

The anomalous dimension of the composite operator A^2 in the Landau gauge

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Abstract

The local composite operator A^2 is analysed in pure Yang-Mills theory in the Landau gauge within the algebraic renormalization. It is proven that the anomalous dimension of A^2 is not an independent parameter, being expressed as a linear combination of the gauge β function and of the anomalous dimension of the gauge fields.

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1 Introduction

Nowadays an increasing evidence has been reported on the relevance of the local composite operator $A_\mu^a A^{a\mu}$ for the nonperturbative regime of Yang-Mills theories quantized in the Landau gauge. That this operator has a special meaning in the Landau gauge can be simply understood by observing that, due to the transversality condition $\partial_\mu A^{a\mu} = 0$, the integrated mass dimension two operator $(VT)^{-1} \int d^4x A_\mu^a A^{a\mu}$ is gauge invariant, VT being the space-time volume. Lattice simulations [1] have indeed provided strong indications for the existence of the pure gluon condensate $\langle A_\mu^a A^{a\mu} \rangle$, confirming its relevance for the infrared dynamics of Yang-Mills. Also, the monopole condensation in compact QED turns out to be related to a phase transition for this condensate [2].

Recently, a renormalizable effective potential for $\langle A_\mu^a A^{a\mu} \rangle$ has been obtained in [3] by using the local composite operator (LCO) technique [4]. This result shows that the vacuum of pure Yang-Mills theory favors a nonvanishing value of this condensate, which provides effective masses for the gluons while contributing to the dimension four condensate $\langle \alpha F^2 \rangle$ through the trace anomaly. It is worth remarking here that this mass has a pure dynamical origin and manifests itself without breaking the gauge group. Both features are indeed expected in a confining gauge theory, being in agreement with the Kugo-Ojima criterion for color confinement [5].

An important ingredient in the analysis of the effective potential for the gluon condensate is the anomalous dimension γ_{A^2} of the operator $A_\mu^a A^{a\mu}$. Till now, γ_{A^2} has been computed up to three loops in the \overline{MS} renormalization scheme [6]. The available expression for γ_{A^2} shows rather interesting properties concerning the operator $A_\mu^a A^{a\mu}$ in the Landau gauge. It turns out indeed that, besides being multiplicative renormalizable, its anomalous dimension can be expressed as a combination of the gauge β function and of the anomalous dimension γ_A of the gauge fields, according to the relation

$$\gamma_{A^2} = - \left(\frac{\beta(a)}{a} + \gamma_A(a) \right), \quad a = \frac{g^2}{16\pi^2}, \quad (1.1)$$

which can be easily checked up to three loops [6]. This feature strongly supports the existence of some underlying Ward identities which should be

at the origin of eq.(1.1), meaning that γ_{A^2} is not an independent parameter of the theory.

The aim of this paper is to provide an affirmative answer concerning the possibility of giving a purely algebraic proof of the relation (1.1), which extends to all orders of perturbation theory. Our proof will rely only on the use of the Slavnov-Taylor identity and of an additional Ward identity, known as the ghost Ward identity, present in the Landau gauge [7]. Furthermore, according to [7], it turns out that also the anomalous dimension γ_c of the Faddeev-Popov ghosts can be written as a combination of β and γ_A , namely

$$2\gamma_c(a) = \frac{\beta(a)}{a} - \gamma_A(a) . \quad (1.2)$$

Both relations (1.1) and (1.2) can be used as a very useful check for higher order computations in Yang-Mills theories quantized in gauges which reduce to the Landau gauge when the gauge parameters are set to zero, as in the case of the nonlinear Curci-Ferrari gauge [6].

The work is organized as follows. In Sect.1 we give a brief account of eqs.(1.1) and (1.2) by making use of the available three loops expressions. Sect.2 is devoted to their algebraic proof.

2 The anomalous dimension of the operator A^2 in the Landau gauge

In order to give a short account of the relations (1.1) and (1.2), let us recall the three loop expressions of the gauge β function and of the gauge and ghost fields anomalous dimensions γ_A and γ_c , as given in [6]. They read

$$\frac{\beta(a)}{a} = -\frac{11}{3}(Na) - \frac{34}{3}(Na)^2 - \frac{2857}{54}(Na)^3 , \quad (2.3)$$

$$\gamma_A = -\frac{13}{6}(Na) - \frac{59}{8}(Na)^2 + \frac{(648\zeta(3) - 39860)}{1152}(Na)^3 , \quad (2.4)$$

and

$$\gamma_c = -\frac{3}{4}(Na) - \frac{95}{48}(Na)^2 - \frac{(1944\zeta(3) + 63268)}{6912}(Na)^3 , \quad (2.5)$$

where N is the number of colors corresponding to the gauge group $SU(N)$. Making use of the relation

$$\gamma_{A^2} = - \left(\frac{\beta(a)}{a} + \gamma_A(a) \right) , \quad (2.6)$$

for the anomalous dimension of $A_\mu^a A^{a\mu}$ one gets, up to the third order,

- first order

$$\gamma_{A^2}^{(1)} = - \left(\frac{\beta^{(1)}}{a} + \gamma_A^{(1)} \right) = \frac{35}{6} (Na) . \quad (2.7)$$

- second order

$$\gamma_{A^2}^{(2)} = - \left(\frac{\beta^{(2)}}{a} + \gamma_A^{(2)} \right) = \frac{449}{24} (Na)^2 . \quad (2.8)$$

- third order

$$\gamma_{A^2}^{(3)} = - \left(\frac{\beta^{(3)}}{a} + \gamma_A^{(3)} \right) = \left(\frac{75607}{864} - \frac{9}{16} \zeta(3) \right) (Na)^3 . \quad (2.9)$$

Expressions (2.7), (2.8), (2.9) are in complete agreement with those given in¹ [6]. Concerning now the ghost anomalous dimension γ_c in eq.(2.5), it is straightforward to verify in fact that the relation

$$2\gamma_c(a) = \frac{\beta(a)}{a} - \gamma_A(a) , \quad (2.10)$$

holds. This equation expresses the nonrenormalization properties of the ghost fields in the Landau gauge and, as shown in [7], follows from the ghost Ward identity. Although we are considering only pure Yang-Mills theory, it is worth mentioning that eqs.(2.6) and (2.10) remain valid also in the presence of matter fields, as one can verify from [6].

¹Notice that the anomalous dimension γ_O for A^2 given in [6] is related to γ_{A^2} in eq.(2.6) by $\gamma_{A^2} = -4\gamma_O$.

3 Algebraic proof

In this Section we provide an algebraic proof of the relation (2.6). We shall make use of a suitable set of Ward identities which can be derived in the Landau gauge in order to characterize the local operator A^2 . Let us begin by reminding the expression of the pure Yang-Mills action in the Landau gauge

$$\begin{aligned} S &= S_{YM} + S_{GF+FP} \\ &= -\frac{1}{4} \int d^4x F_{\mu\nu}^a F^{a\mu\nu} + \int d^4x \left(b^a \partial_\mu A^{a\mu} + \bar{c}^a \partial^\mu D_\mu^{ab} c^b \right), \end{aligned} \quad (3.11)$$

where

$$D_\mu^{ab} \equiv \partial_\mu \delta^{ab} + g f^{acb} A_\mu^c. \quad (3.12)$$

To study the operator $A_\mu^a A^{a\mu}$, we introduce it in the action by means of a set of external sources. It turns out that three external sources J , η^μ and τ^μ are needed, so that

$$\begin{aligned} S_J &= \int d^4x \left[J \frac{1}{2} A_\mu^a A^{a\mu} + \frac{\xi}{2} J^2 - \eta^\mu A_\mu^a c^a - \tau^\mu s(A_\mu^a c^a) \right] \\ &= \int d^4x \left[J \frac{1}{2} A_\mu^a A^{a\mu} + \frac{\xi}{2} J^2 - \eta^\mu A_\mu^a c^a + \tau^\mu \partial_\mu c^a c^a + \frac{g}{2} \tau^\mu f^{abc} A_\mu^a c^b c^c \right], \end{aligned} \quad (3.13)$$

where s denotes the BRST operator acting as

$$\begin{aligned} s A_\mu^a &= -D_\mu^{ab} c^b \\ s c^a &= \frac{g}{2} f^{abc} c^b c^c \\ s \bar{c}^a &= b^a \\ s b^a &= -J c^a \\ s J &= 0 \\ s \eta^\mu &= \partial^\mu J \\ s \tau^\mu &= \eta^\mu. \end{aligned} \quad (3.14)$$

It is easy to check that

$$s(S_{YM} + S_{GF+FP} + S_J) = 0. \quad (3.15)$$

According to the LCO procedure [3, 4], the dimensionless parameter ξ is needed to account for the divergences present in the vacuum Green function $\langle A^2(x)A^2(y) \rangle$, which are proportional to J^2 .

3.1 Ward identities

In order to translate the BRST invariance (3.15) into the corresponding Slavnov-Taylor identity [8], we introduce two additional external sources Ω_μ^a and L^a coupled to the nonlinear BRST variation of A_μ^a and c^a

$$S_{ext} = \int d^4x \left[-\Omega^{a\mu} D_\mu^{ab} c^b + L^a \frac{g}{2} f^{abc} c^b c^c \right], \quad (3.16)$$

with

$$s\Omega_\mu^a = sL^a = 0.$$

The complete action

$$\Sigma = S_{YM} + S_{GF+FP} + S_{LCO} + S_{ext}. \quad (3.17)$$

turns out thus to obey the following identities:

- The Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = 0, \quad (3.18)$$

$$\mathcal{S}(\Sigma) = \int d^4x \left(\frac{\delta\Sigma}{\delta A_\mu^a} \frac{\delta\Sigma}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta c^a} + b^a \frac{\delta\Sigma}{\delta \bar{c}^a} + \partial_\mu J \frac{\delta\Sigma}{\delta \eta_\mu} + \eta^\mu \frac{\delta\Sigma}{\delta \tau^\mu} - J c^a \frac{\delta\Sigma}{\delta b^a} \right) \quad (3.19)$$

- The Landau gauge condition

$$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A^{\mu a}, \quad (3.20)$$

and the antighost Ward identity

$$\frac{\delta\Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta\Sigma}{\delta \Omega_\mu^a} = 0, \quad (3.21)$$

- The ghost Ward identity [7, 8]

$$\mathcal{G}^a \Sigma = \Delta_{\text{cl}}^a, \quad (3.22)$$

where

$$\mathcal{G}^a \Sigma = \int d^4x \left(\frac{\delta \Sigma}{\delta c^a} + g f^{abc} \bar{c}^b \frac{\delta \Sigma}{\delta b^c} - \tau_\mu \frac{\delta \Sigma}{\delta \Omega_\mu^a} \right), \quad (3.23)$$

and

$$\Delta_{\text{cl}}^a = \int d^4x \left(g f^{abc} \Omega_\mu^b A^{c\mu} - g f^{abc} L^b c^c + \eta^\mu A_\mu^a \right). \quad (3.24)$$

Notice that expression (3.24), being purely linear in the quantum fields, is a classical breaking. It is remarkable that the ghost Ward identity can be established also in the presence of the external sources (J, η^μ, τ^μ) . As we shall see, this identity will play a fundamental role for the algebraic proof of the relation (2.6).

3.2 Algebraic characterization of the general local counterterm

We are now ready to analyse the structure of the most general local counterterm compatible with the identities (3.18) – (3.22). Let us begin by displaying the quantum numbers of all fields and sources, namely

	A_μ^a	c^a	\bar{c}^a	b^a	L^a	Ω_μ^a	J	η^μ	τ^μ
Gh. number	0	1	-1	0	-2	-1	0	-1	-2
Dimension	1	0	2	2	4	3	2	3	3

(3.25)

In order to characterize the most general invariant counterterm which can be freely added to all orders of perturbation theory, we perturb the classical action Σ by adding an arbitrary integrated local polynomial Σ^{count} in the fields and external sources of dimension bounded by four and with zero ghost number, and we require that the perturbed action $(\Sigma + \varepsilon \Sigma^{\text{count}})$ satisfies the same Ward identities and constraints as Σ to the first order in the perturbation parameter ε , *i.e.*

$$\begin{aligned} \mathcal{S}(\Sigma + \varepsilon \Sigma^{\text{count}}) &= 0 + O(\varepsilon^2), \\ \frac{\delta(\Sigma + \varepsilon \Sigma^{\text{count}})}{\delta b^a} &= \partial^\mu A_\mu^a + O(\varepsilon^2), \end{aligned}$$

$$\begin{aligned} \left(\frac{\delta}{\delta \bar{c}^a} + \partial_\mu \frac{\delta}{\delta \Omega_\mu^a} \right) (\Sigma + \varepsilon \Sigma^{\text{count}}) &= 0 + O(\varepsilon^2), \\ \mathcal{G}^a(\Sigma + \varepsilon \Sigma^{\text{count}}) &= \Delta_{\text{cl}}^a + O(\varepsilon^2). \end{aligned} \quad (3.26)$$

This amounts to impose the following conditions on Σ^{count}

$$\mathcal{B}_\Sigma \Sigma^{\text{count}} = 0, \quad (3.27)$$

$$\begin{aligned} \mathcal{B}_\Sigma &= \int d^4x \left(\frac{\delta \Sigma}{\delta A_\mu^a} \frac{\delta}{\delta \Omega^{a\mu}} + \frac{\delta \Sigma}{\delta \Omega^{a\mu}} \frac{\delta}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta}{\delta L^a} \right. \\ &\quad \left. + b^a \frac{\delta}{\delta \bar{c}^a} + \partial_\mu J \frac{\delta}{\delta \eta_\mu} + \eta^\mu \frac{\delta}{\delta \tau^\mu} - J c^a \frac{\delta}{\delta b^a} \right), \end{aligned}$$

and

$$\frac{\delta \Sigma^{\text{count}}}{\delta b^a} = 0, \quad (3.28)$$

$$\frac{\delta \Sigma^{\text{count}}}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma^{\text{count}}}{\delta \Omega_\mu^a} = 0, \quad (3.29)$$

$$\mathcal{G}^a \Sigma^{\text{count}} = 0. \quad (3.30)$$

From equations (3.28) and (3.29) it follows that Σ^{count} does not depend on the Lagrange multiplier field b^a and that the antighost \bar{c}^a enters only through the combination $\widehat{\Omega}_\mu^a = (\Omega_\mu^a + \partial_\mu \bar{c}^a)$. As a consequence, Σ^{count} can be parametrized as follows

$$\begin{aligned} \Sigma^{\text{count}} &= S^{\text{count}}(A) \\ &+ \int d^4x \left(\frac{a_1}{2} f^{abc} L^a c^b c^c + a_2 \widehat{\Omega}_\mu^a \partial^\mu c^a + a_3 f^{abc} \widehat{\Omega}_\mu^a A_\mu^b c^c + \frac{a_4}{2} \xi J^2 \right) \\ &+ \int d^4x \left(\frac{a_5}{2} J A_\mu^a A^{a\mu} + a_6 \eta^\mu A_\mu^a c^a + \frac{a_7}{2} \tau^\mu f^{abc} A_\mu^a c^b c^c + a_8 \tau^\mu \partial_\mu c^a c^a \right), \end{aligned} \quad (3.31)$$

where a_i , $i = 1, \dots, 8$, are free parameters and $S^{\text{count}}(A)$ depends only on the gauge fields A_μ^a . From the ghost Ward identity condition (3.30) it follows that

$$\begin{aligned} a_1 &= a_3 = a_6 = a_7 = 0, \\ a_8 &= -a_2. \end{aligned} \quad (3.32)$$

The vanishing of the coefficient a_1 expresses the absence of the counterterm $f^{abc}L^a c^b c^c$. Also, $a_3 = 0$ states the nonrenormalization of the ghost-antighost-gluon vertex, stemming from the transversality of the Landau propagator and from the factorization of the ghost momentum. As shown in [7], these features are related to a set of nonrenormalization theorems of the Landau gauge. Furthermore, the vanishing of a_6 implies the ultraviolet finiteness of the operator $A_\mu c$. It is easy to check indeed that at one loop order the 1PI amputated Green function $\langle A_\mu(x)c(x) A_\nu(y)\bar{c}(z)\rangle_{1PI}$ with the insertion of $A_\mu c$ is not divergent, due to the transversality of the Landau propagator. These finiteness properties extend to all orders, due to the ghost Ward identity (3.22). It remains now to work out the condition (3.27). Making use of the well known results on the cohomology of Yang-Mills theory [8, 9], it turns out that the condition (3.27) implies that the coefficient a_5 is related to a_2 ,

$$a_5 = a_2 , \quad (3.33)$$

and that $S^{\text{count}}(A)$ can be written as

$$\begin{aligned} S^{\text{count}}(A) &= \rho S_{YM}(A) + a_2 \int d^4x A_\mu^a \frac{\delta S_{YM}(A)}{\delta A_\mu^a} , \\ S_{YM}(A) &= -\frac{1}{4} \int d^4x F_{\mu\nu}^a F^{a\mu\nu} , \end{aligned} \quad (3.34)$$

where ρ is a free parameter. In summary, the most general local counterterm compatible with the Ward identities (3.18) – (3.22) contains three independent parameters ρ , a_2 , a_4 , and reads

$$\begin{aligned} \Sigma^{\text{count}} &= \rho S_{YM}(A) + a_2 \int d^4x A_\mu^a \frac{\delta S_{YM}(A)}{\delta A_\mu^a} \\ &+ \int d^4x \left(a_2 \left(\Omega_\mu^a + \partial_\mu \bar{c}^a \right) \partial^\mu c^a + \frac{a_4}{2} \xi J^2 + \frac{a_2}{2} J A_\mu^a A^{a\mu} - a_2 \tau^\mu \partial_\mu c^a c^a \right) . \end{aligned} \quad (3.35)$$

It is apparent from the above expression that the parameters ρ and a_2 are related to the renormalization of the gauge coupling g and of the gauge fields A_μ^a , while the parameter a_4 corresponds to the renormalization of ξ . It should be remarked also that the coefficient of the counterterm $J A_\mu^a A^{a\mu}$ is given by a_2 . This means that the renormalization of the external source J , and thus

of the composite operator $A_\mu^a A^{a\mu}$, can be expressed as a combination of the renormalization constants of the gauge coupling and of the gauge fields. As we shall see in the next section, this property will lead to the eq.(2.6).

3.3 Stability and renormalization constants

Having found the most general local counterterm compatible with all Ward identities, it remains to discuss the stability [8] of the classical starting action, *i.e.* to check that Σ^{count} can be reabsorbed in the starting action Σ by means of a multiplicative renormalization of the coupling constant g , the parameter ξ , the fields $\{\phi = A, c, \bar{c}, b\}$ and the sources $\{\Phi = J, \eta, \tau, L, \Omega\}$, namely

$$\Sigma(g, \xi, \phi, \Phi) + \varepsilon \Sigma^{\text{count}} = \Sigma(g_0, \xi_0, \phi_0, \Phi_0) + O(\varepsilon^2) , \quad (3.36)$$

where, adopting the same conventions of [6]

$$\begin{aligned} g_0 &= Z_g g , \\ \xi_0 &= Z_\xi \xi , \\ \phi_0 &= Z_\phi^{1/2} \phi , \\ \Phi_0 &= Z_\Phi \Phi . \end{aligned} \quad (3.37)$$

As already mentioned, the parameters ρ and a_2 are related to the renormalization of g and A_μ^a , according to

$$\begin{aligned} Z_g &= 1 - \varepsilon \frac{\rho}{2} , \\ Z_A^{1/2} &= 1 + \varepsilon \left(a_2 + \frac{\rho}{2} \right) . \end{aligned} \quad (3.38)$$

Concerning the other fields, it is almost immediate to verify that they are renormalized as follows

$$Z_b = Z_A^{-1} , \quad (3.39)$$

and

$$Z_{\bar{c}} = Z_c = Z_g^{-1} Z_A^{-1/2} . \quad (3.40)$$

Similar relations are easily found for the sources. In particular, for the source J and for the parameter ξ one has

$$Z_J = Z_{A^2} = Z_g Z_A^{-1/2}, \quad (3.41)$$

and

$$Z_\xi = 1 + \varepsilon (a_4 + 2a_2 + 2\rho) = (1 + \varepsilon a_4) Z_g^{-2} Z_A. \quad (3.42)$$

We see therefore that the relation

$$\gamma_{A^2} = - \left(\frac{\beta(a)}{a} + \gamma_A(a) \right), \quad (3.43)$$

follows from eq.(3.41). Concerning now the eq.(2.10) for the ghost anomalous dimension, it is a direct consequence of eq.(3.40).

Summarizing, we have been able to give a purely algebraic proof of the relationship (2.6). This means that the anomalous dimension γ_{A^2} of the composite operator $A_\mu^a A^{a\mu}$ is not an independent parameter for Yang-Mills theory in the Landau gauge.

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