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A numerical scheme for Maxwell's equations in the quasi-static regime with a non-local source

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Abstract

Let Ω be a bounded domain in $\mathbb{R}^3$ with a Lipschitz continuous boundary $\partial\Omega$ and outward unit normal vector $\mathbf{n}$. We consider the following quasi-static Maxwell's system with a non-local field-dependent source term,

$$\begin{cases} \partial_t \mathbf{H} + \nabla \times \nabla \times \mathbf{H} &= \mathbf{J}(\mathbf{H}), & \text{in } (0,T) \times \Omega, \\ \mathbf{n} \times \mathbf{H} &= \mathbf{0}, & \text{in } (0,T) \times \partial\Omega, \\ \mathbf{H}(0) &= \mathbf{H}_0, & \text{in } \Omega, \end{cases} \tag{1}$$

where $\mathbf{J}(\mathbf{H}) = \mathbf{J}_0 \sqrt{\int_{\Omega_0} |\mathbf{H}|^2 \mathrm{d}x}$, with $\overline{\Omega_0} \subset \Omega$ and $\mathbf{J}_0$ a known source. The problem (1) can be regarded as a self-regulating system, wherein the source is automatically adjusted to the present field.

Uniqueness of a solution to (1) is verified. Using variational formulation in $H(\mathrm{curl}, \Omega)$, we construct a recurrent approximation scheme, based on the backward Euler discretization in time. We prove suitable a priori estimates for the discretized fields. The discretized fields are then combined to a semi-discrete approximation of the solution of (1) using Rothe's method [1]. Based on the generalized div-curl lemma for time-dependent problems [2], we prove convergence of the semi-discrete field to the solution of (1). Finally, we study the error estimates for the approximation scheme.

In the next step we construct a fully discretized model, based on the $H(\mathrm{curl}, \Omega)$-conforming Whitney elements and provide error estimates for this approximation. Finally, we perform some computational experiments of our model in the free software package ALBERTA [3].

References

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