Physiological boundary conditions for flow calculations in 3D models of the human vasculature

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I. Introduction

Among other applications, numerical simulation of blood flow can be used in pre-operative planning of vascular surgery and predict its outcome [1]. This requires the implementation of the necessary tools to perform these types of simulations based on patient-specific data, attainable during pre-operative clinical examination. Usually, only a segment of the vascular tree is simulated to make the computation feasible. In doing so, the in- and outlets of this segment need appropriate boundary conditions (BCs) which describe pressures and flows arising from the interaction between the studied segment and the rest of the cardiovascular system. The boundary conditions should be of lower order complexity with a moderate amount of parameters easily tunable to patient-specific data, and foremost independent of any geometrical changes to the computational domain. In this paper, we describe an impedance BC for the distal vasculature (outlets) [2] and a lumped parameter model of the heart (inlet) [3], implemented within a convergence enhancing framework that couples them to our numerical flow solver (Fluent 6.3, Ansys, UK).

II. Methods

A. Impedance Boundary Condition

The vascular network distal to each outlet can be modeled as a linear dynamic system wherein pressure, p, is a result of convolving the input flow with this system's impulse response, as dictated by eq. (1):

$$p(t) = \frac{1}{T}\int_{t-T}^{t} z(\tau)\ q(t-\tau)\ d\tau \qquad (1)$$

T being the duration of a cardiac cycle, q the flow, and z the impulse response function (e.g. Figure 1). The impulse response is primarily determined by the vascular morphology distal to the outlets.

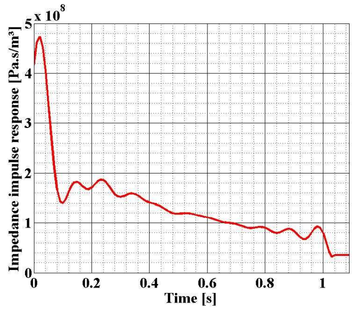

Figure 1. Impulse response at the abdominal aorta, just proximal to the renal artery branching.

B. Lumped parameter heart model

The human left ventricle can be modeled by the electrical circuit analogy depicted in Figure 2. The time varying elastance, E(t), represents the contractile state of the heart muscle. Interestingly the elastance normalized in magnitude and time is patient independent. As a result, any elastance curve is fully defined by the patient's heart rate, the peak elastance and the timing of peak elastance.

The heart valves are modeled as resistances that depend on the overlying pressure drop

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(δP). When δP is negative, the resistance is maximal and acts like a diode.

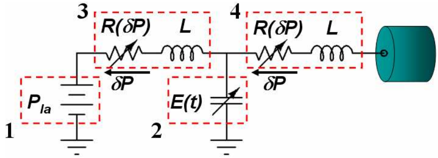

Figure 2. Lumped parameter model of the heart: (1) left atrium, (2) left ventricle, (3) mitral valve, (4) aortic valve (see also [3])

C. Coupling complex BCs to Fluent

Ideally one would solve for the flow field together with the BCs discussed above in an implicit manner. However, since Fluent is used as a blackbox solver, the BC-equations can not be added implicitly and the Jacobian (**J**) that links pressure changes (Δp) at the boundaries to flow changes (Δq) at the outlets, has to be estimated at the beginning of each timestep. For the case of two outlets (indices 1 and 2) and one inlet (index 3), the Jacobian equation writes:

$$\begin{bmatrix} \Delta q_1 \\ \Delta q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_1}{\partial p_2} & \frac{\partial q_1}{\partial p_3} \\ \frac{\partial q_2}{\partial p_1} & \frac{\partial q_2}{\partial p_2} & \frac{\partial q_2}{\partial p_3} \end{bmatrix} \cdot \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \end{bmatrix} = \mathbf{J} \cdot \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \end{bmatrix} \quad (2)$$

After perturbing the outlet pressures and calculating the resulting flow changes, the Jacobian can be estimated through finite differences. Summing the elements on each row of **J** should give zero, since q does not change when the boundary pressures are perturbed uniformly. As a result, the inlet pressure does not need to be perturbed to estimate **J**.

The Jacobian-equation (eq. (2)) and the BC-equations form a closed system of non-linear equations that can be solved with a Newton method. This is repeated every time step.

III. Example: a patient-specific aorta

The BCs discussed here were applied to a patient-specific aortic geometry segmented from MRI scanning data (Figure 3). The aorta was connected to a model of the heart at the inlet (A), while pressure and flow at the outlets (B-E) were linked by different impulse responses similar to the one depicted in Figure 1. The insets in Figure 3 show calculated pressure and flow waves at each outlet.

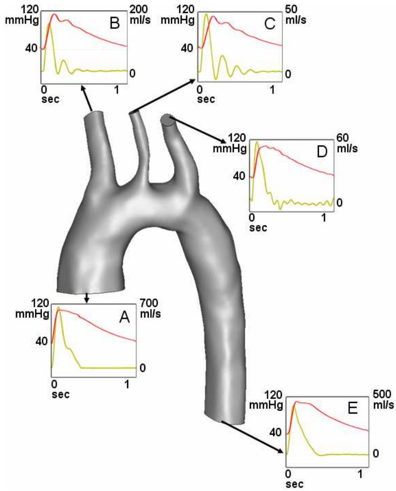

Figure 3. Pressure and flow in a patient-specific aorta: (A) Ascending Aorta, (B) Brachio-Cephalic A., (C) Left Carotid A., (D) Left Subclavian A., (E) Abdominal Aorta.

IV. Conclusions

By using a stable and robust convergence scheme we were able to implement complex, physiological BCs for calculating 3D flow in a patient-specific geometry, giving realistic (physiological) pressure and flow waves at all of the boundaries (A-E).

References

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[2] Vignon-Clementel, I.E., *Outflow boundary conditions for three-dimensional finite element modeling of blood flow and pressure in arteries*, Comput.Methods Appl. Mech. Engrg., 195:3776-3796, 2006

[3] Kim, H.J., *On Coupling a Lumped Parameter Heart Model and a Three-Dimensional Finite Element Aorta Model*, Ann. Biomed Eng., e-pub ahead of print.