The influence of transversal crack geometry on the frequency changes of beams

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Abstract

Global non-destructive testing methods use changes in modal data as damage indicators. Depending on their geometry, damages influence the modal parameters in different way. While breathing cracks influence the natural frequencies due to only stiffness change, open cracks affect supplementary these frequencies due to loss of mass. This paper describes a study devoted to clarify the influence of the damage geometry on the natural frequencies of beams considering the two above mentioned aspects. First, analytical and numerical analyses are performed, in order to highlight the effect of mass changes on the frequency changes. Afterwards, the study focuses on breathing cracks in open and closing stages as well as on open cracks and emphasizes the influence of the geometrical discontinuity. It was demonstrated that, for beams with a breathing crack, a two degrees of freedom model is necessary to characterize its dynamic behavior. Finally, a relation between frequency changes and damage geometry and location is contrived.

1 Introduction

The development of global structural integrity assessment methods gained an increasing attention in last years, vibration-based techniques being the most relevant among them. In order to implement these techniques, the study of the changes in the structural dynamic response due to cracks is required, for better understanding of the connections between these changes and the crack parameters [1]. Depending on the crack geometry, the presence of a crack can not only cause a local variation in the stiffness, but it can affect the mass distribution in the structure. Apparently this makes the crack (or cracked structure) modeling difficult; often considerable simplifications are made, leading to inaccurate results and misunderstandings in vibration analysis.

In damage detection methods, two main categories of crack models are involved: (i) open crack models and (ii) breathing crack models [2]. Obviously, the crack is somehow evaluated separately from the structure, while the mass influence is not considered; in fact, because the crack is usually substituted with a spring, it is impossible to consider other parameters as stiffness. Among the two models above presented, most researchers prefer to assume the open crack, in order to avoid the complexity resulting from the opening and closing process. But assuming this crack model to interpret vibration measurements will lead to incorrect conclusions if the real damage is a breathing crack. It was evidenced that using the use of open crack models in damage assessment lead to undervaluation of the crack severity [3].

For real cracks, at least in the forming stage, during vibration the crack edges come into and out of contact, leading to changes in the dynamic behavior of the structure. These changes influence the dynamic response and can be used for crack detection [4]. However, only a few papers were devoted to the analysis of breathing cracks, due to the difficulty to generate a true fatigue crack; consequently these analyses utilize mainly mathematical tools.

Actual damage identification using inverse methods bases on 'blind' model updating [5-6], while it is almost impossible to create damage scenarios that cover all potential damage cases. Obviously, for open cracks it is a relation between the stiffness decrease and mass loss, imposed by the parameters defining the damage. The purpose of the present paper is to clarify some aspects regarding the dynamic behavior of beam-like structures with various crack types, and to contrive the mathematical relation that consider both stiffness reduction and, if it is the case, mass loss. This approach permits the determination of dynamic characteristics of a beam with given crack parameters, or, the estimation of crack parameters by the known values of corresponding dynamic characteristics.

2 The influence of mass loss upon the frequency changes

Since loss of mass increases the natural frequencies of beams, the phenomenon has to be considered by frequency analysis in damage detection processes. Therefore, the modeling of mass loss due to damage is essential for defining a baseline applicable to open cracks. This paper presents the researches made on a cantilever beam, characterized in healthy state using following geometric parameters: length L, width B and height H. As a consequence, the beam's cross-sectional area is $A = B \cdot H$, while the moment of inertia is $I = (B \cdot H^3)/12$. In the damaged region, for a segment of length ΔL, the beam height is reduced to h, as depicted in Figure 1.

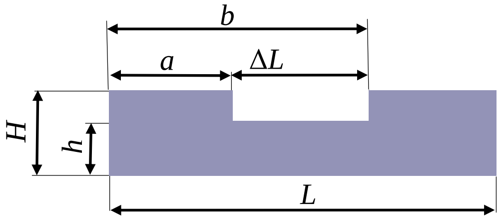


Figure 1: Longitudinal cross-section of a beam with an open crack

The natural frequencies of the healthy beam, fixed at one end and free at the other, can be expressed as:

$$f_{i-U} = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{ML^3}} \tag{1}$$

where M is the beam mass derived as $M = mL = \rho AL$ with ρ the volumetric mass density. The coefficients λ_i are deduced from the well-known characteristic frequency equation, i standing for the vibration mode number.

Let us consider the continuous cantilever beam itself as mass-less, a point load M_P acting at distance x from the fixed end, and a point load M_E at the free end (i.e. at distance L); the two masses have two different transversal displacements in the same period T_i, as shown in Figure 2 for vibration mode 2. These transversal displacements are derived from Eq. (2) as:

$$\phi_i(x) = \cosh\alpha_i x - \cos\alpha_i x - \frac{\cos\alpha_i L + \cosh\alpha_i L}{\sin\alpha_i L + \sinh\alpha_i L}\left(\sinh\alpha_i x - \sin\alpha_i x\right) \tag{2}$$

Herein $\alpha_i = \lambda_i L^{-1}$. Thus, the kinetic energies due to these two masses are:

$$U_{Ki}(x) = \frac{M_P}{2}\left[\frac{\phi_i(x)}{T_i}\right]^2 \tag{3}$$

$$U_{Ki}(L) = \frac{M_E}{2}\left[\frac{\phi_i(L)}{T_i}\right]^2 \tag{4}$$

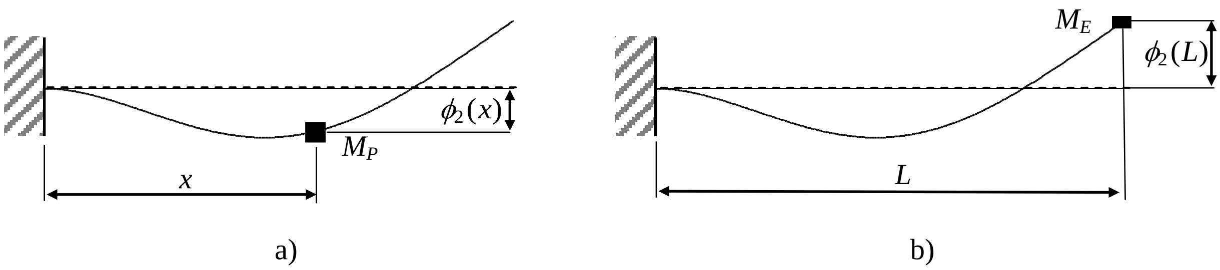


Figure 2: Displacements performed by the point mass M_P located at distance x from the fixed end (a) and its equivalent mass M_E placed at the beam tip (b) for vibration mode 2

To have equal frequencies in both cases, the kinetic energies have to be equal, thus, from Eqs. 3 and 4, one obtains:

$$M_{Ei} = M_P \left[\frac{\phi_i(x)}{\phi_i(L)} \right]^2 = M_P \left[\bar{\phi}_i(x) \right]^2 \tag{5}$$

Here $\bar{\phi}(x)$ is the normalized mode shape, taking values between null and unity.

The uniformly distributed mass m along the entire beam length L represents the total beam mass M. The equivalent beams self-mass M_E can now be determined by considering the influence of the local mass contribution. This is equivalent to weight the beam mass M with the normalized squared of the mode shape, as depicted in Figure 3 for vibration mode two, i.e. multiplying mass M by a coefficient κ_2 representing the ratio between the total area and the area under the curve $\left[\bar{\phi}_2(x) \right]^2$.

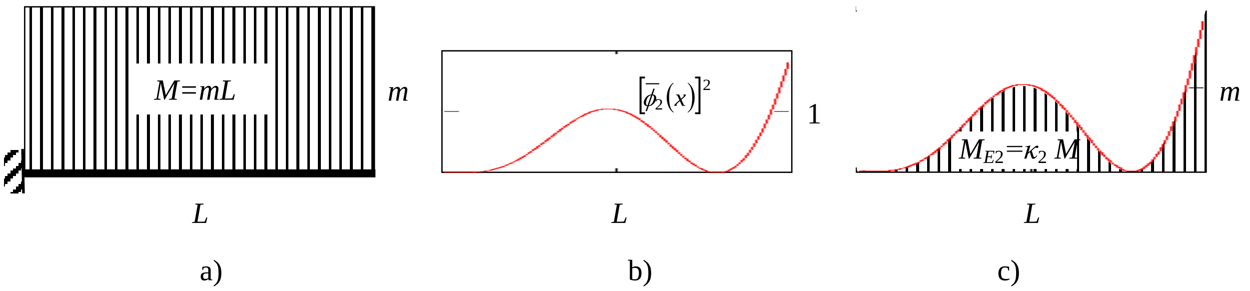


Figure 3: The cantilever beam with: (a) the load scheme due to own mass, (b) the local mass influence for vibration mode 2, and (c) the value of the equivalent mass placed on the free end

The coefficient κ_i is obtained by integrating the function $\left[\bar{\phi}_i(x) \right]^2$ along the whole beam length, as:

$$\kappa_i = \int_0^L \left[\bar{\phi}_i(x) \right]^2 dx = 0.25 \tag{6}$$

It is remarkable that the value 0.25 is obtained for all bending modes. Introducing the equivalent beam mass M_{Ei} in the relation that considers the frequency of the beam with a tip mass, the beam's i-th natural frequency is obtained as:

$$f_{i-U} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{EI}{ML^3}} = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{EI}{4M_{Ei}L^3}} \tag{7}$$

For a cantilever beam with a distributed mass m for the healthy part and a reduced distributed mass m_R for the damaged segment of length $\Delta L \in [a,b]$, the coefficient characterizing the equivalent mass loss $M_{\Delta Ei}$ is

$$\varsigma_i = \int_a^b \left[\bar{\phi}_i(x) \right]^2 dx \tag{8}$$

The equivalent reduced mass M_{REi} becomes therefore $M_{REi} = M_{Ei} - M_{\Delta Ei} = \kappa_i mL - \varsigma_i (m - m_R)L$. Denoting the distributed mass loss as $\Delta m = \rho A - \rho A_R = \rho BH - \rho Bh$ and denoting $1 - h \cdot H^{-1} = \eta$, the distributed mass loss becomes $\Delta m = m\eta$. Consequently, the equivalent mass of the damaged beam is:

$$M_{REi} = \kappa_i mL - \varsigma_i \eta mL = \frac{mL}{4}(1 - 4\varsigma_i \eta) \tag{9}$$

and therefore, the frequency for the *i*-th bending vibration mode of a beam with reduced mass becomes:

$$f_{i-M} = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{4M_{REi}L^3}} = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{mL^4(1-4\varsigma_i\eta)}} = f_{i-U}\sqrt{\frac{1}{1-4\varsigma_i\eta}} \tag{10}$$

From Eq. (7) and (10)one obtains the frequency shift for the *i*-th bending vibration as:

$$\Delta f_{i-M} = f_{i-M} - f_{i-U} = f_{i-U}\left(\sqrt{\frac{1}{(1-4\varsigma_i\eta)}} - 1\right) \tag{11}$$

The highest frequency increase is obtained, for all modes, if the mass loss is present at the cantilever's free end, while the loss of mass at the fixed end does not affect the natural frequencies. Obviously, in this analysis the beam rigidity *EI* is assumed to be constant, i.e. the stiffness reduction is neglected. Normalizing the frequency shift by the frequency of the healthy beam, one obtains:

$$\Delta \bar{f}_{i-M} = \frac{f_{i-M} - f_{i-U}}{f_{i-U}} = \sqrt{\frac{1}{(1-4\varsigma_i\eta)}} - 1 \tag{12}$$

To illustrate the effect of the mass loss upon the natural frequencies, a beam with the geometrical dimensions L = 1 m, B = 0.05 m and H = 0.05 m was subjected to simulations. The mass loss was considered for 20 equidistant intervals along the beam, that is a step of s_1 =0.05 m, for a thickness reduction h/H = 5/6, that gives $\eta \approx 0.15$. The twenty cases are considered one-by-one. Figure 4 shows the relative frequency increase for the first four bending vibration modes.

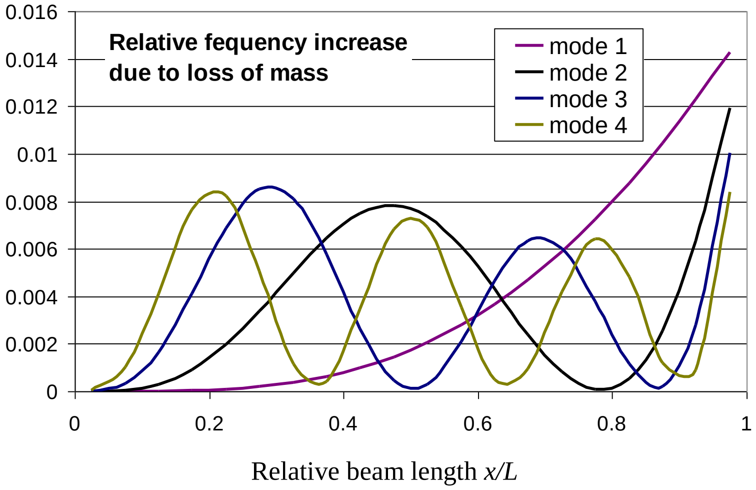
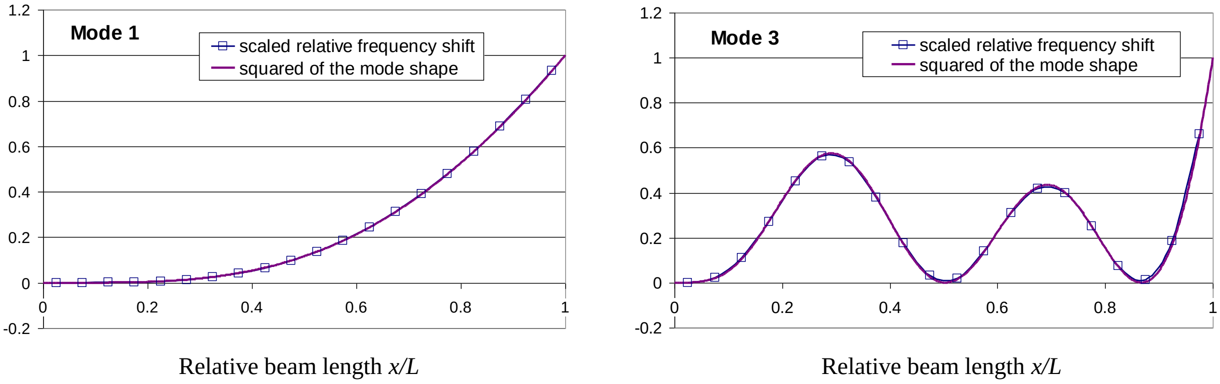


Figure 4: The relative frequency shift due to loss of mass for the first four bending modes.

To highlight the effect of slim transversal damages, the relative frequency shifts are compared with curves representing the squared of the mode shapes. To have comparable values with the squared of the mode shapes curves, the relative frequency shifts are scaled. Figure 5 illustrates the allure of these curves for the first and third vibration mode; the similarity between them is evident. Obviously, the slimmer the damage, the closer the curves.



a) b)

Figure 5: Comparison between the allures of the relative frequency shift due to loss of mass and the squared of the mode shape, for the first (a) and third (b) bending vibration mode

This finding permits us to express the frequency shifts in relation to the squared of the mode shapes. From Figure 5, one can observe that for both curves, the ratio between the values at any location x and the free end of the beam are the same, being identical with the local value of the squared of the mode shape. This means that by knowing the effect of mass loss at the free end of the beam, it is possible to derive the frequency shift as follows:

$$\Delta\bar{f}_{i-M} = \frac{f_{i-M} - f_{i-U}}{f_{i-U}} = \frac{\frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{M_{RE}L^3}} - \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{M_E L^3}}}{\frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{M_E L^3}}} \cdot \left[\bar{\phi}_i(x)\right]^2 = \frac{\sqrt{M_E} - \sqrt{M_{RE}}}{\sqrt{M_{RE}}} \cdot \left[\bar{\phi}_i(x)\right]^2 \qquad (13)$$

It has to be noted that the equivalent reduced mass M_E and the equivalent reduced mass M_{RE} are the same for any bending vibration mode, as it can be deduced from Eq. (5) and (6). The term $c(\beta,w)$, which is actually the relative mass change, i.e. $c(\beta,w) = \frac{\sqrt{M_E} - \sqrt{M_{RE}}}{\sqrt{M_{RE}}}$, depends on the damage width w and depth $\beta = H\text{-}h$. As a conclusion, the relative frequency shift due to loss of mass is

$$\Delta\bar{f}_{i-M} = c(\beta,w) \cdot \left[\bar{\phi}_i(x)\right]^2 \qquad (14)$$

From Eq. (6) the equivalent mass of the healthy beam becomes:

$$M_E = 0.25M \qquad (15)$$

while the equivalent mass of the damaged beam can be found as:

$$M_{RE} = 0.25M - \left(M - M_R\right) = 0.25M - \Delta M \qquad (16)$$

consequently, the relative frequency shift can be deduced from Eqs. (13), (15) and (16) as:

$$\Delta\bar{f}_{i-M} = \frac{\sqrt{M} - \sqrt{M - 4\Delta M}}{\sqrt{M - 4\Delta M}} \cdot \left(\bar{\phi}_i(x)\right)^2 \qquad (17)$$

and therefore the frequency of the beam with a loss of mass is:

$$f_{i-M} = f_{i-U}\left[1 + \Delta\bar{f}_{i-M}\right] = f_{i-U} \cdot \left[1 + \frac{\sqrt{M} - \sqrt{M - 4\Delta M}}{\sqrt{M - 4\Delta M}} \cdot \left(\bar{\phi}_i(x)\right)^2\right] \qquad (18)$$

To prove the validity of the theoretical considerations we present numerical simulations, made in ANSYS, on a steel beam with following dimensions: length L = 0.6 m, width B = 0.05 m and height H = 0.005 m.

The physical characteristics are: mass density ρ = 7850 kg/m^3; Young's modulus $E = 2 \cdot 10^{11}$ N/m^2 and Poisson's ratio μ = 0.3.

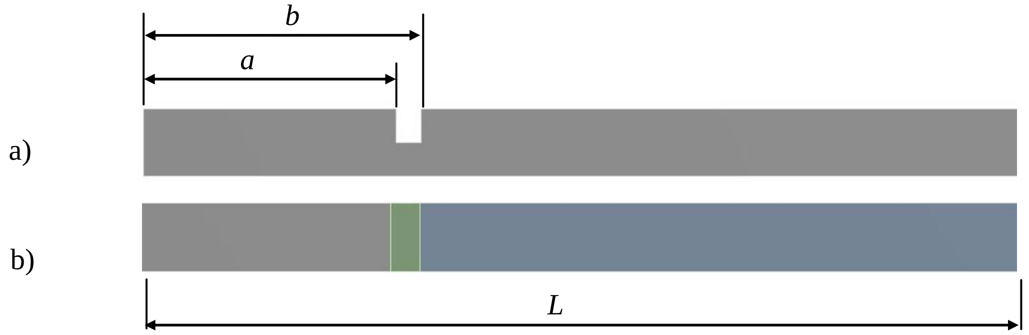


Figure 6: The cantilever beam modeled as structure with loss of mass and stiffness due to damage (a) and the structure with locally reduced density simulating loss of mass alone (b)

A transversal damage, like that presented in Figure 6a, is defined by location x, depth β and width w. The loss of mass due to damage is modeled as a decrease of the volumetric mass density; see Figure 6b. In all damage scenarios we attributed to the damage a depth β = 2.5 mm and a width w = 2 mm. The density ρ = 3925 kg/m^3 corresponds to the damage depth of β = 2.5 mm, which is applied to the slice containing the damage in the real case.

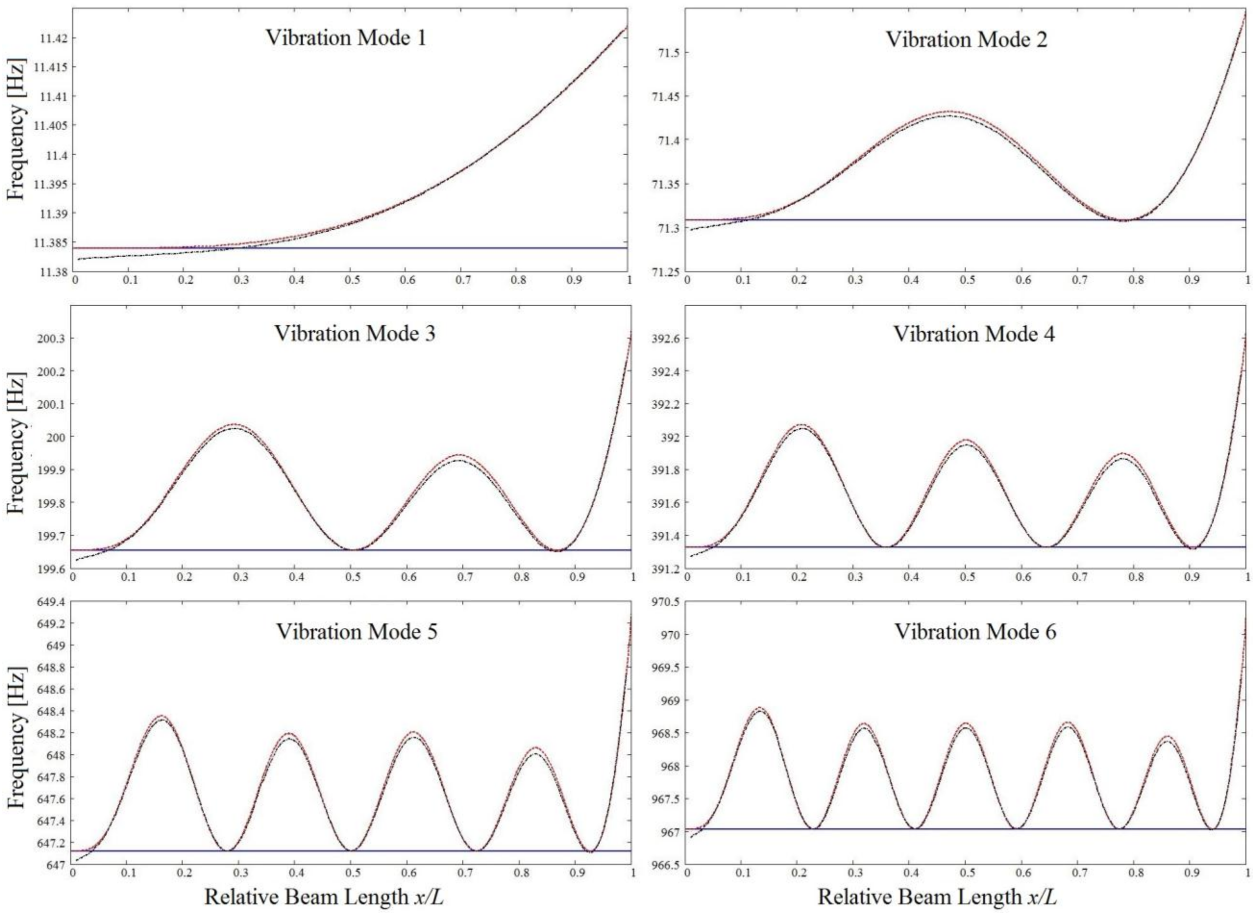


Figure 7: The natural frequency changes due to mass loss derived analytically and by means of the FEA for the first six vibration modes of the cantilever beam

The simulations have lead to the results presented in Figure 7 for the first six bending vibration modes. One can observe the good fit between the frequencies analytically derived using Eq. (18) and the simulated results with the differences being less than 0.2%. The results obtained by numerical simulations confirm the suitability of the proposed relation to calculate the natural frequencies for beams with intact rigidity but presenting a localized loss of mass. These frequency values have to be considered when the frequency changes due to open cracks are evaluated in the process of damage detection.

3 The influence of crack closure

Most papers consider the behavior of structures with open respectively breathing cracks as totally different, even if the loss of mass is neglected. The conclusions base on the crack models: if open cracks are considered to have a 'symmetric' evolution around the equilibrium position (i.e. the two half-periods are similar and increased in comparison with the healthy state), breathing cracks are mainly modeled using the bilinear model, and consequently the two half-periods are different [2,7-9]. The bilinear model, presented in Figure 8, consider two single-degree-of-freedom (SDOF) systems with different stiffness, k_O for the opening stage and k_C for the closing stage. One can remark that the half-period of the opening stage $T_O/2$ is larger that that of the closing stage $T_C/2$.

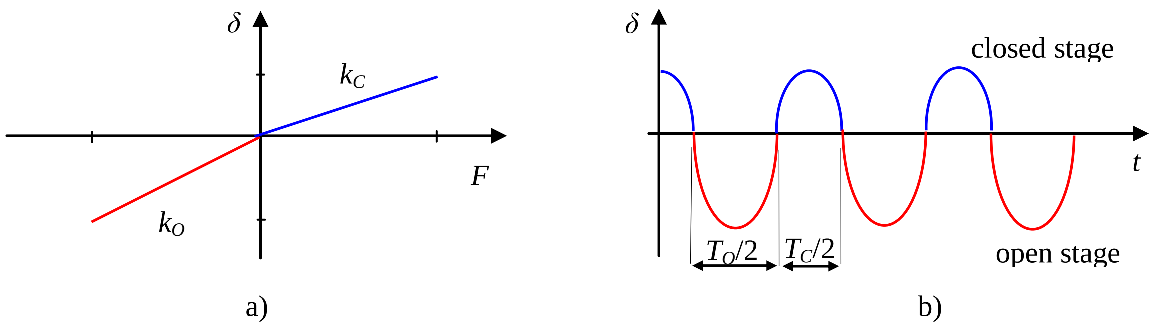


Figure 8: The restoring force model of the bilinear oscillator (a) and the time history (b)

For a cantilever beam the free end is probably considered to be subject of the forces and displacements. Moreover, the cracked beam end behaves in the way described by the bilinear model if it is cinematically conducted, that means that an external energy controls the system. Thus, the interpretation of breathing cracks using this model does not explain completely the phenomenon; some questions remaining open. In this paper we try to answer, so:

1. Is the total energy increasing in the closing stage and decreasing and opening stage for the free undamped vibration? This results from both diagrams in Figure 8. The answer is obviously NO, while the kinetic energy is the same for the zero beam displacement (i.e. the potential energy is null) irrespective to the movement direction.

2. If the total energies are identical in both open and closed stages, is the natural frequency unique for both stages (i.e. are the two half-periods $T_O/2$ and $T_C/2$ identical)? The answer should be YES, while there is a direct relation between the stored energy and the natural frequency.

3. Is the displacement of a point of the healthy beam segment, located between the fixed end and the crack, different in the opening and closing stages? Obviously NO, the difference occurs on the segment between the crack and the free end, where an additional rotation is present [10].

4. Revealed frequencies extracted from measured vibration signals of a structure with a breathing crack two picks, one indicating the sinusoidal component for the healthy beam (closed stage) and one for the crack opening? NO, for instance Yan et al. in [9] found one pick, consequently one frequency (strong sinusoidal component), which they nominated 'bilinear frequency'.

5. If, for the structure having a breathing crack, the half-period for the open stage is $T_O/2$, and for the similar structure with an open crack of same depth both half-period are $T_O/2$, the frequency shift should be almost double. This is NOT the case, the differences between the frequency shifts due to open and breathing cracks are very low [11].

According to these answers, several interesting conclusions can be formulated, that are:

- the global structural stiffness cannot be derived from static analysis, not even for the closed stage, this stiffness being identical for the open ('damaged') stage and closed ('healthy') stage

- the beam stores, for each vibration mode, an amount of energy dependent of the structural stiffness, meaning that there is one single frequency, dependent of the stored energy

- the two half-periods, of the opening stage $T_O/2$ and closing stage $T_C/2$, are identical.

These conclusions are maybe surprising and partially in contradiction to the explanations given by the bilinear model with one degree of freedom, but in perfect concordance with the physical phenomenon. Thus, a two-degrees-of-freedom model is necessary to proper model a beam with one crack.

To find the law that governs the phenomenon, a cantilever beam with one transversal crack is analyzed. For the healthy state, the deflection is given by Eq. (2), while the curvature is expressed as:

$$\phi_i''(x) = \cosh\alpha_i x + \cos\alpha_i x - \frac{\cos\alpha_i L + \cosh\alpha_i L}{\sin\alpha_i L + \sinh\alpha_i L}\left(\sinh\alpha_i x + \sin\alpha_i x\right) \tag{19}$$

Let us consider the healthy beam with a unity force $\bar{1}$ acting at the free end. The deflection at the point where the force is applied is $\bar{\delta}(L)$ and depends on the beam global stiffness. For the similar beam with a crack at location x/L = 0.45, to obtain the same deflection a lower force is needed. This means that the crack diminish the beam capacity to store energy.

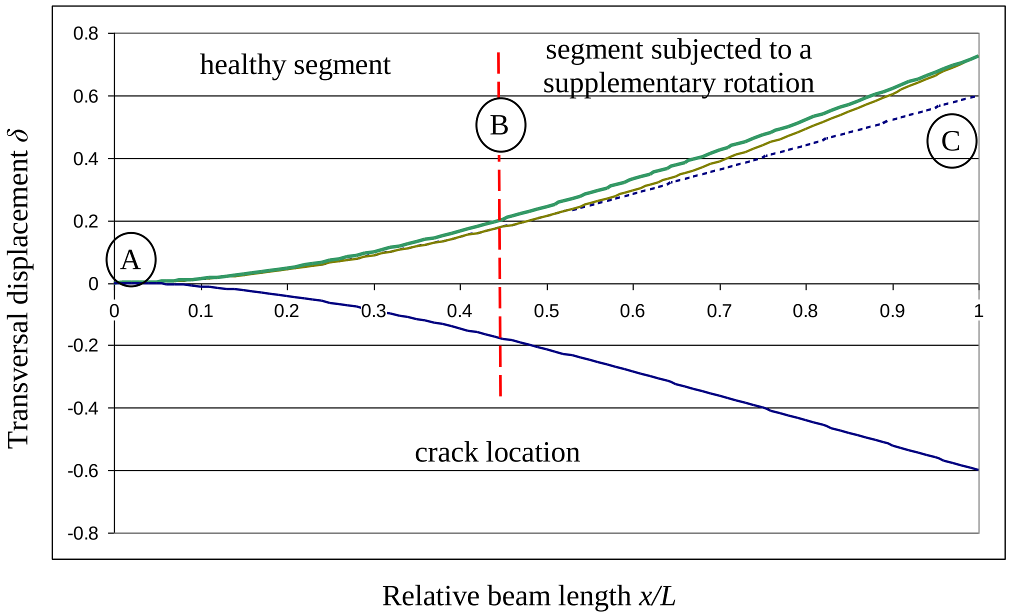


Figure 9: Deflection of the damaged beam in open stage (positive values of δ) and closed stage (negative values of δ)

If the cracked beam is released from this position, namely $\bar{\delta}(L)$, it attains at the opposite side a deflection $\bar{\delta}_U(L)$, like it is shown in Figure 9. Because the beam stiffness is unaltered in the closed stage, and the stored energy is lower than that of the healthy beam, it results $\bar{\delta}_U(L) < \bar{\delta}(L)$. This is also valid in the case that the beam is subjected to other load types, including its own mass.

The opposite is also available, if the healthy beam has a deflection $\delta_U(L)$ under own load, it attempt a higher deflection $\delta_D(L)$ in case that damage occur. Consequently, the damaged beam can be substituted by a beam with lower, but continuous rigidity EI_{eq}.

Let us neglect at this moment the mass loss. In this case, for the cantilever beam subjected alone to its own mass, the deflection $\delta_U(L)$ at the free end is:

$$\delta_U(L) = \frac{\rho g A L^4}{8EI} = \frac{ML^3}{8EI} \tag{20}$$

For the damaged beam, in same conditions, the deflection $\delta_D(L)$ becomes:

$$\delta_D(L) = \frac{ML^3}{8EI_{eq}} \tag{21}$$

Thus, the healthy beam, the stiffness is:

$$EI = \frac{ML^3}{8\delta_U(L)} \tag{22}$$

while for the equivalent continuous beam, behaving as the damaged one, it is:

$$EI_{eq} = \frac{ML^3}{8\delta_D(L)} \tag{23}$$

Therefore, the natural frequencies for the damaged beam, by neglecting mass loss, are:

$$f_{i-S} = \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI_{eq}}{ML^3}} \tag{24}$$

From Eqs. (1) and (24) the relative frequency shift for a beam with stiffness decrease can be calculated as:

$$\Delta\bar{f}_{i-S} = \frac{f_{i-U} - f_{i-S}}{f_{i-U}} = \frac{\frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{ML^3}} - \frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI_{eq}}{ML^3}}}{\frac{\lambda_i^2}{2\pi}\sqrt{\frac{EI}{ML^3}}} = \frac{\sqrt{EI} - \sqrt{EI_{eq}}}{\sqrt{EI}} \tag{25}$$

Substituting the stiffness expression from Eqs. (22) and (23), one can find the relative frequency shift in respect to deflections at the free end, as:

$$\Delta\bar{f}_{i-S} = \frac{\sqrt{\frac{1}{\delta_U(L)}} - \sqrt{\frac{1}{\delta_D(L)}}}{\sqrt{\frac{1}{\delta_U(L)}}} = \frac{\sqrt{\delta_D(L)} - \sqrt{\delta_U(L)}}{\sqrt{\delta_D(L)}} \tag{26}$$

As a consequence, from Eq. (25) and (26) results the mathematical expression for the natural frequency of the damaged beam as:

$$f_{i-S} = f_{i-U} - f_{i-U} \cdot \Delta\bar{f}_{i-S} = f_{i-U}\left[1 - \frac{\sqrt{\delta_D(L)} - \sqrt{\delta_U(L)}}{\sqrt{\delta_D(L)}}\right] \tag{27}$$

We must remark that the mathematical expressions for the natural frequencies of the damaged beam and the relative frequency shifts in Eqs. (27) and (26) are applicable for open and breathing crack as well, irrespective to the crack location. In prior researches we found that the frequency shift due to stiffness loss produced at any location on the beam can be expressed as the frequency shift due to a damage at the fixed end multiplied with the normalized stored energy in that location (i.e. the squared of the normalized mode shape curvature), see for instance references [11-13].

Thus, natural frequency of the damaged beam is:

$$f_{i-S}(x) = f_{i-U}\left[1 - \frac{\sqrt{\delta_D(L)} - \sqrt{\delta_U(L)}}{\sqrt{\delta_D(L)}} \left(\bar{\phi}_i''(x)\right)^2\right] \qquad (28)$$

For a beam with an open crack, thus with mass loss, the frequencies can be derived using Eqs. (18) and (28), as:

$$f_{i-D}(x) = f_{i-U} \cdot \left[1 + \frac{\sqrt{M} - \sqrt{M - 4\Delta M}}{\sqrt{M - 4\Delta M}} \cdot \left(\bar{\phi}_i(x)\right)^2 - \frac{\sqrt{\delta_D(L)} - \sqrt{\delta_U(L)}}{\sqrt{\delta_D(L)}} \left(\bar{\phi}_i''(x)\right)^2\right] \qquad (29)$$

This Eq. (29) can be applied for any crack model and for beams with any support type, by simply choosing the adequate mode shape and curvature relations. The relation is more versatile than that derived from fracture mechanics, while it can be applied to any cross-sectional shapes without any adaption [14].

4 Numerical investigations

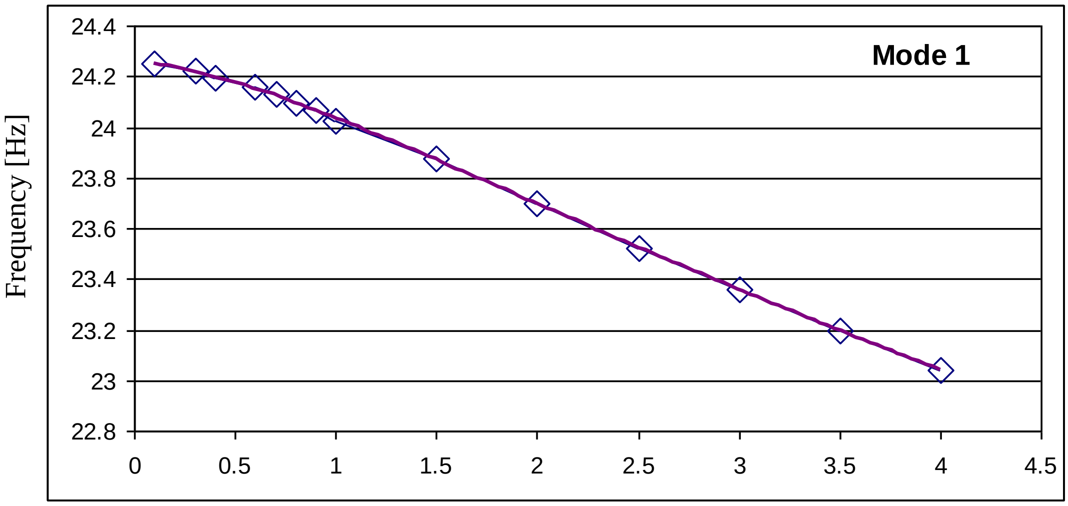
Numerous simulations where made to validate the developed theory and to find the influence of the crack width *w*. In this paper we present first a study regarding the frequency changes produced by a crack located at the clamped end. A finite element analysis using the ANSYS program is performed on a cantilever steel beam similar to that presented in Figure 6a, having length L = 600 mm, and square cross-section with the edge dimension Q = 12 mm. The material parameters are: mass density ρ = 7850 kg/m^3; Young's modulus $E = 2.0 \cdot 10^{11}$ N/m^2 and Poisson's ratio $\mu = 0.3$.

The natural frequencies for six vibration modes are derived, for the healthy state and for a series of damage scenarios. In all cases the damage depth is D = 6 mm, while the considered damage width in the range 0.1…4 mm, as presented in Table 1. The beam deflection at the free end, for all cases, is also found by static analysis. A good agreement between the relative frequency shifts calculated using Eq. (29) and that derived using the frequencies of the damaged and undamaged beam obtained by simulations is found.

The dependency between the cracked beam's frequency for the first mode and the damage width are shown in Figure 10. One can remark the linearity between these parameters for bigger values of the damage width, whereas for lower values the frequencies attempt asymptotically the one representing the breathing crack.

Crack width	Natural frequency [Hz]						Deflection
w [mm]	Mode 1	Mode 2	Mode 3	Mode 4	Mod e5	Mode 6	$\delta(L)$ [mm]
undamaged	27.1973	170.1255	474.9414	926.702	1523.49	2260.70	0.51883
0.1	24.2513	155.6611	441.9725	873.408	1447.91	2202.61	0.65148
0.3	24.2209	155.5393	441.7378	873.079	1447.49	2202.46	0.6531
0.4	24.1948	155.4318	441.5232	872.766	1447.06	2202.30	0.65449
0.6	24.1585	155.2868	441.2439	872.374	1446.55	2202.11	0.65643
0.7	24.1291	155.1664	441.0051	872.027	1446.08	2201.94	0.65802
0.8	24.0977	155.0395	440.7574	871.674	1445.61	2201.78	0.65972
0.9	24.0672	154.9164	440.5163	871.328	1445.15	2201.62	0.66137
1	24.0270	154.7548	440.2012	870.878	1444.55	2201.43	0.66356
1.5	23.8727	154.1428	439.0181	869.199	1442.32	2200.69	0.67206
2	23.7000	153.4715	437.7394	867.406	1439.97	2199.93	0.68177
2.5	23.5234	152.7978	436.4714	865.638	1437.64	2199.16	0.69193
3	23.3595	152.1861	435.3369	864.075	1435.60	2198.51	0.70155
3.5	23.1980	151.5951	434.2554	862.598	1433.68	2197.90	0.71122
4	23.0419	151.0423	433.2738	861.299	1432.04	2197.42	0.72076

Table 1: Natural frequencies of the undamaged beam and the damaged beam



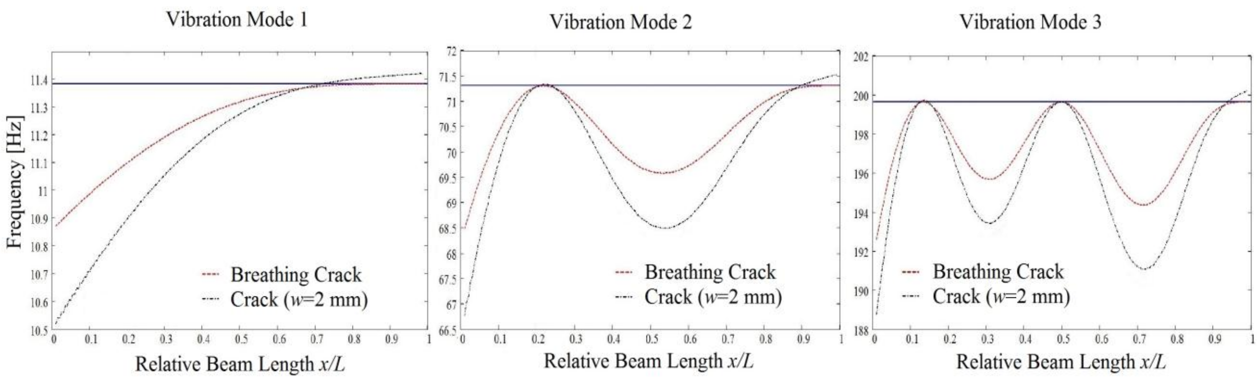
Damage width *w* [mm]

Figure 10: The frequency of the damaged beam vs. damage width

The fact that the frequency of an open crack with a significant width (*w* = 0.3 mm) is quite similar to that of a breathing crack validate the conclusions formulated in the previous section regarding the identity between the half-periods associated to the open and closed stage. We presume, but it is not yet demonstrated, that the frequency shift due to an open crack is the effect of the discontinuity (i.e. the breathing crack, therefore *w* = 0) and the supplementary slope permitted by the cross-section reduction of length *w*.

A second set of simulation is performed to test Eq. (29) for different crack types, located on various locations on a beam. The steel beam has identical length and material parameters like that used in the simulation presented above, but it has a rectangular cross-section with the dimensions: width *B* = 50 mm and height *H* = 5 mm. Open cracks with a width *w* = 2 mm and breathing cracks is simulated The finite element analysis is performed for numerous damage scenarios, involving the healthy structure and the beam with cracks placed one by one on numerous locations along the beam. The mass loss due to damage is also analyzed individually, without considering the stiffness reduction.

The curves representing the frequency shift due to open cracks and breathing cracks are illustrated in Figure 11 for the first three bending vibration modes. The frequencies resulted from the finite element analysis, for damages having a depth *D* = 2.5 mm. One can remark the differences between the frequency changes by low damage depth values.



a) b) c)

Figure 11: Comparison between frequency shift curves of the open and breathing crack, or the first three bending vibration mode

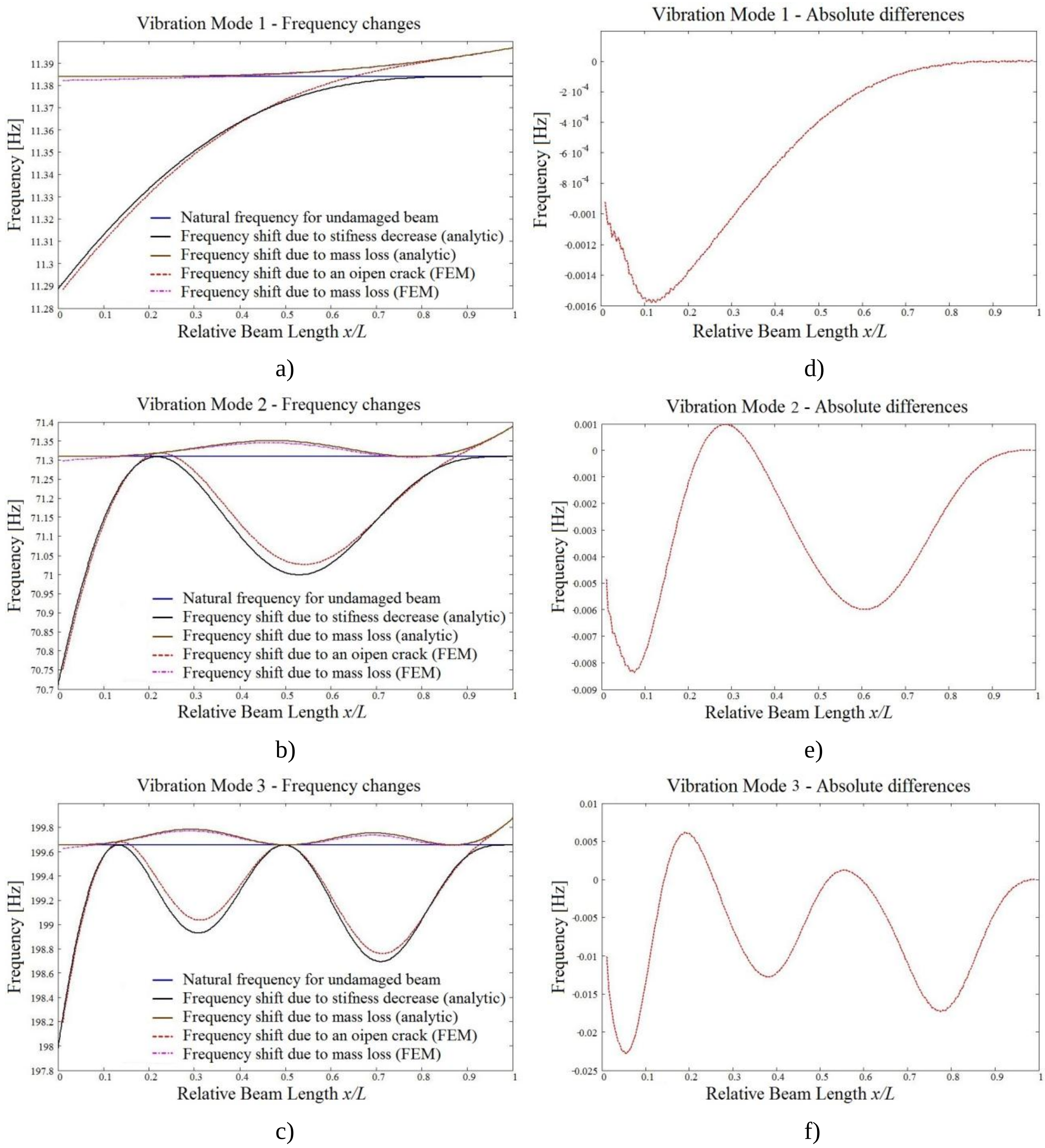
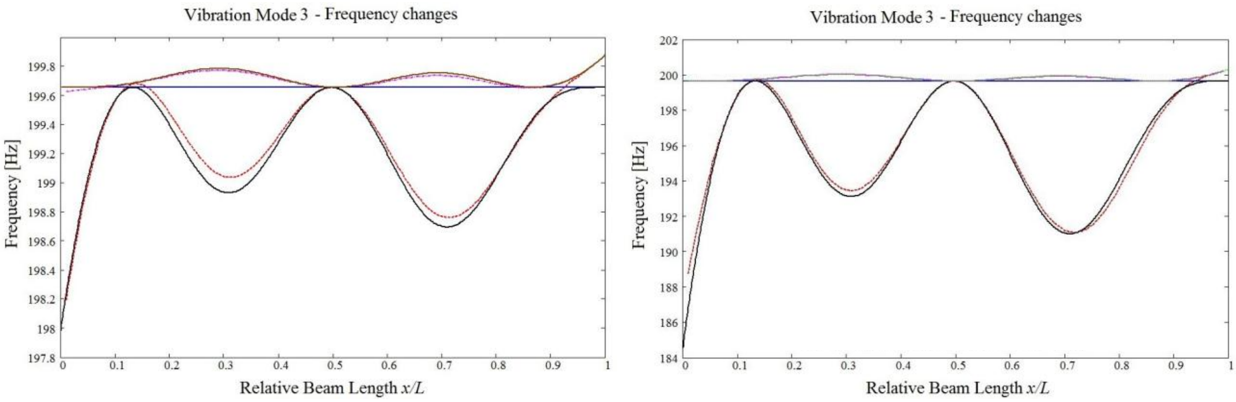


Figure 12: The frequency shift curves for an open crack, the mass loss influence and the frequency changes analytically derived with mass loss compensation (a), and the differences between results obtained theoretically and by means of FEA (b)

In Figure 12.a-c the curves representing the frequency shift due to open cracks are plotted, as well as those presenting the influence of the mass loss. In this analysis a damage depth D = 0.85 mm is considered. First, one can observe the good concordance between the curves representing the frequency increase due to loss of mass only, drawn using Eq. (18) and by means of FEA.

Comparing the frequency shift due to the open crack and the changes due to mass loss only, it is obvious that for incipient damage the mass loss is significant. This means that, when the damage depth is relatively small, the mass loss impose another baseline to be used in damage detection in comparison to breathing cracks where the frequency of the undamaged beam is relevant.

Analyzing the frequency shift curves derived from Eq. (29), which compensates the mass loss, and those traced for the open crack using values from the FEA, one can observe that the difference between them is given strictly by the mass loss. Subtracting the mass loss influence from the frequency shift curves of the open crack, a perfect similitude is found with the curves plotted using Eq. (29); Figure 12.d-f present the absolute differences between these curves, highlighting the perfect fit (errors less then 0.5%).



a) b)

Figure 13: Frequency shift curves for an open crack: damage depth D = 0.85 mm (a) and D = 2.5 mm (b)

The loss of mass influence is less important if the damage depth increases, because the stiffness decrease is more accentuated than the mass loss decrease. This can be observed in Figure 13, which present the frequency shift curves in case of a damage depth D = 0.85 mm and a damage depth D = 2.5 mm respectively. For breathing cracks Eq. (28) predict frequency changes with high accuracy, the frequency shift curves for the analytic solutions and the FEA results being identical for all bending vibration modes.

5 Conclusion

The researches presented in this paper revealed that two-degrees-of-freedom-models have to be utilized to proper describe and interpret the dynamic behavior of beams with one crack. Using this model, some characteristics of the dynamic behavior of cracked beams are clarified and presented in this paper. One of them is the displacement relation, which does not impose a frequency change by increasing the deflection of the free end. In this way it was demonstrated that for open cracks the two half-periods, of the opening stage $T_O/2$ and closing stage $T_C/2$, are identical.

Another conclusion regards the stored energy, that cannot be derived from the stiffness resulted from static analysis. If a discontinuity is present in the structure, the global structural stiffness decreases, being identical for both open and closed stage; therefore a 'dynamic equivalent' stiffness has to be considered.

The paper proposes mathematical expressions that predict frequency shifts due to structural changes in beams, as open and breathing cracks and mass loss. It is highlighted that, especially for small damages, the mass loss produce an important frequency increase exactly at the locations where sensible frequency decreases occur due to stiffness loss; therefore it is the risk of damage depth undervaluation. Thus, a new baseline for open cracks is proposed, that takes in consideration the frequency changes due to mass loss.

The relations between the crack geometry and the modal parameters can be used to define patterns characterizing the damage location and geometry, usable in damage detection procedures. It simplifies damage detection in beams while all damage scenarios are easily defined.

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