Ewald-Transform Accelerated PML-Based Series Expansions for the 1-D Periodic 3-D Green's Functions of Multilayered Media

Hendrik Rogier (FWO-V)*
Dept. of Information Technology, Ghent University, St.-Pietersnieuwstraat 41, Gent, Belgium.
E-mail: hendrik.rogier@intec.UGent.be

Introduction

Periodic structures are of great practical use in many applications in antenna systems, microwave electronics and optics. Efficient modelling techniques rely on the Floquet-Bloch theorem to limit the simulation domain to a single unit cell. Up to now, little has been published about the 1D periodic 3D Green's function, especially when also considering the presence of a stratified dielectric background medium. In [4] the 1D periodic 3D Green's functions for a microstrip substrate are derived in the spectral domain first, and the corresponding spatial-domain quantities are obtained through an efficient sum of inverse Fourier transforms.

We propose a Perfectly Matched Layer (PML) based formalism to derive a fast converging series expansion for the 1D periodic 3D Green's function of layered media. The PMLs are used to transform the open layered medium into a closed waveguide configuration. This results in an efficient expansion for the 3D Green's function of a point source in the stratified background medium in terms of a set of discrete modes of the closed waveguide containing the PML, while the PMLs mimic the open character. As both the spectral and spatial domain series suffer from slow convergence, the Ewald transform is applied to accelerate the PML-based series.

Theory

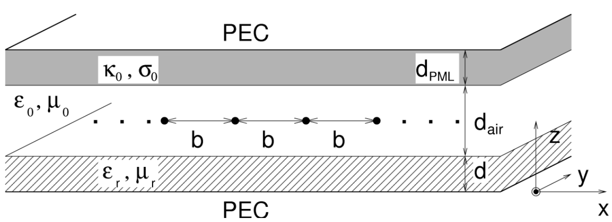

Figure 1: 1D periodic set of point sources on a PML-terminated microstrip substrate.

Consider a planar multilayered dielectric background medium in which we place a 1-D grid of point sources (Fig. 1), resulting in a periodic problem in the *x*-direction with the period given by *b*; two adjacent point source excitations differ by at most a phase factor $e^{-jk_x b}$. For a faster evaluation of the 1D periodic 3D Green's functions, we construct a parallel plate waveguide by terminating the free space with two perfect electrically conducting plates backed by a Perfectly Matched Layer (PML) with thickness d_{PML} and with material parameters κ_0 and σ_0 [1].This results in a series expansion for the 3D Green's function for

978-1-4244-2042-1/08/$25.00 ©2008 IEEE

a 1D periodic grid of point sources:

$$G_{A,V}^{\text{per,1D}}(x,y,z;x',y',z') = \frac{1}{4j}\sum_{n=1}^{+\infty} A_n(\beta_n,z|z') \sum_{m=-\infty}^{+\infty} e^{-jk_x mb} H_0^{(2)}\left(\beta_n\sqrt{(x-x'-mb)^2+(y-y')^2}\right), \tag{1}$$

with β_n and the expansion coefficients $A_n(\beta_n,z|z')$ given in [2]. Applying of the Poisson transform yields following equivalent series expansion

$$G_{A,V}^{\text{per,1D}}(x,y,z;x',y',z') = \frac{1}{2jb}\sum_{n=1}^{+\infty} A_n(\beta_n,z|z') \sum_{m=-\infty}^{+\infty} \frac{1}{\beta_{n,m}} e^{-j\left(\frac{2m\pi}{b}+k_x\right)(x-x')} e^{-j\beta_{n,m}|y-y'|}, \tag{2}$$

with $\beta_{n,m} = \sqrt{\beta_n^2 - \left(\frac{2m\pi}{b}+k_x\right)^2}$.

For a successful acceleration of convergence by means of the Shanks algorithm, (1) or (2) must asymptotically behave as a geometric series. In general, this is not the case for (1) as a function of m, when considering small values of the PML-based mode order n and hence, the Shanks algorithm is not very effective in accelerating the slow convergence. Therefore, an Ewald transform is applied for mode orders $n < N$. To this end, we rewrite the periodic Green's functions as a sum of the modified spectral domain series (2)

$$G_{A,V,1}^{\text{per,1D}}(x,y,z;x',y',z') = \frac{1}{2jb}\sum_{n=1}^{N} A_n(\beta_n,z|z') \sum_{m=-\infty}^{+\infty} \frac{1}{\beta_{n,m}} e^{-j\left(\frac{2m\pi}{b}+k_x\right)(x-x')} \left[e^{+j\beta_{n,m}|y-y'|}\operatorname{erfc}\left(\frac{j\beta_{n,m}}{2\mathcal{E}}+|y-y'|\mathcal{E}\right) + e^{-j\beta_{n,m}|y-y'|}\operatorname{erfc}\left(\frac{j\beta_{n,m}}{2\mathcal{E}}-|y-y'|\mathcal{E}\right)\right], \tag{3}$$

and the modified spatial domain series (1)

$$G_{A,V,2}^{\text{per,1D}}(x,y,z;x',y',z') = \frac{1}{4j}\sum_{q=0}^{+\infty} \frac{\sum_{n=1}^{N} \beta_n^{2q} A_n(\beta_n,z|z')}{q!4^q\mathcal{E}^{2q}} \sum_{m=-\infty}^{+\infty} e^{-jk_x mb} E_{q+1}\left[\mathcal{E}^2\left((x-x'-mb)^2+(y-y')^2\right)\right], \tag{4}$$

in which $E_{q+1}[z] = \int_1^{+\infty} \frac{e^{-zt}}{t^q}\,dt$ is the qth order exponential integral. A suitable choice for the Ewald splitting parameter $\mathcal{E}$ has to be made. Based on the theory developed in [3] for the periodic 2D Green's function series in free space, a suitable choice is $\mathcal{E} = \max\left\{\frac{\sqrt{\pi}}{b}, \frac{\max[\beta_{n,0}]_{n=1}^{N}}{2H}, \frac{\max[\beta_n]_{n=1}^{N}}{2(\varepsilon Q!)^{\frac{1}{2Q}}}\right\}$, where H^2 is the maximum exponent permitted in the spectral series (3), ε the desired error and Q the number of q-terms necessary to achieve convergence in the spatial series (4). Typical choices are $H^2 = 9$, $\varepsilon = 10^{-7}$, and $Q = 13$ [3]. In practice, we choose $N = 4$ for $G_A^{\text{per,1D}}(x,y,z;x',y',z')$ and $N = 8$ for $G_V^{\text{per,1D}}(x,y,z;x',y',z')$. The two series are to be complemented with either the remaining spectral domain series, i.e. the part of series (4) starting from $n = N+1$ instead of $N = 1$, or the remaining spatial domain series, i.e. the part of series (3) starting from $n = N+1$ instead of $N = 1$.

Examples

Consider a microstrip substrate with thickness $d = 9$ mm, $\varepsilon_r = 3$ and $\mu_r = 1$. To obtain an expansion into PML-based modes, a closed waveguide is formed by adding a perfect electrically conducting plate above the substrate, such that $d_{\mathrm{air}} = 5$ mm, $d_{\mathrm{PML}} = 3.5$ mm. A strongly absorbing PML is obtained for $\kappa_0 = 15$ and $\frac{\sigma_0}{\omega\varepsilon_0} = 10$. The free-space wavelength at the operating frequency is chosen to be $\lambda_0 = 2$ cm. We determine the Green's function $G_V^{\mathrm{per,1D}}(x,y,z;x',y',z')$ for a 1D periodic set of point sources with spacing $b = 1.5$ cm (Fig. 1). Fig. 2 presents the Ewald-transform accelerated series expansion for $\left|G_V^{\mathrm{per,1D}}(0,y,9\text{ mm};0,y',9\text{ mm})\right|$ with $k_x = 0$, as a function of $k_0|y-y'|$, which is in excellent agreement with the classic Sommerfeld integrated spectral series, accelerated following [4]. On a Pentium T7400 Centrino Duo 2.16 GHz machine with 2GB RAM, the evaluation of 200 points based on the Ewald-transform accelerated series expansion takes 3 s, whereas the accelerated classic Sommerfeld integrated spectral series requires 1 min 38 s of CPU time. Fig. 3 shows the Green's function series $\left|G_V^{\mathrm{per,1D}}(0,y,7\text{ mm};0,y',7\text{ mm})\right|$ inside the dielectric substrate, calculated as a function of $k_0|y-y'|$ for $k_x = 0$. Again, an excellent agreement is found between the Ewald-transform accelerated series expansion and the accelerated [4] classic Sommerfeld integrated spectral series, together with a significant speedup (3 s for the new approach versus 5 m 47 s). Finally, Fig. 4 proposes the Green's function series $\left|G_V^{\mathrm{per,1D}}(0,y,9\text{ mm};0,y',7\text{ mm})\right|$ for the excitation at the substrate-air interface and the observation point inside the dielectric substrate.

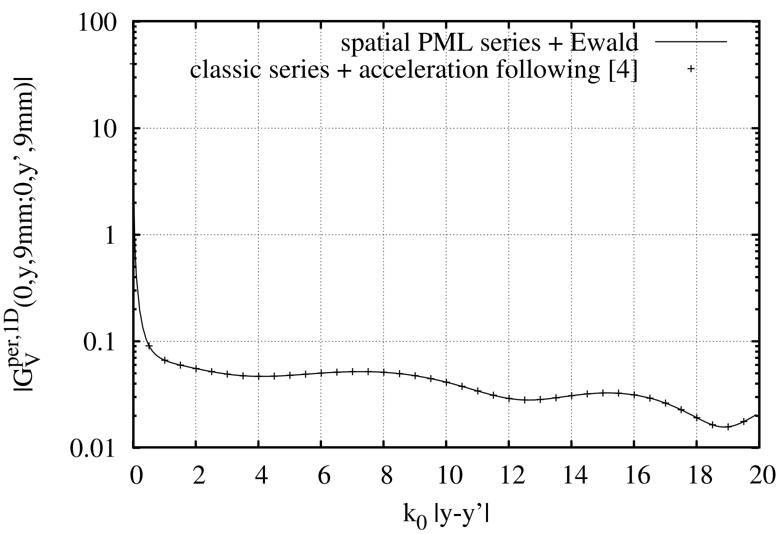

Figure 2: 1D periodic 3D Green's function $\left|G_V^{\mathrm{per,1D}}(x,0,9\text{ mm};x',0,9\text{ mm})\right|$.

References

[1] Henk Derudder, Frank Olyslager, and Daniël De Zutter, "An efficient series expansion for the 2D Green's function of a microstrip substrate using perfectly matched layers", *IEEE Microwave Guided Wave Lett.*, vol. 9, no. 12, pp. 505–507, Dec. 1999.

[2] H. Rogier, "New series expansions for the 3D green's function of multi-layered media with 1D periodicity based on Perfectly Matched Layers", *IEEE Trans. Microwave Theory Tech.*, vol. 55, no. 8, pp. 1730–1738, Aug. 2007.

[3] F. Capolino, D.R. Wilton, and W.A. Johnson, "Efficient computation of the 2-D green's function for 1-D periodic structures using the Ewald method", *IEEE Trans. Antennas Propag.*, vol. 53, no. 9, pp. 2977–2984, Sep. 2005.

[4] P. Baccarelli, C. Di Nallo, Simone Paulotto, and D. R. Jackson, "A full-wave numerical approach for modal analysis of 1-D periodic microstrip structures", *IEEE Trans. Microwave Theory Tech.*, vol. 54, no. 4, pp. 1350–1362, Apr. 2006.

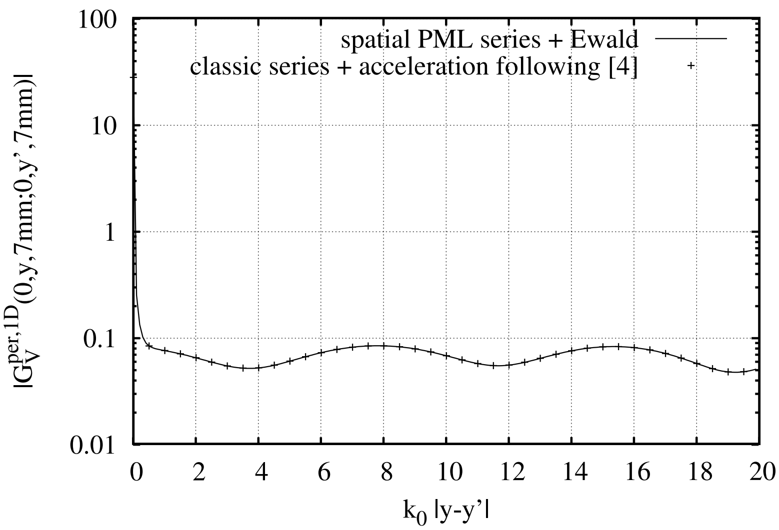

Figure 3: 1D periodic 3D Green's function $\left|G_V^{\mathrm{per,1D}}(0,y,7\text{ mm};0,y',7\text{ mm})\right|$.

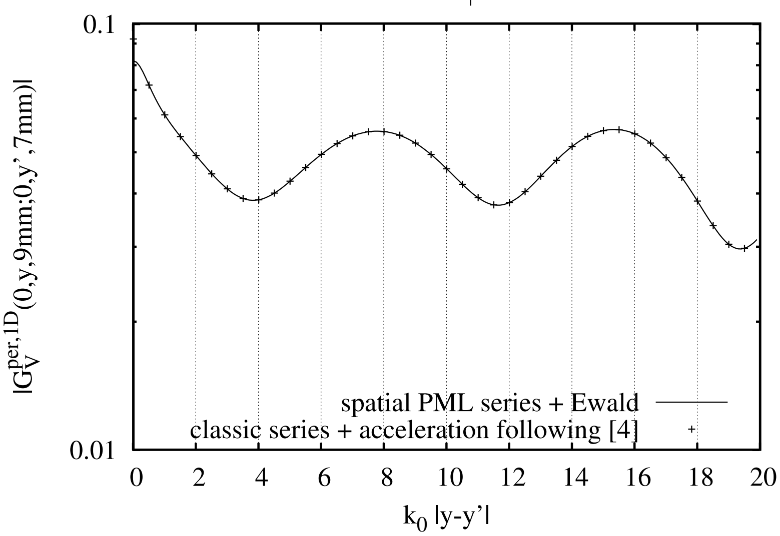

Figure 4: 1D periodic 3D Green's function $\left|G_V^{\mathrm{per,1D}}(0,y,9\text{ mm};0,y',7\text{ mm})\right|$.

2008 IEEE AP-S International Symposium
and
USNC/URSI National Radio Science Meeting

Preliminary Version

July 5-12, 2008
San Diego, CA

Support: If you have problems or questions related to the installation of this disc, please contact the 3WAIsmen at FAX: (818) 952-0183 or e-mail: wais3men@yahoo.com