Polynomial Chaos forward models in Bayesian inference to solve inverse problems

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Abstract. In this paper we introduce polynomial chaos in the forward model used to solve the inverse problem through Bayesian inference. We validate our approach with three different methods that construct the stochastic forward model, to treat the TEAM Workshop Problem 8.

1 Introduction

A model is accurate if its input data (dimensions, behavior law, source) are well known. In some applications, these input data are not available or at least are not known accurately. For example the situation can arise when it is not possible to have access to a material sample without destroying the device under consideration. This may occur for example in eddy current Non Destructive Testing [1] in a nuclear power plant where the physical properties of clogging material are unknown. In the biomedical problem of locating a current dipole assumed to be responsible for seizures using electroence-phalography [2].

2 Inversion

One way to invert the problem is to use Bayesian inference. Observations or experimental data are used to estimate a probability of verification of a certain hypothesis. Bayesian inference [3] offers a rigorous foundation for inference from uncertain forward models and noisy data. It is a natural procedure for incorporating prior information, and provides a quantitative assessment of uncertainty in the results. Contrary to other methods, the output of Bayesian inference is not solely a value, but a probability distribution that captures all available information about the parameters. From this distribution, one may compute marginal distributions, estimate moments or make other predictions. Bayesian inference has many benefits but a major drawback is the computational effort [4]. Especially when the forward model is complex, the number of computations becomes rapidly prohibitive.

Under the squared error loss function, the Bayesian estimator of the input parameters is found as the mean of the posterior density. The latter is constructed as the product of a likelihood function and a prior density of the input parameters. This incorporates all prior knowledge, i.c. a feasible set of input parameters. The likelihood is obtained by assuming the output undergoes error – model error and noise. The error is typically chosen a zero mean normal variable with a predefined standard error. This allows to write the likelihood as a normal density with argument the difference between reality and measured data.

Using a Finite Element (FE) model as a forward model may lead to high computational cost, as mentioned above. In this communication, we present a method to overcome this difficulty by using a Polynomial Chaos (PC) [5] surrogate model instead of the FE model. Uncertainty quantification based on the polynomial chaos expansion are widely used in many domains [6]. These very competitive methods enable to obtain polynomial forms of the stochastic solution. The arguments of the surrogate model are then the input random variables.

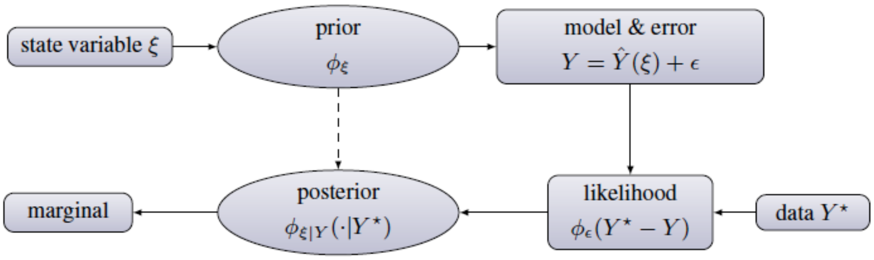

Figure 1: Different components in the Bayesian procedure

3 The application

The Bayesian inference procedure is applied by considering as forward stochastic model the PC surrogate obtained by the vector potential stochastic spectral finite element method (SSFEM) [7], by the non intrusive spectral projection (NISP) and by regression method. The SSFEM forward problem leads to a very large linear system, requiring huge memory space to store it, even if we take advantage of the Kronecker product form of the linear system matrix. However, the linear system matrix can be written under Kronecker product form. Therefore, the Nearest Kronecker Product pre conditioner can be applied to reduce computational cost. Performance of the Bayesian inference is also studied with regard to SSFEM with double orthogonal polynomials. The latter technique leads to a decoupled stochastic system requiring only to solve P_{out} times the deterministic system - P_{out} being the number of coefficient in the PC expansion. On other hand, the NISP with adaptive sparse grid technique and regression method are also efficient and comparable when they are used to build the forward surrogate model. The Bayesian procedure is used with the three methods above, to treat the TEAM Workshop Problem 8.

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