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COMPARISON OF NUMERICAL PREDICTION TECHNIQUES FOR SOUND PROPAGATION IN COMPLEX OUTDOOR ENVIRONMENTS

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Abstract

In realistic outdoor environments, deterministic, full-wave numerical techniques are needed to capture multiple reflections and diffractions near complex objects like noise barriers and buildings. Since propagation distances are large, the computational cost of such techniques often exceeds the available computer resources. In this study, a number of deterministic prediction techniques are evaluated for the analysis of traffic-induced noise propagation in such complex environments. The goal is to identify the particular strengths and weaknesses of the various methods, encouraging cross-fertilisation and ultimately yielding the development of hybrid solution schemes which combine the strengths of the different methods. A specific point of attention is the treatment of non-reflective boundaries in simulating an unbounded atmosphere, which is often a big challenge in full-wave prediction techniques. The numerical simulations in this paper are limited to a homogeneous and still atmosphere.

1. INTRODUCTION

Outdoor sound propagation applications are usually characterized by rather large propagation distances (up to 1 km) and frequencies of interest in the mid-frequency range. The presence of multiple diffractions and reflections, together with atmospheric influences like refraction of sound by gradients in wind speed and temperature further complicate sound propagation calculations in realistic outdoor environments. Due to the increase in computing power, full-wave techniques are finding their way to this kind of applications as well. In recent years, accurate and mature techniques have been developed with their own typical strengths and weaknesses. By coupling the positive aspects of different techniques in hybrid techniques, more powerful methods are developed. The coupling between Finite-Difference Time-Domain (FDTD) method and the Parabolic Equation (PE) method was shown to be successful. The FDTD-PE method

was applied to sound propagation in an urban area [1], to sound propagation over noise barriers [2] and to investigate optimal tree canopy shape to improve downwind sound propagation over highway noise barriers [3]. These applications have in common that there is a complicated source region, where FDTD is applied, and a less complex receiver region, where the much faster PE is used. The PE method in its base form, however, cannot handle reflections. When the receiver region is complex as well, this method cannot be applied. An example of practical interest is a road with barriers, followed by grassland, and a receiver located in a street canyon.

A FDTD-PE-FDTD model would be inefficient. In its final step, such a model would need the reconstruction of a time signal, which needs the calculation of a very large number of frequencies with the frequency-domain technique. This is much less efficient than the forward coupling between FDTD and PE: a single simulation with a pulse-like source in time-domain provides a broad frequency response. Furthermore, a limited amount of frequencies is usually sufficient to calculate e.g. 1/3 octave band sound pressure levels at the receiver.

Therefore, it is more efficient to remain in the frequency-domain in the final step of such a "complex-easy-complex" configuration. For this frequency-domain technique, the Wave Based Method (WBM) [4] is a good candidate. The WBM is a novel deterministic prediction technique for steady-state acoustic problems, based on an indirect Trefftz approach. The use of wave functions satisfying the Helmholtz equation as basis functions enables efficient calculation of large acoustic domains. The WBM has been applied successfully for many steady-state structural dynamic problems [5], interior acoustic problems [6], interior vibro-acoustic problems [7] and exterior vibro-acoustic problems [8]. It is shown that, due to the small model size and the enhanced convergence characteristics, the WBM has a superior numerical performance as compared to the conventional element based methods. As a result, larger problems at higher frequencies may be tackled, making the WBM very suitable for modelling of a relatively complex receiver zone.

In this paper, as an initial step to develop a FDTD-PE-WBM coupled method, a thorough comparison between the accuracy of FDTD and the WBM method is considered.

2. NUMERICAL MODELLING TECHNIQUES FOR OUTDOOR SOUND PROPAGATION

In this section, various mathematical models and numerical modelling techniques for the study of sound propagation problems are detailed. First, the underlying assumptions of the applied partial differential equations for the description of acoustic wave propagation are discussed. Next, two deterministic full-wave techniques for solving these equations are described: the Finite-Difference Time Domain approach and the Wave Based Method, which is a frequency domain method.

2.1. Governing equations for sound propagation problems

The dynamic behaviour of a general fluid is described by the Navier-Stokes equations. The use of these equations as a mathematical model for sound waves is however cumbersome since the acoustic pressure amplitudes are orders of magnitude smaller than those which are involved with macroscopic aerodynamic phenomena. The Linearized Euler Equations (LEE), see equation (1), involving a (non-)uniform mean flow, are a more suited set of model equations. They are obtained by starting from the Navier-Stokes equations and assuming negligible viscosity and

small amplitudes.

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{A}_r \mathbf{q}}{\partial \mathbf{x}_r} + \mathbf{C} \mathbf{q} = \mathbf{s} \quad (1)$$

The vector \mathbf{q} contains the acoustics quantities: the density ρ , the velocities u_i and the acoustic pressure p . The mean flow is represented by the mean density ρ_0 , the mean velocities u_{i0} and the mean pressure p_0 . The matrices \mathbf{A}_r are due to the fluxes, the matrix \mathbf{C} results from the non-uniform mean flow. The vector \mathbf{s} includes the acoustic sources in the problem domain.

It is usually assumed that acoustic responses in a fluid occur under adiabatic conditions. By expressing the resulting pressure-density relations as a Taylor-series expansion and retaining only the first-order terms, this equation simplifies to the linear acoustic wave equation, which governs the adiabatic propagation of longitudinal waves in a homogeneous, inviscid fluid:

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{\partial q}{\partial t} \quad (2)$$

where ∇^2 is the Laplace operator and $c = \sqrt{\frac{\gamma p_0}{\rho_0}}$ denotes the phase speed of a longitudinal wave and γ represents the specific heat ratio.

Most of the acoustic design studies may be confined to the analysis of the steady-state acoustic response to a time-harmonic excitation at a certain circular frequency $\omega = 2\pi f$. In this case, the steady-state acoustic response also has a harmonic time dependence at the same frequency, i.e. $p(\mathbf{r}, t) = \hat{p}(\mathbf{r})e^{j\omega t}$, and the acoustic wave equation (2) transforms into the linear Helmholtz equation:

$$\nabla^2 p + k^2 p = -j\rho_0 \omega q \quad (3)$$

with $k = \frac{\omega}{c}$ the acoustic wavenumber at frequency ω .

2.2. Time-domain model: FDTD

The finite-difference time-domain technique, solving the LEE (eq. 1), has become a reference solution in non-trivial outdoor sound propagation calculations [9, 10, 11]. An important advantage is the possibility to incorporate the effects of a moving and inhomogeneous atmosphere in detail. The accuracy of the FDTD method has been tested against frequency-domain models [12], analytical solutions [11] and wind tunnel experiments at scale [10]. The large need for computational resources limits applications to (mainly) 2D sound propagation problems.

A popular implementation uses both the particle velocities and acoustic pressures in a staggered grid, where the fields are updated in a leap-frog way (staggered-in-time). Such a scheme has a very limited stencil, is second-order accurate, and allows an easy implementation of boundaries [13]. An important advantage is the possibility to use in-place-computation: the new values replace the old ones in computer memory. In a moving atmosphere, these unique properties are lost. The PSIT scheme, as described in [14], was developed to be a compromise between the need for computational resources, numerical stability and accuracy.

Good Perfectly Absorbing Boundary conditions are essential in the application envisaged. The Perfectly Matched Layer (PML) approach [15] has shown to be very accurate and efficient. Real-valued impedances are easily implemented using the approach as described in [13]. This approach is chosen here to keep the comparison between the chosen numerical models simple,

although frequency-dependent impedance planes, as well as non-locally reacting materials, can also be modelled.

2.3. Frequency-domain model: WBM

Frequency-domain techniques are the most commonly used method for analysis of interior acoustic problems. These techniques are also very suitable for treatment of radiation and scattering problems, if the nature of the problem allows it to be modelled as a still and homogeneous acoustic medium. This is often the case if the considered problem is relatively small, or if no significant flow is present.

Within this group of methods, the deterministic element-based finite element (FE) and boundary element (BE) method are frequently applied for the study of acoustic problems [6]. However, the simple polynomial shape functions used require the element size to become increasingly small if higher frequencies are to be tackled. The associated large models and high computational demands limit the practical applicability of these methods to lower frequencies.

The Wave Based Method (WBM) is developed as an alternative to the deterministic element-based prediction techniques, and is based on a Trefftz approach. The use of wave functions satisfying the Helmholtz equation as basis functions removes the need to discretize the problem domain. Only a partitioning in a small number of large subdomains is required. This yields relatively smaller models, resulting in a numerically much more efficient technique as compared to the element based methods [16]. For a detailed description of the functions used and the construction of the system matrices, the reader is referred to DESMET [4] and PLUYMERS [17].

For the problem at hand, it is essential to have an accurate representation of the semi-unbounded problem domain. In the WBM, unbounded problems are divided into a bounded and an unbounded part through the introduction of an artificial truncation curve. The unbounded subdomain is modelled using a special set of basis functions satisfying both the Helmholtz equation and the Sommerfeld radiation condition. In the case of a semi-infinite problem domain, as considered here, the applied set of basis functions is chosen to satisfy also the velocity boundary condition on the baffle plane (the ground). These functions are described in more detail in BERGEN [18].

3. APPLICATION CASE

3.1. Problem description

As a first step towards the coupling of the FDTD and WBM, both techniques are applied on the same problem. This allows to compare and validate the results obtained by the different approaches. The considered problem examines the sound propagation from a road to an urban area. The problem geometry is shown in figure 1. The acoustic fluid is air ($c = 340\text{m/s}$, $\rho_0 = 1.2\text{kg/m}^3$) at rest. The source region consists of a road flanked on both sides by a barrier with a 1m wide top. The receiver zone is an urban canyon constituted of two buildings. Two cases are considered: a first case assumes all surfaces rigid. While not particularly realistic, this is a good starting point for comparison of the two different techniques. In a second case, the sound barriers, grassland plane and buildings are given a more realistic surface impedance value.

The response points are located on two lines at a height of 2 and 4 meters above the

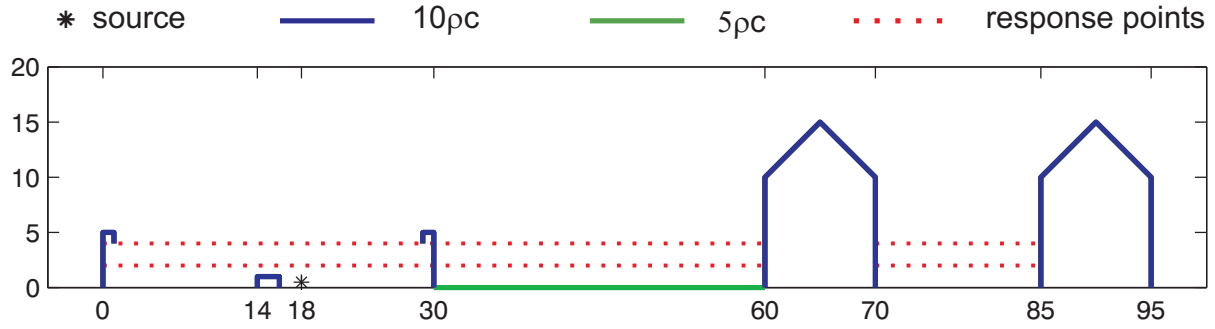


Figure 1. Problem layout. Case 1: all surfaces rigid, Case 2: impedance as indicated. All distances in meters

ground. All point results are expressed as relative sound pressure levels:

$$P_{rel} = 20 \log_{10}(|p|/|p_{free}|), \quad (4)$$

where p is the complex sound pressure at the receiver and p_{free} is the free-field complex sound pressure at the receiver positions due to the acoustic point source. The results are evaluated in 1/3 octave bands.

3.2. Numerical results

Figure 2 shows the contours of the real part of the pressure. It can be clearly seen from this figure that the considered frequencies are well in in the mid-frequency range for this problem.

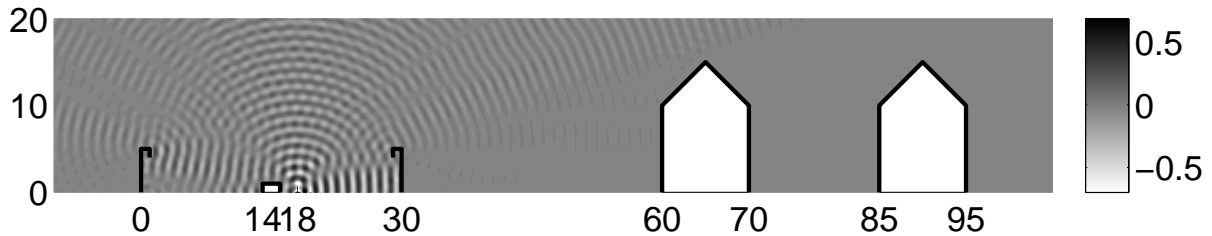


Figure 2. Pressure contours calculated with the WBM at 275 Hz, real part [Pa]

Figure 3 shows the relative pressure results in the response points at 2m height for Case 1. In general, a good correspondence between the FDTD and WBM can be observed. Due to the rigid surfaces, modal behaviour can strongly influence the frequency results resulting in more pronounced spikes for the WBM as compared to the FDTD result.

The results for Case 2 are shown in figures 4 and 5, for response points at 2m and 4m height respectively. Due to the presence of impedance surfaces, unrealistic resonances disappear from the WBM response, yielding a very good match between the results obtained with the different techniques.

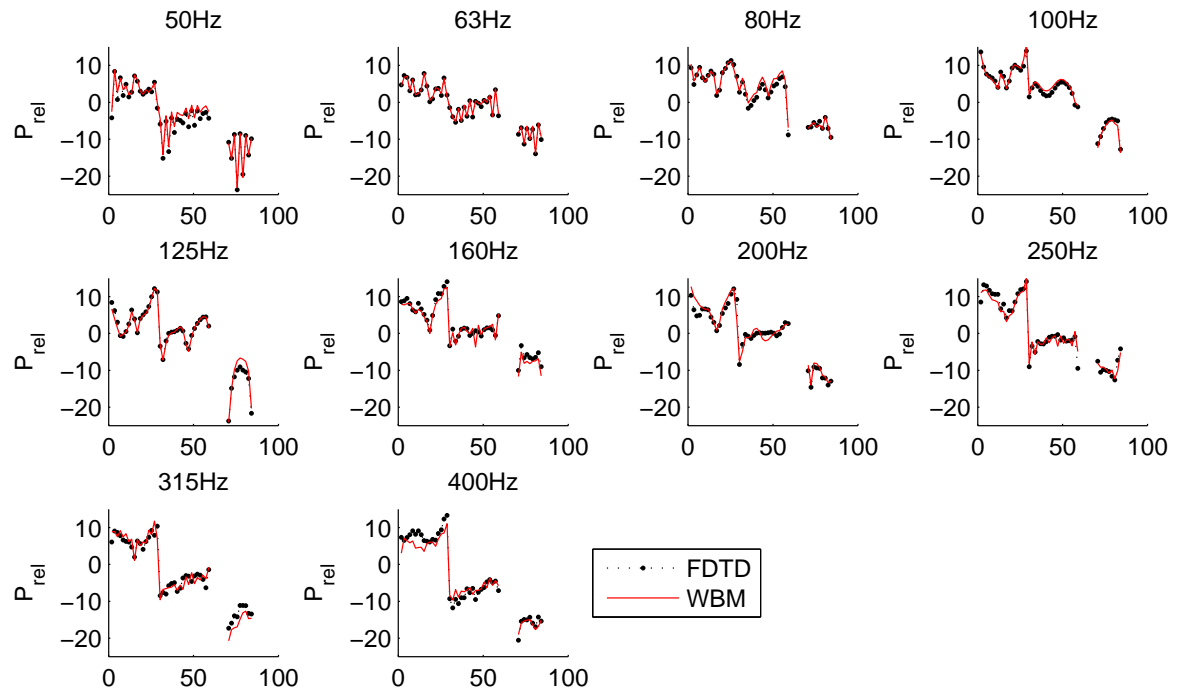


Figure 3. Relative Sound Pressure level in 1/3 octave bands, calculated with FDTD and WBM for case 1, receivers at 2m height

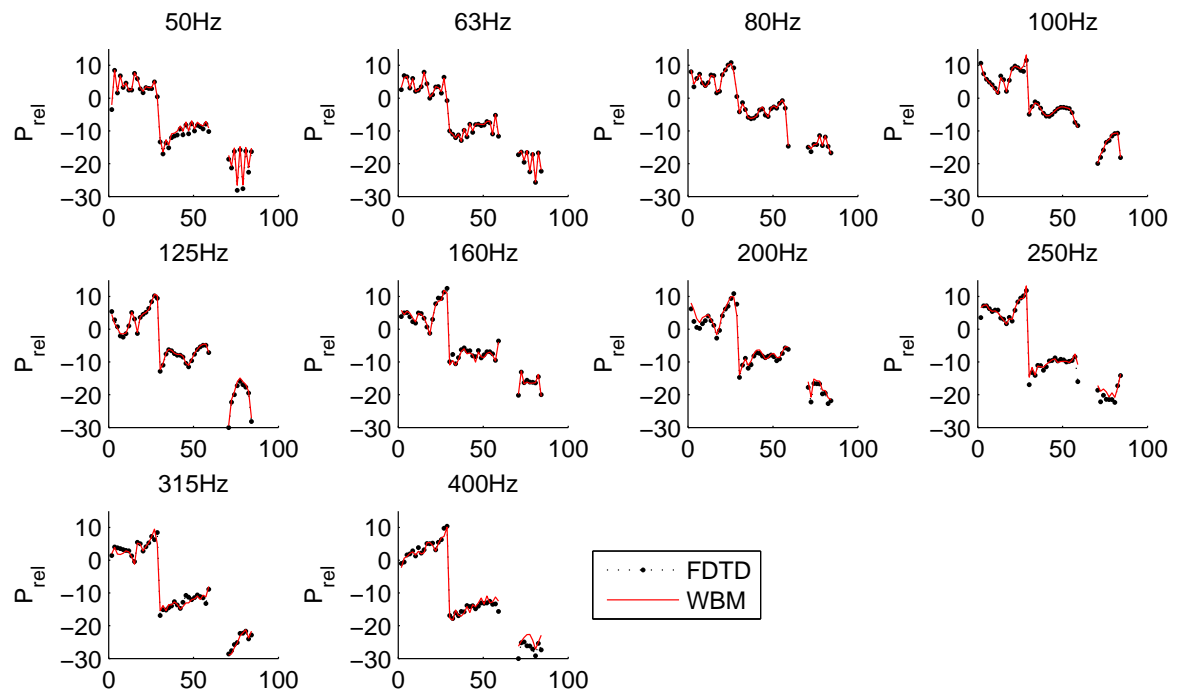


Figure 4. Relative Sound Pressure level in 1/3 octave bands, calculated with FDTD and WBM for case 2, receivers at 2m height

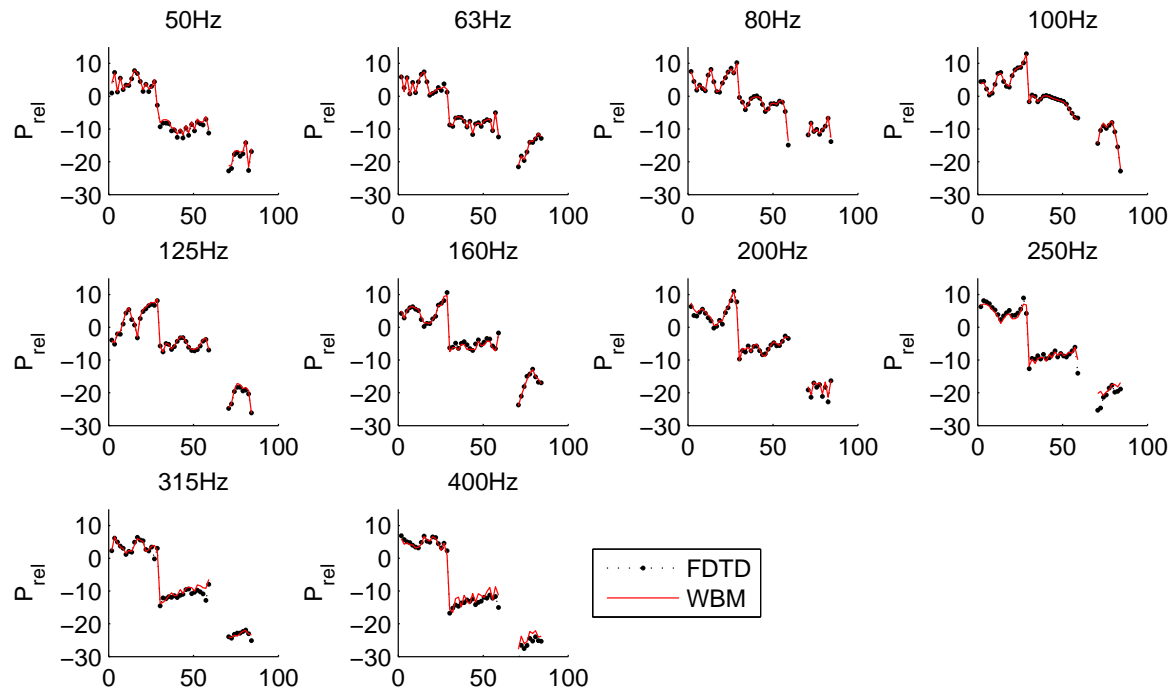


Figure 5. Relative Sound Pressure level in 1/3 octave bands, calculated with FDTD and WBM for case 2, receivers at 4m height

4. CONCLUSION

This paper examines both time-domain and frequency-domain techniques for the application to an outdoor sound propagation problem. Time-domain techniques are commonly used for this application, due to their ability to cope with a moving and inhomogeneous atmosphere. However, their use is best limited to a relatively small source zone, where this aspect is important. For the receiver zone, frequency domain methods are a very suitable modelling method. These techniques also allow a detailed geometrical model, incorporating reflection and refractions, but at a generally lower computational cost. As a first step towards combining these two different approaches, with the time-domain technique used for the source region and the frequency-domain technique for the receiver region, this paper compares the results obtained by the FDTD, a time-domain technique, and the WBM, a frequency-domain technique, for the same problem of sound propagation.

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