New Factor Graph Based Multiuser Detector for Spectrally Efficient CPM

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***Abstract*—This paper presents a new iterative multiuser detection algorithm for asynchronous spectrally-efficient M-ary continuous-phase modulation in additive white Gaussian noise. This detection algorithm is closely related to another algorithm that was recently proposed by the same authors, but it follows from applying the sum-product algorithm to a different factor graph of the same multiuser detection problem. This, in turn, results in a different way to approximate the marginal bit a-posteriori probabilities that are used to perform minimum bit error rate multiuser detection. The girth of the factor graph considered in this contribution is twice as large, which is known to be potentially beneficial for the accuracy of the a-posteriori probabilities. The size of the largest factor graph variable alphabets also multiplies with M, rendering the straightforward application of the sum-product algorithm more complex. Through approximating a suitable set of sum-product messages by a Gaussian distribution, this complexity is significantly reduced. For a set of system parameters yielding high spectral efficiency, the resulting algorithm significantly outperforms the previously proposed solution.**

***Keywords*— Factor Graph; Multiuser Detection; Spectral Efficiency; Continuous Phase Modulation.**

I. Introduction

Continuous-phase modulation (CPM) is a modulation method commonly used in wireless modems [1]. It is attractive because of its high power and spectral efficiency, and because of its robustness to non-linearities. Although the optimal detection [2] of a bit interleaved coded CPM signal is prohibitively complex, there exist approximate iterative detectors with reasonable complexity that yield a very good performance. Such practical detectors can be derived from the sum-product (SP) algorithm and the factor graph (FG) framework [3]. In the past few years, several advanced techniques have been proposed for multiuser (MU) detection of *spectrally efficient* (SE) CPM systems [4], [5], [6]. In such systems, the spacing between the carrier frequencies that are assigned to different users is kept small, such that the leakage of the neighboring signal energy into the desired frequency band cannot be ignored. This leakage signal is referred to as inter-user interference (IUI).

In this paper, we derive a new FG based MU detector for SE CPM. The proposed receiver algorithm will be referred to as g6-MU-FG-GA. Numerical results indicate that, for a set of system parameters yielding high spectral efficiency, the novel g6-MU-FG-GA algorithm significantly outperforms the existing solutions in terms of reliability and/or complexity.

The structure of the paper is as follows. Section II reports on the state-of-the-art in MU bit detection of SE CPM systems. Section III describes the considered system model. The proposed g6-MU-FG-GA detector is derived in Section IV. Its computational complexity and memory requirements are adressed in Section V. Section VI presents numerical results on the packet error rate (PER) performance of the SU-FG, the g10-MU-FG-2IU, the g3-MU-FG-GA and the g6-MU-FG-GA detectors. The main conclusions are summarized in Section VII.

II. State-of-the-art

In [6], an intuitive approach to MU detection of SE CPM is proposed. The considered ad-hoc receiver executes a practical form of MU detection by iterating between conventional FG-based single user (SU) detectors and a separate module for IUI cancellation. A similar solution is also considered as a reference system in [5]. This receiver is further referred to as SU-FG.

A more fundamental approach to MU detection is to consider a FG of the actual MU detection problem. Such a FG is not unique. It represents one particular way to factorize the joint a-posteriori probability (APP) of the bit sequences from all the users, given the observed received MU signal. Running the SP algorithm on this graph yields an approximation of the marginal bit APPs. Using the latter APPs to perform maximum a posteriori (MAP) bit detection results in minimum bit error rate MU detection. Choosing a FG is a matter of trading off the accuracy of the obtained marginal bit APPs against the computational complexity associated with their evaluation. An important parameter for the accuracy of the SP algorithm is the minimum cycle length (i.e., girth) of the graph, while the complexity of the procedure primarily depends on the number of edges and the size of the variable alphabets in the graph.

Optimal MU detection is obtained when the employed FG corresponds to a tree (cycle-less graph). In that case, running the SP algorithm is straightforward and yields the exact marginal bit APPs. Using these APPs to perform MAP bit detection results in minimum bit error rate performance. Unfortunately, the complexity of this optimal MU detector is extremely high and exponential in the number of users [5]. It is therefore not suited for use in practice and one has to resort to approximations.

The practical MU detector proposed in [5] is based on another FG, which contains cycles. The complexity of this receiver is lower than that of the optimal MU detector, but still increases exponentially with the number of users, which becomes impractical as the number of users increases. As an option to further decrease the complexity, the authors propose to apply the simplifying assumption that only the two most adjacent users significantly contribute to the IUI. This results in a FG with girth 10, with a significantly reduced the degree of connectivity. The corresponding receiver will be referred to as g10-MU-FG-2IU. It has a good performance and a linear complexity in the number of users, but it still involves quite a large number of computations and it also requires a considerable amount of memory.

Another low-complexity MU FG-based detection algorithm was proposed in [4]. It results from approximating, in a FG with girth only 3, a suitable set of SP messages by Gaussian distribution functions. The computational complexity and memory requirement of this detector is considerably lower than for the g10-MU-FG-2IU detector from [5]. A significant performance improvement over the SU-FG detector was demonstrated for a particular SE CPM scheme, in [4]. This receiver algorithm will be referred to as g3-MU-FG-GA.

In this paper, we derive a new variant of the algorithm from [4]: g6-MU-FG-GA. The idea of approximating a suitable set of SP messages by a Gaussian distribution is applied to a different FG of the same detection problem. This FG is slightly more complex than the one used to derive the g3-MU-FG-GA detector from [4], but it has cycles of minimum length 6, which is twice as long as the shortest cycles in [4]. This is potentially beneficial for the accuracy of the marginal bit APPs that result from applying the SP algorithm.

III. System Model

The transmitter of user u encodes N_b information bits $\mathbf{b}^{(u)} = \left\{b_k^{(u)}\right\}$ into N_c coded bits $\mathbf{c}^{(u)} = \left\{c_l^{(u)}\right\}$. These coded bits are subsequently interleaved and mapped to N symbols $\mathbf{a}^{(u)} = \left(a_0^{(u)}, ..., a_{N-1}^{(u)}\right)$, with $a_n^{(u)}$ taking values from the M-ary alphabet $\Omega_M = \{\pm 1, \pm 3, ..., \pm (M-1)\}$. The resulting symbols are then used to generate the complex envelope $s_{SU}^{(u)}(t)$, for $0 \leq t < NT$, of the CPM signal from user u:

$$s_{SU}^{(u)}(t) = e^{j\psi\left(t;\mathbf{a}^{(u)}\right)}, \tag{1}$$

$$\psi\left(t;\mathbf{a}^{(u)}\right) = 2\pi h \sum_i a_i^{(u)} q\left(t - iT\right). \tag{2}$$

Here, T is the symbol period, $h = K/P$ is the modulation index and $q(t)$ is the phase-smoothing response. Taking into account that $q(t)$ = 0 for $t \leq 0$ and $q(t)$ = 0.5 for $t \geq LT$, we can rewrite (2) for $nT \leq t \leq (n+1)T$ as:

$$\psi\left(t;\mathbf{a}^{(u)}\right) = \Psi\left(t - nT; \mathbf{S}_n^{(u)}\right), \ \ nT \leq t \leq (n+1)T, \tag{3}$$

where n = 0,1, ..., $N-1$. The quantity $\mathbf{S}_n^{(u)}$ in (3) describes the CPM state transition during the nth symbol interval $[nT, (n+1)T[$ of the transmitter signal $s_{SU}^{(u)}(t)$ from user u:

$$\mathbf{S}_n^{(u)} = \left(\boldsymbol{\sigma}_n^{(u)}, a_n^{(u)}\right), \tag{4}$$

with $\boldsymbol{\sigma}_n^{(u)}$ an L element vector $\left(\sigma_{n,0}^{(u)}, \sigma_{n,1}^{(u)}, ..., \sigma_{n,L-1}^{(u)}\right)$ denoting the CPM state at time instant n and

$$\begin{aligned}\Psi\left(\mathtt{t};\mathbf{S}_n^{(u)}\right) &= \sigma_{n,0}^{(u)} + 2\pi h \sum_{i=1}^{L-1} \sigma_{n,i}^{(u)} q\left(\mathtt{t} - (L-i)T\right) \\ &\quad + 2\pi h a_n^{(u)} q(\mathtt{t}), \quad 0 \leq \mathtt{t} < T.\end{aligned} \tag{5}$$

Given the symbol vector $\mathbf{a}^{(u)}$ and starting from a given initial CPM state $\boldsymbol{\sigma}_0^{(u)}$, the vectors $\boldsymbol{\sigma}_n^{(u)}$, with $n = 1, 2, ..., N$, can be computed recursively according to the following equations:

$$\sigma_{n,0}^{(u)} = \left[\sigma_{n-1,0}^{(u)} + \pi h \sigma_{n-1,1}^{(u)}\right]_{2\pi}, \tag{6}$$

$$\sigma_{n,i}^{(u)} = \sigma_{n-1,i+1}^{(u)}, \quad 1 \leq i \leq L-2, \tag{7}$$

$$\sigma_{n,L-1}^{(u)} = a_{n-1}^{(u)}, \tag{8}$$

where $[x]_{2\pi}$ denotes modulo-2 reduction of x to the interval $[0, 2\pi[$. At each time instant n the CPM transmission scheme has PM^{L-1} possible states. The complete set of CPM state vectors $\boldsymbol{\sigma}_n^{(u)}$, $u = 1, 2, ..., U$, is grouped in the vector $\boldsymbol{\sigma}_n = \left(\boldsymbol{\sigma}_n^{(1)}, \boldsymbol{\sigma}_n^{(2)}, ..., \boldsymbol{\sigma}_n^{(U)}\right)$. The complete sequence of CPM state transition identifiers $\mathbf{S}_n^{(u)}$, $n = 0, 1, ..., N-1$, is grouped in the vector $\mathbf{S}^{(u)} = \left(\mathbf{S}_0^{(u)}, \mathbf{S}_1^{(u)}, ..., \mathbf{S}_{N-1}^{(u)}\right)$.

A group of U users are simultaneously active. They asynchronously transmit frequency-shifted versions of their signals $s_{SU}^{(u)}(t)$, $u = 1, 2, ..., U$ over an additive white Gaussian noise (AWGN) channel, which is typical for satellite communications. The short-hand notations $\mathbf{b}$, $\mathbf{c}$, $\mathbf{a}$, $\boldsymbol{\sigma}$ and $\mathbf{S}$ respectively collect the information bit sequences, the coded bit sequences, the data symbol sequences, the CPM state vector sequences and the CPM state transition identifier sequences from all the users: $\mathbf{b} = \left\{\mathbf{b}^{(1)}, \mathbf{b}^{(2)}, ..., \mathbf{b}^{(U)}\right\}$, $\mathbf{c} = \left\{\mathbf{c}^{(1)}, \mathbf{c}^{(2)}, ..., \mathbf{c}^{(U)}\right\}$, $\mathbf{a} = \left\{\mathbf{a}^{(1)}, \mathbf{a}^{(2)}, ..., \mathbf{a}^{(U)}\right\}$, $\boldsymbol{\sigma} = \left\{\boldsymbol{\sigma}^{(1)}, \boldsymbol{\sigma}^{(2)}, ..., \boldsymbol{\sigma}^{(U)}\right\}$ and $\mathbf{S} = \left\{\mathbf{S}^{(1)}, \mathbf{S}^{(2)}, ..., \mathbf{S}^{(U)}\right\}$. The phase offset, frequency shift and time delay associated with user u ($u = 1, 2, ..., U$) are equal to $\theta^{(u)}$, $f^{(u)}/T$ and $\tau^{(u)}T$, with $f^{(u)}$ and $\tau^{(u)}$ denoting the dimensionless relative (to the symbol interval T) frequency shift and time delay, and are assumed to be known at the receiver. We further assume that, within a given group of users, all signals are received with the same power. The complex baseband representation $s(t)$ of the received signal component, which aggregates the contributions of all users from a same group is given by $s(t) = \sum_{u=1}^{U} s^{(u)}(t)$, with

$$s^{(u)}(t) = s_{SU}^{(u)}\left(t - \tau^{(u)}T\right) e^{j2\pi f^{(u)}\frac{t}{T}} e^{j\theta^{(u)}}. \tag{9}$$

The latter contribution differs from zero only for $t \in \left[\tau^{(u)}T, \left(\tau^{(u)} + N\right)T\right[$. It is assumed that $s(t)$ is band-limited (although this is not strictly true in the case of CPM signals whose spectrum has an infinite support) with bandwidth lower than $R_s/2T$, where R_s is a proper integer value (determined by the spectral shape of the considered CPM scheme and the

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number of users in a group). The received baseband signal is applied to a low-pass anti-aliasing filter and sampled at R_s samples per symbol interval; the corresponding sampling interval is $T_s = T/R_s$. It is assumed that the spacing between groups of U users is sufficiently large, such that the leakage of the signal energy from neighboring groups into the considered frequency band $\left[\frac{R_s}{2T}, \frac{R_s}{2T}\right]$ can be safely ignored. The resulting samples r_k can then be modeled as follows:

$$r_k = \sum_{u=1}^{U} s_k^{(u)} + n_k, \tag{10}$$

where $s_k^{(u)}$ are samples of the receiver signal $s^{(u)}(t)$ from (9), taken at $t = kT_s$ and n_k are zero-mean complex AWGN samples with variance equal to $N_0 R_s / E_s$, with N_0 the noise power spectral density and E_s the energy per symbol period. The samples r_k are conveniently grouped into vectors of the type $\mathbf{r}_l = \left(r_{lR_s}, r_{lR_s+1}, ..., r_{(l+1)R_s-1}\right)^T$ and $\mathbf{r} = \left(\mathbf{r}_0^T, \mathbf{r}_{R_s}^T, ..., \mathbf{r}_{(N_s-R_s)}^T\right)^T$.

IV. Proposed g6-MU-FG-GA Receiver: Derivation

When an information bit is detected erroneously at the receiver, a bit error occurs. Optimal detection, which minimizes the bit error probability is achieved by performing MAP bit detection [2]:

$$\hat{b}_k^{(u)} = \arg\max_{b\in\{0,1\}} p\left(b_k^{(u)} = b \,|\mathbf{r}\right), \forall k. \tag{11}$$

The APPs $p\left(b_k^{(u)} |\mathbf{r}\right)$ involved in (11) are the marginals of $p(\mathbf{b}|\mathbf{r})$, with $p(\mathbf{b}|\mathbf{r})$ the probability of $\mathbf{b}$, given $\mathbf{r}$. An efficient way to jointly compute these marginals is to apply the SP algorithm to a FG representing a suitable factorization of the joint probability $p(\mathbf{b}, \mathbf{x}|\mathbf{r})$ of $\mathbf{b}$ and $\mathbf{x}$, where $\mathbf{x}$ represents a convenient set of additional variables [3].

In the following, we derive the proposed g6-MU-FG-GA detector.

We construct a FG for deriving $p\left(b_k^{(u)} |\mathbf{r}\right)$ by employing $(\mathbf{c}, \mathbf{a}, \boldsymbol{\sigma}_0, \boldsymbol{\sigma}_N, \mathbf{S}, \mathbf{s})$ as additional variables. The vector $\mathbf{s}$ results from stacking the U vectors $\mathbf{s}^{(u)}$, $u = 1, 2, ..., U$, where $\mathbf{s}^{(u)}$ consists of all the samples (both zero and non-zero valued) of $s^{(u)}(t)$ from (9), i.e., taken at instants kT_s. The vector $\mathbf{s}^{(u)}$ itself results from stacking the vectors $\mathbf{s}_x^{(u)}$, where $\mathbf{s}_x^{(u)}$ contains the R_s samples $s^{(u)}(iT_s + xT)$, $i = 0, 1, ..., R_s - 1$, that correspond to the xth symbol interval $[xT, (x+1)T[$ of the receiver signal $s^{(u)}(t)$ from (9). In general, these samples are spread over the $\left(x - N_\tau^{(u)} - 1\right)$th $\left[\left(x - N_\tau^{(u)} - 1\right)T, \left(x - N_\tau^{(u)}\right)T\right[$ and the $\left(x - N_\tau^{(u)}\right)$th $\left[\left(x - N_\tau^{(u)}\right)T, \left(x - N_\tau^{(u)} + 1\right)T\right[$ symbol interval of the signal $s_{SU}^{(u)}(t)$ from (1), with $N_\tau^{(u)} = \left\lfloor \tau^{(u)} \right\rfloor$, being the smallest integer value smaller than or equal to $\tau^{(u)}$. The joint APP $p(\mathbf{b}, \mathbf{c}, \mathbf{a}, \boldsymbol{\sigma}_0, \boldsymbol{\sigma}_N, \mathbf{S}, \mathbf{s}|\mathbf{r})$ can be factorized as follows, by taking into account the specific structure of the transmitted CPM signal:

$$\begin{aligned} & p(\mathbf{b}, \mathbf{c}, \mathbf{a}, \boldsymbol{\sigma}_0, \boldsymbol{\sigma}_N, \mathbf{S}, \mathbf{s}|\mathbf{r}) \qquad (12) \\ & \propto \prod_x F_{O,x}(\mathbf{s}_x) \prod_{u=1}^{U} F_C^{(u)}\left(\mathbf{c}^{(u)}, \mathbf{b}^{(u)}\right) F_M^{(u)}\left(\mathbf{a}^{(u)}, \mathbf{c}^{(u)}\right) \\ & \cdot p\left(\boldsymbol{\sigma}_0^{(u)}\right) p\left(\boldsymbol{\sigma}_N^{(u)}\right) F_T^{(u)}\left(\boldsymbol{\sigma}_0^{(u)}, \boldsymbol{\sigma}_N^{(u)}, \mathbf{S}^{(u)}, \mathbf{s}^{(u)}\right), \end{aligned}$$

with

$$\begin{aligned} F_T^{(u)}\left(\boldsymbol{\sigma}_0^{(u)}, \boldsymbol{\sigma}_N^{(u)}, \mathbf{S}^{(u)}, \mathbf{s}^{(u)}\right) &= F_{T,0}^{(u)}\left(\boldsymbol{\sigma}_0^{(u)}, \mathbf{S}_0^{(u)}, \mathbf{s}_{N_\tau^{(u)}}^{(u)}\right) \qquad (13) \\ & \cdot \prod_{n=1}^{N-1} F_{T,n}^{(u)}\left(\mathbf{S}_{n-1}^{(u)}, a_{n-1}^{(u)}, \mathbf{S}_n^{(u)}, \mathbf{s}_{N_\tau^{(u)}+n}^{(u)}\right) \\ & \cdot F_{T,N}^{(u)}\left(\mathbf{S}_{N-1}^{(u)}, a_{N-1}^{(u)}, \boldsymbol{\sigma}_N^{(u)}, \mathbf{s}_{N_\tau^{(u)}+N}^{(u)}\right) \end{aligned}$$

and

$$F_{O,x}(\mathbf{s}_x) = e^{\frac{E_s}{N_0 R_s}\left[2\sum_{u=1}^{U} \Re\left\{\mathbf{r}_x^H \mathbf{s}_x^{(u)}\right\} - \sum_{k=0}^{R_s-1}\left|\sum_{u=1}^{U} s_{xR_s+k}^{(u)}\right|^2\right]}. \tag{14}$$

Again $F_C^{(u)}\left(\mathbf{c}^{(u)}, \mathbf{b}^{(u)}\right) = p\left(\mathbf{c}^{(u)} \left|\mathbf{b}^{(u)}\right.\right)$, $F_M^{(u)}\left(\mathbf{a}^{(u)}, \mathbf{c}^{(u)}\right) = p\left(\mathbf{a}^{(u)} \left|\mathbf{c}^{(u)}\right.\right)$ and $F_T^{(u)}\left(\boldsymbol{\sigma}_0^{(u)}, \boldsymbol{\sigma}_N^{(u)}, \mathbf{S}^{(u)}, \mathbf{s}^{(u)}\right)$ impose the constraints from the encoder, the mapper and the CPM state transitions. The factors $F_{T,n}^{(u)}(\cdot)$ in (13) equal one when their arguments satisfy equations (1)-(8) and (9), and zero otherwise. The notation $\mathbf{s}_x$ is used to denote the set of vectors $\mathbf{s}_x^{(u)}$, $u = 1, 2, ..., U$. This set contains the contributions from all the users from a same group to the samples of the received signal $r(t)$ taken in a given symbol interval $[xT, (x+1)T[$.

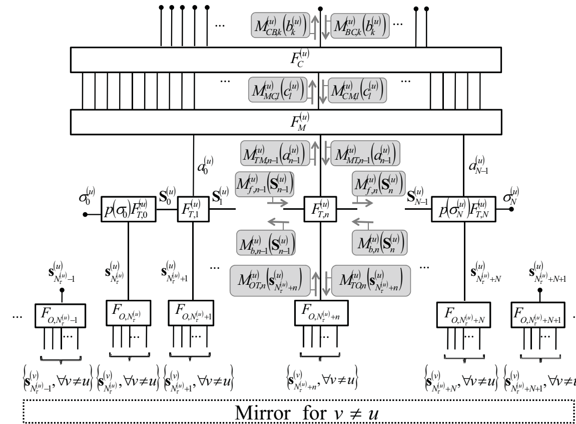

Figure 1. FG representing the factorization of $p(\mathbf{b}|\mathbf{r})$ from (12).

The FG representing the function factorization (12) is shown in Figure 1. Only the part related to user u is detailed. The upper three rows of nodes need to be repeated for every user $v \neq u$ and suitably connected to the observation nodes $F_{O,x}$, in order to obtain the complete graph. The cycles with minimum length are between the trellis constraint nodes of two interfering users; they are of length 6. This is relatively short, but twice as long as for the FG employed to derive the low-complexity MU detector from [4].

Algorithm 1 SP initialization and scheduling strategy.

Initialization. $i=0$; $\left\{M^{(u)}_{CM,l}\right\}$=$\left\{M^{(u)}_{MT,n}\right\}$=$\left\{M^{(u)}_{TO,n}\right\}$=1.

Iterative procedure. For $i=1,2,...i_{max}$:

-Multiuser processing. Compute messages $\left\{M^{(u)}_{OT,n}\right\}$.

-Per user processing. For $u=1,2,...,U$, compute

- forward/backward messages $\left\{M^{(u)}_{f,n}\right\}$ and $\left\{M^{(u)}_{b,n}\right\}$.
- upward messages $\left\{M^{(u)}_{TM,n}\right\}$, $\left\{M^{(u)}_{MC,l}\right\}$ and $\left\{M^{(u)}_{CB,k}\right\}$.
- downward messages $\left\{M^{(u)}_{CM,l}\right\}$, $\left\{M^{(u)}_{MT,n}\right\}$, $\left\{M^{(u)}_{TO,n}\right\}$.

-The messages $\left\{M^{(u)}_{CB,k}\right\}$ provide an estimate of the marginal information bit APPs. Use these estimates to compute $\left\{\hat{b}^{(u)}_k\right\}$ using (11). **If** all detected bits are error-free: end iterations.

The notation for the SP messages traveling along the FG edges is also introduced in the figure. The rules for computing the FG messages are the SP rules from [3], followed by a normalization step such that all messages communicated along the edges of the FG can be interpreted as probability mass functions. Because the graph contains cycles (paths from a node to itself), the SP algorithm is an iterative procedure that, after convergence, yields only an approximation of the APPs $p\left(b^{(u)}_k|\mathbf{r}\right)$. The initialization and message passing scheduling strategy is outlined in Algorithm 1. In most practical scenarios and without further approximations, the multiuser processing step in Algorithm 1 is prohibitively complex. This follows directly from applying the SP algorithm and it is an inevitable consequence of the large amount of variable edges entering the observation function node and the large size of the associated variable alphabet. It follows from (1)-(4), (6)-(8) and (9) that, for a given value of $\left(f^{(u)},\theta^{(u)},\tau^{(u)}\right)$ and for $x=N^{(u)}_\tau+1,N^{(u)}_\tau+2,...,N^{(u)}_\tau+N-1$, $\mathbf{s}^{(u)}_x$ is fully determined by the consecutive state transitions $\mathbf{S}^{(u)}_{x-N^{(u)}_\tau-1}$ and $\mathbf{S}^{(u)}_{x-N^{(u)}_\tau}$, with $\mathbf{S}^{(u)}_{x-N^{(u)}_\tau}$ itself fully determined by $\mathbf{S}^{(u)}_{x-N^{(u)}_\tau-1}$ and $a^{(u)}_{x-N^{(u)}_\tau}$ (see (4) and (6)-(8)). The vectors $\mathbf{s}^{(u)}_x$, $x=N^{(u)}_\tau+1,N^{(u)}_\tau+2,...,N^{(u)}_\tau+N-1$, can, therefore, take on PM^{L+1} different values. It is further easily verified that

- $\mathbf{s}^{(u)}_x\equiv\mathbf{0}_{R_s\times1}$, if $x<N^{(u)}_\tau$ or $x>N+N^{(u)}_\tau$.
- $\mathbf{s}^{(u)}_{N^{(u)}_\tau}$ ($\mathbf{s}^{(u)}_{N+N^{(u)}_\tau}$) is fully determined by the state transition $\mathbf{S}^{(u)}_0$ ($\mathbf{S}^{(u)}_{N-1}$), and, therefore, the vector $\mathbf{s}^{(u)}_{N^{(u)}_\tau}$ ($\mathbf{s}^{(u)}_{N+N^{(u)}_\tau}$) can take on only PM^L different values.

A common approach to simplify the SP algorithm is to approximate messages by a canonical distribution. We will apply this approach to the FG from Figure 1, which will lead to a novel receiver structure. Based on the discussion in [7], we propose to approximate the messages $\left\{M^{(u)}_{TO,n}\right\}$ in Figure 1 by the product of R_s univariate Gaussian distribution functions of complex-valued circularly symmetric random variables with means $\left\{\mu^{(u)}_{\left(N^{(u)}_\tau+n\right)R_s+k}\right\}$ and variances $\left\{\upsilon^{(u)}_{\left(N^{(u)}_\tau+n\right)R_s+k}\right\}$:

$$M^{(u)}_{TO,n}\left(\mathbf{s}^{(u)}_{N^{(u)}_\tau+n}\right)\approx e^{-\sum_{k=0}^{R_s-1}\frac{\left|s^{(u)}_{\left(N^{(u)}_\tau+n\right)R_s+k}-\mu^{(u)}_{\left(N^{(u)}_\tau+n\right)R_s+k}\right|^2}{\upsilon^{(u)}_{\left(N^{(u)}_\tau+n\right)R_s+k}}} \tag{15}$$

with

$$\begin{aligned}&\mu^{(u)}_{\left(N^{(u)}_\tau+n\right)R_s+k} \\ &=\sum_{\mathbf{s}^{(u)}_{N^{(u)}_\tau+n}} s^{(u)}_{\left(N^{(u)}_\tau+n\right)R_s+k}M^{(u)}_{TO,n}\left(\mathbf{s}^{(u)}_{N^{(u)}_\tau+n}\right),\end{aligned} \tag{16}$$

$k=0,1,...,R_s-1$, and

$$\upsilon^{(u)}_l=\begin{cases}1-\left|\mu^{(u)}_l\right|^2 & ,\ \tau^{(u)}R_s\le l<\left(\tau^{(u)}+N\right)R_s\\ 0 & ,\ \text{otherwise}\end{cases} \tag{17}$$

The approximation (15) significantly simplifies the multiuser processing step of Algorithm 1. Exploiting the property that the sum of independent Gaussian variables (signal samples from different users $v\neq u$) is Gaussian with mean equal to the sum of the means and variance equal to the sum of the variances, the SP messages $M^{(u)}_{OT,n}$ are easily obtained in closed-form. Applying the SP rules from [3], we find:

$$M^{(u)}_{OT,n}\left(\mathbf{s}^{(u)}_{N^{(u)}_\tau+n}\right)\propto e^{2\Re\left\{\left(\mathbf{y}^{(u)}_{N^{(u)}_\tau+n}\right)^H\mathbf{s}^{(u)}_{N^{(u)}_\tau+n}\right\}}, \tag{18}$$

where $\mathbf{y}^{(u)}_x$ is a size-R_s column vector with components $y^{(u)}_q$, for $q=xR_s,xR_s+1,...,xR_s+R_s-1$, given by:

$$y^{(u)}_q=\frac{r_q-\left(\mu_{MU,q}-\mu^{(u)}_q\right)}{\frac{N_0}{E_s}R_s+\left(\upsilon_{MU,q}-\upsilon^{(u)}_q\right)}, \tag{19}$$

where $\mu_{MU,q}=\sum_{u=1}^U\mu^{(u)}_q$ and $\upsilon_{MU,q}=\sum_{u=1}^U\upsilon^{(u)}_q$. Expression (19) indicates that a soft interference estimate $\left(\mu_{MU,q}-\mu^{(u)}_q\right)$ is subtracted from the observation r_q, and the estimation error variance $\left(\upsilon_{MU,q}-\upsilon^{(u)}_q\right)$ is added to the noise variance N_0R_s/E_s. The particular structure of (18) indicates that the proposed multiuser detector can be decomposed into an equivalent set of U iterative single user detectors, with a separate module for IUI parameter estimation (means and variances). The single user detectors are operated in parallel, with the uth detector accepting the samples $\left\{y^{(u)}_q\right\}$ from (19) as equivalent observations. At each iteration, new values for the messages $\left\{M^{(u)}_{TO,n}\right\}$ are computed at the end of the per user processing step in Algorithm 1, according to [3]:

$$\begin{aligned}&M^{(u)}_{TO,n}\left(\mathbf{s}^{(u)}_{N^{(u)}_\tau+n}\right)\\ &=M^{(u)}_{f,n-1}\left(\dot{\mathbf{S}}^{(u)}_{n-1}\right)M^{(u)}_{MT,n-1}\left(\dot{a}^{(u)}_{n-1}\right)M^{(u)}_{b,n}\left(\dot{\mathbf{S}}^{(u)}_n\right)\\ &\quad\cdot F^{(u)}_{T,n}\left(\dot{\mathbf{S}}^{(u)}_{n-1},\dot{a}^{(u)}_{n-1},\dot{\mathbf{S}}^{(u)}_n,\mathbf{s}^{(u)}_{N^{(u)}_\tau+n}\right),\end{aligned} \tag{20}$$

with $\left(\dot{\mathbf{S}}_{n-1}^{(u)}, \dot{a}_{n-1}^{(u)}, \dot{\mathbf{S}}_{n}^{(u)}\right)$ the only possible value of $\left(\mathbf{S}_{n-1}^{(u)}, a_{n-1}^{(u)}, \mathbf{S}_{n}^{(u)}\right)$ for which $F_{T,n}^{(u)}\left(\mathbf{S}_{n-1}^{(u)}, a_{n-1}^{(u)}, \mathbf{S}_{n}^{(u)}, \mathbf{s}_{N_{\tau}^{(u)}+n}^{(u)}\right)$ differs from zero. These messages are subsequently used to update the IUI parameters $\left\{\mu_q^{(u)}\right\}$ and $\left\{\upsilon_q^{(u)}\right\}$, which in turn are used to update the equivalent observation samples $\left\{y_q^{(u)}\right\}$. The messages $\left\{M_{OT,n}^{(u)}\right\}$ can then efficiently be obtained from (18), using (1)-(3), (9).

V. Proposed g6-MU-FG-GA Receiver: Complexity

The proposed g6-MU-FG-GA algorithm involves the same steps as the g3-MU-FG-GA detection algorithm from [4], but the execution of these steps it is slightly more complex. In particular, the messages $M_{f,n}^{(u)}$, $M_{b,n}^{(u)}$, $M_{TO,n}^{(u)}$ and $M_{OT,n}^{(u)}$ computed in the g6-MU-FG-GA detector take M times more values than the corresponding messages computed in the g3-MU-FG-GA detector from [4]. All these values have to be re-computed and stored at each iteration. The number of operations that is required to update the messages $M_{TM,n}$ according to the SP rules and the number of additions that is needed to evaluate the means $\mu_q^{(u)}$ from (16) is also M times larger for g6-MU-FG-GA than for g3-MU-FG-GA.

The SU-FG detection algorithm from [5], [6] has about the same computational load and memory requirement as the g3-MU-FG-GA detector. The structure of the g10-MU-FG-2IU detector from [5] differs substantially from that of the proposed detectors g3-MU-FG-GA and g6-MU-FG-GA. Per user, per iteration and per symbol period, 2 additional messages have to be evaluated and stored. Both these additional messages take as much values as the messages $M_{OT,n}^{(u)}$ in the proposed g6-MU-FG-GA detector. Besides, the number of operations involved in the computation of these additional messages is large and contributes significantly to the overall complexity of the g10-MU-FG-2IU detector.

Table I compares the memory requirements (expressed in number of real values to be stored, MEM) and the computational complexity (expressed in the number of elementary operation between two real arguments, OP) of the proposed detection algorithm to those of the g3-MU-FG-GA and the g10-MU-FG-2IU detector. Only the contributions to the required memory and the number of operations that result from parts in which the g6-MU-FG-GA, the g3-MU-FG-GA and the g10-MU-FG-2IU detector differ from each other are taken into account. These contributions provide a solid basis for comparing the considered algorithms because they dominate the detector's total memory requirements and total computational complexity, respectively. Operations that are executed only once, at the start of the iterative process, are also not taken into account. General closed form expressions are provided, as well as numerical results for the simulation set-up in Section VI. For the g10-MU-FG-2IU detector, we distinguish between memory that needs to be allocated dynamically (first term) and static memory (second term). For the g6-MU-FG-GA and the g3-MU-FG-GA detectors, the amount of static memory that is needed is negligibly small as compared to the dynamic memory resources they require, and therefore only the latter is reported.

The g10-MU-FG-2IU detector requires a considerably larger amount of memory than the proposed g6-MU-FG-GA detector, which in turn requires about M times as much memory as the g3-MU-FG-GA detector. The complexity of the g6-MU-FG-GA detector is also significantly less complex than the g10-MU-FG-2IU detector. As opposed to the complexity of the g10-MU-FG-2IU detector, the complexity of the proposed g6-MU-FG-GA detector increases less than proportional with the number of users in a group. Finally, the complexity of the g6-MU-FG-GA detector is about M times as large as that of the g3-MU-FG-GA detector.

A fair complexity comparison of the different algorithms must also take into account the number of iterations that actually needs to be performed by the receiver (in order to meet some given performance specifications). The required number of iterations for the different algorithms will be considered in Section VI.

VI. Numerical Performance Results

Computer simulations have been run to evaluate the performance of the proposed g6-MU-FG-GA detector. For comparison, the performance of the simple ad-hoc SU-FG detector (also used as a reference system in [5]), the performance of the g3-MU-FG-GA algorithm from [4] and the performance of the overly complex graph-based g10-MU-FG-2IU receiver from [5] are evaluated as well. We consider one of the MU BIC-CPM schemes with the highest asymptotic (for $U \rightarrow \infty$) spectral efficiency reported in [8]. Each user asynchronously transmits an information bit vector of size 1024. Gray mapping and pseudo-random bit interleaving are employed. The outer code is a (128,115) extended BCH code. The CPM parameters are $M = 4$, $L = 2$, $h = 1/3$ and $q(t) = \frac{t}{2LT}\left(1 - \cos\left(\frac{2\pi t}{LT}\right)\right)$, $t \in [0, LT]$ (raised-cosine pulse shaping). Equal normalized nominal frequency spacings of 0.8 are used such that $f^{(u)} = 0.8\left(u - \frac{U+1}{2}\right)$. Groups of $U = 17$ active users are considered. In each simulation new normalized time delays $\tau^{(u)}$, $u = 1, 2, ..., 17$, are taken independently from a random uniform distribution over $[0, 8.5]$. The influence of adjacent user groups is neglected. The received signal is sampled $R_s = 16$ times per symbol period. In this case, the complexity (per user, per iteration and per transmitted symbol) of the proposed g6-MU-FG-GA detector is about 4 times as large as that of the g3-MU-FG-GA detector from [4], but only about 1/8 times as large as that of the g10-MU-FG-2IU detector from [5]. At every iteration, hard decisions about the information bits are made from the corresponding a posteriori information bit probabilities, after which a genie checks for bit errors; the receiver stops iterating after a maximum number of iterations $i_{\max}$, or when all information bits have been detected correctly. Figure 2 shows the PER of the middle user ($u = 9$) for several values of $i_{\max}$, at a given value of E_b/N_0, with

	g3-MU-FG-GA	g6-MU-FG-GA	g10-MU-FG-2IU
MEM	$2NUPM^{L-1}(1+M)$	$2NUPM^{L}(1+M)$	$[2NUPM^{L}(1+3M)]$ + $[2\left(R_s PM^{(L+1)}\right)^2]$
MEM	1 197 480	4 789 920	14 253 792 + 18 874 368
OP	$15R_s + 8R_sPM^L + 13PM^L$	$15R_s + 8R_sPM^{L+1} + 13PM^{L+1}$	$13PM^{L+1} + 8P^2M^{2(L+1)}$
OP	7008	27312	297408

Table I
MEM: NUMBER OF REAL VALUES TO BE STORED, ASSUMING THE TRANSMISSION OF 1024 INFORMATION BITS. OP: NUMBER OF ELEMENTARY OPERATIONS BETWEEN TWO REAL ARGUMENTS, PER USER, PER ITERATION AND PER TRANSMITTED SYMBOL.

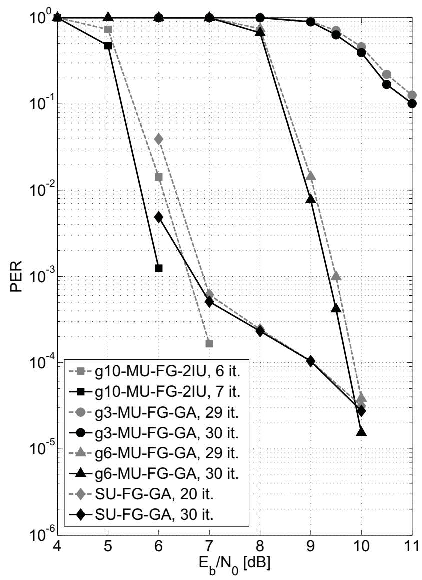

Figure 2. PER versus E_b/N_0.

$E_b = NE_s/N_b$ (with 1 packet containing 1024 information bits).

Values of $i_{\max}$ up to 30 are considered for SU-FG, g3-MU-FG-GA and g6-MU-FG-GA. The latter two algorithms are reasonably close to convergence after 29 iterations. The SU-FG detector converges faster; for E_b/N_0 above 6.5 dB, the PER of this algorithm hardly decreases between the 20th and the 30th iteration. For this particular simulation set-up, the proposed g6-MU-FG-GA performs much better than g3-MU-FG-GA from [4]. The gain in terms of PER amounts to several dB. The g6-MU-FG-GA also outperforms the ad-hoc SU-FG for PERs smaller than 3.10^{-5}. Above $PER = 3.10^{-5}$, SU-FG yields a better performance. Values of $i_{\max}$ up to 7 are considered for g10-MU-FG-2IU. This is too little to reach convergence. This overly complex algorithm results in a lower PER than g6-MU-FG-GA and SU-FG, after significantly less iterations. However, 5 or more iterations of g10-MU-FG-2IU require considerably more computation time than 20 iterations of SU-FG or 30 iterations of g6-MU-FG-GA.

VII. CONCLUSION

In this paper, we have derived a new MU detection procedure for asynchronous spectrally efficient CPM. In practice, the technique employs simple IUI cancellation, yet its derivation from the SP algorithm and a FG of the MU detection problem with girth 6 is theoretically sound. It is characterized by a relatively low computational complexity. For a set of system parameters yielding very high spectral efficiency, it is shown to outperform the existing solutions either in terms of PER performance (SU-FG from [6] and g3-MU-FG-GA from [4]) or computational complexity (g10-MU-FG-2IU from [5]). Overall, the proposed detection algorithm can be a valuable alternative to more complex algorithms for systems where the main concerns are the computational complexity and the memory requirements.

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