Multivariable Fractional Order Controller Design for a Non-minimum Phase System

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<u>Summary</u>. Two control strategies for multivariable processes are proposed that are based on a decentralised and a steady state decoupling approach. The designed controllers are fractional order PIs. The efficiency and robustness of the proposed strategies is tested and validated using a non-minimum phase process. Previous research for the same non-minimum phase process has proven that simple decentralised or decoupling techniques do not yield satisfactorily results and a multivariable IMC controller has been proposed as an alternative solution. The simulation results presented in this paper show that the proposed fractional order multivariable control strategies ensure an improved closed loop performance and disturbance rejection, as well as increased robustness to modelling uncertainties, as compared to traditional multivariable IMC controllers.

Introduction

The large majority of chemical processes are multivariable in nature, exhibiting some strong couplings and occasionally a non-minimum phase character that makes the control design problem a challenging task [1,2]. In general, for such systems, the objective of a control system is to maintain several controlled variables at independent set points. Despite the coupling problems associated with multivariable systems, a non-minimum phase system is even more difficult to control. None of the techniques that are based upon model inversion can be used since such an inversion leads to an unstable closed loop system. Multivariable controllers have been previously designed for such systems. However, simplified algorithms are generally preferred. In contrast to the centralised multivariable control, decentralised control is widely preferred in practice and industrial applications especially because of its main advantage that allows for an easy implementation and tuning, if a sufficient number of sensors and actuators exist. It is also highly reliable.

For highly interacting processes, a decoupling control is usually preferred instead of a decentralized algorithm. Decoupling is a procedure that reduces multivariable interactions and sets the premises for an improved design of the decentralized control. The mathematical procedure to decouple a MIMO system consists in a transformation of the original transfer function matrix of the process into a diagonal one, achieved by using an additional controller, also called a decoupler, which is designed in order to compensate for process interactions. Then, for the resulting pseudo-plant, consisting of the original model of the multivariable process and the decoupler, SISO techniques can directly be used in designing the controllers.

The quadruple tank process, considered as a case study in this paper, is a multivariable process with a multivariable zero located in the right half plane. For this particular process, a decentralised, decoupling and multivariable IMC strategy have been proposed [3], however the experimental results obtained showed the necessity of more complex control algorithms when stringent performance is envisaged and coupling, as well as RHP zeros need to be tackled efficiently. For this particular process, both decentralized and decoupling controls achieved poor performance for disturbance rejection tests, which motivated the application of the more advanced IMC control and even a possible future work regarding model predictive control.

The purpose of this paper is to design a simple control algorithm that is based on combining fractional order controllers with a decentralised as well as decoupling approach that allow for a SISO interpretation of the controller tuning, but that can also achieve improved performance compared to the multivariable IMC control (MIMO IMC). The use of fractional order controllers is expected to enhance the performance of the closed loop system and increase the robustness of the system [4,5]. Several fractional order techniques have been proposed in literature for controlling multivariable processes, such as the extension of the CRONE algorithm [4], MIMO-QFT robust synthesis methodology combined with CRONE control [5], sliding mode control based on the selection of a special fractional-order sliding variable [6] or fractional PID formulated as an H_{∞} problem with a controller structure constraint [7]. Contrary to these multivariable fractional order control algorithms, the present paper proposes simpler approaches, also based on robust fractional order control algorithms that enable the use of SISO control techniques for multivariable processes.

Alternative designs of a fractional order controller for multivariable processes

Decentralised approach

The decentralised approach in controlling MIMO systems consists in a proper selection of the input-output pairings, with the purpose of dividing the initial control problem into several SISO control loops, while aiming to reduce the amount of interaction. The first step in the decentralised approach consists in a RGA analysis of the multivariable

process that allows for a proper pairing of the input-output signals. The next step consists in the design of the individual fractional order PI controllers for each input-output pairing by neglecting the effect of the interaction loop. The transfer function of the fractional order PI controller, proposed in this paper, is given as:

$$H_{FO-PI}(s) = k_p \left(1 + \frac{k_i}{s^{\mu}} \right)$$
(1)

with $\mu \in (0 \div 2)$ the fractional order. To tune the fractional order PI controller, three performance specifications are imposed: a) a certain gain crossover frequency - ω_{gc} , b) a phase margin - φ_m - of the open loop system, denoted H_d(s) and c) a robustness condition to gain variations. Considering that the open loop transfer function is written as:

$$H_d(s) = H_{FO-PI}(s)H_P(s)$$
⁽²⁾

where $H_p(s)$ is the process transfer function, the tuning of the controller is done based on the following set of equations [6,7,8]:

$$\left|\frac{1}{K+jL}\right|_{\omega_{gc}} \left|k_{p}\left[1+k_{i}\omega_{gc}^{-\mu}\left(\cos\frac{\pi\mu}{2}-j\sin\frac{\pi\mu}{2}\right)\right]\right|=1$$

$$k_{c}\sin\left(\frac{\pi\cdot\mu}{2}\right)$$
(3)

$$\frac{w_i \sin\left(\frac{2}{2}\right)}{\omega_{gc}^{\mu} + k_i \cos\left(\frac{\pi \cdot \mu}{2}\right)} = tg\left(\pi - \varphi_m - a \tan\left(\frac{L}{K}\right)\right)$$
(4)

$$\frac{\mu k_i \omega_{gc}^{-\mu-1} \sin \frac{\pi \mu}{2}}{1 + 2k_i \omega_{gc}^{-\mu} \cos \frac{\pi \mu}{2} + k_i^2 \omega_{gc}^{-2\mu}} - \frac{\dot{L}K - L\dot{K}}{L^2 + K^2} = 0$$
(5)

where *K* is the real part and *L* is its imaginary part of the process $H_P(j\omega_{gc})$. To simplify the computation of the fractional order PI controller parameters, the values for k_i and μ are determined graphically using (4) and (5) [6,7,8], while k_p is then computed using (3).

Decoupling approach

In case of a highly coupled MIMO system, the decentralised approach may result in poor closed loop performance due to the multiple input-output interactions. A decoupling solution could then be used instead. In this paper, a steady state decoupling is employed. Given the *nxn* MIMO system:

$$G_{p}(s) = \begin{pmatrix} H_{p11}(s) & H_{p12}(s) & \dots & H_{p1n}(s) \\ H_{p21}(s) & H_{p22}(s) & \dots & H_{p2n}(s) \\ \dots & \dots & \dots & \dots \\ H_{pn1}(s) & H_{pn2}(s) & \dots & H_{pnn}(s) \end{pmatrix}$$
(6)

the steady state decoupler is the inverse of the process transfer function gain matrix in (6), denoted as $G_m^{\#}$. The steady state decoupled process is then computed as:

$$G_{D}(s) = G_{m}^{\#} \begin{pmatrix} H_{p11}(s) & H_{p12}(s) & \dots & H_{p1n}(s) \\ H_{p21}(s) & H_{p22}(s) & \dots & H_{p2n}(s) \\ \dots & \dots & \dots & \dots \\ H_{pn1}(s) & H_{pn2}(s) & \dots & H_{pnn}(s) \end{pmatrix}$$
(7)

The tuning of the fractional order PI controllers is then performed for each diagonal element in the decoupled process $G_D(s)$ using the same tuning procedure based on (3)-(5). The final multivariable FO-PI controller computed as:

$$G_{C}(s) = G_{m}^{\#} \begin{pmatrix} H_{FO-PI_{1}}(s) & 0 & \dots & 0 \\ 0 & H_{FO-PI_{2}}(s) & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & H_{FO-PI_{n}}(s) \end{pmatrix}$$
(8)

Case study. Control strategies for non-minimum phase quadruple tank system

The schematic representation of the quadruple water tanks system is given in Fig. 1. The system is a multivariable one, with two inputs, the voltages applied to the two pumps, denoted as $V_{p1}(t)$ and $V_{p2}(t)$, and two outputs, the water levels of Tank 2 and Tank 4, denoted as $L_2(t)$ and $L_4(t)$, respectively. The model transfer function matrix was determined experimentally to be [3]:

$$G(s) = \begin{bmatrix} \frac{1.64}{18.43s+1} & \frac{2.49}{178.8s^2+26.74s+1} \\ \frac{2.56}{172.2s^2+27.6s+1} & \frac{1.28}{15.92s+1} \end{bmatrix}$$
(9)

The transmission zeros for the quadruple water tanks system are: z_1 =-0.26; z_2 = 0.07; z_3 = -0.06; z_4 = -0.05. Due to the positive zero z_2 = 0.07, the system is non-minimum phase.



Fig. 1. Schematic diagram of the quadruple water tanks system

Based on the RGA analysis [7] ($\Lambda = \begin{bmatrix} -0.49 & 1.49 \\ 1.49 & -0.49 \end{bmatrix}$), the 1-2/2-1 pairing was selected and two FO-PI controllers are then computed. The following performance specifications are imposed for the two loops: $\omega_{gc1} = 0.025$, $\varphi_{m1} = 60^{\circ}$ and $\omega_{gc2} = 0.02$, $\varphi_{m2} = 60^{\circ}$. The resulting fractional order PI controllers, to be used in the decentralised approach are:

$$\begin{cases} H_{FO-PI_1}(s) = 0.22 \left(1 + \frac{0.023}{s^{1.23}} \right) \\ H_{FO-PI_2}(s) = 0.17 \left(1 + \frac{0.02}{s^{1.24}} \right) \end{cases}$$
(10)

To tune the fractional order controllers for the decoupling control strategy, the decoupler was first computed as:

$$G_m^{\#} = \begin{pmatrix} -0.3 & 0.58\\ 0.6 & -0.38 \end{pmatrix} \tag{11}$$

Similar performance specifications were imposed to design the fractional order controllers for the decoupling strategy, $\omega_{gc1} = 0.02$, $\varphi_{m1} = 70^{\circ}$ and $\omega_{gc2} = 0.015$, $\varphi_{m2} = 60^{\circ}$, in order to obtain similar closed loop performance in terms of overshoot and settling time. The two fractional order controllers are:

$$H_{FO-PI_1}(s) = 0.68 \left(1 + \frac{0.013}{s^{1.24}} \right)$$
(12)

$$H_{FO-PI_2}(s) = 0.4 \left(1 + \frac{0.015}{s^{1.25}} \right)$$
(13)

with the final multivariable FO-PI controller determined using (8).

To compare the results, a multivariable IMC strategy has been designed according to [3], to yield similar closed loop performance in terms of settling time, as compared to the decentralised and decoupling fractional order control algorithms given by (10) and (14), respectively. The closed loop simulation results, considering step changes in the reference signals for the levels L_2 and L_4 , are given in Fig. 2 and 3. Since the simplified model in (9) was obtained by linearizing a nonlinear model around the operating point of 10cm [3], the results in Fig. 2 and 3 are regarded as nominal operating conditions. The decentralised and decoupling fractional order control strategies ensure no overshoot and 300 seconds settling time. The MIMO IMC algorithm ensures the same settling time, but with an overshoot of 20%. It must be noted here that zero overshoot for the MIMO IMC strategy is possible to be obtained at the expense of a major increase in the settling time. In terms of interaction, the MIMO IMC offers the best results, however this is valid under the assumption of a perfect model. Among the fractional order control strategies, the decoupling approach provides better interaction responses than the decentralised control algorithm.



Fig. 2. Comparative nominal closed loop simulation results considering a step change in the reference signal for L_2

Fig. 3. Comparative nominal closed loop simulation results considering a step change in the reference signal for L_4

To test the robustness of the designed controller, similar step changes in the reference signals were considered, but with a variation of 30% of the gains and time constants of the process in (9):

$$G(s) = \begin{vmatrix} \frac{2.14}{23.96s+1} & \frac{3.24}{232s^2+26.74s+1} \\ \frac{3.33}{230s^2+27.6s+1} & \frac{1.66}{21s+1} \end{vmatrix}$$
(14)

The closed loop simulation results are indicated in Fig. 4 and 5, for the fractional order control strategies. As noted form the two figures, the 30% change in the modeling parameters do not affect significantly the closed loop performance results. Also, the robustness of the decentralized and decoupling strategies are almost identical.

Fig. 6 and 7 show the same robustness results for the MIMO IMC strategy. It can be easily observed that in the case of the MIMO IMC, there is a significant change in the settling time (over 500 seconds) and in the overshoot (35%). Overall, the proposed fractional order decentralised and decoupling strategies offer an increased robustness as compared to the previously proposed MIMO IMC algorithm.

Previous results [3] showed that poor disturbance rejection performance was achieved when using classical integer order PID controllers in a decentralised or decoupling approach, which justified the application of the more advanced MIMO IMC control. Fig. 8 and 9 present the disturbance rejection tests, considering both the nominal conditions and modelling errors in (14). The simulation results show that the MIMO IMC and the decoupling fractional order controller are outperformed in terms of settling times by the decentralised fractional order controller. Also, the MIMO IMC is more oscillating with increased amplitudes.



Fig. 4. Comparative robust closed loop simulation results considering a step change in the reference signal for L₂ for the proposed fractional order control strategies



Fig. 6. Comparative robust closed loop simulation results considering a step change in the reference signal for L_2 for the MIMO IMC algorithm



Fig. 8. Comparative disturbance rejection tests considering nominal conditions and modelling errors



Fig. 5. Comparative robust closed loop simulation results considering a step change in the reference signal for L₄ for the proposed fractional order control strategies



Fig. 7. Comparative robust closed loop simulation results considering a step change in the reference signal for L_4 for the MIMO IMC algorithm



Fig. 9. Comparative disturbance rejection tests considering nominal conditions and modelling errors

To evaluate the disturbance rejection tests, the following performance index was used:

$$J = \sum_{t=0}^{\infty} (r_i(t) - y_i(t))^2 \text{, with } i=1,2$$
(16)

The computed values are given in Table 1. The computed values for the performance index in (16) show that the proposed fractional order control strategies outperform the MIMO IMC in terms of disturbance rejection, both under nominal as well as modelling errors.

Control strategy	Output y ₁		Output y ₂	
	Nominal	Modelling errors	Nominal	Modelling errors
Decentralised	J = 16.2384	J = 13.9362	J = 19.2701	J = 16.0638
fractional order				
control				
Decoupling fractional	J = 27.5339	J = 21.9440	J = 40.8953	J = 32.5505
order control				
MIMO IMC	J = 41.0897	J = 42.8995	J = 41.0900	J = 41.9865

Table 1. Performance index for the disturbance rejection tests

Conclusions

This paper presented two alternative solutions for controlling non-minimum phase systems and significant coupling. The previous traditional decentralised and decoupling strategies applied for the presented case study, the quadruple tank system, have shown the necessity for an advanced control solution, such as the MIMO IMC. The alternative solutions proposed in this paper consist in decentralised and decoupling fractional order control strategies. The simulation results prove that the proposed multivariable fractional order control algorithms outperform the MIMO IMC solution previously proposed, in terms of closed loop performance, disturbance rejection, both under nominal conditions, as well as modelling errors. Further research includes the implementation of the designed controllers on the actual plant and the validation of the results considering real life experiments.

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