Starkey and Cooper (1980) used the paradigm on 4-month olds, investigating the viewing time to a set criterion of two or three dots. The spacing of the dots was varied. Viewing times were computed in response to the different arrangements of the dots. Infants looked significantly longer towards the numerically different displays (i.e. two dots expecting three dots) than they did toward the numerically familiar displays (i.e. three dots expecting three dots). Infants were also shown to habituate to 2-object visual displays and dishabituate to 3beat sound stimuli (Starkey et al. 1980, 1990), indicating the recognition of numerical sequences across modalities. Wood and Spelke (2005) used the same paradigm with fivemonth old infants in order to investigate the ability of infants to discriminate quantities in visual arrays which were very briefly presented. The infants were made to watch a succession of dot arrays, with a new array appearing every $1 \mathrm{sec}, 1.5 \mathrm{sec}$ or 2 sec . Infants were habituated to a sequence of visual displays. Van Loosbroeck and Smitsman (1990) habituated infants of

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5 months, 8 months and 13 months to small sets of rectangular figures moving continuously on a computer screen (see Figure 1).

Insert Figure 1 about here.

There has been some evidence of subitizing at the age of 5 months. Wynn (1996) used the habituation paradigm with 6-month-old infants to investigate differentiation between twojumps from three-jump sequences of a puppet. Infants were presented with six test trials in which the puppet alternated between jumping two and three times. Prior to this experiment the infants were shown the puppet and allowed to hold it. On habituation trials, the curtain was raised to reveal the stationary puppet. Approximately 1 second later, the experimenter executed the specified jump sequence, measuring tempo and numeration of jumps with a metronome. Upon completion of the jump sequence ( $1 / 2$ second after the descent of the final jump), it was timed how long the infant was looking at the now-stationary puppet. Jumps were soundless. At the end of the trial, the curtain dropped to obscure the display briefly (1.5 second), and then rose to initiate the next trial. The trial would be ended when (a) after 2 or more seconds of continuous looking, the infant looked away for at least 2 continuous seconds or (b) the infant looked for 30 sec cumulatively. Habituation was achieved when (a) the viewing time of 3 consecutive trials excluding the initial 3 was less than half of the infant's viewing time during those 3 initial trials or (b) the infant completed 14 trials without meeting the criterion (a). Upon habituation the infant received a 40 -second break, during which he or she was turned away from the display and allowed to interact with a parent. Test trials followed the same procedure. Jumps were of 1 second duration ( $1 / 2$ second ascent, $1 / 2$ second descent), with a pause between jumps. Infants were able to differentiate between sequences

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with a previous number of jumps and those with a different number of jumps. Starkey, Spelke and Gelman (1990) used a variety of photographs (e.g., lions, frogs, horses, birds, trees, leaves, etc.), with either two or three items on them, to show to 7 -month-olds (see Figure 2).

Insert Figure 2 about here.

Infants looked significantly longer toward the numerically novel displays (i.e. expecting 2 items and looking at 3 items) than they did toward the numerically familiar displays (i.e. expecting 2 items and looking at 2 items). Strauss and Curis (1981) presented 10- to 12-months-olds with sets of color drawings (e.g., chicks, dogs, houses, etc.) that were arranged randomly on an imaginary four by four matrix. During the trial, infants were shown arrays of a new quantity that consisted of either unfamiliar items (for infants in the heterogeneous condition with dogs, houses etc.) or chicks (for infants in the homogenous condition with chicks). Infants in both conditions differentiated two from three items. More recently, using a cross modal transfer task from touch to vision, Féron, Gentaz and Streri (2006) showed that 5-month-old infants were able to detect numerical correspondence between sets of objects presented in the tactile (two or three objects presented one by one in the infant's right hand) and visual modalities (visual displays with 2 or 3 objects presented either simultaneously or sequentially). Moreover, this ability was no longer limited to small number sets, but allowed infants to differentiate between larger sets as well, if the ratio between the two numerosities was $2: 1$. By the age of 9 months this ratio of numerosities that can be differentiated had developed to 3:2 (Lipton \& Spelke, 2003).

Familiarization is a variation of the habituation paradigm (e.g., Caron et al., 1997). Familiarization relies on changes in viewing time as measures for understanding. Identical

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objects are presented to the left and right of a focal point, for a set number of trials. After a fixed number of trials, the familiar object is presented either to the right or left of fixation, paired with a new stimulus. Now children look longer at the new stimulus. Unlike with habituation, the overall length of time the stimuli are presented is determined by the experimenter (e.g., 12 trials), rather than by how long the infant is looking. This paradigm is used to investigate whether infants are able to learn to anticipate the 'number' of pictures appearing on the left. The stimulus content consisted of a randomly ordered heterogeneous set of five animated stimuli (smiling face, turning head, running puppy, rotating wheel, and jumping stick figure). After a trial period, the pictures on the left appeared according to one of three different timing schedules: fast (for 500 ms each), average (for 1000 ms each) and slow (for 1500 ms each). The target picture (R) was always preceded by a 1000 ms interstimulus interval (ISI) and stayed for 1500 ms . During the 2 to 4 min period of data collection, the infants saw on average 95 pictures. Infants' eye movements were recorded while they viewed either a 'numerically predictable' or a 'numerically unpredictable' repeated sequence of pictures (Canfiled \& Smith, 1996). Children looked longer at the new 'numbers', which indicates that the longer observation time is linked with subitizing and not merely with differentiation between expected and unexpected stimuli.

Another eye movement paradigm that is used in the study of future goal-oriented or prospective behaviour is the visual expectation paradigm or surprise test procedure pioneered by Haith (1993). This paradigm relies on changes in viewing time as measures for understanding. Objects are presented to the infant. Then the scene is hidden by a screen, whilst an action is performed (e.g., adding or removing objects from behind the screen). Afterwards the screen is removed to reveal a display either consistent or inconsistent with the action. Children look longer at inconsistent test displays. This kind of set-up was applied to

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infants at the age of 4-5 months by Wynn (1992), using a number of faces hidden behind a screen. It was shown that infants are able to add and subtract small numbers. In the study by Simon, Hespos and Rochat (1995) children expected to see two horses but got to see i.e. one horse and a frog etc. Children were found to look longer in the unexpected number condition (2 horses instead of 3) than in the unexpected stimuli condition (a horse and a frog instead of 2 horses). Eye-tracking was also used as an indicator by Lécuyer and colleagues (2004), in a situation where a teddy bear was successively hidden behind a screen and appeared to be present or not when the screen was removed. Feigenson and Carey (2003) explored the representation of small sets of objects in 12 to 14 month-old infants in a similar way. Infants observed ping pong balls being hidden in a black foam-core box. In the anticipated empty condition infants searched for the ball in an anticipated empty box. Onishi, Baillargeon and Leslie (2007) applied this paradigm to investigate whether 15-month-old infants also detected inconsistencies in these kinds of scenarios.

Psychological studies on enumeration have typically measured reaction times and error rates. A number of objects are presented to a subject and the subject must identify as quickly as possible how many objects there are on the screen. Typical results obtained in these tasks show an increase in latency with the number of digits to be identified. The results show a discontinuity in the increase. For one to three or four items the slope of increase is much shallower (50-80 ms/item) than for higher numbers where a much sharper and linear increase is shown (by about $200-275 \mathrm{~ms} /$ item), leading to the hypothesis that subitizing and counting are two distinct processes. Fast and accurate enumeration was found for random configurations of a small amount of moving targets even when presented among static distractions. There are inconsistent results with regard to the effect of target color heterogeneity on subitizing efficiency. Watson and Maylor (2006) found that observers were

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no less efficient at subitizing displays containing red and green items than they were at subitizing displays of a single color. The enumeration approach was also used with 2 to 5 year old children. These children were found to enumerate small arrays of objects accurately and reliably irrespective of spatial arrangement and with a presentation duration ( 200 ms ) that precluded verbal counting (Starkey \& Cooper, 1995). Turner syndrome patients too were asked to state out loud, as accurately as possible, the number of dots in a randomly organized square display, which remained on screen until a response was given (Bruandet, Molko, Cohen, \& Dehaene, 2004). They were told the numbers ranged from 1 to 8 . All numbers were presented 10 times each, for a total of 80 trials. Response times were recorded via a voice key, and the vocal responses were later scored for accuracy. Impairments were observed in cognitive assessment, subitizing and calculation compared to a control group. Finally number assessment tasks were used involving split-brain patients. A 47 year old split-brain patient had to indicate how many items were in each stimulus set by pressing one of the four response keys. Each response key was numbered, beginning with 1 on the left key and proceeding to four on the key furthest to the right. There were 8 sets of each of the four quantities of circles (Colvin, 2005). Both hemispheres proved capable of performing the subitizing tasks. The same tasks were used by Piazza et al. (2006) on human adults. There were two different categories per modality: red and green lights for the visual presentation, and high and low tones for auditory presentation. The number estimation task consisted of deciding which of the categories contained more items.

Finally number inhibition tasks with a Stroop paradigm have been used to test number comparison skills (Cohen Kadosh et al., 2007; Roussell \& Noel, 2007). In these tasks people are instructed to pay attention to the physical size of a number (i.e. small size ' 3 ' $<$ large size ' 2 ') and to ignore its numerical value (i.e. $3>2$ ). In general, participants need more

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time to carry out incongruent tasks (e.g., a physically large 2 and a physically small 3) than for congruent ones (e.g., a physically small 2 and a physically large 3 ) because in the incongruent condition participants need to inhibit irrelevant spontaneous magnitude comparison $(2<3)$. The size-congruity effect or the difference in response time between incongruent (e.g., 2 3), neutral (e.g., 2 2) and congruent (e.g., 23 ) conditions is being studied (Henik \& Tzelgov, 1982; Landerl, Bevan, \& Butterworth, 2004; Girelli, Lucangeli, \& Butterworth, 2000; Rubinstein \& Henik, 2005). Currently enumeration and stroop number inhibition tasks are often integrated in screeners and tests for MLD. For example the dyscalculia screener is a computer-based assessment test that indicates tendencies towards MLD by measuring pupils' from 6-14 years response times as well as the accuracy of their answers. The dyscalculia screener includes a simple reaction time test and three MLD tests: a number stroop, a dot counting task and an item-timed arithmetic task. Results are given as standard scores. Butterworth (2003) aims to differentiate children with MLD (lowest $10 \%$ on all tests) from low performers due to poor learning/teaching (low on arithmetic but not on the other tests). In the ZAREKI (Von Aster, 2002) screener similar tasks are included. The TEDIMATH (Van Nieuwenhoven, Grégoire and Noël, 2001) test is not a screener, but measures magnitude comparison (subtest 4 in Appendix A) as well as procedural and conceptual counting (subtest 1 and 2 in Appendix A) are measured (Desoete \& Grégoire, 2007; Stock, Desoete, \& Roeyers, 2007).

## 4. Counting and subitizing in MLD and other patient data

There are studies on counting skills of subjects with MLD. Dowker (2001) showed that children who had difficulties in any particular aspect of counting had overall below

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average mathematical performances. In addition, it was shown that toddlers who lacked adequate and flexible counting knowledge went on to develop deficient numeracy skills which resulted in MLD (Aunola et al., 2004; Gersten et al., 2005). Furthermore, Geary, BowThomas and Yao (1992) found that small children with MLD were more likely to make procedural errors in counting and still had conceptual difficulties at the age of six. Desoete and Grégoire (2007) also showed that children with MLD in grade 1 already had encountered problems on numeration in nursery school. They also found some evidence of dissociation of numerical abilities in children with MLD in grade 3 (certain skills appeared to be developed whereas others were not, which made it necessary to investigate them separately and independent of one another). About 13\% of the MLD-children still had processing deficits in number sequence and cardinality skills in grade 3 . About $67 \%$ of the MLD-children in grade 3 had a lack of conceptual knowledge. Finally in this field of research Porter (1998) contributed the finding that the acquisition of procedural counting knowledge did not automatically lead to the development of conceptual understanding of counting in children with MLD. Taking into account the complex nature of mathematical problem solving, it may be useful to assess procedural as well as conceptual counting procedures in young children at risk and in children with MLD, in order to focus on these factors and their role in mathematics learning and development.

There are far less studies on subitizing skills of subjects with MLD. However, Reeye and Reynolds (1994) found that $6 \%$ of the babies had a subitizing deficit. Later on these children were also found to be slower in reading three digit numbers in comparison to children who had good subitizing skills as a baby. The authors however did not report whether or not these children actually developed a MLD. The importance of subitizing in MLD was pointed out by Koontz and Berch (1996) who found that children with MLD were

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slower to process numbers and slower in subitizing tasks, in comparison to children without MLD. This finding was confirmed by Landerl, Bevan, and Butterworth (2004) when they found that children with MLD were slower at numerical differentiation in comparison to control groups and that they showed deficits in subitizing.

People with MLD scored faster on number inhibition tasks than subjects without MLD because they did not spontaneously compare magnitudes which otherwise might have delayed their response. Participants with MLD showed a smaller size-congruity effect. The same MLD-pattern was induced in healthy volunteers with fMRI-guided Transcranial Magnetic Stimulation disrupting left- or right-intraparietal sulcus activation clusters (Cohen Kadosh et al., 2007). The study revealed that it was possible to evoke MLD-performances in subjects without MLD. In addition, recent fMRI studies confirmed an under-activation in the parietal areas (important for number functions), and in the frontal regions (dominant for executive working memory and attention functions) in children with MLD (Von Aster \& Shalev, 2007).

However, not all subjects with a clinical diagnosis of MLD were found to have subitizing problems. Desoete and Grégoire (2007) found a more severe subitizing deficit in $33 \%$ of the children of 8 and a half years old with a clinical diagnosis of MLD. Fischer, Gebhart and Hartnegg (in press) found that between $43 \%$ and $79 \%$ of the subjects in the age range of 7 to 17 years with MLD performed below the 16 -th percentile of the peer control groups on subitizing tasks.

Nevertheless, subitizing and estimation tasks are not trivial. Because of a certain degree of plasticity and modifiability of the spatially-oriented number line (Von Aster \& Shalev, 2007), Wilson, Revkin, Cohen, Cohen and Dehaene (2006) studied if numerical cognition skills could be trainable, in order to improve them, even in 7 to 9 year old children with MLD. Over a period of five weeks, they used a computer-assisted training (number race

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software) and found an increased speed of subitizing by 300 msec , no change in the speed of counting range and increased subtraction accuracy by an average of $23 \%$. These results showed that subitizing could be enhanced in children with MLD.

The concept of separate processes for subitizing and counting, in connection with the involvement of spatial serial processing, has been supported by studies carried out with patients affected by acquired brain damage (Dehaene \& Cohen, 1994). Simultagnosic patients were excellent at subitizing tasks but made $100 \%$ errors in counting tasks. In patients with visuospatial neglect, dissociation between localizing and enumeration processes was observed, whereas subitizing remained operationally intact even in the field affected by the neglect (Vuilleumier \& Rafal, 1999). Deaf and hearing adults showed evidence of similar patterns of performance on subitizing tasks (Bull, Blatto-Vallee, \& Fabich, 2006). Studies with cerebral-palsy children aged 4 - to 8 years revealed that they subitized significantly slower than their peers in the control group, suggesting that subitizing and counting are dependent processes during development, arguing in favour of a subitizing model that supports a global process for handling magnitude (Arp \& Fagard, 2001; Arp, Taranne \& Fagar, 2006). In split-brain patients it was found that results for both hemispheres did not differ on the internally represented analogical number lines. Both hemispheres were able to rapidly enumerate small sets of items and make magnitude judgments regardless of the way in which numerical information was presented. The functional specialization of each hemisphere had an impact on the performance in specific tasks, but both hemispheres showed comprehension of quantity and magnitude (Colvin et al., 2005). Impairments were observed in subitizing but also in cognitive estimation and calculation in patients with MLD due to Turner syndrome (Bruandet et al., 2004). Limited subitizing ranges were also found in

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patients with Alzheimer's disease (Maylor et al., 2005) and in children with chromosome 22q11.2 (Simon, Bearden, Mc-Ginn, \& Zackai, 2005).

To sum up, it may be stated that there is currently a strong focus on the assessment of counting (e.g., Le Fevre et al., 2006; Gersten et al., 2005; Stock, Desoete \& Roeyers, 2007). Stock et al. (2007) indicated that it is important to assess both procedural counting knowledge as well as the ability to distinguish essential from inessential counting characteristics (or the conceptual counting knowledge of young children) in nursery children. The inability to carry out tasks, requiring these skills, at the age 5 to 6 , proved to be a marker for MLD at a later age. Clinicians should, however, not trivialise the importance of response time based estimation and subitizing tasks in nursery and grade 1 till 3 (Geary, Bailey \& Hoard, in press; Schneider et al., 2008). These should also be included as measures in a screening or an assessment battery for children at-risk. According to Wilson et al. (2006) the core deficit hypothesis promotes the need for an early assessment of the understanding of the meaning of numbers, and a response time based assessment of number sense, by means of a wide variety of tasks, including response time based both non symbolic (e.g., subitizing), as well as symbolic numerical comparison and approximation tasks, controlling for alternative stimulus cues such as cumulative surface area, density, or contour length. Moreover, according to Wilson et al. (2006) subtraction provides a more sensitive measure of an inborn deficit in the number module in grade 1 children than either addition or multiplication. Taking into account the complex nature of mathematical problem solving, we are of the opinion that it may be useful to assess counting and subitizing skills in young children with MLD and focus on their role in mathematics learning and development. We expect that the skill to differentiate between magnitudes develops more slowly among children with MLD compared to typically developing peers. Specific remedial programs can be set up when deficits are detected in these

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skills. Magnitude comparison and subitizing can be assessed in small children using the habituation, the familiarization or the visual expectancy paradigm, always controlling for alternative stimulus cues. In older children these paradigms can be applied as well but enumeration tasks and the number-stroop inhibition paradigms are further possibilities.

In addition, research has shown that subitizing is not a constant, static ability but that it has a certain plasticity (Wilsson et al., 2006), allowing children with MLD to make improvements in how quickly they are able to take in quantities, by means of 10 hours training over a five week period with computer games. For children who start school with limited subitizing skills, this can offer some useful perspectives. In addition to the more traditional counting training programs, it can be useful and helpful to focus on number comparison and subitizing in children who give evidence of limited skills in these areas. It is important however that such programs are continued to be critically analyzed and checked on their effectiveness by using peer control groups. It seems worthwhile to investigate whether the allocation of extra time to subitizing skills can make MLD less pervasive.

## 5. Conclusions

As we mentioned in the introduction, the aim of this article was to focus on magnitude processing, particularly subitizing, which has a distinct characteristic, namely the automatic assessing of numerical magnitude. In addition, the relationship with counting has also been highlighted.

Counting proved to be a suitable predictor for at-risk arithmetic performances. 'Procedural knowledge about counting' can be assessed on the basis of how accurate children are in counting objects. 'Conceptual knowledge about counting' can be assessed by asking

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children to make judgments about unusual right-to-left counts and reflections on why counting procedures work.

In addition, it was established that subitizing occurs in preverbal human infants (e.g., Xu , 2003), animals (e.g., Phillips \& Santos, 2007), as well as in adults (e.g., Watson, Maylor, \& Bruce, 2007) and a growing body of research evidence of problems was recorded in patient samples (e.g., Bull et al., 2006). In order to assess subitizing and number processing, psychologists have turned to a diversity of behavioural methods. In pre-verbal infants the habituation (e.g., Wood \& Spelke, 2005), familiarization (e.g., Caron et al., 1997) or visualexpectancy paradigm (e.g., Wynn, 1992) can be used, possibly in combination with eye or gaze tracking recording (e.g., Canfiled \& Smith, 1996). Recent studies demonstrate the need to control for alternative stimulus cues such as cumulative surface area, density, or contour length. For older children enumeration tasks (e.g., Starkey \& Cooper, 1995) and strooprelated inhibition paradigms (Cohen Kadosh et al., 2007), measuring reaction times and error rates, can be used, again controlling for continuous alternative cues. For adults the same methods can be applied (e.g., Bruandet, Molko, Cohen, \& Dehaene, 2004). It is however not implied that the use of one paradigm, in exclusion of another, leads to more accurate results. We consider it advisable not to restrict testing according to just one paradigm and one modality (eg., visual displays) but to compare results of poor subitizing, obtained by a particular technique and modality (e.g., vision) to results obtained with another technique and modality (e.g., touch, sounds) in order to find whether they are consistent or not. Multimethod techniques are the obvious way to investigate whether the choice of paradigm is affecting the outcome of the assessment, in other words whether the method of testing is determining what is found.

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All of this implies that we should not only assess how accurate young children can count (procedural knowledge) but also whether they have mastered the counting principles of Gallistel and Gelman (1992) and how fast and accurate these children can subitize. It may be interesting to study a sufficiently big enough group of children from birth on in order to see if their counting skills in e.g. the third grade can be predicted by means of scores obtained at the age of six months. A controlled intervention focusing on counting knowledge in children atrisk can prevent MLD from occurring later on in these vulnerable children.

There is currently interest in early indicators which can predict arithmetic development. This interest stems from the belief that, if we are able to succeed in detecting children at risk at an early stage, we may be capable of averting a deficient development and as such able to prevent later problems in arithmetical development. If such early indicators for MLD can be translated into key components of remediation programs, it may stop children from falling further and further behind.

Geary and Hoard (2005) found that children at-risk had less developed counting knowledge and especially lacked conceptual counting knowledge.

Within the explanatory models for MLD, subitizing is sometimes considered as a low-level symptom of the core deficit in magnitude processing (e.g., Landerl, Bevan, \& Butterworth, 2004). Others, however, make light of the role subitizing may play in MLD and attribute quantity processing deficits in children with MLD to either a deficit in working memory or speed processing (e.g., Geary et al., 2007), or problems with visual spatial elaboration (e.g., Shalev, 2004) or executive deficits (Passolunghi \& Siegel, 2004). On the other hand, Wilson and colleagues (2006) revealed the plasticity of subitizing speed and subtraction accuracy by means of a short-term intervention on children with MLD.

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It is important however to be aware of the benefits and pitfalls of focusing on subitizing. Subitizing is applicable to sets of entities which are presented for a short while, at a rate of about 50 msec per item, and which have to be enumerated within this time. This works accurately with little or no error (Simon \& Vaishnavi, 1996). Recent studies revealed that total luminance, total occupied area, item size and free area around each item, as nonnumerical parameters, have to be controlled to prevent infants responding to a change in contour length instead of to a change in number (Brannon et al., 2004; Dehaene, Izard \& Piazza, 2005). In addition, to figure out the amount in larger sets of objects without a ratio of 3:2 (e.g., 25 vs. 30 ), subitizing does not work. It requires counting, grouping or estimation. Therefore, it is not sufficient to practise and train subitizing skills of children with MLD. Besides some researchers (e.g., Geary et al., 2007) remain opposed to the subitizing deficit hypotheses. After all, the Wilsson et al. (2006) study was conducted without a control group. Additional research with a different design is required in order to investigate the effect of the plasticity and trainability of subitizing in other tasks (generalization) and the durability of the progress made.

To sum up, we would like to state that, since subitizing and counting are both important for mathematical problem solving and since subitizing can not be simply reduced to counting, both have to be assessed separately, especially when problems occur in mathematical problem solving. In this paper, the paradigms to assess counting and subitizing were reviewed. Furthermore, we pointed out that we cannot just assume that subitizing skills will develop further on their own as children with MLD grow older and have had more practice of counting and increased experience of mathematics. It is likely that the lack or restriction of the ability of 'rapid enumeration of small sets of items' (subitizing) in early childhood is responsible for the fact that the 'mathematical language' (e.g. more, bigger than),

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which becomes engrained in other children on their subitizing experiences (e.g. 3 is more than 2; 3 is bigger than 2), remains problematic in children with MLD. It could also provide an explanation for the fact that a substantial amount of children at risk are initially reported with a language disorder in their early childhood, which eventually turns out to develop into a substantial and persistent MLD in primary school. Prevention programs and therapy may be able to offer these children basic visual-spatial experiences and focus on the strengths and weaknesses in counting and subitizing of children with MLD, in order to make them more aware of how they can efficiently deal with magnitudes. These kinds of studies are definitely an important challenge for future research on MLD.

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Figure 1: Habituation items without control over non-numerical parameters

| Item 1 |  |  |  |
| :--- | :--- | :--- | :--- |

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Figure 2: Habituation items
Item 1

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Appendix A: Examples of test-items of the TEDI-MATH

| Subtest | Content and example of item |
| :--- | :--- |
| 1 Procedural <br> counting <br> knowledge | Counting up from a lower bound to an upper bound ("from 5 up to 9") |
| 2. Conceptual <br> counting <br> knowledge | Counting random pattern of items ("how many turtles are there? How many <br> turtles are there in total?"). <br> Understanding of the cardinal ("How many hats do I have in my hand, when all <br> the snowman in this picture have had a hat?") |
| 4. Estimation of <br> the size | Comparison of dot sets <br>  <br>  <br>  <br>  <br> Where do you have most dots? Here or here? Show me. <br> Estimation of size: Comparison of distance between numbers |
| -Target number is 3. Which number is closest to this number (8 or 2)? |  |

