Multiple Objective Shortest Path Algorithms for Transportation Problems

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I. INTRODUCTION

Multiple objective shortest path algorithms (MOSP) are indispensable in the logistic world, as the 'optimal' route is often a route which optimizes multiple criteria at the same time. The optimal route is for example both the fastest and the shortest route, which at the same time minimizes the impact on the environment.

In this article an exact MOSP algorithm is given together with a number of speedup measures. It is shown that these measures indeed realize a speedup, while maintaining the optimality of the solution.

II. THE BASIC MOSP ALGORITHM

The basic exact MOSP algorithm is an algorithm presented by Martins [1] , which is similar to the algorithm of Dijkstra [2] .

A label now contains multiple values, namely one for each objective. Moreover, nodes have assigned to them a set of labels instead of a single label. This set is called to be Pareto optimal, which means that none of the labels is dominated by any of the other. A label is called to be dominated by another one iff all its objective values are equal or worse than the objective values of the other label and there is at least one objective value which is worse.

Similar to Dijkstra's algorithm, the MOSP algorithm searches the network from the origin node and updates the label set of the neighboring nodes. This search process continues until the temporary set is empty. Instead of working on node level, the MOSP algorithm works at label level, which means that labels have references to previous labels instead of previous nodes, and that the temporary set contains labels instead of nodes.

III. SPEEDING UP MOSP CALCULATIONS

A. A Novel Stop Condition

As the algorithm of Martins always investigates the complete network, this can be sped up by finishing the search as soon as the label set of the destination node is permanent and will not change anymore. Denote the minima of each objective in the temporary set by $[\min_i(T)]$ and the maximum (according to a certain ordering) label of the destination label set by $\max(L_d)$, then following stop condition can be defined.

Theorem 1 (Stop condition) The search process can be aborted as soon as $[\min_i(T)]$ is dominated by $\max(L_d)$.

It can be shown, both experimentally and theoretically, that this stop condition speeds up the calculations while maintaining the optimality of the solution.

B. Searching the Network Bidirectionally

Searching bidirectionally is one of the methods which are commonly used in order to speed up calculations.

In the bidirectional MOSP algorithm 2 search processes are carried out simultaneously: one from the origin and one

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from the destination. Labels are added to nodes, which now contain two Pareto optimal label sets instead of one. When a node contains labels in both its forward and its backward label set, these labels are combined to new labels/paths, which are added to the Pareto optimal solution set.

The search processes are finished when the pointwise sum of the minima of the temporary sets $[\min_i(T_F)+\min_i(T_B)]$ is dominated by one of the result labels of the solution set.

Again, it can be shown, both experimentally and theoretically, that this stop condition speeds up the calculations while maintaining the optimality of the solution.

IV. Problem Example

Consider for example a multimodal logistic planner. Goods are first transported by truck from their origin to a harbor where a ship transports them to their final destination. As ships are bound to a time schedule, the trucks arriving too late cause a major delay, while trucks taking the fastest route often are more expensive. So, the least expensive route needs to be found which allows the truck to arrive on time. This can only be done by making use of MOSP algorithms.

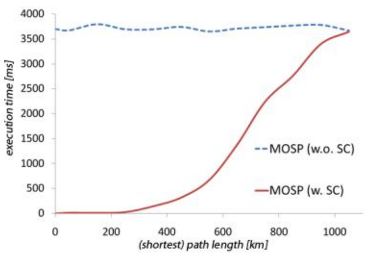

Figure 1. Execution time of MOSP algorithm with and without stop condition.

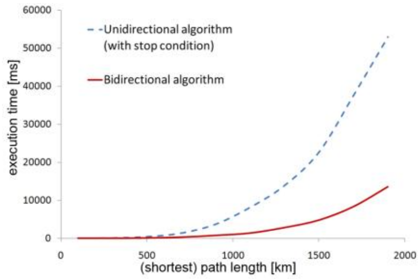

Figure 2. Execution time of unidirectional and bidirectional MOSP algorithm

V. Results

Experiments were performed on a European road network, where the calculation times of the original MOSP algorithm, the MOSP algorithm with stop condition and the bidirectional MOSP algorithm were measured. Average results can be seen in Figure 1 and Figure 2.

It can be seen that the execution time is dependent upon the length of the path, except for the original algorithm, which always searches the complete network. The stop condition can realize a remarkable speedup, especially for shorter paths. The bidirectional algorithm (BMOSP) realizes a greater speed up for longer paths, compared to the MOSP algorithm with stop condition (SC).

VI. Conclusions

An exact MOSP algorithm was presented together with two speedup measures. It is shown experimentally that these measures indeed speed up the calculations. Furthermore a problem example from the logistic world was given, to which this algorithm can be applied.

References

[1] Martins EQV, *On a multicriteria shortest path problem*, EJOR, 1984, 16, 236-245.

[2] Dijkstra EW, *A note on two problems in connexion with graphs*, Num. Mat. 1959, 1, 269-271.