7 - 12 SEPTEMBER 2014, KRAKÓW
FORUM ACUSTICUM

A 2.5D method for the prediction of structure-borne noise from multi-span concrete rail transit bridges

Xiaodong Song, Dingjun Wu, Qi Li
Department of Bridge Engineering, Tongji University, Shanghai, China.

Dick Botteldooren
Acoustics Group, Department of Information Technology, Ghent University, Ghent, Belgium.

Summary

Bridge-borne low-frequency noise has aroused a general interest of many researchers since it forms an important contribution to the rail traffic noise annoyance. Only a single span bridge model was developed to simulate the structure-borne noise in most of the existing literature, which ignores the sound pressure radiating from adjacent spans. This paper presents a two-and-a-half dimensional (2.5D) boundary element method (BEM) based procedure to predict the multi-span bridge-borne noise induced by moving vehicles. The proposed 2.5D model reduces significantly the computational time compared with the three dimensional (3D) BEM. The numerical results match the measured results in both time and frequency domain, regardless of the distance between bridge and measurement points. It is concluded that sound radiating from all the segments of multi-span bridges should be included when predicting the far-field sound pressure.

PACS no. 43.40Rj, 43.40.At

1. Introduction

Low frequency noise has been found harmful to human health and well being. A lot of researchers have focussed their efforts on the low frequency structure-borne noise problem since it is difficult to control. The three-dimensional (3D) boundary element method (BEM) has been extensively used in structure-borne noise prediction [1,2,3]. Zhang et al. [1] presented a numerical procedure to simulate the concrete bridge-borne noise by applying 3D BEM in the frequency domain and vehicle-bridge coupling vibration analysis in the time domain. Li et al. [3] proposed the acoustic transfer vectors (MATVs) method to predict the bridge-borne noise, which was appropriate for the parametric analysis. However, it is time consuming to apply the 3D BEM to compute the MATVs. In our previous work [4], we presented a 2.5 dimensional (2.5D) BEM to calculate the MATVs of the bridge, with much higher efficiency, yet no loss of accuracy. However, the numerical results only agree well with the measured results at near-field points; the computed sound pressures are smaller than the measured ones at far-field due to the neglect of sound radiation from adjacent spans, which is similar as shown in Ref.[2]. In this paper, we extend our work on multi-span bridge-borne noise prediction using the 2.5D BEM. First, the MATVs of a three span rail transit U-shaped bridge are obtained using the 2.5D BEM. Then, the dynamic response of the vehicle-train-bridge system is calculated. Finally, the MATV method is used to predict the bridge-borne noise, and the simulated sound pressure is compared with measured one.

2. 2.5D BEM theory

The 2.5D BEM theory is briefly introduced, with a boundary element (BE) model shown in Fig. 1. The 3D Helmholtz equation can be written in the form of velocity potential [5],

$$(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2)\hat{\psi}=q(r) \qquad (1)$$

where $\hat{\psi}$ is the velocity potential for the fluid, $q(r)$ are the fluid sources, and k is the wavenumber.
Applying a spatial Fourier transform along the z direction to Eq. (1) in the absence of no sources yields the 2D Helmholtz equation,

---

$$[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+(k^2-k_z^2)]\tilde{\psi}=0 \qquad (2)$$

where k_z is the wavenumber in z direction, and $\tilde{\psi}$ is the Fourier transform of $\hat{\psi}$.

The sound pressure, $\tilde{p}(k_z)$, and the component of the fluid particle velocity orthogonal to the structure surface, $\tilde{v}(k_z)$ on the surface of the structure and in the Fourier domain are given by,

$$\begin{cases}\tilde{v}(k_z)=\dfrac{\partial\tilde{\psi}}{\partial n},\\ \tilde{p}(k_z)=j\omega\rho\tilde{\psi},\end{cases} \qquad (3)$$

where ρ is the fluid density, n is the coordinate in the direction orthogonal to the surface, and ω is the angular frequency.

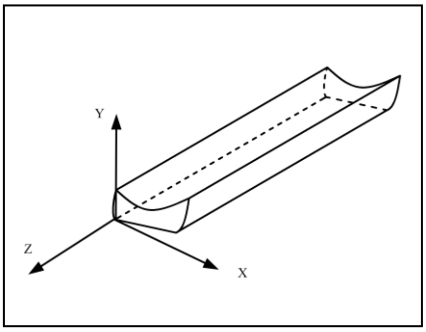


Figure 1. Geometry of boundary element model.

The 2D result $\tilde{p}(k_z)$ can be derived from the combination of Eqs.(2) and (3) using 2D BE model. Then, the 3D sound pressure $\hat{p}$ generated by one surface velocity point located at (x_s, y_s, z_s) can be easily calculated after an inverse Fourier transform,

$$\hat{p}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{p}(k_z)e^{-jk_z(z-z_s)}dk_z \qquad (4)$$

3. Bridge-borne noise prediction

3.1. Vehicle-track-bridge interaction analysis

The modal superposition method proposed by Li et al.[6] can be used for analyzing vehicle-track-bridge interaction. The equations of the vehicle-track-bridge systems' motions can be expressed as follows,

$$\begin{cases}\ddot{\mathbf{q}}_v+2\boldsymbol{\xi}_v\boldsymbol{\omega}_v\dot{\mathbf{q}}_v+\boldsymbol{\omega}_v^2\mathbf{q}_v=\boldsymbol{\Phi}_v^T\mathbf{f}_v,\\ \ddot{\mathbf{q}}_t+2\boldsymbol{\xi}_t\boldsymbol{\omega}_t\dot{\mathbf{q}}_t+\boldsymbol{\omega}_t^2\mathbf{q}_t=\boldsymbol{\Phi}_t^T\mathbf{f}_t,\\ \ddot{\mathbf{q}}_b+2\boldsymbol{\xi}_b\boldsymbol{\omega}_b\dot{\mathbf{q}}_b+\boldsymbol{\omega}_b^2\mathbf{q}_b=\boldsymbol{\Phi}_b^T\mathbf{f}_b,\end{cases} \qquad (5)$$

where $\mathbf{q}$, $\boldsymbol{\Phi}$, $\boldsymbol{\omega}$, $\boldsymbol{\xi}$, and $\mathbf{f}$ are the modal coordinate vector, modal shape matrix, modal frequency matrix, modal damping matrix, and force matrix, respectively; the subscripts v, t, and b stand for vehicle, track, and bridge models, respectively. The linear/nonlinear spring elements and dashpot elements in the coupled vehicle-track-bridge system are treated as pseudo-forces so that the vehicle, track and bridge subsystem can be modeled separately. The force matrix $\mathbf{f}$ represents the combination of the pseudo-force vector produced by these springs and dashpots, and the wheel-rail contact force vector.

3.2. MATV algorithm

The bridge-borne sound pressure spectrum $\mathbf{P}(\boldsymbol{\omega})$ can be predicted using the MATV algorithm [7],

$$\begin{cases}\mathbf{P}(\boldsymbol{\omega})=\mathbf{MATV}(\boldsymbol{\omega})^T\mathbf{Q}_b(\boldsymbol{\omega}),\\ \mathbf{MATV}(\boldsymbol{\omega})^T=j\boldsymbol{\omega}\mathbf{ATV}(\boldsymbol{\omega})^T\mathbf{T}_n\boldsymbol{\Phi}_b,\end{cases} \qquad (6)$$

where $\boldsymbol{\omega}$ is the angular frequency vector; $\mathbf{ATV}(\boldsymbol{\omega})$ is the acoustic transfer vector, which relates the sound pressure at field points to normal surface velocity of the bridge; $\mathbf{Q}_b(\boldsymbol{\omega})$ is the modal coordinate spectra; $\boldsymbol{\Phi}_b$ is the mode shapes; and $\mathbf{T}_n$ is the matrix projecting these mode shapes to outward normal direction displacement of the bridge surface.

The MATVs of the 3D bridge model can be obtained using the aforementioned 2.5D BEM; for details, see Ref.[4]. The sound pressure spectrum can be obtained from Eq. (6) after $\mathbf{Q}_b(\boldsymbol{\omega})$ and $\mathbf{MATV}(\boldsymbol{\omega})$ are computed, and an inverse Fast Fourier Transform (IFFT) can be applied to obtain the time history of the sound pressure.

4. Case study

A simply supported U-shaped concrete girder was adopted in Shanghai elevated metro line 8, as shown in Fig. 2. The webs and bottom slab of the bridge are 240 mm thick, with a standard span of 25m. Five microphones were installed at the mid-span of the test bridge to measure the sound pressure, as shown in Fig. 3

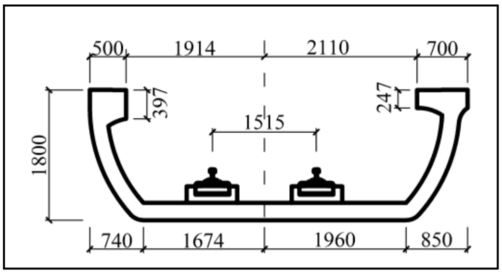


Figure 2. Geometry of the U-shaped girder (unit: mm).

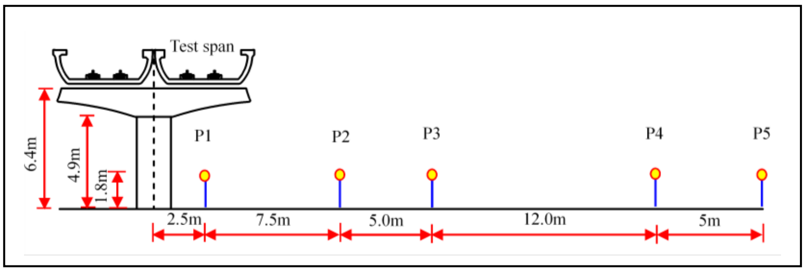


Figure 3. Microphone installation at mid-span.

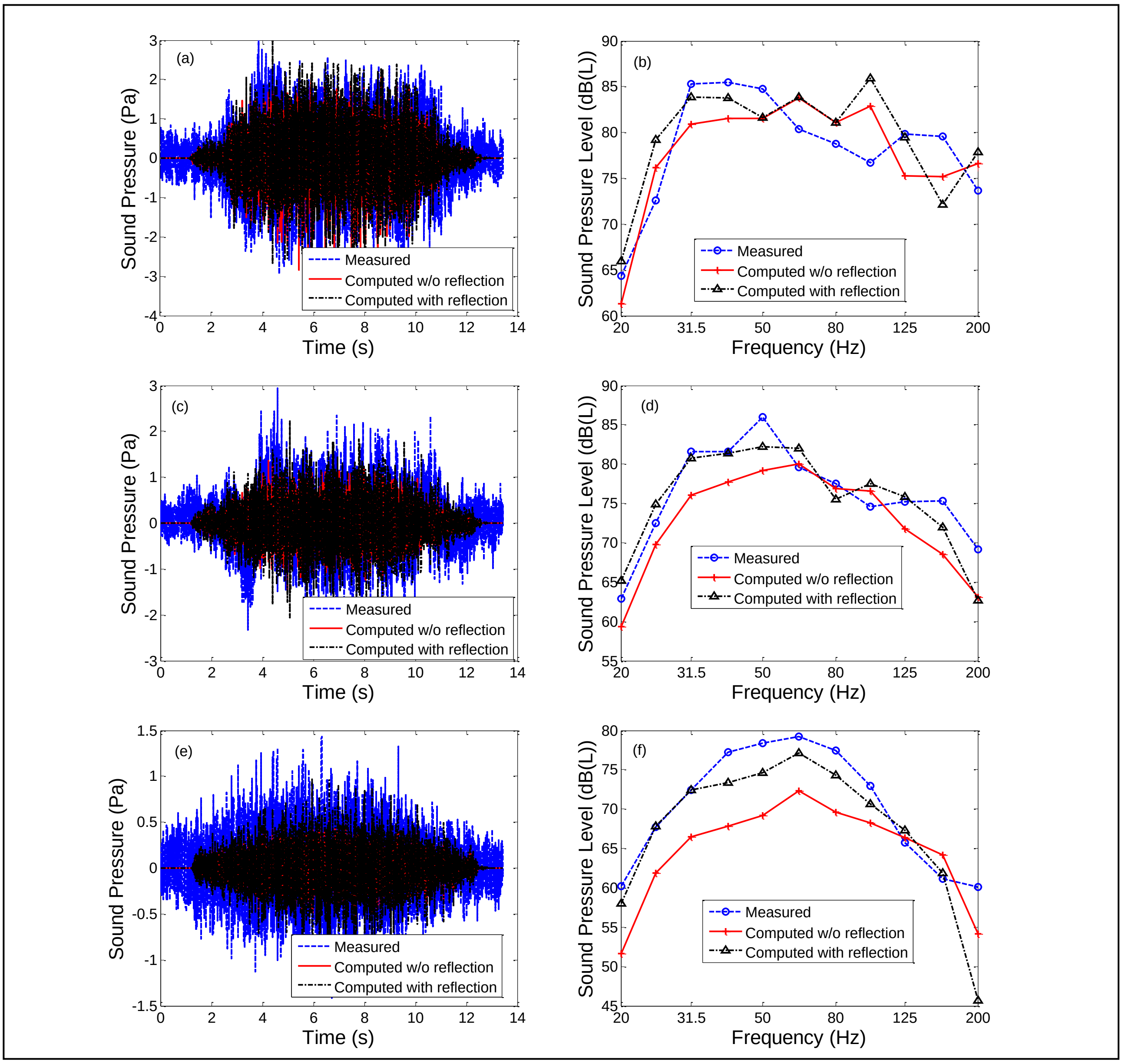


Figure 4. Computed and measured sound pressure of multi-span bridges: (a) time histories, P1; (b) spectra, P1; (c) time histories, P2; (d) spectra, P2; and (e) time histories, P5; and (f) spectra, P5.

4.1. Bridge and vehicle model

The vehicle in the field test comprised seven passenger cars, configured by one trailer, five motor cars, and a trailer in sequence. The detailed parameters of the vehicles can be found in Ref.[3]. The piers are not modeled in this paper because the influence of the piers on bridge vibration and

Table I. Measured and simulated pressure level.

Model	P1			P2			P5		
	Measured	W/o reflection	With reflection	Measured	W/o reflection	With reflection	Measured	W/o reflection	With reflection
One span	92.07	90.61	91.31	89.93	84.65	85.04	85.14	72.76	77.35
Three		90.51	92.17		86.15	88.83		77.82	82.34

noise above 32Hz was insignificant. Thus, multi-span bridges can be treated as identical single span substructure. A single U-shaped girder with simply supported boundary conditions was modeled in the ANSYS software by the eight-node solid element. A modal analysis was then conducted, with the modal frequency ranging from 4.36Hz to 231.16Hz. All of the 126 modes were extracted for the vehicle-track-bridge interaction analysis and the corresponding MATVs were computed.

4.2. Multi-span bridge borne noise

A three span bridge model was developed and the bridge-borne noise was simulated using the aforementioned 2.5D BEM. The concrete girder and ground are assumed to be acoustically perfectly rigid. The computed structure-borne sound pressures were compared with the measured ones, as shown in Fig.4. It can be seen that the simulated and measured results agree well in both time and frequency domain, regardless of near- or far-field points. It should be noted that the numerical results at far-field points increased significantly compared with the results obtained from the single span bridge model in Ref.[4], and the simulated results using the three span bridge model match better with the measured ones, especially at far-field points.

The unweighted equivalent continuous sound pressure level on the pass-by time is used to evaluate the bridge-borne noise level. The sound pressure levels under 200Hz obtained from simulation and measurement were listed in Table I. Table I shows that the single bridge model can be used to predict the structure-borne noise at near-field points with high accuracy. However, the simulated results calculated from the single span model are smaller than the measured results at far-field points because the influence of adjacent span bridge on the sound radiation is more significant at far-field points. In contrast, the three span bridge model can predict the far-field pressure with higher accuracy.

5. Conclusions

A 2.5D BEM-based procedure was presented to predict the low-frequency bridge-borne noise for both single span and three span bridge models. The simulated sound pressures are compared with measurements. The conclusions of the study in this paper can be drawn as follows:
(1) The influence of adjacent spans on the sound pressure prediction for near-field points is insignificant. The single span bridge model can be adopted to simulate the near-field pressure.
(2) The simulated sound pressures using the three span bridge model agree well with its measured counterparts for both near- and far-field points. The sound pressure radiating from multi-span bridges should be included when predicting the far-field sound pressure.

(3) The effect of ground reflection is insignificant at near-field points, but the influence should be considered for far-field points.

Acknowledgement

The authors are grateful to the financial supports from the National Natural Science Foundation of China (No.50908178) and the Fundamental Research Funds for the Central Universities.

References

[1] X. Zhang, X. Z. Li, Q. M. Liu, J. F. Wu, Y. D. Li: Theoretical and experimental investigation on bridge-borne noise under moving high-speed train. Sci. China Tech. Sci. **56**(2013) 917-924.

[2] Y. Kozuma, K. Nagakura: An Investigation on vibratory and acoustical characteristics of concrete bridge for Shinkansen. Proc. 2010. Int. Workshop on Railway Noise, 185-192.

[3] Q. Li, Y. L. Xu, D. J. Wu: Concrete bridge-borne low-frequency noise simulation based on vehicle-track-bridge dynamic interaction. J. Sound Vib. **331** (2012) 2457-2470.

[4] Q. Li, X.D. Song, D. J. Wu: A 2.5-dimensional method for the prediction of structure-borne low-frequency noise from concrete rail transit bridges. J. Acoust. Soc.Am. **135** (2014) 2718-2726.

[5] C.M.Nilsson, C.J.C.Jones, D.J.Thompson, J.Ryue: A waveguide finite element and boundary element approach to calculating the sound radiated by railway and tram rails. J. Sound Vib. **321**(2008) 813-836.

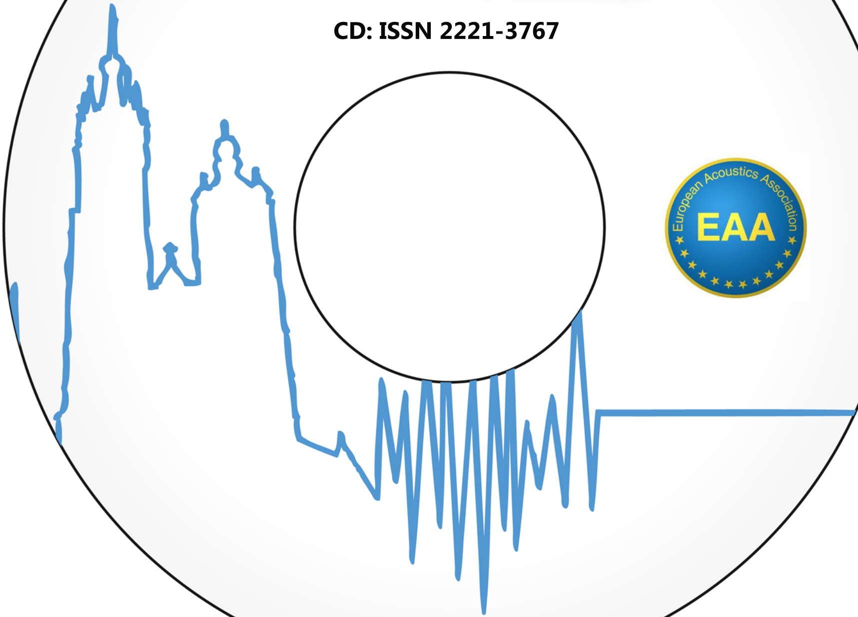
[6] Q. Li, Y. L. Xu, D. J. Wu, Z. W. Chen: Computer-aided nonlinear vehicle-bridge interaction analysis. J. Vib. Control. **16** (2010) 1791-1816.

[7] R.Citarella, L.Federico, A.Cicatiello: Modal acoustic transfer vector approach in a FEM-BEM vibro-acoustic analysis. Eng. Anal. Bound. Elem. **31**(2007) 248-258.

FORUM ACUSTICUM

KRAKÓW, 7-12 SEPTEMBER 2014

CD: ISSN 2221-3767



European Acoustics Association

EAA