Jedrzej Banaszczyk,
Gilbert De Mey,
***Anne Schwarz,**
***Lieva Van Langenhove**

**Department of Electronics
and Information Systems,
Ghent University,**
Sint Pietersniewstraat 41, 9000 Gent, Belgium
E-mail: jedrzej.banaszczyk@elis.ugent.be
demey@elis.ugent.be

***Department of Textiles,
Ghent University,**
Technologiepark 907, 9052 Zwijnaarde-Ghent, Belgium
E-mail: anne.schwarz@ugent.be
lieva.vanlangenhove@ugent.be

Current Distribution Modelling in Electroconductive Fabrics[1]

Abstract
In this paper the current distribution within woven electroconductive textile sheets is investigated. The iterative method of solving very large resistor networks, which is used to model current density in textile sheets, is discussed. Factors taken into account are the conductivity of the fibres, the contact resistance between the fibres, the contact angle between the fibres and supply electrodes, as well as the size and aspect ratio of textile sheets. The influence of these factors on sheet resistivity is discussed. Maps of the current distribution generated with a computer program are included.

Key words: *textile, current distribution, electroconductivity, modelling.*

Introduction

Electroconductive textiles are a relatively new but promising idea. The range of possible applications they could have in the future is very wide and varies from medicine [1] to telecommunications [2] and wearable computing [3]. Fibres can be used as conductors, which can be easily incorporated into clothing as well as form connections between electronic components [4], but they can also be made into textile sheets by means of e.g. weaving or knitting, to form antennas [2] or electrodes [1], for example. In order to model the operation of such devices it is necessary to know their internal current distribution.

At first glance any piece of textile can be considered as a thin conducting sheet, which may be described by just one parameter - sheet conductivity. The main purpose of this paper is to show that this statement no longer holds for electroconductive textiles. Due to the contact resistance, a sheet of textile cannot always be considered as a thin, uniform and isotropic conductor.

An electroconductive textile sheet, be it woven or knitted, can be treated as a combination of series and parallel resistor connections that correspond to the resistivity of the fibres themselves and to the resistance between fibres which mutually cross in the fabric (contact resistance). These parameters will depend on numerous external factors, such as the temperature, humidity, pressure and extension [5 - 8]. Furthermore, these may vary as a result of the device heating up during operation. This is why it is difficult to create a consistent model of the current density.

Here an attempt was made to generate a simplified model by taking into account the conductivity of the fibres, the contact resistance between them, the angle between the fibres and the supply voltage electrodes (anisotropy), as well as the size and aspect ratio of the textile sheet. With this model we wanted to verify how these parameters influence the current distribution in a textile and whether it is possible to define sheet resistivity for woven electroconductive textile sheets irrespective of fibre orientation (contact angle).

Electrical model

A woven electroconductive textile is presented in ***Figure 1.a***. A voltage V is applied to the textile by means of two electrodes with perfect contact assumed. The angle α between the fibres and electrodes, as seen above, is called the contact angle. The sheet resistance R_{sheet} of the textile sheet, expressed in Ω, is defined as

$$R_{sheet} = \frac{V_{el}}{I_{el}}, \qquad (1)$$

where V_{el} and I_{el} are the voltage and current between the electrodes. The value of R_{sheet} will depend on the shape and dimensions of the sample investigated . On the other hand, the sheet resistivity ρ_{sheet}, is defined as

$$\rho_{sheet} = \frac{\dfrac{V_{el}}{L}}{\dfrac{I_{el}}{D}}, \qquad (2)$$

where L and D are the length and width of the sheet, respectively (***Figure 1.a***), expressed in Ω/□, which is characteristic
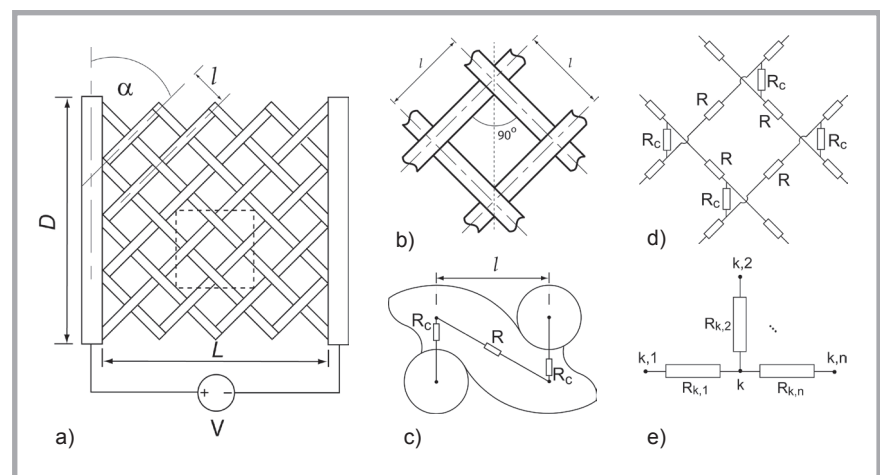

Figure 1. *Woven electroconductive textile; a) General view, b) Enlarged fragment of a woven textile, c) Schematics of the contact resistance, d) Equivalent electrical model for a fragment of woven textile, e) Explanation of the iterative method.*

for the material itself, irrespective of the sample's size and shape [9].

An electroconductive textile can be regarded as an electric circuit consisting solely of resistors connected in series and in parallel. An enlarged fragment of the textile from *Figure 1.a* and its electrical model equivalent are shown in *Figures 1.b* to *1.d*. The electrical resistance of a piece of fibre of length l, which is the distance between two parallel fibres, is denoted as R; for the purpose of our considerations $R = 10\ \Omega$ was chosen. The contact resistance between two intersecting fibres is denoted as R_c. Both of these resistances have to be determined experimentally for every type of fibre investigated . Their values will vary significantly depending on the material of which the fibres are made, the structure of the fibres (e.g. they could consist of many thinner fibres), as well as external conditions, such as the pressure between contacting fibres or the humidity of the textile (e.g. the influence of human sweat) [5 - 8]. Creating a consistent method for determining the fibre resistivity and contact resistance is one of our most important goals for the nearest future. In the model discussed here, R_c values ranging from $0\ \Omega$ to $100\ \text{k}\Omega$ have been assumed.

Solution for the equivalent circuit

The idea of the electrical model is simple in itself, but the main problem is the large size of the circuit, which could contain up to one hundred thousand elements, making it difficult to simulate using standard electrical circuit simulators. An analytical solution based on Kirchhoff's current and voltage laws is not possible due to the large number of unknowns. Such a solution would become unstable, and the amount of memory required to store the coefficient matrix would be enormous. Therefore a computer program was written that builds the electric circuit out of elementary sub-circuits and then solves it using the iterative method. In each iteration the new potential of every node in the circuit is calculated according to the formula [10]:

$$\phi_k^{new} = \phi_k^{old} + \tau \left(\frac{\dfrac{\phi_{k,1}}{R_{k,1}} + \dfrac{\phi_{k,2}}{R_{k,2}} + \cdots + \dfrac{\phi_{k,n}}{R_{k,n}}}{\dfrac{1}{R_{k,1}} + \dfrac{1}{R_{k,2}} + \cdots + \dfrac{1}{R_{k,n}}} - \phi_k^{old} \right),$$

(3)

where ϕ_k^{new} is the new potential of the k-th node after the iteration, ϕ_k^{old} is the potential of the k-th node before the iteration, τ is the relaxation coefficient, $\phi_{k,1}$, $\phi_{k,2}$, ... $\phi_{k,n}$ are the potentials of the neighbouring nodes of the k-th node, and R_1, R_2 ... R_3 are the resistances of the branches connected to the k-th node, as shown in *Figure 1.e*, which can be equal to either R or R_c.

Iterations are repeated until the difference between ϕ_k^{new} and ϕ_k^{old} for every node in the circuit is smaller than the value ε, which can be chosen as a trade-off between the accuracy and minimum number of iterations required. We chose $\varepsilon = 10^{-9}$. The minimum number of iterations required to solve the circuit with a given accuracy depends on the size of the circuit and the relaxation coefficient τ. In general τ may be chosen from the interval $(0, 2)$, but the choice can be optimized for a given circuit size. In *Figure 2* we can see how the number of iterations depends on the relaxation coefficient for a square sheet of size $L = D = 4\sqrt{2}l$. As we can see there exists an optimum value τ_{min} for which the number of iterations is the lowest. For the smallest sheets $\tau_{min} = 1.3$, which will converge to 2 for large sheets, as shown in *Figure 3*. The minimum
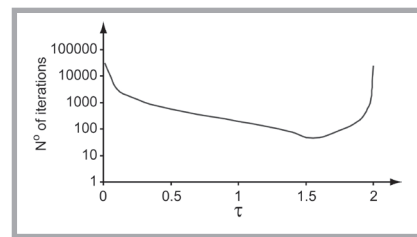

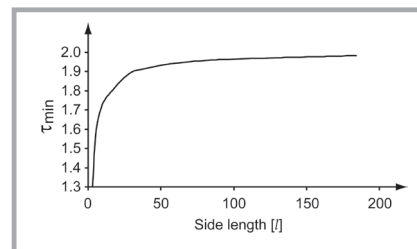*Figure 2. Number of iterations vs. τ, $4\sqrt{2} \times 4\sqrt{2}l^2$, ε = 10⁻⁹.*

Figure 3. τ_{min} vs. sheet size, ε = 10⁻⁹.

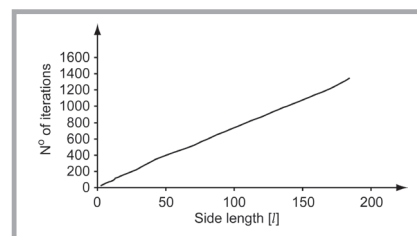*Figure 4. Minimum number of iterations vs. sheet size, ε = 10⁻⁹.*

number of iterations increases linearly with the size of the sheet, as presented in *Figure 4*.

Current distribution

The current distribution within a textile is an important property that must be taken into consideration when designing e.g. electrodes or antennas. We want the current to spread uniformly over the surface of the electrode, rather than crowd in some particular fibres or regions, which
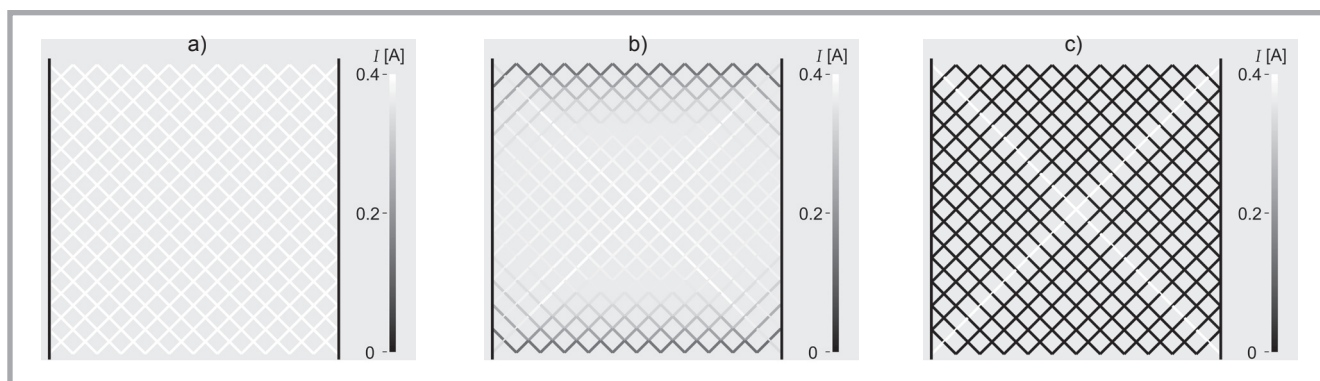

Figure 5. Current distribution for a square shaped sheet with dimensions $D = L = 12\sqrt{2}l$, (sheet length (ls) = 16.9706 l, sheet width (ws)= 16.9706 l), α = 45°; a) $R_c/R = 0$, $R_{sheet} = 10\ \Omega$, b) $R_c/R = 10$, $R_{sheet} = 11.558\ \Omega$, c) $R_c/R = 10000$, $R_{sheet} = 103.486\ \Omega$.
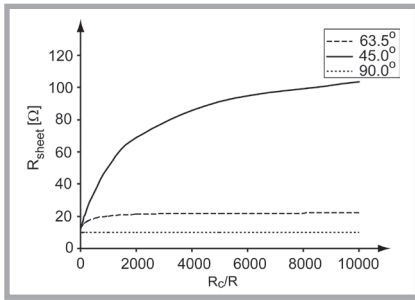
Figure 6. Sheet resistance R_{sheet} vs. contact resistance ratio R_c/R for a fabric sheet of size $D = L = 12\sqrt{2}l$.

could cause hot spots and reduced performance. In our model four factors influence the current distribution within the textile sheet, namely the contact ratio, meaning the ratio of the contact resistance R_c to the fibre resistance R, the contact angle α, the size and aspect ratio of the sheet. These parameters shall now be discussed in the following subsections.

Contact resistance

For the purpose of our investigations, we have assumed contact ratios ranging from 0 to 10000. In **Figure 5.a** (see page 29) a sheet with 0 Ω contact resistance is pictured. The magnitude of the current in all the fibres is equal. As the contact ratio is increased to 10 (**Figure 5.b,** see page 29) the current shows a tendency to flow through the middle of the sheet. A further increase in the contact ratio to 10000 (**Figure 5.c,** see page 29) causes the current to flow almost only through the fibres directly connecting the electrodes. Later the increase in the contact ratio also increases the overall sheet resistance R_{sheet}. The relationship between the contact ratio and sheet resistance for various contact angles for a sheet whose approximate dimensions are $L = D = 17\,l$ ($12\sqrt{2} \approx 8\sqrt{5} \approx 17$) is shown in **Figure 6**.

Contact angle

Simulations for sheets with the same dimensions and contact resistance, as in the previous example, were carried out, but the contact angle was changed from 45° to 63.5°. The resulting current distributions are depicted in **Figures 7**. The first observation we can make is that the current distribution over the surface of the sheet is less uniform than for the 45° contact angle. The reason is that this time only horizontal fibres can provide a direct connection between the electrodes. However, since for the same sheet size more fibres are directly connected between the electrodes, the sheet resistance will be lower for the 63.5° contact angle. This can be verified by reffering back to **Figure 6**. The curve in the case of α = 45° climbs much steeper than for α = 63.5°.

A contact angle of 90° also occurs, i.e. the fibres are perpendicular and parallel to the electrodes. The situation is shown in **Figure 8**. The potentials ϕ_1 to ϕ_5 are all equal, hence no current flows in the vertical fibres. For this reason the sheet resistance for 90° remains constant irrespective of the contact ratio. In this case it is easy to calculate R_{sheet} using the simple formula:

$$R_{sheet} = \frac{aR}{b+1}, \qquad (4)$$

where a and b are the length and width of the sheet, respectively, expressed in l. So for a square sheet of dimensions $L = D = 18l$ and contact angle α = 90°, the sheet resistance would be 9.47368 Ω, irrespective of the contact ratio, which agrees perfectly with the result of our simulation (see **Figure 9**).

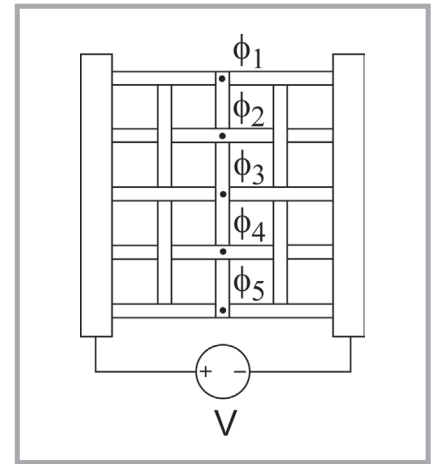To conclude, the most uniform current distribution within the textile sheet is

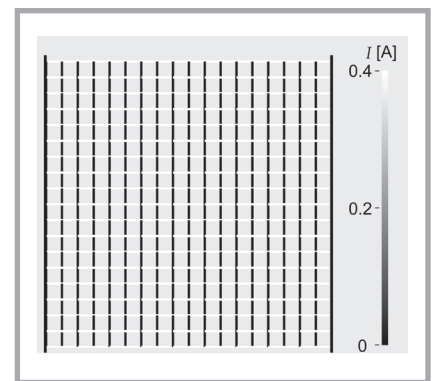**Figure 8.** Textile with 90° contact angle.

Figure 9. Current distribution for a square shaped sheet with dimensions $L = D = 18\,l$, ($ls= 18\,l$, $ws= 18\,l$) α = 90°, $R_c/R = 10$, $R_{sheet} = 9.47368$ Ω,

obtained for 45°. For a 90° contact angle, the current only flows through the horizontal fibres, and the contact resistance has no influence on R_{sheet} and the current distribution. For small contact ratios, sheets with α = 63.5° have a worse current spread than those with α = 45°; however, for large contact ratios they show a lower sheet resistance and better uniformity of the current distribution.
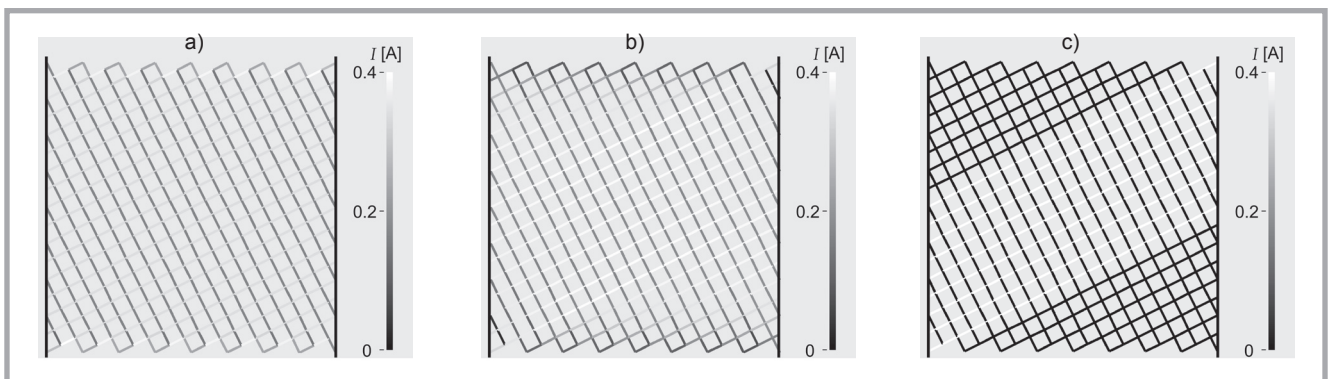

Figure 7. Current distribution for a square shaped sheet with dimensions $D = L = 8\sqrt{5}l$, (sheet length (ls) = 17.8784 l, sheet width (ws)= 17.8784 l), α = 63.5°; a) $R_c/R = 0$, $R_{sheet} = 10.1099$ Ω, b) $R_c/R = 10$, $R_{sheet} = 11.197$ Ω, c) $R_c/R = 10000$, $R_{sheet} = 21.9518$ Ω.

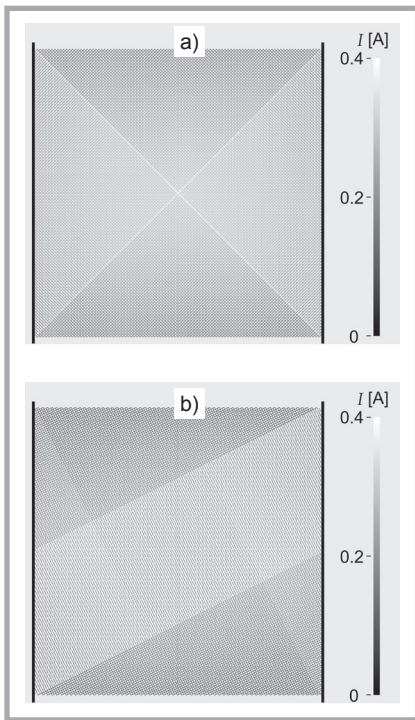**Figure 10.** Current distribution for a square shaped sheet with dimensions $R_c/R = 10,000$; a) $L = D = 98\sqrt{2}l$ (ls= 138.593 l, ws= 138.593 l), $\alpha = 45°$, $R_{sheet} = 22.8384\ \Omega$, b) $L = D = 65\sqrt{5}l$ (ls= 138.558 l, ws= 138.558 l), $\alpha = 63.5°$, $R_{sheet} = 16.6599\ \Omega$.

It should be mentioned again that if the sheet discussed were uniform, the contact angle would not have any influence at all.

■ Sheet size

In the previous subsections we observed that fibre orientation has an impact on current distribution and sheet resistance, hence the sheets are anisotropic. However, this does not mean that the conductivity can be represented by a conductivity tensor for a uniform and linear anisotropic material. The samples we investigated were, however, relatively small. We

would like to know whether there exists a certain ratio of the area of a square sheet to its contact ratio for which the impact of the fibre orientation is so small that the isotropic sheet resistivity can be defined according to equation (2).

By comparing **Figure 7** with **Figure 12.a**, a significant improvement in the uniformity of the current spread is observed. The highest current still flows through the two fibres directly connecting the electrodes, but only fibres located close to the edge of the sheet will remain unused. Similarly, in the case of $\alpha = 63.5°$ the current tends to spread better for bigger sheets, as shown in **Figure 12.b**.

In **Figure 10** R_{sheet} is plotted for square sheets against the length of the sides of the sheets, expressed in l. In the case of small sheets the sheet resistance for $\alpha = 45°$ clearly surpasses that for $\alpha = 63.5°$ and $\alpha = 90°$. However, when the size is increased, the sheet resistance converges to 10 Ω, which is the resistance of a fibre piece of length l. The phenomenon can be further observed in **Figures 11**, with the contact ratio used as the parameter. The contact angle and contact ratio only determine how fast the sheet resistance will converge. As the R_{sheet} converges to R, the current density in the sheet becomes more uniform.

Hence, the question is how big exactly must the square sheet be for a given contact ratio, so that the current density is uniform, and we can speak of an isotropic sheet resistivity. We assumed that the relative difference between the simulated sheet resistance and R must be smaller than or equal to 5%. Based on this criterion, we plotted the minimum lengths of the sides against the square root of the contact ratio for all three fibre orientations
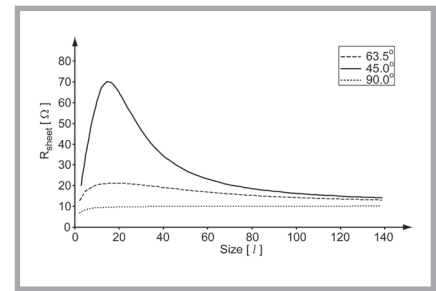

Figure 11. Sheet resistance R_{sheet} vs. side length L (where l is taken as the unit of length), $R_c/R = 2000$.

and linear dependencies obtained. This indicates that for every fibre orientation there exists a minimum ratio of the sheet's area to the contact ratio for which sheet resistivity can be defined. The results are shown in **Figure 13** (see page 32).

■ Sheet aspect ratio

So far only square sheets have been discussed; however, other aspect ratios are also possible. Let us now verify whether it is possible to calculate the sheet resistances of rectangular textile sheets based on the sheet resistivity discussed in the previous subsection.

Let us take a look at the current distribution in rectangular sheets with an aspect ratio greater than 1 (electrode sides are the shorter ones). In previous examples there have always been some fibres connecting the electrodes directly. This caused the current to flow mostly through these connections when contact ratios were high. In such a case an aspect ratio of 3 is chosen, with the sheet length and width equal to approximately 204l and 68l. This ensures that there are no direct links between the electrodes. When the contact ratio is low (e.g. 10) the current spreads uniformly (**Figures 14** and **15**) as
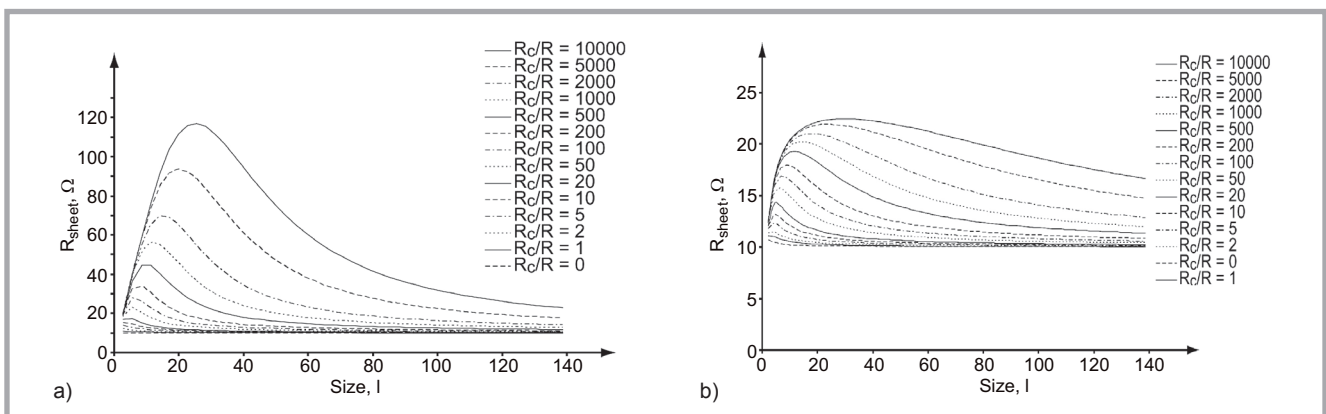

Figure 12. Sheet resistance R_{sheet} vs. side length L (where l is taken as the unit of length), a) $\alpha = 45°$, b) $\alpha = 63.5°$.
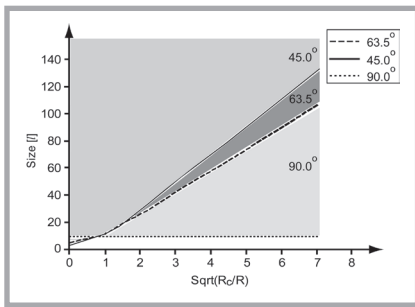
Figure 13. *Minimum side length of a square sheet (where l is taken as the unit of length), whose sheet resistivity can be defined vs. the square root of the contact ratio.*

in the square sheets discussed before. By observing **Figure 13**, we can confirm that for a contact ratio of 10, a square sheet with a side length of 68l is big enough for it to have isotropic resistivity. Indeed, according to the simulations for contact angles of 45° and 63.5° (**Figures 14.a** and **14.c**), the sheet resistance of the long sample is only slightly over 30 Ω, yielding a difference of 5.1% and 4.4%, respectively. Thus the long sheet can be considered a series connection of three square sheets with resistivities equal to 10 Ω/□. When the contact ratio is increased, we observe a change in the pattern of the current distribution. For α = 45° the current tends to crowd at two points where the fibres connected to the top and bottom of where the electrodes intersect (**Figure 14.c**). For α = 63.5° the current seems to spread better (**Figure 14.d**); however, regions of higher current density remain. From **Figure 11.a** we can observe that for a square sheet with a side length equal to 68l and a 45° contact angle, the sheet resistance is equal to approximately 50 Ω. The result of our simulation for a sheet that was 3 times longer yielded a sheet resistance of 201 Ω, which is much more than 3 ×50 Ω. We repeated the same calculation for a sheet with a contact angle of 63.5°. From **Figure 11.b** we can observe that the resistance of a square sheet of dimensions $L = D = 68l$ is 21 Ω. However, the simulation result we obtained for a sheet of dimensions of approximately $L = 204l$ and $D = 68l$ is 132 Ω which is obviously more than 3 ×21 Ω. According to **Figure 13**, the size of the square sheets comprising the sample in both cases is not big enough for the sheets to have isotropic resistivity at this contact ratio. Hence the sample sheet cannot be treated as a series connection of 3 square sheets. The last simulations were devoted to rectangular sheets with aspect ratios smaller than 1 (electrode sides are the longer ones), in which there are a lot of fibres
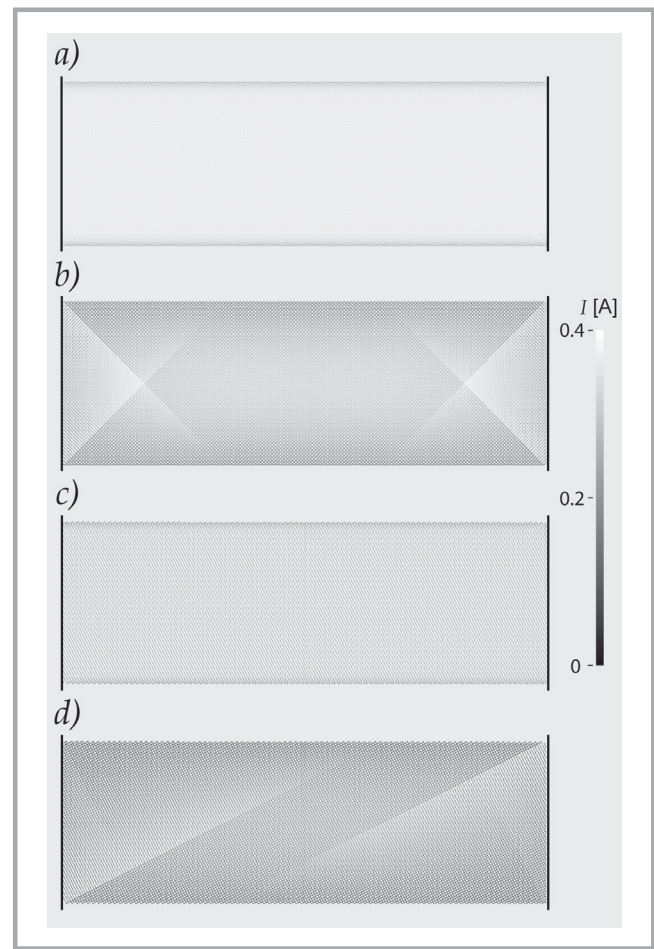
Figure 14. *Current distribution for a square shaped sheet with dimensions:*
a) L =144√2l ,
D = 48√2l,
α = 45°,
R$_c$/R = 10;
R$_{sheet}$ = 31.16 Ω,
ls= 203.65 l,
ws = 67.88l;

b) L =144√2l ,
D = 48√2l,
α = 45°,
R$_c$/R = 10000;
R$_{sheet}$ = 200.93 Ω,
ls = 203.65 l,
ws = 67.88l,

c) L =90√5l ,
D = 30√5l,
α = 63.5°,
R$_c$/R = 10;
R$_{sheet}$ = 31.33 Ω,
ls = 203.37 l,
ws = 67.04l;

d) L =90√5l ,
D = 30√5l,
α = 63.5°,
R$_c$/R = 10000;
R$_{sheet}$ = 132.22 Ω,
ls = 203.37 l,
ws = 67.04l

directly connecting the electrodes. We selected an aspect ratio of 1/3. The current distribution for a contact ratio of 10 is comparably uniform with the distribution of sheets with an aspect ratio of 3, but the current magnitude is much higher.

What is also important is the fact that the sheet resistance is very low. The R_{sheet} is equal to about 3.33 Ω, hence the textile can be considered as a parallel connection of three square sheets with sheet resistivities equal to 10 Ω/□ each. Then
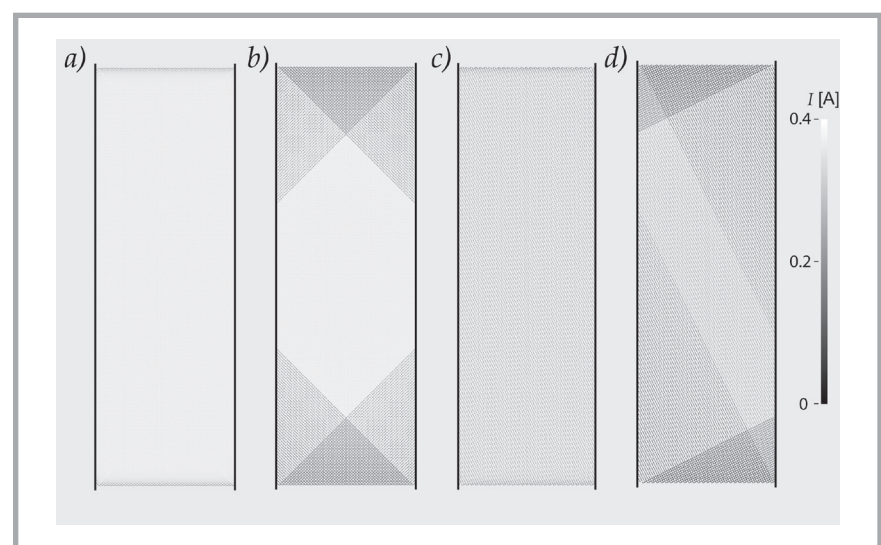

Figure 15. *Current distribution for a square shaped sheet with dimensions: a) L =48√2l, D =144√2l, α = 45°, R$_c$/R = 10; R$_{sheet}$ = 31.16 Ω, ls= 203.65 l, ws=67.88l; b) L =48√2l, D =144√2l, α = 45°, R$_c$/R = 10000; R$_{sheet}$ = 4.54 Ω, ls= 67.88 l, ws = 203.65l; c) L =30√5l, D =90√5l, α = 63.5°, R$_c$/R = 10; R$_{sheet}$ = 3.33 Ω, ls= 67.04 l, ws=203.37l; d) L =30√5l, D =90√5l, α = 63.5°, R$_c$/R = 10; R$_{sheet}$= 4.13 Ω, ls = 67.04 l, ws=203.37l.*

we increase the contact ratio to 10000. The current spread is very uniform in the middle part of the sheet with a 45° contact angle, in which the current density decays only at the edges. In the case of $\alpha = 63.5°$, the current distribution is significantly worse. A stripe of higher current is visible in the middle part of the sheet. However, the overall sheet resistance in both cases increased only by about 25% compared to the sheet resistance of the same sheets with a contact ratio of 10. Similar to before, it can be observed that the resistivity of the material in $\Omega/\square$ can be defined only if the size of the sheet is big enough for a given contact ratio, in which case sheets with aspect ratios smaller than 1 may be treated as parallel connections of square sheets.

Experimental verification of the model

So far we have been able to verify our model experimentally using a woven fabric made of stainless steel yarns (Bekintex). As it is impossible to measure the current distribution directly, an indirect method was adopted. Because the heat generation is proportional to the square of the current, one can expect, as a first approximation, that the temperature rise will also be proportional to the square of the current. We compared the temperature distribution measurements obtained with infrared thermography with our simulations and found that they are in very good agreement. Thorough analysis of these results is beyond the scope of this article and is to be published elsewhere [11]. We are currently using our model to design textile electrodes for blood flow electrostimulation.

Conclusions

It is clear that woven electroconductive fabrics cannot be considered homogenous structures and that the contact resistance R_c is a crucial factor in current distribution. The simulations show that there exists a minimum ratio of the area of a square sheet to the contact ratio for which an isotropic sheet resistivity, expressed in $\Omega/\square$, can be defined irrespective of the fibre orientation in the sample. Therefore the resistivity of a sheet only depends on the electrical resistivity of the fibres. The aim of our future investigations is to develop a reliable system of measuring and characterising the electrical properties of electroconductive fibres, with a special emphasis on the measurement of their contact resistance.

References

1. Lawrence M., Kirstein T., Keller T., "Textile Electrodes for Transcutaneous Electrical Stimulation", EMPA conference talk, Dubendorf (Switzerland), 2004.
2. Locher I., Klemm M., Kirstein T., Troster G., "Design and Characterization of Purely Textile Patch Antennas", Transactions on Advanced Packaging, vol. 29, No. 4, November 2006.
3. Locher I., Troster G., "Fundamental Building Blocks for circuits on Textiles", IEEE Transactions on advanced packaging, vol. 30, NO. 3, AUGUST 2007.
4. Bonderover E., Wagner S., "A Woven Inverter Circuit for e-Textile Applications", IEEE Electon Device Letters, vol. 25, No. 5, MAY 2004.
5. Hersh S. P., Mongomery D. J., "Electrical Resistance Measurements on Fibers and Fiber Assemblies", Textile Research Journal, Vol. 22, No. 12, 805-818 (1952).
6. Zieba J., Frydrysiak M., "Textronics-Electrical and Electronic Textiles. Sensors for Breathing Frequency Measurement", Fibres & Textiles in Eastern Europe January/December 2006, Vol. 14, No. 5(59).
7. Oh K. W., Park H. J., Kim S. H., "Strechable Conductive Fabric for Electrotherapy", Journal of Applied Polymer Science, Vol. 88, 1225-1229 (2003).
8. Asanovic K. A., Mihajlidi T. A., Milosavljevic S. V., Cerovic D. D., Dojcilovic J. R., "Investigation of the electrical of some textile materials", Journal of Electrostatics, Vol. 65, Issue 3, 162-167, March 2007.
9. Halperin S. A., "The Difference Between Surface Resistance and Surface Resistivity", Evaluation Engineering, (6), 49-50 (1996).
10. Collins G. W., II, "Fundamental Numerical Methods and Data Analysis", pp. 44-46, http://ads.harvard.edu/books/1990fnmd.book/, 1990.
11. Banaszczyk J., Anca A., De Mey G.: "Infrared Thermography of Electroconductive Woven Textiles", accepted for publication in the Proceedings of the 9th International Conference on Quantitative InfraRed Thermography, July 2-5 2008, Krakow, Poland.