Capital constraints and net present value optimization in project scheduling

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1 Problem description

The resource–constrained project scheduling problem with discounted cash flows (RCP-SPDC) is an extension of the well–known resource–constrained project scheduling problem (RCPSP). Both problems are subject to precedence and renewable resource constraints, but whereas the RCPSP aims to minimize the project duration, the RCPSPDC maximizes the project NPV based on a net cash in– or outflow ($c_{i,net}$) associated with each activity. Conceptually, the RCPSPDC can be formulated as follows:

$$\text{Maximize} \sum_{i=1}^{n} c_{i,net} \cdot e^{-\alpha f_i} \tag{1}$$

Subject to:

$$f_i \le f_j - d_j, \quad \forall (i,j) \in A, \tag{2}$$

$$\sum_{i \in S(t)} r_{ik} \le a_k, \quad \forall k \in R, \ t = 1, \ldots, \delta_{n+1}, \tag{3}$$

$$f_{n+1} \le \delta_{n+1}, \tag{4}$$

$$f_i \in int^+, \quad \forall i \in N \tag{5}$$

The objective function (1) maximizes the project NPV and discounts the cash flows to the activity finish times f_i based on a discount rate α. Constraints (2) include the precedence constraints, with A the set of arcs and d_j the duration of activity j. The renewable resource constraints are enforced in constraints (3), with r_{ik} the resource demand of activity i for resource k, a_k the availability of resource k and $S(t)$ the set of activities in progress at time t. Constraint (4) imposes a project deadline δ_{n+1} to avoid activities with a negative cash flow from being delayed indefinitely, and constraints (5) state that the decision variables f_i should be positive integers (N is the set of activities).

The capital– and resource–constrained project scheduling problem with discounted cash flows (CRCPSPDC) extends the work of Smith–Daniels *et. al.* (1996) by including renewable resources. Alternatively, the CRCPSPDC extends the RCPSPDC by introducing additional capital constraints, which state that the cumulative cash cannot be negative at any time during the project. The cumulative cash or available capital at any time t is defined as the sum of the initial capital C_0 minus any cash outflows paid and plus any cash inflows received until that time t. Just like for the RCPSPDC, we assume cash inflows occur upon activity completion. For the cash outflows, on the contrary, we consider three cases:

1. Both cash in– and outflows are incurred upon activity completion. As a result, changes in capital only occur at activity finish times. The objective function (1) of the model remains the same, although we explicitly distinguish between a cash in– ($c_{i,in}$) and outflow ($c_{i,out}$) per activity in function (6). Constraints (7) are added to enforce the limited capital availability, with $Q_f(t)$ the set of activities which have been completed by time t.

$$\text{Maximize} \sum_{i=1}^{n} c_{i,in} \cdot e^{-\alpha f_i} - \sum_{i=1}^{n} c_{i,out} \cdot e^{-\alpha f_i} \qquad (6)$$

$$\sum_{i \in Q_f(t)} c_{i,out} \leq C_0 + \sum_{i \in Q_f(t)} c_{i,in}, \quad t = 0, \dots, \delta_{n+1} \qquad (7)$$

2. The cash outflows are paid on a per time unit basis during the activity duration. The per time unit cash outflows are set equal to $c_{i,out}/d_i$, which means that an equal portion of the total activity cash outflow is paid at each time unit during the planned activity duration (function (8)). The capital constraints are adjusted to (9) with $Q_s(t)$ the set of activities which have been started by time t.

$$\text{Maximize} \sum_{i=1}^{n} c_{i,in} \cdot e^{-\alpha f_i} - \sum_{i=1}^{n} \sum_{t=1}^{d_i} cf_{i,out} \cdot e^{-\alpha(f_i - t)} \qquad (8)$$

$$\sum_{i \in Q_s(t)} \sum_{w=f_i-d_i}^{min(t,f_i-1)} c_{i,out}/d_i \leq C_0 + \sum_{i \in Q_f(t)} c_{i,in}, \quad t = 0, \dots, \delta_{n+1} \qquad (9)$$

3. The cash outflows are paid at the activity start times, which implies that reductions in the available capital occur at the start of an activity. The adjusted objective function and capital constraints are shown in functions (10) and (11) respectively.

$$\text{Maximize} \sum_{i=1}^{n} c_{i,in} \cdot e^{-\alpha f_i} - \sum_{i=1}^{n} c_{i,out} \cdot e^{-\alpha(f_i - d_i)} \qquad (10)$$

$$\sum_{i \in Q_s(t)} c_{i,out} \leq C_0 + \sum_{i \in Q_f(t)} c_{i,in}, \quad t = 0, \dots, \delta_{n+1} \qquad (11)$$

2 Methodology

We propose a genetic algorithm (GA) with a specialized local search to solve the three variants of the CRCPSPDC. The local search consists of three parts:

– **Initial schedule**: the initial schedule is constructed by the serial schedule generation scheme of Kolisch (1996) and starts from a priority list (PL) provided by the GA. Additionally, this step ensures that the schedule is feasible with respect to the project deadline.

– **Capital feasibility improvement**: activities are delayed in sets to reduce capital shortages. The improvement method evaluates the capital feasibility at every time instance between 0 and the project deadline. If the capital is negative at a time instance t, an evaluation is made of the activities whose delay can reduce the capital shortage. Subsequently, one or more of these activities is delayed within their feasible range. The method explicitly distinguishes between the three models discussed, and terminates once a feasible schedule has been found, or feasibility cannot be obtained based on the initial schedule.

– **NPV improvement**: this part of the local search is an adaptation of the activity move rules of Leyman and Vanhoucke (2015) for the RCPSPDC and delays sets of activities to improve the project NPV. The NPV improvement is only applied if the previous step was able to find a capital feasible schedule.

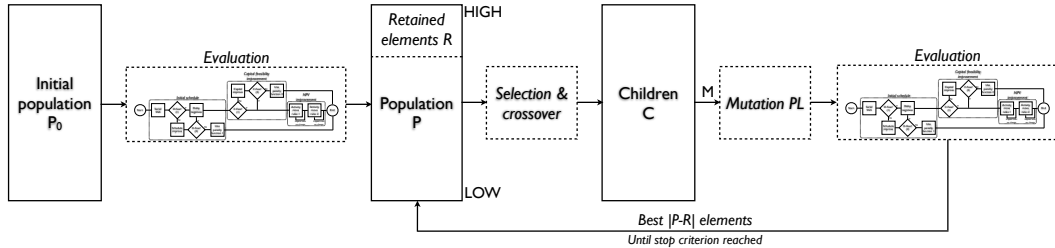Figure 1 gives an overview of the proposed GA, with the local search included in the evaluation part.

Fig. 1. Genetic algorithm flow

3 Results

We have extended the data of Vanhoucke (2010) for the RCPSPDC with C_0 values and have included two cash flow parameters, namely the profit margin percentage (PMP) and the cash flow distribution (CFD). The PMP defines the proportion between the total cash inflows and the total cash outflows, with a higher (lower) value corresponding with a higher (lower) total cash inflow for a constant total cash outflow. The CFD constitutes the distribution of cash inflows over the different activities in terms of the project network. A higher (lower) value means that larger cash inflows are received relatively early (late), whereas smaller cash inflows are received later (earlier).

Table 1 displays the added value of the capital feasibility improvement in terms of the percentage of instances which are capital feasible. LS_{nocap} are the results of our proposed method without the capital feasibility improvement, whereas LS_{full} are the results with the local search. Based on the results in the table, including the corresponding p–values, it can be concluded that the capital feasibility improvement step has a strong added value.

Table 1. Effect of capital feasibility improvement (%capital feasible)

	LS_{nocap}	LS_{full}	p–value
Model 1	77.71	95.31	<0.001
Model 2	32.10	89.44	<0.001
Model 3	30.39	87.42	<0.001

Figure 2 shows more detailed results for the complete method based on the parameters order strength (OS), PMP and CFD for the three models. The graphs go into detail about the capital feasibility. Based on the figure, the following can be concluded:

– An increase in OS decreases the capital feasibility for all three models, but the decreases are considerably larger for models 2 and 3 compared to model 1. The reasoning behind these results is that a lower OS value allows for more flexibility in the project schedule and as such makes it easier to solve capital shortages.

– The capital feasibility increases for a higher PMP. A higher PMP implies that cash outflows can be more easily compensated by cash inflows since the latter are on average larger. The effect is again larger for models 2 and 3.

– The effect of the CFD parameter is similar to the effect for the PMP factor, but larger (consider the different values on the vertical axis of the bottom and top right graphs).
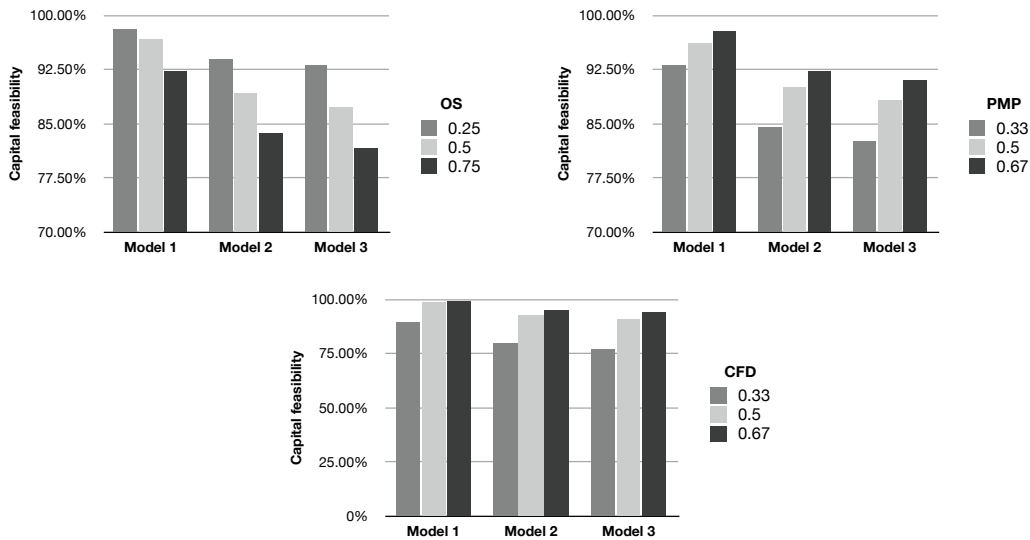

Fig. 2. Analysis results in terms of data parameters

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