Stability analysis of the coupling iterations in fluid-structure interaction simulations

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Abstract—**Fluid-structure interaction problems can be solved with separate solvers for the fluid and the structure. Equilibrium of the forces on the common boundary of the fluid and the structure is obtained by performing coupling iterations between the two problems in every time step. This article gives insight into the parameters that influence the number of coupling iterations by means of Fourier analysis on a one-dimensional model of an elastic artery.**

Keywords— **fluid-structure interaction, stability, coupling iterations, reduced-order model, artery**

I. INTRODUCTION

THE simulation of fluid-structure interaction and other coupled problems has gained in interest over the last decade. This due to the increase in computer power and the maturity of the codes to simulate fluid or structure problems separately. Interesting applications are parachute dynamics [1], biomedical applications [2] and flutter analysis [3], [4].

Two main approaches exist to solve a coupled problem.

• Monolithic approach: a new code that solves both problems simultaneously is developed.

• Partitioned approach: existing codes for both problems are coupled by performing iterations in every time step to bring the forces of both problems on their common boundary into equilibrium.

The former approach requires no coupling iterations every time step but the development and maintenance of such a code is very expensive. A new code has to be created for every combination of physical problems. The equations of two different physical problems can have diverse mathematical properties so it can be hard to solve them simultaneously.

The partitioned approach reuses existing codes in various combinations, but both problems have to be solved multiple times during the coupling iterations in a time step. However, a code for a single physical problem can be more optimized and the partitioned approach makes it easier to use a different mathematical formulation for each problem, e.g. finite volumes for the fluid and finite elements for the structure.

This article focuses on the partitioned approach to fluid-structure interaction for which several coupling algorithms exist. These algorithms can be classified according to their need to access the source code of the solvers. In this article, only Aitken underrelaxation and coupling with reduced-order models will be discussed further. These algorithms treat the codes for the fluid and the structure as *black boxes* and thus do not require access to the source code, enabling the use of commercial codes that are common in industry.

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However, the coupling iterations in a partitioned calculation are not automatically stable. The number of iterations must also be as low as possible to minimize the total calculation time. Therefore the parameters that influence the stability and the number of coupling iterations are investigated with Fourier analysis on a linearized model of a one-dimensional elastic artery. The results of this analysis are verified with non-linear simulations, which indicate the shortcomings of the current coupling techniques but also how they can be improved.

II. PROBLEM DESCRIPTION

A. Fluid problem

The unsteady blood flow in an artery is analyzed with a one-dimensional model. The blood is represented by an incompressible and inviscid fluid and gravity is neglected. The governing equations are the conservation of mass and momentum which can be written in conservative form as

$$\frac{\partial a}{\partial t} + \frac{\partial au}{\partial x} = 0 \tag{1a}$$

$$\frac{\partial au}{\partial t} + \frac{\partial au^2}{\partial x} + \frac{1}{\rho}\left(\frac{\partial ap^*}{\partial x} - p^*\frac{\partial a}{\partial x}\right) = 0 \tag{1b}$$

with a the cross sectional area of the artery, u the velocity along the axis of the artery and $\frac{\partial}{\partial t}$ the time derivative. x is the spatial coordinate, ρ is the density of the blood, p^* the pressure and $p = p^*/\rho$ the kinematic pressure.

The Navier-Stokes equations are discretized on a one-dimensional equidistant mesh with N cells and mesh size Δx. The time discretization scheme is implicit Euler and the time step is indicated with Δt. Solution of the fluid problem can be summarized as $Y = F(X)$, with X a list with the coordinates of the points on the fluid-structure interface and Y a list with the components of the fluid load (pressure and viscous force) on the interface.

B. Structure problem

The behaviour of the elastic artery wall is described by a Hookean constitutive relation without mass, as the inertia of the artery wall is neglected with regards to that of the fluid. An axisymmetric model is used in coordinate system (x, r, ϕ), with r the inner radius of the artery and ϕ the angle in the cross sectional plane. $\sigma_{\phi\phi}$ is approximated as

$$\sigma_{\phi\phi} = E\frac{r - r_0}{r_0} + \sigma_0 \tag{2}$$

with E Young's modulus and r_0 the radius where $\sigma = \sigma_0$. This model allows only radial motion of the nodes.

Under the assumption that only pressure forces act on the boundary, the force balance reads $p^* r = \sigma_{\phi\phi} h$ with h the thickness of the artery wall. Equation (2) and the definition of the kinematic pressure are substituted in the force balance, which can be rewritten as

$$a = a_0 \left(\frac{p_0 - 2c_{mk}^2}{p - 2c_{mk}^2} \right)^2 \qquad (3)$$

by using $a = \pi r^2$ and by introducing the Moens-Korteweg wave speed, $c_{mk}^2 = \frac{Eh}{2\rho r_0}$. Similar to the fluid problem, $X = S(Y)$ represents the solution of the structure problem.

III. COUPLING TECHNIQUES

A. Coupling iterations

Coupling iterations between the fluid and structure in a time step can be performed by the following scheme where a subscript k indicates the coupling iteration.
1. Calculate the fluid load on the interface by solving the fluid problem, $Y_k = F(X_k)$.
2. Calculate the position of the interface by solving the structure problem, $X_{k+1} = S(Y_k)$
3. Increase k and go to 1 if the L1-norm of $X_k - X_{k+1}$ is bigger than the convergence criterium.

B. Aitken underrelaxation

Aitken's method [5] applies a variable underrelaxation factor on the difference between the current interface position (X_{k+1}) and the previous one (X_k) after step 2. This factor is calculated from the differences in the position of the interface during the last two coupling iterations.

C. Reduced-order models

After the solution of the fluid or structure problem, an input and output of that code is known. With this new input/output pair and all the ones from the preceding iterations, a linear reduced-order model of the solver is created which is an approximation for the real solver. The coupled solution of those simplified models is calculated and given to the other solver [2]. This procedure is iterated until convergence is obtained.

IV. FOURIER STABILITY ANALYSIS

By linearizing the discretized fluid and structure equations, one can perform a Fourier stability analysis of the error between the interface's current position and load and the exact solution in a coupling iteration. The amplification factor of the error must be smaller than one for all spatial frequencies for an iteration scheme to be stable. The influence of the structure's stiffness and the time step on the stability is shown in Figure 1 for coupling iterations without underrelaxation or reduced-order models. Perturbations with a low spatial frequency cause instability and a decreasing stiffness or decreasing time step further destabilizes the coupling iterations. A smaller time step or lower structural stiffness thus increase the number of coupling iterations. The difficulties arising from the decrease in time step is highly unwanted, but this result gives a theoretical basis for the findings in previous work [2]. None of the simulations shown
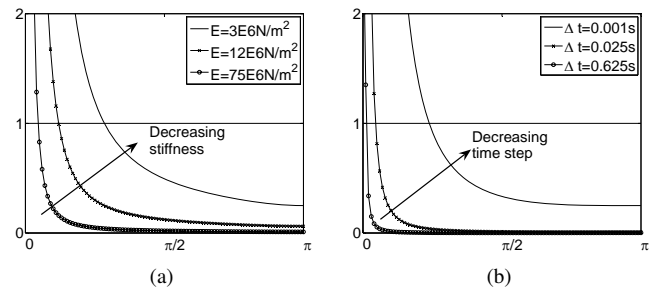

Fig. 1. Error amplification from Fourier analysis (a) for different values of the structural stiffness and (b) for different values of the time step.

in Figure 1 is stable without underrelaxation as the error amplification is bigger than one for the lower spatial frequencies.

V. NON-LINEAR RESULTS

The configurations studied with the Fourier analysis have been simulated with the non-linear fluid and structure equations using Aitken underrelaxation and coupling with reduced-order models. Both techniques require more coupling iterations if the time step or the structural stiffness decreases, which extends the conclusions from the Fourier analysis to non-linear problems. Coupling with reduced-order models outperforms Aitken underrelaxation in every configuration.

VI. CONCLUSIONS

The partitioned approach to fluid-structure interaction enables the reuse of existing codes but requires a coupling algorithm to obtain force equilibrium on the fluid-structure interface. If the mass of the structure is small with respect to the mass of the fluid then more coupling iterations are required when the time step or structural stiffness decreases. New coupling techniques without this unwanted behaviour should be developed by using the information of the previous time steps.

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