Stability analysis of different combinations of time-integration schemes in fluid-structure interaction simulations

L. Taelman[†]*, J. Degroote[‡], P. Segers[†], J. Vierendeels[‡]

[†]IBiTech-bioMMeda, Faculty of Engineering and Architecture, Ghent University
De Pintelaan 185, Ghent, Belgium
Liesbeth.Taelman@UGent.be, Patrick.Segers@UGent.be

[‡]Department of Flow, Heat and Combustion Mechanics,
Faculty of Engineering and Architecture, Ghent University
Sint-Pietersnieuwstraat 41, Ghent, Belgium
Joris.Degroote@UGent.be, Jan.Vierendeels@UGent.be

Keywords: Stability, fluid-structure interaction, scheme, partitioned simulation.

ABSTRACT

Coupled problems, such as partitioned fluid-structure interaction (FSI) simulations, often use different time-integration schemes to discretize the different sub-problems. This approach allows the flow equations and the structural equations to be solved with schemes that are particularly suited to solve each individual sub-problem. However, using incompatible time-integration schemes, these simulations can encounter stability problems. The temporal stability of a problem with one degree of freedom has been studied before for a one-dimensional damped spring-mass system [1] and the motion of a rigid body in a moving fluid [2]. In this paper the number of degrees of freedom is extended and a stability analysis is performed for a simplified model of blood flow in an artery.

First, an analytical study is presented in which the temporal stability of the one dimensional unsteady flow in a straight, flexible tube is studied. The governing equations are discretized in space and time and subsequently linearized. The backward Euler scheme (BE) is used for the time discretization of the flow equations. For the temporal discretization of the structure two schemes are used: (1) the compatible backward Euler scheme and (2) the operator defined by Hilber, Hughes and Taylor (HHT) in which the numerical damping is controlled by a single parameter α [3]. In the linearized equations the pressure, the fluid velocity and the radial displacement are substituted by the sum of N Fourier modes. The resulting equations are written in matrix form

$$Ax^{n+1} = Bx^n \tag{1}$$

with x containing the radial displacement of the wall, the radial velocity, the radial acceleration, the fluid velocity and the pressure. Stability requires that the absolute value of all eigenvalues λ_i of the amplification matrix $A^{-1}B$ are less than or equal to one. For a good damping of the spurious modes, the amplitude of the eigenvalues corresponding to these spurious modes must be close to zero. The influence of the numerical parameter α and some physiological parameters on the stability and the damping of the spurious modes are studied.

According to this analysis, the combination of the BE and HHT scheme is stable if the geometrical and the mechanical parameters are chosen within the physiological range. As can be seen in Figure 1, the simulation of a wave with a large wave number (small wave length) will experience a better damping of the spurious modes than the simulation of a wave with a small wave number (large wave length).

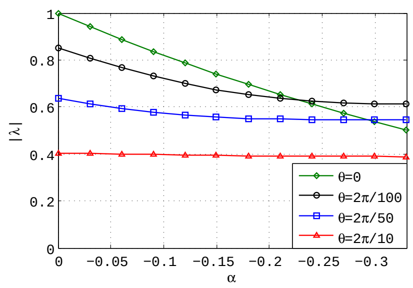

Figure 1: Maximal absolute value of the eigenvalues corresponding to the spurious modes as a function of the numerical damping factor α for different wave numbers θ.

Only for small wave numbers, the damping of the spurious modes can be improved by increasing the numerical damping. Besides the wavenumber, also the density of the fluid and the structure, the wall thickness and the tube radius affect the damping of the spurious modes considerably.

To verify the analytical results, a numerical study is performed in which the propagation of a sinusoidal flow wave in a straight tube is simulated using nonlinear two-dimensional axisymmetric FSI simulations. In Figure 2 the influence of the numerical damping factor α on the damping of the spurious modes is illustrated. For the simulation using the BE scheme, the spurious modes are damped immediately and this simulation can thus be used as a reference for the simulations using the HHT scheme. Large oscillations in the wall acceleration and small oscillations in the pressure are found if no artificial damping is used (α=0). In case of maximal artificial damping (α=-0.3) both oscillations are damped well.

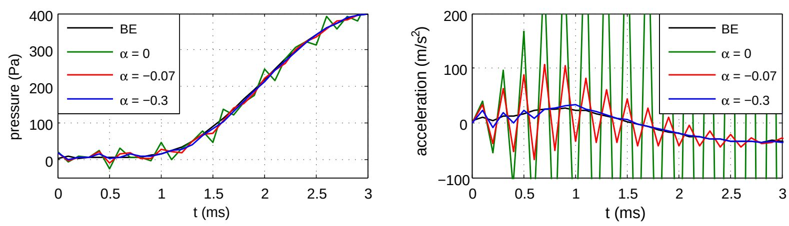

Figure 2: Comparison of the evolution of (left) the pressure halfway the tube and (right) the acceleration of the wall, as a function of the numerical damping factor α.

References

[1] M. M. Joosten, W. G. Dettmer, D. Perić: On the temporal stability and accuracy of coupled problems with reference to fluid-structure interaction. *Numerical Methods in Fluids*, 64 (2010), 1363-1378.

[2] J. Vierendeels, K. Dumont, E. Dick, P. Verdonck: Analysis and stabilization of fluid-structure interaction algorithm for rigid-body motion. *AIAA journal*, 43 (2005), 2549–2557.

[3] H. M. Hilber, T. J. R. Hughes, R. L. Taylor: Improved numerical dissipation for time integration algorithms in structural dynamics. *Earthquake Engineering & Structural Dynamics*, 5 (1977), 283–292.