Symmetry breaking: how vector calculus can violate the associative law of addition

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Abstract. Standard vector calculus based upon properties of non-composite particles violates the associative law of addition when applied to composite particles. The proof starts with a redistribution of number 1 as $1 = x_1 + x_2$, where $x_2 = 1\text{-}x_1$.Physical unit hydrogen mass m_H is redistributed as $m_H=m_e + M_p$, with $M_p = m_H - m_e$. With the associative law of addition, species H can occur naturally in 2 forms: an atom state H, related to the standard distribution $x_1+(1\text{-}x_1)$ and an anti-atom state or anti-hydrogen $\underline{H}$-state, related to the mirror of the equivalent distribution of H or $-x_1 +(1+x_1)$.The difference between the 2 forms of species H, both with total mass m_H, lies in their reduced mass: (a) $m_e(1\text{-}m_e/m_H) \equiv m_e/(1+m_e/M_p)$, the Bohr value for the H-state and (b) its super-symmetrical equivalent $m_e(1+m_e/m_H)$ for the anti-H or $\underline{H}$-state The ratio is 1.0011 known as an anomaly for m_e but an equivalent solution gives 2 proton radii, a problem for bound state QED theory as well as for experimentalists. Consequences for CERN-based artificial anti-hydrogen experiments are given.

Keywords: generic symmetry breaking, associative law of addition, antihydrogen, Mexican hat curve

Introduction

Symmetry breaking exists where symmetry is *expected*, but *that expectation is not met*. Why and how symmetry is broken remains an intriguing question. Algebraic symmetry between numbers +a and –a can *never be broken* but algebraic symmetry between *particle properties* (masses +m and –m) can.

Algebraic symmetry in a hydrogen/anti-hydrogen transition is generated by charge-inversion from +1 in H [electron(-); proton(+)] to –1 in $\underline{H}$ [positron(+); antiproton(-)], based upon Dirac's view on antimatter. H-$\underline{H}$ conversions are fundamental for physics, as they refer to conservation of lepton and baryon number [1-3] as well as to matter-antimatter (a)symmetry in the Universe [4].QFT can deal with H-$\underline{H}$ conversions but cannot reach an unambiguous solution [5].To solve this enigma, one is now trying to produce large amounts of artificial anti-hydrogen $\underline{H}$ [6-8] in order to measure its spectrum, especially the $\underline{H}$ 1S-2S interval, known already with great precision (parts in 10^{14} [9]) for H. Today, everything seems set to *measure* even the smallest symmetry breaking effect in H and $\underline{H}$.

We argue that symmetry breaking is due to *a violation of the associative law of addition when standard vector calculus* is applied to a composite system with an origin in its center of mass. Dimensionless numbers in physics, generated by scaling *properties of material particles* with a dimension (mass m, length ℓ…) must be *positive* ($+m/m_0$, $+\ell/\ell_0$…).*We can simply not measure a zero system or a negative system* [10].

To prove this rather unconventional viewpoint we use (i) the validity of the commutative and associative laws of addition; (ii) the algebra of numbers in physics (the distinction between a positive (mass) world and the negative (mass) or anti-world is due to a *convention*, based upon physical measurements. Perfect symmetry between positive and negative mass worlds would *never* be broken, just like the symmetry between algebraic numbers); (iii) the distinction between scalar and vector properties, important for the higher order laws of algebra (multiplication); (iv) a category of symmetries like SU(n), SO(n), Sp(n) and classical symmetry operations (inversions, reflections, rotations…); (v) the numerical effects associated with the transition from linear x and inverse 1/x representations (T-duality) and finally (vi) the effects of fields (repulsive and/or attractive *inter*-particle and/or *intra*-particle effects).

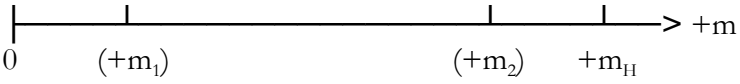
This paper clarifies some of the points raised recently in connection with mirror symmetry as it is hidden in the observed terms of the Lyman series of natural species H,

known for many a decade but left unnoticed thus far [11,12].In particular, we present a quantitative explanation for the difference between hydrogen- and antihydrogen-states, as already given in [12] without further explanation.

Theory

Positive world: the division or redistribution of hydrogen mass m_H (scaled number +1= m_H/m_H). The center of mass of a composite particle and position-vectors for elementary particles.

We start with a common 1D representation of hydrogen mass on a semi-axis as in (1) by means of two vertices 0, $+m_H$ of which only the second is a *material point*.



(1)

Neglecting the internal mass-points m_1 and m_2 between brackets, 1D model (1) is physically unrealistic, if not impossible, to arrive at a rotation of m_H around the center of this system (zero), since there cannot be an *attraction* between material point $+m_H$ (system H) and mass-less origin 0.Just like with algebra, only a rotation by 180°, an inversion or a reflection in a mirror plane at the origin would lead to $-m_H$ in the negative world (see further below), the basis of talking about symmetry effects for (1).With the internal material points, we would be conditioned by Newton's law, which states that heavier particle m_H should be closer to the origin than less heavy particle m_1.

Also, *scalar* hydrogen mass in (1) is treated as a *vector* with length m_H. Avoiding a preferential direction in space for $+m_H$ means we must place the origin for H *within* the system itself, e.g.at its center of mass, to arrive at rotational (field-) invariance for $+m_H$. *Hence, we first abandon the material point-model for neutral H and then, artificially or not, divide its total mass in two sub-units m_1 and m_2, respecting the law of addition, as indicated in generic scheme (1).*In the positive mass world (1), this gives

$$+m_H = +m_1 + m_2 = +m_1 +(m_H - m_1) \qquad (2a)$$

since m_2 is *automatically* defined as

$$m_2 = m_H - m_1 \qquad (2b)$$

For the Bohr-like H system,

$$M_p = m_2 = m_H - m_e \qquad (2c)$$

as indicated in (1) by the two points between brackets.

Speaking about positive and negative number worlds in physics nevertheless introduces a certain ambiguity, realizing that numbers generated in physics derive from scaling (dividing) particle properties by means of a reference value for the same property, having the same physical dimension. For instance, positive number +1 can be generated by means of two mass ratios: (i) $+m_0/(+m_0) = +1$ deriving from the positive mass world, using a positive mass reference value $+m_0$ but also (ii) by $-m_0/(-m_0)=+1$ in the negative mass world, using the appropriate negative reference mass[1] $-m_0$.*Although one usually believes this ambiguity can be solved by convention by allowing only one (say a positive) world, the situation is more complex, as illustrated in footnote 1.*Let us therefore analyze first the situation with positive numbers.

[1] The common sense idea behind this distinction is best illustrated with a simple example. The (positive) number of cars N, made by the same manufacturer, driving from A to B can be exactly the same as the number of the same cars going from B to A. Yet, the physical characteristics of the cars must be completely different: opposite direction, different drivers, different circumstantial conditions (wind, slope) and, above all, separate lanes, to avoid calamities (such as crashes, if not *annihilation*).

In terms of positive numbers, i.e.positive masses in (1) scaled by $+m_H$, and *of the associative law of addition*, the two equivalents of (2a) in the positive number world for +1 are

$$+1 = +x + (1-x) = -x + (1+x) \qquad (3a)$$

This creates a set of *two equivalent partition functions* for positive number +1, which has become a binary mixture with simple partition functions. The outer right hand side solution in (3a), *not yet illustrated in (1)*, is obtained with *the associative law of addition* (and is discussed further below).

In terms of physics, (2a) can be read as *a zero perturbation of mass* m_H since in (2a)

$$m_1 - m_1 = 0 \qquad (3b)$$

by definition, reproducing *unperturbed* $+m_H = +m_H$, as it should. *Whether or not these redistributions of H-mass are real or not, we are obliged to admit that real or virtual particles with mass* m_1 *and* m_2 *must always attract in order that its conglomerate H with mass* m_H *remains the same stable particle*. Leaving out m_1 and m_2 in (1) would mean that, exactly as in the kinetic theory for the ideal gas, H is just a *material point* somewhere on an axis. The inclusion of m_1 and m_2, however, makes H *a linear additive structure* in the first place instead of a material point, as depicted in (1).

In terms of graphs, the remedy to reproduce (1) without the internal mass points between brackets in (1), which means a reproduction of vertices 0 and $+m_H$ only in (1), a number of discrete options are available, which we can classify simply by introducing *two secondary axes*. Schematically, we get either

(a) co-linear or parallel graphs (biaxial representations)

⌊⌋ or ⌐¬ (3c)

generating *a non-zero field component* along the secondary axes, scaled with m_1 or

(b) anti-linear or anti-parallel graphs (biaxial representations)

⌊¬ or ⌐⌋ (3d)

whereby there is *a zero-field component* along the secondary axes. These models will be discussed further below but we remark already here that these structures are basically *4-particle structures*, if the origin is regarded as *a virtual mass-less point-particle*. As argued elsewhere [13,14], the *classical 19th century treatment requires at least 4-particles (points) for chiral structures*, or slightly distorted out-of-plane structures like those in (3c)-(3d).

Despite the internal linear 1D division in (1), the absolute center of the 1D axis (the reference frame) is still outside the H-system. With respect to the origin, m_1 and m_2 are *in conjunction not in opposition*. To get the origin *within* the system, i.e. to get immunity towards external field effects, we use vector calculus and an inverse law for the mass of non-composite particles X in the standard way by using the linear-inverse (m_x to $1/r_x$) transformation[2]

$$m_x c^2 = e^2/r_x \qquad (4a)$$

without *external* algebra[3]. Equation (4a) is the result of equilibrium in a central force model, classically given by $mv^2 = e^2/r$ for Coulomb systems. In the following, we will put the scale factor between real velocity v and absolute velocity c in (4a) equal to one (although α could be used too), since its value does not affect the *procedure* developed below. Therefore (4a) is the *absolute formulation* for the linear-inverse transition we need to discuss. This absolute procedure gives unit particles X (an indivisible particle -ατομος- instead of composite H) with mass m_X a constant moment (*position vector*), deriving from a (central) field effect

$$m_x r_x = e^2/c^2 \qquad (4b)$$

One effect of this (inverse) field notation for m_1 and m_2 in (1) is to secure that H, with mass m_H as given in (1) and (2) and despite its division in two particles, remains a stable

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[2] Using equation (4a) does not exclude the possibility that extra numerical scale factor ½ may be needed.The equally possible alternative and important expression $2m_Xc^2 = e^2/r_X$ is not discussed in this paper.

[3] At this stage, we leave algebra out of (4a).Stable X requires $-e^2/r_X$, unstable X $+e^2/r_X$ *but this is just convention.*

unit. An advantage is that we can speak of particles using position vectors instead of so-called *mass vectors* as implicit in (1).We must now evaluate the effect of the linear-inverse transformation (4a) for indivisible non-composite particles upon the law of addition (1) and (2) for composite particle system H. Using the same conversion factors c^2 and e^2 as in (4a) for *linear* mass system H, we can rewrite (2a) as

$$m_H c^2/e^2 = 1/r_{H'} = 1/r_1 + 1/r_2 = (r_1+r_2)/r_1r_2 \qquad (5a)$$

which defines the corresponding *position vector* for H in 1D, $r_{H'}$, as *a reduced distance*, equal to

$$r_{H'} = r_e r_p/(r_e+r_p) \qquad (5b)$$

if r_e is the so-called electron- and r_p the proton-radius, reminding (4a).This is at variance not only with the conventional Bohr but also with the vector-based quantum field theory solution that

$$r_H = r_e + r_P \qquad (6a)$$

or, in vector notation

$$\mathbf{r}_H = \mathbf{r}_e - \mathbf{r}_P \qquad (6b)$$

which is identical *in absolute value* with (6a).

These transformations are valid for the redistribution of mass H as in (1) and (2a) as long as the classical equilibrium condition

$$m_e r_e = M_p r_p \quad (=e^2/c^2) \qquad (7a)$$

holds[4], on the basis of (4b), since e^2/c^2 is a universal constant. One of the reasons to use (6a) rather than (5b) is that, for system H, the moment (mass times radius) is equal to the universal constant in (4b) pending a scale factor, *if and only if* one goes over to *a reduced mass for hydrogen* too or

$$\mu_H r_H = e^2/c^2 = m_X r_X \qquad (7b)$$

Only in this way the laws for scaling and the determination of physical constants can proceed consistently. In fact, *with a reduced mass for system H,* defined as

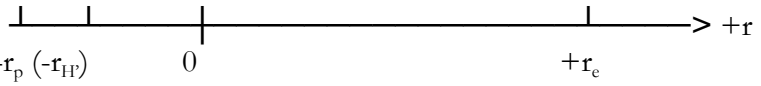
$$\mu_H = m_e M_p/(m_e+M_p) = 1/(1/m_e + 1/M_p) = (e^2/c^2)/(r_e+r_p) \qquad (8a)$$

the identity $\mu_H r_H = e^2/c^2$ in (7b) is still obeyed for composite system H also. Using identity (2c), we easily verify that this reduced mass (8a) is *numerically identical* with

$$\mu_H = m_e/(1+m_e/M_p) \equiv m_e\,(1-m_e/m_H) \qquad (8b)$$

Although (8b) is perfectly allowed, *this linear form is never used in the analysis of system H.*

Using this information on the transformed *inverse* model for H using position vectors $\mathbf{r}_e$ and $\mathbf{r}_p$ for particles m_e and M_p, we get at the linear 1D (*but in reality inverse mass*) representation in r (9)



(9)

to be compared with equivalent linear mass representation (1).Electron and proton are now described by position vectors indeed and, as required, the origin of the H system is *naturally* replaced within the system (*center of mass*), securing the system is rotationally invariant to external field effects as intended. The inverse mass representation leads to vector calculus, culminating in (6b), meaning essentially the same as (6a) but whereby an addition (+ sign) is formally replaced by a subtraction (- sign), although the picture is *qualitatively* the same as that given in (1).

Quantitatively, however, there is a difference. If we choose to place r_p at the negative semi-axis, the implicit effect is that $r_{H'}$, defined in (5b), must also be placed in the *negative world*, only it must be closer to the origin than r_p, since mass M_p is smaller than m_H. This means that going over to the standard vector model, implies that we shifted the mass of neutral system H from a positive to a negative world in a 1D model also. To restore this

[4] Like in a classical balance

seemingly illegitimate rotation or inversion, we should rotate 1D model (9) also by exactly 180° or by π (or invert it).But *if we do, we are obliged to displace or linearly shift, instead of particle with mass m_2 or M_p, the properties of particle with mass m_1, or m_e, from a positive to a negative world in turn.*

Conflict with the associative law of addition in going from (1) to (9) or from the linear mass to the inverse mass or r-representation for a composite system like H. Going to a negative world

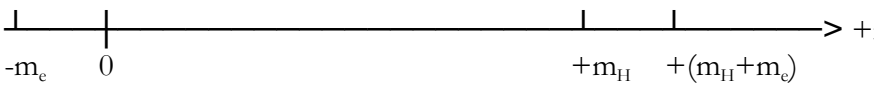
Whatever transformation we choose, (9) or its mirror, one of the two sub-particles must be placed in an algebraically conjugated world. Then these particles are in *opposition*, since the center of mass of the total neutral system H is situated between the two. This is as far as we can go for the *external* algebra of particle masses m_1 and m_2 (or m_e and M_p) and their center of mass. Nevertheless, the equivalence of (1) and (9) is, however, *false*: *the associative law of addition has been violated* tacitly, as is easily demonstrated. In (10), the *commutative law of addition* for H mass or

$$m_H = m_e + M_p = M_p + m_e = m_e + (m_H - m_e) = (m_H - m_e) + m_e \qquad (10)$$

is still obeyed but *the associative law of addition is violated in going from (1) to (9).*This law says that x+(y+z) = (x+y) +z, which means that, *in an addition*, we are completely free to change the order as well the combination (*association*) of all terms to be added, since all operations (*additions*) are equivalent. If we apply this law to the right hand side of (2a), we get 2 different *associations*

$$+m_H = +m_e + (m_H - m_e) = +m_e + m_H - m_e$$
$$= -m_e + (m_H + m_e) \qquad (11)$$

for $+m_H$, which are both perfectly valid, as illustrated numerically in (3a). In the linear 1D mass representation we now obtain for the last identity in (11) the 1D mass representation



(12)

instead of (1).The effect of this representation is that also here the center of the system is *internal*, exactly as with inverse mass based vector model (9). However, in the vector notation $\mathbf{r}_e$ as well as $-\mathbf{r}_e$ (related to mass m_e) remain correlated with the same vector with *length* r_p (related to mass $M_p = m_H - m_e$) *irrespective of its orientation in space*, due to (4a) and (7a). Equation (11) and representation (12) both clearly show that negative mass particle $-m_e$ *must and can only be correlated* (*associated*) with a particle with total mass m_H+m_e instead of with the less heavy particle with mass $m_H-m_e=M_p$, the proton, only valid for unit $+m_e$. This explains the title above[5].

Instead of the proton with mass m_H-m_e, the associative law of addition *demands* the existence of another (*real of virtual*) particle with total mass m_H+m_e, which, at least, seems strange. Since a particle with negative mass appears ($-m_e$) as a unit or reference mass, it seems straightforward to forbid representation (12) in a positive world or for positive mass system H (1). Nevertheless, *mass being a scalar and not a vector*, *an absolute exclusion may be too severe*. For a scalar like energy, positive and negative values for unstable (unbound) and stable (bound) systems cannot be excluded a priori.

Distinguishing between bound and unbound states can be done by considering reduced mass for the two mathematically allowed and correct representations (11) or (12).If reduced mass (8) is denoted as $\mu_H(+,+)$, referring to (1) with $+m_e$ and $+M_p$, we get analytically for distribution (11) or (12)

$$\mu_H(-,+) = (-m_e)(m_H+m_e)/m_H$$

[5] Classical macroscopic cases, subject to the same phenomenon, are dealt with elsewhere [15].

$= - m_e(1+m_e/m_H) = -m_e(M_p+ 2m_e)/(m_e+M_p)$
$= - \mu_H(+,+)(1+2m_e/M_p)$ (13)

not only *a difference in absolute value* with (8) but, obviously with *the wrong algebraic sign* to be applicable in an exclusively positive (mass) world. Result (13), deriving from the associative law for particle masses, must be rejected in a positive mass world. This reflects the problems associated with presentation (9) and the seemingly arbitrary choice to use its inverted or rotated equivalent, as discussed above.

To reach a conclusion, the problem is to find out exactly what happens with mathematically valid representations and associations (3a) *in a negative world*. Is it possible to distinguish between (9) and its rotated equivalent (*not shown*) and if so, is there a *quantitative* measure available to do so unambiguously? To do so, all models above are now repeated for *a negative mass hydrogen species*.

Equivalent representations (associations) in the negative (mass) world

Without duplicating details, we give the additive mass equations in the negative mass world for system H, with mass $-m_H$ instead of $+m_H$, using (11) as reference. Again, we obtain two different but perfectly allowed *associations starting from the same zero-mass perturbation*

$- m_H = -m_H + m_e - m_e$ (14a)

in the negative mass world. Analytically, these associations are now

$- m_H = (-m_H + m_e) - m_e$ (14b)
$- m_H = (-m_H - m_e) + m_e$ (14c)

First, (14b) is the *negative* equivalent of representation (1) or *the electron-proton model in the negative world*, which generates a *negative* reduced mass

$\mu_H(-,-) = (-m_e)(-M_p)/(-m_e-M_p) = - \mu_H(+,+)$ (15a)

numerically equal to Bohr's (8a).It is a *forbidden* version of Bohr's reduced mass (8) for species H but, by symmetry, it is only allowed in the symmetrical negative mass world.

Next, (14c) is the *negative* equivalent of distribution (12).Despite the fact that it derives from an inverted or mirrored negative mass world, this model generates a *positive* reduced mass, equal to

$\mu_H(+,-) = m_e(-M_p)/(m_e-M_p) = m_eM_p/(M_p-m_e) >0$
$= m_e(1+m_e/m_H) = - \mu_H(-,+)$
$= \mu_H(+,+)(1+2m_e/M_p)$ (15b)

Although this reduced mass derives from a negative (mass) world, *where it is forbidden*, it must be allowed in a positive (mass) world. It is, however, *numerically different from the standard reduced mass* $\mu_H(+,+)$ or simply μ_H as derived above in (8).Result (15b), compared with (8a), perfectly *illustrates the violation of the associative law of addition through an incorrect application of vector calculus*, as mentioned in the title.

The consequence is that, as soon as we decide by convention that only one positive world is allowed for composite particle masses like m_H, which is exactly as we did a long time ago, this convention no longer holds for the masses of the components of that particle due to the associative law of addition, as illustrated in (3a).

Symmetry breaking. ART (algebraic recoil theory) [16] and supersymmetry. Physical number theory.

The perfect symmetry between algebraic numbers suggests that this same symmetry should exist between the positive world ($+m_H$) in physics and the negative antiworld ($-m_H$). Unfortunately, this expectation is not met, due to the difference between (15b) and (8).In physics, one of the two mathematically possible conjugated worlds must be forbidden (like

for *scalar* particle mass), *since our measurements are bound or restricted to the positive number world exclusively*. Nevertheless, as soon as the idea of the point-like mass particle[6] is abandoned by using a Bohr model like in (2b), the positive/negative symmetry must be looked at differently, as we have shown above. For a composite neutral particle model, *an internal algebraic symmetry breaks the external algebraic symmetry conform a process, which is even easily assessable quantitatively*. In fact, given the derivations above for allowed positive reduced masses $\mu_H(++)$ and $\mu_H(-,+)$, the result is that *a numerical symmetry breaking correction factor* appears, equal to

$$\mu_H(-,+)/\mu_H(+,+) = 1+ 2m_e/M_p \qquad (16a)$$

This is the first *quantitative* result of the correction needed for standard vector calculus, according to the associative law of addition, applied to a composite physical system like species H with mass m_H. *This is the so-called simple electron-proton Coulomb bond.* When transposed to the positive world only, it appears that the two associations simply follow *a (slightly) different scaling law*, which is easily deduced.

Using (8b) and (15b), the anomaly (16a) may be rewritten in a super-symmetric form using a number A, *a reduced difference*, which leads to familiar expressions for (chiral) symmetry breaking processes, since from (15b) we obtain

$$\begin{aligned} A &= \mu_H(+,+)/\mu_H(-,+) \\ &= (1-m_e/m_H)/(1+m_e/m_H) \\ &= (m_H - m_e)/(m_H+m_e) \end{aligned} \qquad (16b)$$

With (16b), we have an *algebraic recoil theory*, instead of a conventional *non-algebraic* one, expressible as

$$\mu_H (\pm) = m_e(1\pm m_e/m_H) \qquad (16c)$$

as proposed earlier (ART, Algebraic Recoil Theory) [16]. It can be expected that this super-symmetrical version (16c) of reduced mass is the equivalent of a chiral symmetry breaking (CSB) theory [11,12]. In fact, the physical mechanism behind the sign-inverted equivalent of (12), which leads to a positive reduced mass (15b) for the *inverted* H system, deriving from (3a) and the associative law of addition, must be identified. If this transformation is real, it must be retraceable *experimentally* (this is presented elsewhere [18]). As remarked in the Introduction, the simplest interpretation possible is *a charge inversion* (*a mirror symmetry*) between two charge conjugated particles. If confirmed by experiment, this must now lead to two physically different forms or states for species H: the hydrogen- and the anti-hydrogen-state, as we argued in [12]. If so, ART is also a CSB theory, conform (16b).

In this hypothesis, the difference between the two states, allowed by a law of association (internal algebra) and by an inversion (external algebra), can be quantified using the two different reduced masses in (16c). Numerically, this distinction reduces to the difference between

(a) a baryon with mass M_p= 1836.1526675m_e (proton mass [19]) for the *hydrogen-state* and

(b) a (*not yet identified*) baryon with mass M_x= 1838.1526675m_e for the *antihydrogen-state*

while the total mass of the neutral system H remains invariant and equal to m_H=1837.1526675m_e. These results were given in [12] without proof. If confirmed, symmetry in neutral species H is broken by mass m_H, both conceptually and quantitatively.

It is now easily verified that algebraic symmetry for non-material points (i.e. finite systems) like system H must always be broken, since correction factors are needed when the composite material system is described by unconventional mass unit $-m_e$ instead of conventional $+m_e$. Quantitatively, generic symmetry breaking is determined by the ratio of the unit mass $|m_e|$ and the total mass of the composite system $|m_H|$ or $|m_e/m_H|$, as illustrated by equations (16). This ratio is close to, but not identical with classical

[6] A point-model was also the basis of the famous ideal gas law, the first ever equation of state (EOS).Vanderwaals succeeded in deriving a much better EOS by giving up the point-model.The analogy of the Vanderwaals-Maxwell model for unit particles and its refined EOS with the Bohr model (2b) for H is discussed in full elsewhere [15].

recoil m_e/M_p, building upon (7a). Although we would expect a perfect (algebraic) symmetry between $+m_e$ and $-m_e$, just like between pure numbers +1 and –1, *this expectation is not met* for physical numbers for a composite neutral system (see Introduction). And since this generic symmetry breaking or scaling effect is simply overlooked in bound state QED, this theory can not yet be validated, as we argued before [12]. Only in the limit of very small, nearly zero unit m_1, the less useful material point model for perturbed m_H in (1) is reproduced, since this is only valid for unperturbed system H [13]. In [15] we show that the switch from a point model to a non-point model is exactly what Vanderwaals had to do to account for easily observable phase- or order-disorder-transitions in macroscopic systems.

Mechanical symmetry breaking models

For *mechanical models* to describe a linear 1D phase transition, allowed by the associative law of addition, we can only make some reasonable guesses.

(a) For linear mass representations, a first model is offered by comparing (1) and (12). To go over from association (1) to association (12) in the same world, a double rotation of m_e is needed. In a 2D Cartesian frame, a biaxial system is generated, with one of 2 secondary axes with scale m_e crossing at the origin 0 of the +m semi-axis in (1), the other crossing at point $+m_H$. Only *the perpendicular configuration* can reproduce the simple non-composite model for H, with just two vertices 0 and $+m_H$ on the semi-axis (1), since only then the projection of m_e at either position is zero. These *4-particle structures* were already sketched in (3c) and (3d) and, using a different approach and their importance was stressed in [13].

(b) For inverse mass or vector models with a common origin for both vectors $\mathbf{r}_e$ and $\mathbf{r}_p$, *any rotation of vector* $\mathbf{r}_e$ *must be accompanied by small variations(oscillations) in the length of vector* $\mathbf{r}_p$*, as a result of the above*. This is exactly the effect, overlooked in standard vector calculus in quantum theories for composite particles. If both internal particles rotate independently with variable position vectors, *chaotic behavior* may be the result (see below and [10]).Unlike Bohr's model, particles cannot only be in opposition but also in conjunction. *The length of* $\mathbf{r}_p$ *for electron and proton when in opposition must be smaller than when in conjunction*, for the reasons set out above around (16).Both models use m_e as a unit to describe m_H. To describe this model using a unitary triangle, we should go over to what we already called *the engine-model* [16,18] and to illustrate this, we can switch from the Bohr triangle (the standard vector model)

$$r_H = r_e\sqrt{[1+ (r_p/r_e)^2 - 2(r_p/r_e)\cos\theta]} \qquad (16d)$$

with fixed angle $\theta=\pi$ and constant r_p, which cannot rotate, to *the engine triangle*

$$r_H = r_e\cos\eta + r_p\cos\chi \qquad (16e)$$

This model uses the two other angles in the same Bohr unitary triangle, obeying $\eta+\chi+\theta=\pi$ [16].The analytical treatment of this mathematical (trigonometric) transition is given in [16]. The correlation between the two is unambiguous but the analytical dependence on recoil becomes slightly different [16].

However, a *common feature of both mechanical models* (a) and (b) is a critical value for the angle of rotation, invariantly equal to 90° or ½π.With this value, the simplest model of all is returned: the undivided material point-like H system, either with 1D vertices 0, $+m_H$ (or 0, $-m_H$) and 0, $+r_H$ (or 0, $-r_H$).

The generic picture behind all this is the transformation of a left-handed 3D Cartesian reference frame into a right-handed one, and where the mirror plane, situated exactly at this critical angle too, is being crossed [12]. This brings in chiral behavior in a classical way, since a least four particles (vertices) are needed in a 3D-structure to arrive at permanent and stable left-right structures as we know them since the 19th century [13].

Results and discussion

A theory which tries to explain and account for generic symmetry breaking must be put to the test in as many ways as possible to look for any ambiguity. Fortunately, there are a number of equivalent tests available, pending the experiments and the interpretation of anomaly A (16a) or (16b).We discuss some, not all, pertinent straightforward examples.

(a) *Electron mass*. A first straightforward result is in terms of electron mass, when the correction in (16a) is incorporated in electron mass. The generic symmetry breaking effect above for the *electron* with mass m_e can be rewritten such as if there were two different electron masses m_e and m'_e, with m'_e given by

$$m'_e = m_e(1+2m_e/M_p) = 1.0011m_e \qquad (17a)$$

the validity of which should be verified experimentally. Numerically, the anomaly is $2m_e/M_p$. An *anomalous* electron mass has been observed a long time ago [20], giving 0.0011596521869 [19] as most recent value for this anomaly. Here, (17a) gives $2/1836.1526675 = 0.001089234 \approx 0.0011$, a divergence of -6 %. The *alternative* QED explanation for the anomaly starts with the leading term $\alpha/2\pi$ in a series expansion in the *fine structure constant* [21].

(b) *Proton radius*. Another test is in terms of two different proton radii r_p and r'_p, invisible in the standard treatment of reduced mass and in the conventional vector-model for H. If we adhere to a model with *constant* electron mass m_e and *overlook* its anomaly (17a), the effect of (16b) will have to show elsewhere. Since r'_p is *associated* with a particle, with absolute mass larger than absolute proton mass (in casu $M_p + 2m_e$), we get as an alternative for (17a)

$$r'_p = r_p/(1+2m_e/M_p) \qquad (17b)$$

Despite its accuracy, modern bound state QED is indeed faced with the problem of two slightly different proton radii [22]. Since ART is missing (*overlooked*) in bound state QED and if the transition process described above is real, this error of vector calculus must, sooner or later, show in the QED results, when put to the test with experiment. The reality of having to deal with two *slightly* different proton radii [22], is a second, be it indirect proof for the validity of (16b). Mechanical models (a) and (b) above rely on implications of (17b).

(c) *Number theory, dimensions and symmetry*. A next issue is to transform the indirectly measurable reduced mass into a number, obtained with scaling by another mass. In the case of H, the most pertinent mass scale factor is m_H itself. Numbers $N_\pm$, showing *internal* algebra, generated with (16c) are

$$N_\pm = \mu_H(\pm)/m_H = (m_e/m_H)(1\pm m_e/m_H)$$
$$= (1/n)(1\pm 1/n) = 1/n^2 \pm 1/n \qquad (18a)$$

whereby we introduced a subsidiary *number* n, defined as

$$n = m_H/m_e \qquad (18b)$$

The equivalent expression with inverse number $1/n = n'$ (nearly *recoil*) gives numbers $N'_\pm$ obeying

$$N'_\pm = n'(1\pm n') = n'^2 \pm n' \qquad (18c)$$

instead of (18a).It is easily verified that the numbers $N'_\pm$, generated with (18c), are of Hadamard-type, rather than of Heisenberg-type, as discussed in full elsewhere [10]. As a consequence of these number generating processes, (18c) with positive sign is related to Sp(n), whereas with negative sign it correlates with SO(n). Any product of the two correlates with SU(n).Fig.1 gives the behavior of $N'_\pm$ in function of n', in line with an

INSERT Fig.1 around here

earlier analysis [10]. For large n' (*not shown*), the two parabola nearly coincide near the minimum but zooming in at small n' as in Fig.1 separates the two sets and a double well or Mexican hat potential is generated, illustrating the left-right asymmetry hidden in (16b). This aspect is discussed further below. As remarked elsewhere, N'_+ goes over into N'_- by applying the simple linear substitution $\ell = n' - 1$ [10]. Since both principal quantum number n as well as secondary quantum number $\ell = n-1$ are needed to interpret the H-spectrum, the analysis above turns out to be self-consistent [10].

It is also obvious that (18a) is an idealization of the famous Kratzer[23]-Fues[24] potential, we discussed recently in great detail [25]. In a generalized form due to Varshni [26], this potential is useful in molecular spectroscopy [25] and provided with a first hint for natural *atom-antiatom transitions*, using information from molecular band spectra [27].

(d) *Chaos and fractal behavior*. When number 1 is a critical value for numbers like $N_\pm$, we get from (18a)

$$1 = (1/n)(1 \pm 1/n)$$
$$= 1/n^2 \pm 1/n \qquad (18d)$$

In this case, we arrive at chaos and fractal behavior for the numbers n and their inverse n' [28]. Using (18d), we can also remove the inverse (mass based) notation by multiplying first by n and then by n^2 to get at the *linear number* equivalent of reduced masses, deriving from distribution (1) in the *positive* world and from (12) in the (inverted) *negative* world. We obtain two additional relations

$$n = 1/n \pm 1 \qquad (19a)$$
$$n^2 = 1 \pm n \qquad (19b)$$

intimately related to *chaotic and fractal*, if not *chiral* behavior, of pure numbers [28], as discussed in [10]. For instance, (19a) shows that linear n is equal to inverse 1/n pending algebraic constant 1. Other number theoretical applications and implications, including the case of *harmonic* behavior, are in [10].

(e) *H-H conversion: a phase-transition in species H*. Despite the great accuracy claimed by modern and highly sophisticated bound state QED [22], the analysis given here cannot be retraced in the QED framework. The simple left-right phase-transition between different states of species H is lacking in bound state QED, which is why we said that QED as it stands cannot yet be validated [12]. The issue is important, as the possibility of a natural H-H conversion can have profound consequences for the (non-) conservation of lepton and/or baryon number [1-3]. Recently [15], we analyzed the H spectrum and even found that there is evidence for a Vanderwaals-Maxwell-type phase transition within species H (when compressed or expanded).The critical n-values, as well as the binodal are easily computed, which provides additional evidence for an internal change (an internal phase-transition) in the H configuration, as derived above.

(f) *Mirror symmetry*. Whatever mechanical model (a) or (b) above we would choose to describe this phase transition, the critical angle is invariantly 90° or ½π for the position of the electron mass (the electron), a value for which the undivided model or material-point model for system H (1) is returned. It is also the generic angle for a mirror plane, placed somewhere on one axis of the 3D Cartesian reference frame. A recent analysis of the terms of the H Lyman ns-series shows that a critical value for principal quantum number n indeed appears, exactly equal to ½π [12].

(g) *Mexican hat curve (binodal) for system H*. The shape of a binodal, characteristic for a classical Vanderwaals-Maxwellian phase transition [15], is essentially that of a Mexican hat or double well potential. These latter are characteristic for *chiral symmetry*, as argued a long time ago by Hund [29]. Exactly this type of potential (Mexican hat) is easily extracted from the H-spectrum too [13,30,31] and suggests the applicability and the validity of the mathematics behind Fig.1 for system H.

(h) *H mass*. Above, we showed that hydrogen mass m_H is important for a description of the H system. This is in line with particle physics, where a crucial –if not the most important- characteristic of a particle, is its mass. Bohr-Schrödinger theory as well as bound state QED, only *indirectly* refer to m_H when describing H, except for the small recoil correction. As shown here and also elsewhere [13], this is a serious neglect indeed.

(i) *CERN's artificial antihydrogen*. The fact that an artificial charge-inverted antihydrogen is made with positron and antiproton, producing *numerically* the same reduced mass as in Bohr theory for hydrogen is sufficient ground to show that the generic internal symmetry effects, discussed above, are insufficiently accounted for in the ongoing CERN-AD experiments [6-8]. Given the conjecture that only a positive mass world is allowed, *artificial antihydrogen* is doomed to get annihilated, which seems in line with observation [6-8]. We suspect that reports [6-8] on the mass production of artificial antihydrogen are premature as argued elsewhere on different grounds [13] or, at least, require a different interpretation, along the lines set out above.

(j) *Recoil corrections in QED*. Formally adapting Dirac-based bound state QED for recoil is a very complicated task ([32], see also [22]). For H, additional recoil corrections contribute less than 1 kHz for terms or energies. The standard or classical recoil correction $1/(1+m_e/M_p)$ leads to an expansion in recoil m_e/M_p, like $1 - m_e/M_p + (m_e/M_p)^2 - \ldots$. However, using the equivalent linear notation $1-m_e/m_H = 1/(1+m_e/M_p)$ in (8b) simply avoids this expansion and its higher correction terms. Eventually, using the linear form (8b) instead may lead to a considerable simplification of QED-calculations, *without loss of accuracy*. Moreover, until today, ART result (16c) has never been considered in this type of highly sophisticated QED-calculations. This places question marks on attempts to correctly describe analytically and/or to compute recoil corrections in a Dirac-theory based framework [32] for a bound system like the electron-proton Coulomb bond in species H.

(k) *Woodward-Hoffmann symmetry rules*. Finally, we must return to the perpendicular biaxial representations (3c) and (3d) for the material point m_H on the linear axis. To arrive at the linear 1D representations (1) and (12) allowed by the associative law of addition, rotations are required in these biaxial structures. The main point thereby is to find out if these double rotations are coupled by additional symmetry rules and if so, if these rotations are con- or disrotatory. For parallel structures like ⌊⌋ in (3c) producing a subsidiary field effect on the secondary axes, only *disrotatory* ring-closure mechanisms can be allowed to reproduce (1) and (12).For the anti-parallel structures like ⌊¬ in (3d), which do not invoke subsidiary field effects, only *conrotatory* mechanisms can reproduce the states (1) and (12), allowed by the associative law of addition. Although, to the best of my knowledge, in the context of the H problem these mechanisms are completely new, the analytical treatment was given a long time ago by Woodward and Hoffman [33], since they represent a very common type of reaction mechanism in *organic chemistry. The connection between these rules and the atom-antiatom transition* was discussed already a long time ago [34].

Some of these results may be new but they amply illustrate the implications of the present deductions for a number of topics, of interest for both theoretical and experimental physics and chemistry. These summary of tests provides sufficient, almost conclusive, evidence for the validity of the generic symmetry breaking mechanism in this work.

Conclusion

Symmetry breaking seems to be an unnecessary problem. The *internal* symmetry of a composite system is already broken at the 1D level due to a violation of *the associative law of addition*. The error produced by standard vector calculus using two position vectors of *fixed* length, is easily pinpointed, is simply assessable quantitatively and can therefore easily be

accounted for. Whether or not this solution for the artificial problem of symmetry breaking is consistent with other tests than those already mentioned above, remains to be determined. Additional tests and quantitative verifications will be presented shortly. Maybe the current CERN-AD experiments on *artificial antihydrogen* and their outcome can provide an additional test.

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Fig. 1. $N'_{\pm}$ according to (18c) versus n' (N'_{+} full, N'_{-} dashed).