Title: Advances in broadband and time-domain fast multipole research

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Abstract

The Multilevel Fast Multipole Method (MLFMA) allows to solve boundary integral equations involving several millions of unknowns. In this presentation some advanced MLFMA techniques in time and frequency domain will be discussed. First, attention will be devoted to a new broadband technique allowing to extend the classical technique to the low frequencies. Next, we will turn to the time-domain version of the MLFMA and show how the Calderón identities can used to obtain a much more efficient preconditioning. Finally, an asynchronous parallelization strategy for the MLFMA will be outlined. The obtained results will be illustrated by a collection of numerical results, both in 2D and in 3D.

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Advances in Broadband and Time-domain Fast Multipole Research

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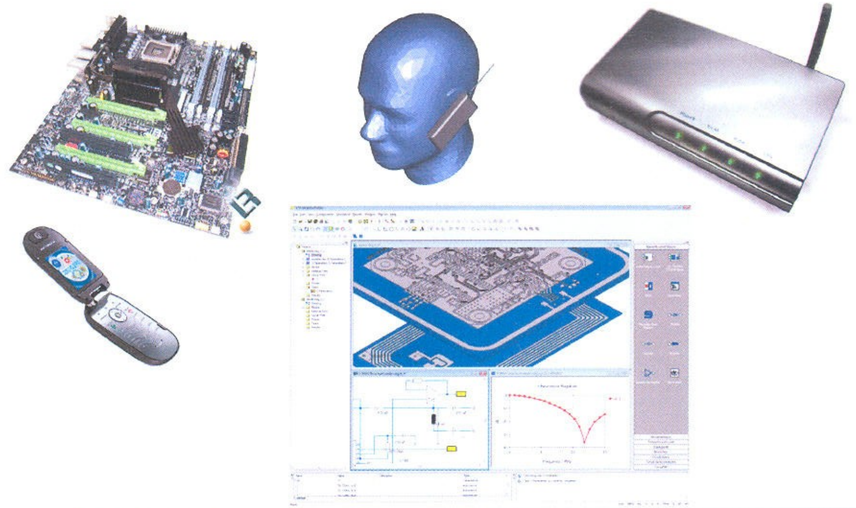
EM applications and simulations



Overview

- Introduction to the MultiLevel Fast Multipole Algorithm (MLFMA)
- Broadband MLFMA
- Parallelization of MLFMA
- Calderòn preconditioning
- Examples
- Conclusions

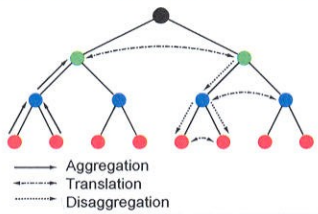
Fast methods

- **classical methods will (most) always lead to a solution**
- **however, for large problems…**
  - very large amounts of memory are needed
  - CPU time can become prohibitive

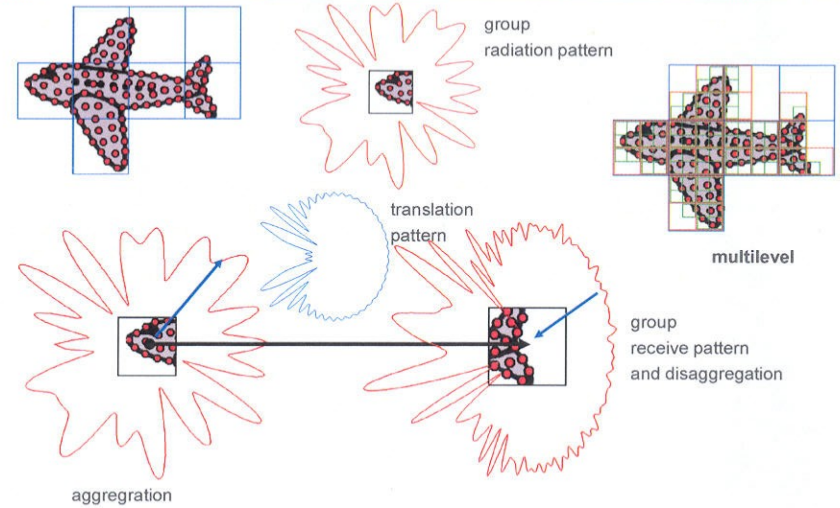
- **→ solution: use iterative methods**

Introduction to MLFMA

- **MoM integral equation techniques yield linear system of dim N**
- **Linear system is solved iteratively**
- **In each iteration a matrix-vector product is required**
  - Is equivalent with the calculation of fields due to N sources in N points
- **Matrix-vector product is accelerated with MLFMA**
  - Sources are grouped in groups (aggregation)
  - Field by groups at groups is calculated (translation)
  - From field in groups field in points is calculated (disaggregation)
  - Groups are grouped in large groups etc ...
  - Purpose: make solution time proportional to N logN



Introduction to MLFMA



Broadband MLFMA

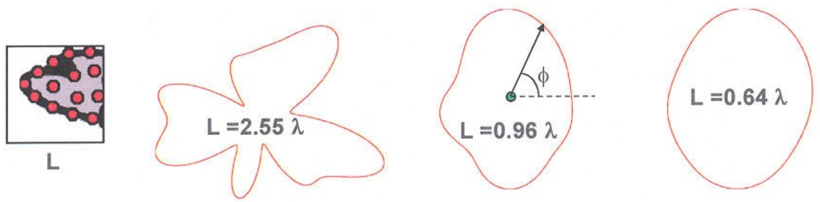
- **MLFMA breaks down at low frequencies**
  - Does not incorporate evanescent field information only propagating plane waves
- **Existing solutions**
  - Use multipole expansion at low frequencies
    - **Non-diagonal translation matrices**
    - **Difficult to combine with MLFMA**
  - Use evanescent plane wave spectrum
    - **6 different translation matrices**
- **New techniques**

  Non-directive/analytical stable plane wave MLFMA (NSPWMLFMA)
    - **Diagonal translation matrices**
    - **Easy to combine with MLFMA**
    - **Closed form translation operators**
    - **Breaks down at high frequencies**

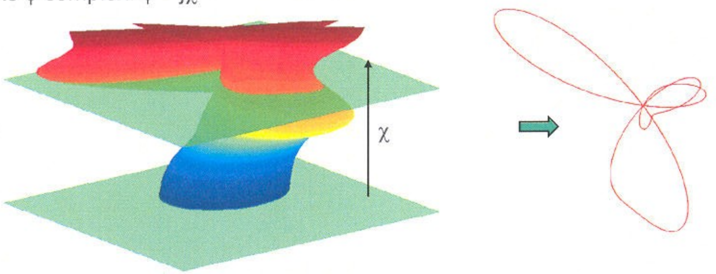
Broadband MLFMA

- **Highlights of the NSPWMLFMA**
  - It employs the MLFMA addition theorem
    - **This theorem is valid for all frequencies!**
    - **But not numerically stable at LF**
    - **NSPWMLFMA makes it numerically stable**
  - Properties
    - **Non-directional: only one radiation pattern needed**
    - **Combines seamsless with MLFMA**
    - **Yields only stable translations along z-axis**
    - **Use rotation for translation to other directions**
    - **Interpolations and anterpolations are dense and numerical**
    - **Limits the use of NSPWMLFMA to low frequencies**

complex plane radiation pattern



make ϕ complex: $\phi + j\chi$

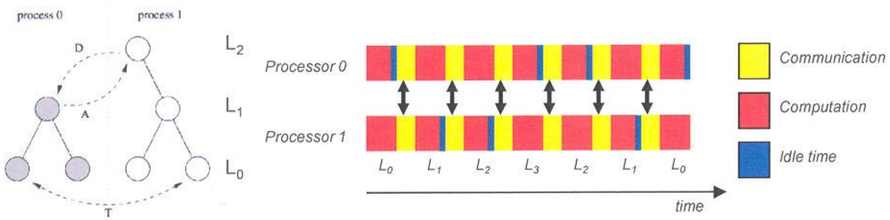


Parallelization of MLFMA

- **Previous efforts:**
  - Simulation of large single 3D objects
  - Allows good load balancing
  - Synchronous algorithms
  - Fast interconnection environments (Infiniband)
  - Problems up to 85.000.000 unknowns (10^6 cubic wavelengths)
- **Our efforts:**
  - Simulation of complex geometries consisting of multiple objects
  - Difficult to obtain good load balancing
  - Cheap computer networks
  - Slow interconnection environments (Ethernet)
  - Asynchronous algorithm
  - Problems up to 15.000.000 unknowns (10^9 square wavelengths)

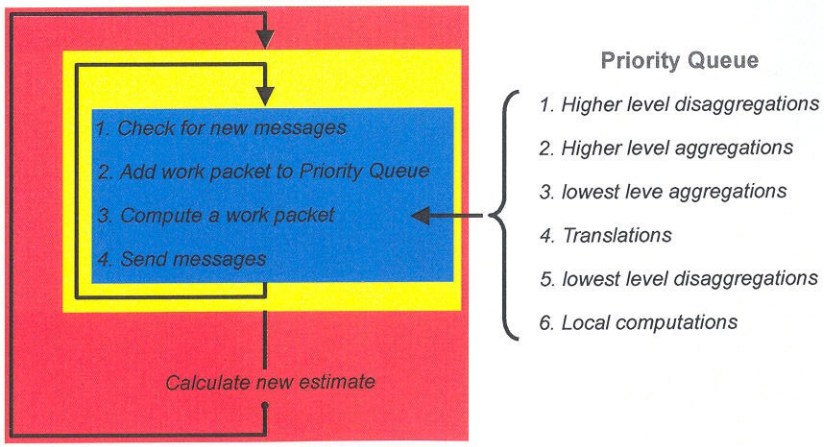
Parallelization of MLFMA

- **Properties of a synchronous algorithm:**
  - Each processor is doing the same type of calculations at the same moment
  - Alternating computation and communication phases
    - **Each processor performs calculations on level L_i**
    - **Each processor performs necessary communication for level L_{i+1}**
  - Synchronization on each level results in idle time for some processors
  - Communication comes into bursts
    - **Risk of saturation of the interconnection network (switch)**
    - **Inefficient use of communication bandwidth**

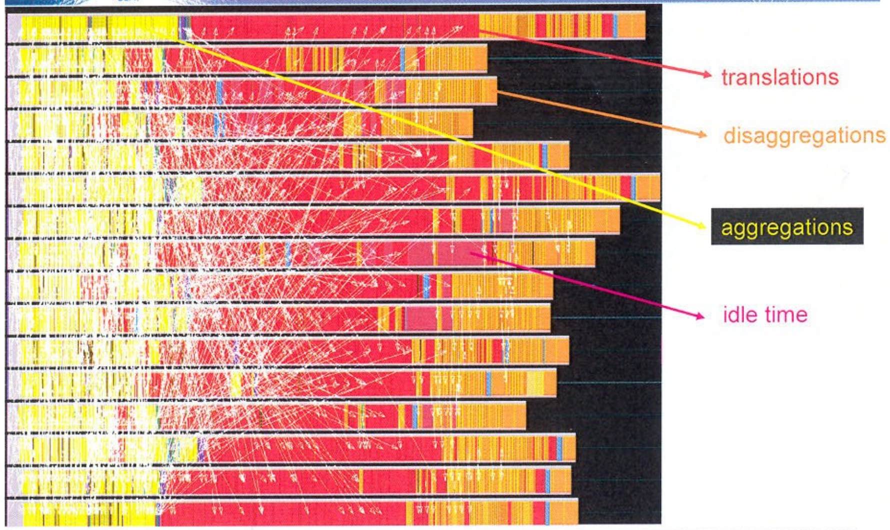


Parallelization of MLFMA

- **Asynchronous algorithm**



Parallelization of MLFMA



Preconditioning

- **Causes of large condition numbers**
  - Geometry – strong interactions between parts of geometry
    - **e.g. photonic crystals, resonators**
  - Materials
    - **e.g. negative index materials**
  - Type of integral equation
    - **e.g. EFIE at low frequencies**
- **Types of preconditioners**
  - General preconditioners:
    - **e.g. LU and block-Jacoby**
  - Dedicated preconditioners:
    - **e.g. SVD based preconditioners for photonic crystals or thin wires**
  - Spectral preconditioners:
    - **e.g. Calderòn preconditioners**

Preconditioning

- **Calderòn preconditioning of EFIE**
  - EFIE: 0 = $\mathbf{u}_n \times \mathbf{e}^i + \eta \mathcal{T}(\mathbf{j})$ with $\eta = \sqrt{\mu/\varepsilon}$ and $\mathcal{T} = \mathcal{T}_s + \mathcal{T}_h$

$$\mathcal{T}_s\left[F\right](r,t) = -\frac{1}{4\pi c}\hat{n} \times \int_\Gamma dS' \frac{\dot{F}(r',t-R/c)}{R}$$

$$\mathcal{T}_h\left[F\right](r,t) = \frac{c}{4\pi}\hat{n} \times \int_\Gamma dS' \nabla \frac{\int_0^{t-R/c} \nabla'_s \cdot F(r',t')dt'}{R}$$

  - Operator T(ω) becomes unbounded at low frequencies
  - MFIE: 0 = $\mathbf{u}_n \times \mathbf{h}^i + \{½ - \mathrm{K}\}(\mathbf{j})$

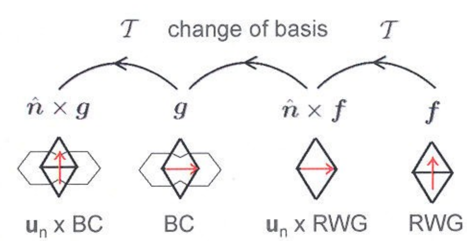
$$\mathcal{K}\left[F\right](r,t) = -\hat{n} \times \frac{1}{4\pi}\int_\Gamma dS' \nabla \times \frac{F(r',t-R/c)}{R}$$

  - Operator K remains bounded at low frequencies
  - Calderòn identity: $T^2 + K^2 = ¼$
  - Operator T^2 remains bounded at low frequencies

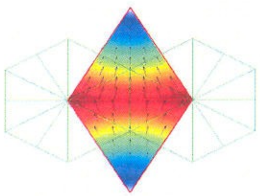
Preconditioning

- Preconditioning of EFIE: 0 = T[$\mathbf{u}_n \times \mathbf{e}_i$] + $\eta T^2(\mathbf{j})$

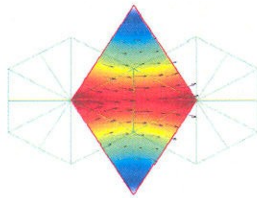
How to discretize T^2 such that spectral properties remain?



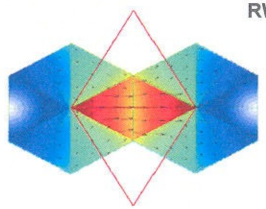
relevant basis functions (1)



RWG div-conforming



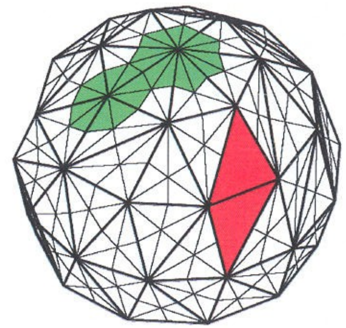
RWG curl-conforming

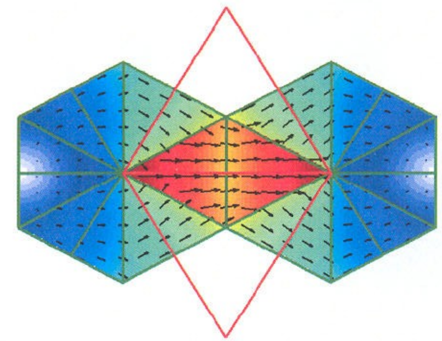


Buffa-Christiansen quasi curl-conforming

basis functions (2)

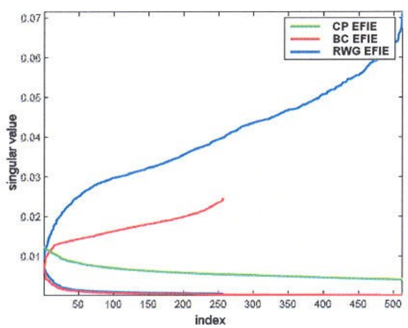
- Buffa-Christiaensen (BC) basis functions

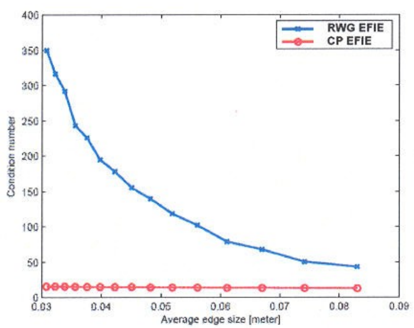




Preconditioning

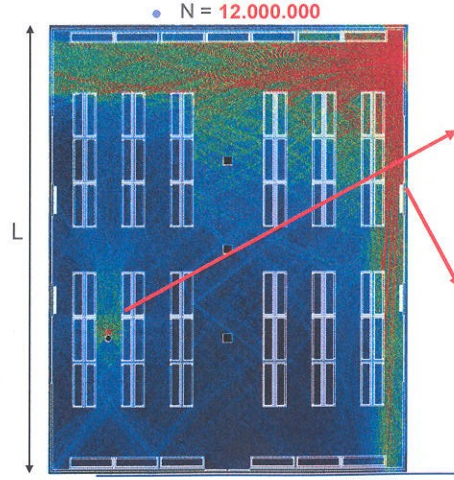
- Effect of Calderòn preconditioning (CP)

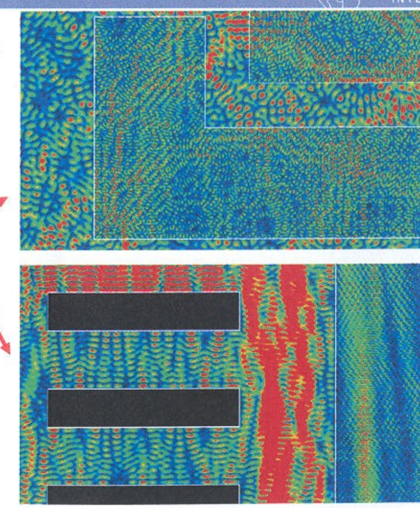




2D Library

- Library @ 60 GHz L = 4000 λ
  - N = 12.000.000

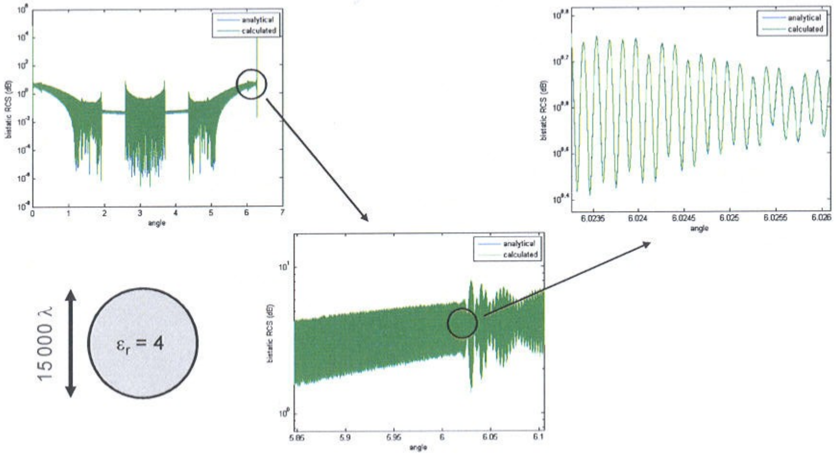




Close-up

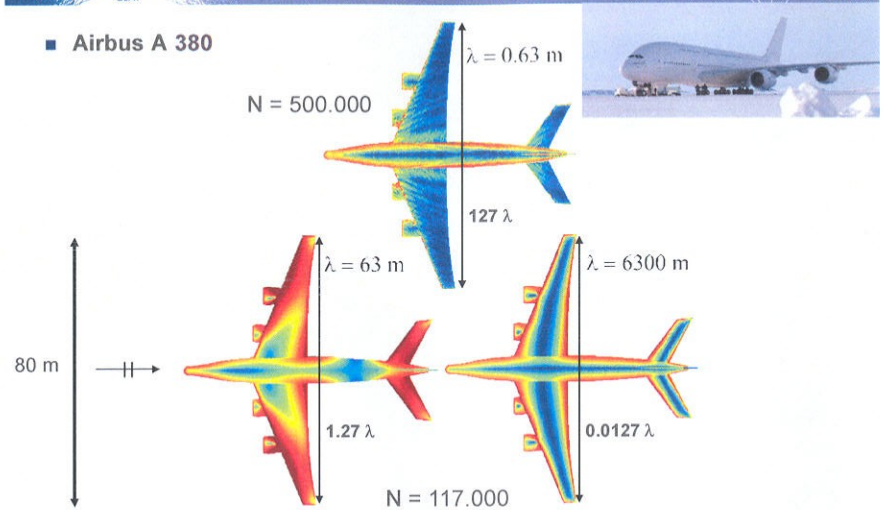
Scattering by a dielectric cylinder

- **RCS dielectric cylinder of 15.000 wavelengths in diameter**



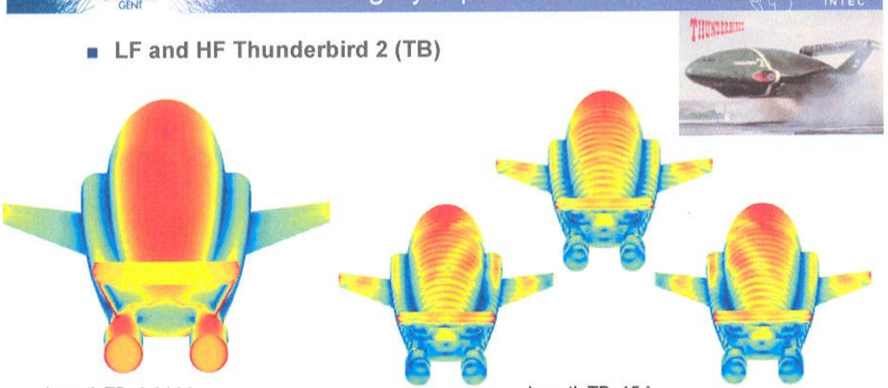
Scattering by a plane

- **Airbus A 380**



Scattering by a plane

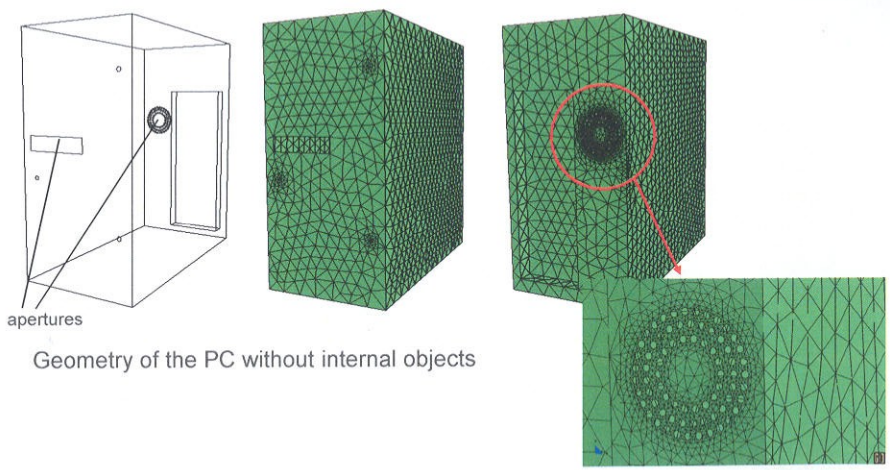
- **LF and HF Thunderbird 2 (TB)**



Length TB: 0.014 λ
N = 101.466
21 iterations
Accuracy: 10^{-3}
20s per iteration
12 AMD Opteron 270 processors

Length TB: 15 λ
N = 1.025.559, 1.2GByte
28 iterations
Accuracy: 10^{-3}
28s per iteration
20 AMD Opteron 270 processors

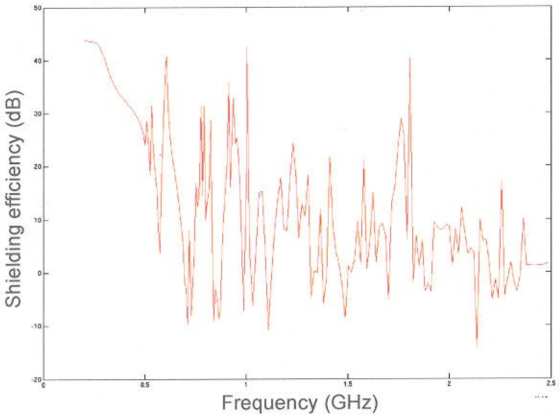
Personal computer

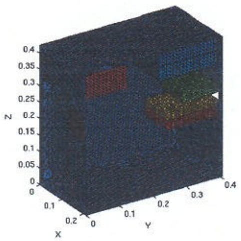


Geometry of the PC without internal objects

Personal computer

- **Shielding efficiency**



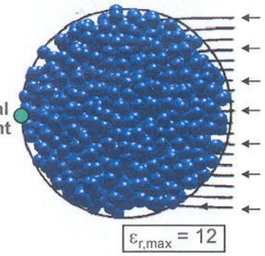


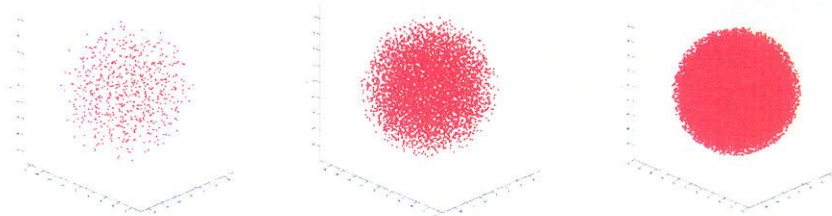
@ 250MHz

- 3.2 GByte (with symmetry extr.)
- 16 processors
- 526 s setup time (with st)
- 251 iterations
- 0.5 s per iteration

artificial medium

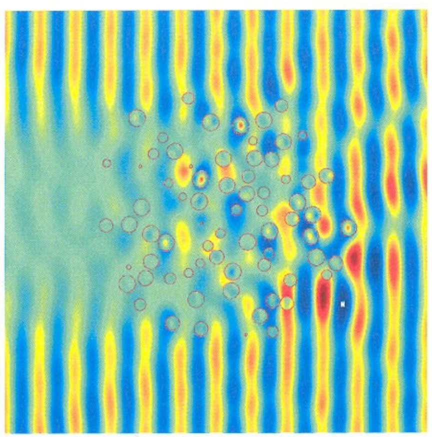
- **Lüneburg lens (radius = 8λ =80 cm)**
  - inhomogenious refractive index
    $\varepsilon_r = (2 - R/R_{sphere})\ \varepsilon_{r,max}$
    modelled by identical spheres ($\varepsilon_r = 12$)
    but denser near the centre
- **3 test geometries**
  - R_{ss} = 2.4cm (669 small spheres)
  - R_{ss} = 1.2cm (5362 small spheres)
  - R_{ss} = 0.6cm (42899 small spheres)





Lüneburg lens

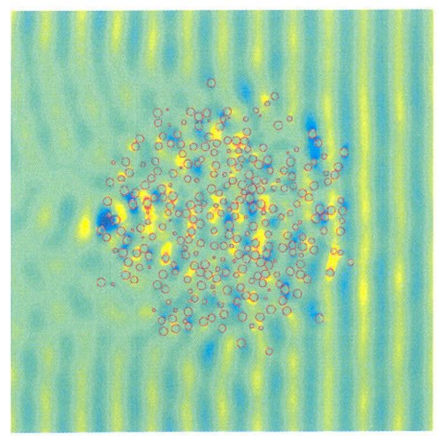
- **669 spheres, 2 007 currents**



note:
+ the small spheres are indentical but their centre is not necessarily in the plane of the cross-section
+ each small sphere is modelled by a single equivalent electric current i.e. 3 scalar unknowns
+ all the interactions between the spheres are taken into account

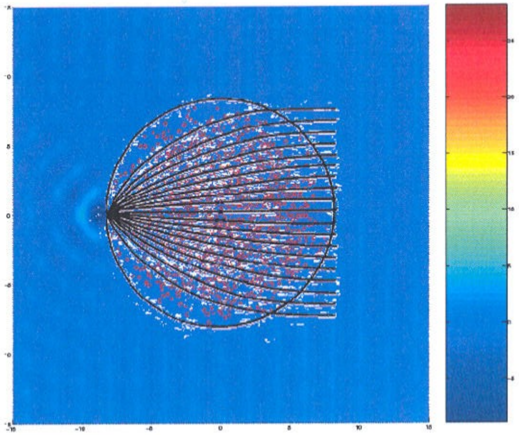
Lüneburg lens

- **5362 spheres, 16 086 currents**



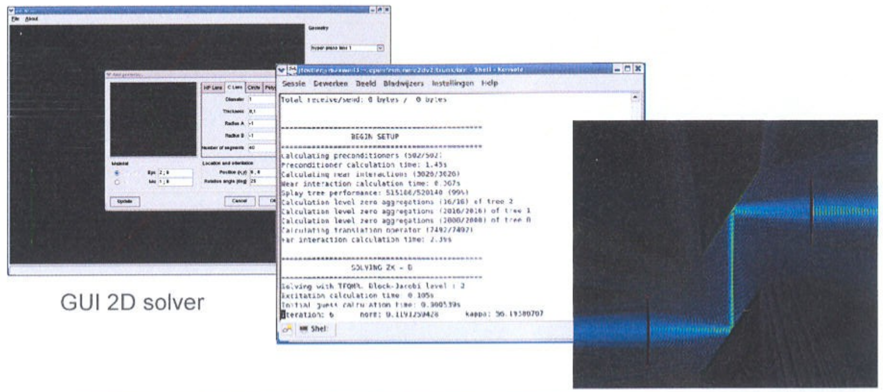
Lüneburg lens

- 42899 spheres, 128 697 currents



Try it yourself: Open FMM

- **Software is available for free (GPL license)**
  - **http://openfmm.intec.ugent.be**
- **Consists of: 2D EM solver (Nero) en 3D EM solver (Cassandra)**



GUI 2D solver

Conclusions

- **Future:**
  - a truly broadband MLFMA
  - a scalable asynchronous parallel MLFMA
  - good more widely applicable preconditioners
  - complex interconnect problems in layered media
  - Can we handle 10.000.000.000 unknowns? soon?
  - ....

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