Non Intrusive Polynomial Chaos–based Stochastic Macromodeling of Multiport Systems

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Abstract

We present a novel technique to efficiently perform the variability analysis of electromagnetic systems. The proposed method calculates a Polynomial Chaos-based macromodel of the system transfer function that includes its statistical properties. The combination of a non-intrusive Polynomial Chaos approach with the Vector Fitting algorithm allows to describe the system variability features with accuracy and efficiency. The results of the variability analysis performed with the proposed method are verified by means of comparison with respect to the standard Monte Carlo analysis.

Introduction

The analysis of the effects of geometrical and electrical parameters variability on the performance of modern electronic integrated circuits has become crucial in the recent years. The Monte Carlo (MC)-based techniques represent the standard for the variability analysis (VA) due to their accuracy and ease of implementation, at the cost of performing a large number of simulations due to the slow convergence rate of the MC analysis. Considering that both the operative bandwidth and complexity of modern electrical systems are constantly increasing, the MC analysis has a very high computational cost.

In this scenario, different techniques have proposed PC-based methodologies for the VA of specific systems: multiconductor transmission lines [1] - [3] or lumped elements circuits [4]. Recently, the technique [5] has proposed the application of the PC expansion to the VA of generic linear multiport systems. In particular, the PC model of the system transfer function is computed using the PC model of the system state-space description and the Galerkin projections [5]. However, the PC model of the system transfer function in [5] must be computed for each frequency of interest solving a suitable linear system and the stability of the calculated PC-based model might not be guaranteed.

The proposed contribution overcomes the previously mentioned limitations by first computing the system scattering parameters on a discrete set of values of the frequency and the parameters chosen for the variability analysis. Once the PC model of the system transfer function for the chosen frequencies is calculated using the linear regression method, the desired frequency-domain stochastic macromodel is obtained as weighted summation of frequency-dependent rational functions of the PC matrix coefficients by means of the Vector Fitting (VF) algorithm [6], [7].

Polynomial Chaos Properties

The PC expansion describes a stochastic process in matrix form $\boldsymbol{Y}$ with finite variance [8] as a series of orthogonal polynomials $\varphi_i(\boldsymbol{\xi})$ depending on a vector of N normalized random variables $\boldsymbol{\xi}$ with suitable coefficients $\boldsymbol{\alpha}_i$ as [8], [9]:

$$\boldsymbol{Y} = \sum_{i=0}^{\infty} \boldsymbol{\alpha}_i \varphi_i(\boldsymbol{\xi}) \tag{1}$$

In particular, the polynomials in equation (1) are orthogonal with respect to a probability measure $W(\boldsymbol{\xi})$ with support $\boldsymbol{\Omega}$ according to [10]:

$$< \varphi_i(\boldsymbol{\xi}), \varphi_j(\boldsymbol{\xi}) >= \int_{\boldsymbol{\Omega}} \varphi_i(\boldsymbol{\xi})\varphi_j(\boldsymbol{\xi})W(\boldsymbol{\xi})d\boldsymbol{\xi} = a_i\delta_{ij} \tag{2}$$

where a_i are positive numbers and δ_{ij} is the Kronecker delta.

In the following, without loss of generality, the random variables $\boldsymbol{\xi}$ are supposed to be independent, since the PC expansion can be calculated also for correlated random variables as described in [9], [10]. The infinite expansion (1) must be truncated to a limited number of $M+1$ basis functions for practical implementation, leading to the finite PC model:

$$\boldsymbol{Y} \approx \sum_{i=0}^{M} \boldsymbol{\alpha}_i \varphi_i(\boldsymbol{\xi}) \tag{3}$$

Since the global uncertainty probability density function (PDF) is the product of the PDFs of the single random variables, it can be proven that the orthogonal polynomials in (3) (also referred to as basis functions) can be computed as product combinations of orthogonal polynomials corresponding to each individual random variable ξ_i [9]. In particular, the basis functions can be obtained numerically for independent random variables with arbitrary PDFs [10], while for random variables with specific PDFs the corresponding basis functions are the polynomials of the Wiener-Askey scheme [11].

Furthermore, it can also be proven that the total number of basis functions $M+1$ used in the PC model (3) can be expressed as a function of the number of random variables N and the highest degree of the polynomials used in the PC model P as [9]:

$$M + 1 = \frac{(N+P)!}{N!P!} \tag{4}$$

Upon determination of the basis functions, the corresponding PC coefficients $\boldsymbol{\alpha}_i$ for $i = 0, \ldots, M$ can be computed following one of the two main methods described in the literature: the spectral projection and the linear regression technique [9].

The main advantage of the PC model is the efficient and accurate representation of the system variability. Indeed, the mean $\boldsymbol{\mu}$ and the variance $\boldsymbol{\sigma}^2$ of the stochastic process $\boldsymbol{Y}$ can be analytically computed. Apart from all moments, also stochastic functions of $\boldsymbol{Y}$, such as the probability density function (PDF) and the cumulative density function (CDF), can be computed following standard analytical formulas or numerical schemes [12]. For a complete reference on PC theory, the reader is referred to [8] - [12].

Macromodeling Strategy

Many techniques described in the literature use an intrusive PC-based approach to perform the VA [1] - [3], [5]. Hence, the PC expansion is applied to a deterministic model (i.e per-unit-length parameters, state-space representation) that describes the system under study, leading to an augmented PC-based model that can be computed using the Galerkin projections [1] - [3], [5]. The proposed contribution describes a non-intrusive PC-based approach: the PC model (3) is computed directly for the system transfer function expressed by means of scattering parameters. The scattering parameters are widely used to describe the broadband frequency behavior of microwave systems. Also, the scattering parameters have in general a smoother and more bounded behavior with respect to the impedance, admittance and hybrid parameters. This makes the scattering parameters particularly suitable to be efficiently modeled with a PC-based approach.

First, the scattering parameters $\mathbf{S}$ of the systems are computed for a discrete set of values of the frequency f_l for $l = 1, \ldots, L$ corresponding to the Laplace variable $s_l = j2\pi f_l$ for $l = 1, \ldots, L$ and the normalized random variables $\boldsymbol{\xi}_j$ for $j = 1, \ldots, K$. Hence, equation (3) can therefore be written as

$$\mathbf{S}(s_l, \boldsymbol{\xi}) \approx \sum_{i=0}^{M} \boldsymbol{\alpha}_i(s_l)\varphi_i(\boldsymbol{\xi}) \tag{5}$$

where the only unknown are the coefficients $\boldsymbol{\alpha}_i(s_l)$ and the number of basis functions $M + 1$. The basis function can be computed as described in the previous Section, considering that for independent random variables the number $M + 1$ of basis functions in (8) can be chosen upfront according to (4), since for practical application, the order of expansion P is limited [1] - [5], [11].

Next, the linear regression technique [9] is used to obtain the desired PC coefficients. The latter approach allows to calculate the PC coefficients in (5) solving, for each value of the Laplace variable s_l for $l = 1, \ldots, L$, a least-square system [9] of the form

$$\boldsymbol{\Phi\alpha} = \mathbf{R} \tag{6}$$

where $\boldsymbol{\alpha}$ contains the matrices of the unknown PC coefficients $\boldsymbol{\alpha}_i(s_l)$ for $i = 0, \ldots, M$, the $j-$th row of the matrix $\boldsymbol{\Phi}$ contains the multivariate polynomial basis functions φ_i for $i = 0, \ldots, M$ evaluated in $\boldsymbol{\xi}_j$ for $j = 1, \ldots, K$ multiplied by the identity matrix of the same dimension as the matrix $\boldsymbol{\alpha}_i(s_l)$ for $i = 0, \ldots, M$, and the matrix $\mathbf{R}$ collects the corresponding set of scattering parameters values $\mathbf{S}(s_l, \boldsymbol{\xi}_j)$ for $j = 1, \ldots, K$.

Since the system (6) must be over-determined to be solved in a least-square sense, the number of basis functions $M + 1$ is chosen according to the following relation [9]

$$K \approx 2\,(M + 1) \tag{7}$$

Finally, the VF algorithm is applied to calculate a rational model for each PC coefficient matrix $\boldsymbol{\alpha}_i(s_l)$ for $i = 0, \ldots, M$ and s_l for $l = 1, \ldots, L$.

Hence, the desired PC model for the scattering parameters of a generic multiport system has been calculated as a weighted sum of frequency-dependent rational functions in the form

$$\mathbf{S}(s, \boldsymbol{\xi}) \approx \sum_{i=0}^{M} \boldsymbol{\alpha}_i(s)\varphi_i(\boldsymbol{\xi}) \tag{8}$$

It is important to notice the stability of the proposed macromodel can be ensured by calculating a stable rational model for each PC coefficient matrix $\boldsymbol{\alpha}_i(s_l)$ for $i = 0, \ldots, M$ [13], using the VF algorithm. Finally, the loads can be included in the variability analysis by means of the Galerkin projections [1] - [3], as shown in [5].

The technique described in this contribution is flexible, since it can be applied to any generic linear multiport system, it offers an immediate implementation, since only an initial set of samples of the system transfer function for s_l for $l = 1, \ldots, L$ and $\boldsymbol{\xi}_j$ for $j = 1, \ldots, K$ are required, and it allows to perform the VA with accuracy and efficiency thanks to the property of the PC expansion. Furthermore, the novel proposed method presents two major advantages with respect to the technique [5]:

- the calculation of a deterministic model, e.g. state-space models as in [5], is not required prior to the application of the PC expansion;

- the evaluation of the PC-based macromodel in the form (8) is immediate and do not require to solve a linear system over a discrete set of frequencies as in [5];

Numerical Results

A folded stub microwave notch filter on a substrate with relative permittivity $\epsilon_r = 9.6$ and a thickness $T = 635\mu$m is modeled in this example. The layout of this filter is shown in Fig. 1. The lengths D and L are considered as independent random variables with uniform PDFs, varying by $\pm 10\%$ with respect to the nominal value of $D_0 = 3.5$mm and $L_0 = 7$mm, respectively. The corresponding basis functions are products of the Legendre polynomials [11].

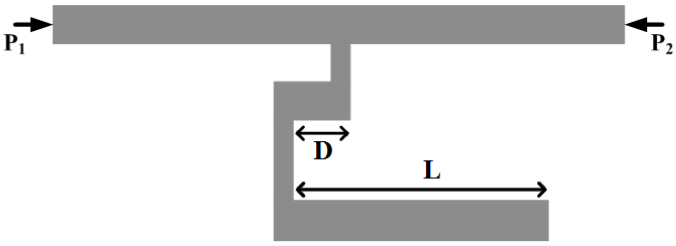

Figure 1: Geometry of the stub microwave notch filter

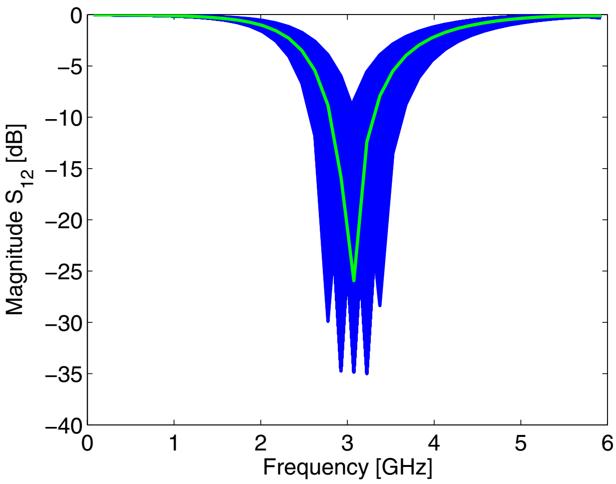

Figure 2: Variability of the magnitude of S_{12}. The green thick line corresponds to the central value for $(D_0, L0)$, while the blue lines are the results of the MC simulations.

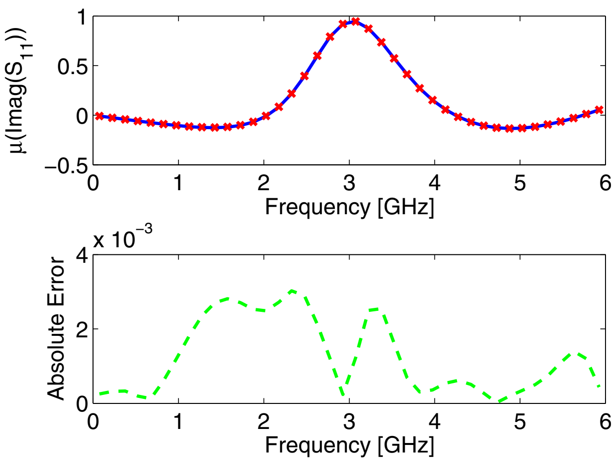

Figure 3: The top plot shows a comparison between the mean of the imaginary part of S_{11} obtained with the MC analysis and the proposed PC-based method (red exs: $(\times)$) for the validation frequencies. The lower plot shows the corresponding absolute error.

ADS Momentum[1] is used to compute the filter scattering parameters over a regular grid composed of 81 samples for the frequency and 8×8 (D, L) samples for the geometrical parameters. The number K of initial samples for the geometrical parameters is chosen according to the relation (7), considering a PC model of order $P = 6$ and $M + 1 = 28$ basis functions, according to (4). Next, the frequency samples are divided in modeling points (41 samples) and validation points (40 samples). Figure 2 shows an example of the variability of the scattering parameters with respect to the chosen random variables: it is important to notice that the two parameters chosen for the VA have a strong influence on performances of the filter. The rational model of the PC coefficients are computed using the VF algorithm, assuming 0.01 as a threshold for the following relative error measure

$$Err = max_{r,c,l} \left(\frac{|\alpha_i^{rc}(s_l) - \tilde{\alpha}_i^{rc}(s_l)|}{\frac{1}{A^2 L} \sum_{r=1}^{A} \sum_{c=1}^{A} \sum_{l=1}^{L} |\alpha_i^{rc}(s_l)|} \right) \quad (9)$$
$$for\; r, c = 1, \ldots, A;\; l = 1, \ldots, L;$$

where the symbol $\alpha_i^{rc}(s_l)$ represents the element in the r-th row and c-th column of the i-th PC coefficient matrix $\boldsymbol{\alpha}_i$ of dimension $A \times A$ evaluated in the frequency s_l for $l = 1, \ldots, L$ and $i = 1, \ldots, M$, while $\tilde{\alpha}_i^{rc}(s_l)$ is the corresponding value of the rational model of the i-th PC coefficient matrix $\boldsymbol{\alpha}_i$ for $i = 0, \ldots, M$.

The PC-based frequency-domain macromodel obtained with the proposed technique shows a superior efficiency and high accuracy compared with the standard MC analysis in computing system variability features. Indeed, Figs. 3 - 5 show an example of the comparison results for the proposed technique and the MC analysis, while the computational time required by the two approaches is described in Table 1. In particular, Figs. 3 - 4 show the mean and the standard deviation of the imaginary part of S_{11} for the validation frequencies obtained with the proposed technique and a MC analysis performed using 6000 (D, L) samples. Then, Fig. 5 describes the PDF and the CDF of S_{12} for the frequency $f = 3.225$GHz obtained with the proposed method and the MC analysis. Similar results can be obtained for the other entries of the scattering matrix.

[1]Momentum EEsof EDA, Agilent Technologies, Santa Rosa, CA.

Table 1: Efficiency of the Proposed PC-based Technique

Technique	Computational time
Monte Carlo Analysis (6000 ADS samples)	10 h 8 min, 37 s
PC-based technique	6 min 53.39 s
Details PC-based technique	**Computational time**
Initial simulations (64 ADS samples)	6 min 39.26 s
PC model scattering parameters	14.13 s

Conclusion

In this contribution, an innovative non intrusive PC-based approach is presented to calculate frequency-domain macromodels for efficient VA of general multiport systems. It is based on the calculation of the PC model of the system transfer function over a discrete set of frequencies in combination with rational identification of the computed PC coefficients in the frequency-domain. The proposed technique offers a straightforward implementation and a high flexibility, since it can be applied to a large range of microwave systems, combined with the accuracy and efficiency of the PC expansion in performing the VA. The validation of the proposed technique is performed by means of comparisons with the standard MC approach for a pertinent numerical example.

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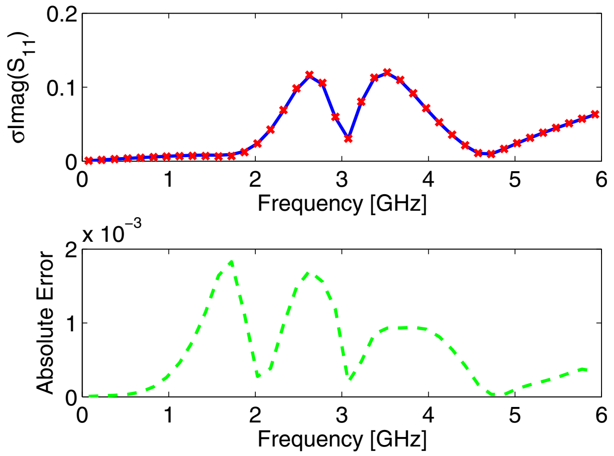

Figure 4: The top plot shows a comparison between the standard deviation of the imaginary part of S_{11} obtained with the MC analysis and the proposed PC-based method (red exs: $(\times)$) for the validation frequencies. The lower plot shows the corresponding absolute error.

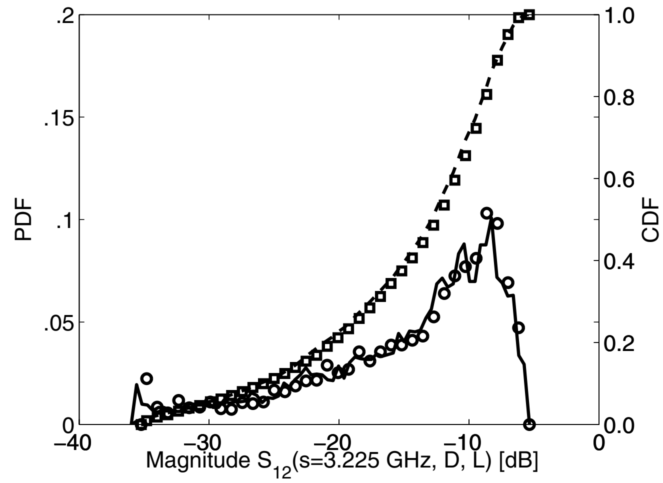

Figure 5: PDF and CDF of the magnitude of S_{12} at 3.225GHz. Full black line: PDF computed using the novel technique; Dashed black line: CDF computed using the novel technique; Circles ($\circ$): PDF computed using the MC technique; Squares ($\square$): CDF computed using the MC technique.

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