REGULARIZED NEWTON-RAPHSON METHOD FOR SMALL STRAIN CALCULATION

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ABSTRACT: Digital Image Correlation (DIC) has been proven to be a highly reliable framework for the full-field displacement and strain measurement of materials that undergo deformation when subjected to physical stresses. This paper presents a new method that extends the popular Newton-Raphson algorithm through the inclusion of spatial regularization in the minimization process used to obtain the motion data. The basic principle is that the motion data is calculated between corresponding blocks in the reference and deformed images using adaptively previously obtained motion estimates in the immediate vicinity of the respective location along with the local block-based image information. The results indicate significant accuracy improvements over the classic approach especially when the block sizes and strain calculation windows used for motion and strain estimation decrease in size.

1. INTRODUCTION

Digital image correlation techniques have been increasingly employed in recent years in surface deformation analysis experiments and received a lot of attention since their introduction [1]. Previous studies [2, 3] clearly evidenced that the Newton-Raphson method of partial differential correction [4, 5] performed best in terms of accuracy although at a higher computational cost compared to the other methods.
One of the principal drawbacks of DIC methods in general resides in the difficulty of finding an optimal size for the blocks used to calculate each motion vector. Spatial consistency can be improved by increasing the block size however this might lead to over-smoothed results if the assumed motion model is simpler than the real underlying displacements. Decreasing the block size reduces the undesired smoothing effect however accuracy is reduced because less data is effectively used to calculate each motion vector. This paper presents a natural and flexible extension of the Newton-Raphson method through the addition of a regularization term [6, 7] in the error criterion that is minimized in order to obtain the desired motion data. Each motion parameter is iteratively calculated with respect to both greyscale image levels and the values of previously calculated neighbouring motion parameters. The regularization term, based on the Geman-McClure robust function is adaptive with respect to neighbouring data so that only the neighbours with close values will influence the final estimate.

2. THE REGULARIZED NEWTON-RAPHSON METHOD

Two corresponding blocks $f(x,y)$ and $g(x',y')$ of size $M\times M$ pixels belonging to the reference and deformed speckle images respectively are considered with $x' = x + u(x,y)$ and $y' = y + v(x,y)$ where:

$$u(x,y) = P_1 + P_3(x - x_0) + P_5(y - y_0)\,;\;\; v(x,y) = P_2 + P_4(x - x_0) + P_6(y - y_0) \tag{1}$$

are the horizontal and vertical linear displacement components with the parameters $(P_i)_{i=1..6}$ and, (x_0, y_0) represent the center coordinates of the reference block. The new regularized correlation criterion for the two blocks based on the sum of the square differences (SSD) becomes:

$$E(\mathbf{P}) = \sum_{x=1}^{M}\sum_{y=1}^{M}\left(f(x,y) - g(x,y,\mathbf{P})\right)^2 + \lambda\cdot\sum_{i=1}^{6}\sum_{j\in N_i^8}\rho\left(P_i - P_j, \sigma_{S_i}(P_i,P_j)\right) \tag{2}$$

where $\mathbf{P} = (P_i)_{i=1..6}$, λ is a regularization parameter associated with the amount of smoothing done, $\rho = x^2/(x^2+\sigma)$ is the Geman-McClure robust function and N_i^8 denotes the 8-connected spatial neighbourhood of the parameter P_i. The robust parameter σ_{S_i} which regulates the influence of neighbouring parameters is calculated separately for each of the six motion parameters as a function of the standard deviation of neighbouring parameter values and updated in every iteration of the minimization process.
The minimization of Eq. (2) is the same as in the original method without regularization term and involves iterations of the form:

$$\mathbf{P}^{(k)} = \mathbf{P}^{(k-1)} - \frac{\nabla E(\mathbf{P}^{(k-1)})}{\nabla\nabla E(\mathbf{P}^{(k-1)})} \tag{3}$$

where (k) and $(k-1)$ represent the current and previous iterations, $\nabla E(\mathbf{P})$ and $\nabla\nabla E(\mathbf{P})$ the Jacobian and Hessian matrices associated with the correlation criterion. The algorithm updates the motion information across the whole displacement field at every iteration with the exception of the locations where convergence has been reached. Here, convergence is considered to be reached when a variation smaller than 10^{-5} of every motion parameter between consecutive iterations is found.

3. RESULTS AND CONCLUSIONS

In the evaluation of the method's performance a set of artificial ground truth images [3] were used in which the numerically deformed image was obtained using radial basis function interpolation and a theoretical displacement model of a plate with hole under biaxial stress [8]. The underlying displacements had variations between -0.1 and 0.1 pixels for the horizontal component and -1 and 1 pixels for the vertical one respectively. The strain magnitudes varied between 1.2×10^{-3} and -6.86×10^{-4} for the horizontal strains, 2.8×10^{-3} and -2.8×10^{-3} for the shear strains and 4.7×10^{-3} and 1.73×10^{-5} for the vertical strains. The displacements were obtained using blocks of 21×21 pixels with an 11 pixel step between each block while the strains were calculated using strain windows *W* varying between 3×3 and 11×11 motion vectors. In Fig. 1 the mean of the absolute strain errors for each of the three strains are presented.

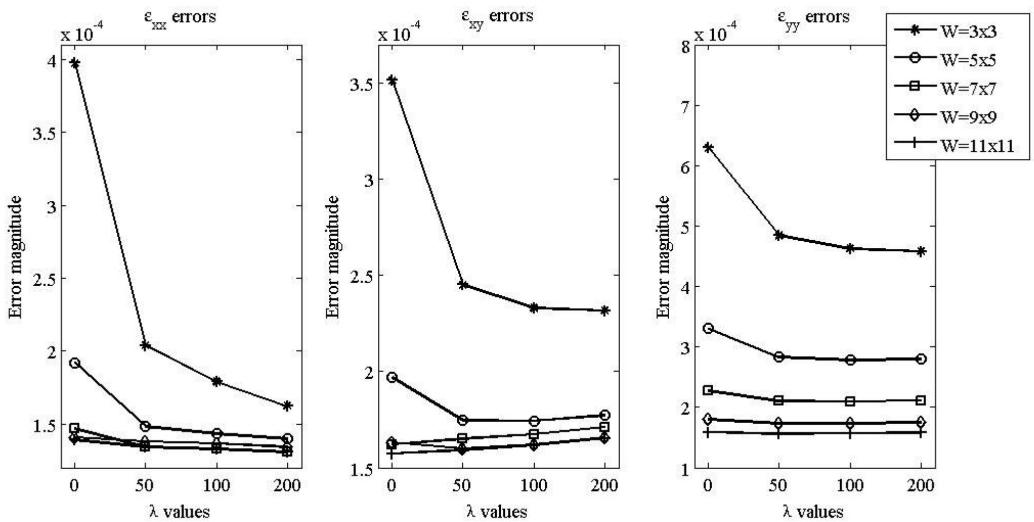

Figure 1- Mean absolute strain errors for varying strain windows

The main advantage of the new method resides in the fact that increased accuracy can be obtained using smaller windows making possible the accurate detection of higher frequency discontinuities in the measured strain fields. The initial results suggest that spatial regularization brings significant improvements to the displacement and strain accuracy of present DIC algorithms and constitutes a viable choice especially when localized displacements and strains are sought.

4. REFERENCES

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