SIMULATION STUDY OF A BASIC INTEGRATED INVENTORY PROBLEM WITH QUALITY IMPROVEMENT AND LEAD TIME REDUCTION

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ABSTRACT

In this paper, we investigate the trade-off between investing in product quality improvement and lead time reduction in a B2B single-supplier, single-buyer environment. To this end, a discrete event simulation model is proposed, based on the integrated inventory model defined in Ouyang et al. (2006). Furthermore, the possibility of using alternative failure mechanisms and capital investment functions are studied.

INTRODUCTION

Supply chain management -as defined by the Council of Supply Chain Management Professionals- encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers. In essence, supply chain management integrates supply and demand management within and across companies.

Supply chains have existed since man began to trade, but only during the last decades have they grown globally and have they taken the enormous proportions they do today. This of course has to do with economic globalization. An overview of the impact and effects of economic globalization is depicted in Friedman's bestseller "The World is Flat". Globalization brings opportunities as well as threats: in the context of supply chains we could state that the possible competitive advantage to be gained from efficient supply chaining becomes more important, but global supply chains are also more vulnerable. This heightened vulnerability stems from increased security risks, increased product quality risks and increased probabilities of late delivery.

Lee and Whang (2005) comment on the impact of increased security measures after 9/11 on supply chain safety and cost, while Robinson and Malhotra (2005) define the concept of supply chain quality management. Batson and McGough (2007) introduce the concept of supply chain quality modeling. They state that on one hand there has been a lot of attention for supply chain optimization, but mostly from an operational or financial point of view and not from a quality point of view; and on the other hand there are lots of quality management models but they are all geared towards optimizing quality in one entity in the supply chain and not in the supply chain as a whole. But as the quality of a firm's product depends not only on its own actions but also on the actions of its suppliers, Batson and McGough (2007) suggest a supply chain quality model to measure, predict and follow up the evolution of product quality throughout a supply chain.

In this paper the trade-off between investing in product quality improvement and lead time reduction in a B2B single-supplier, single-buyer environment is investigated. Thereto the integrated inventory model proposed by Ouyang et al. (2006), is transformed into a simulation model which is used to analyze the dynamics of the system and to study the effect of several modifications to the model. Two types of modifications to Ouyang's original model are introduced: modifications concerning the failure mechanism of the production process and modifications concerning the quality improvement (investment) function.

THE BASIC MODEL

Following Baiman et al. (2000), we assume a risk neutral buyer and a risk neutral supplier in a B2B single supplier-single buyer network. The buyer sells units to the market, which it first purchases from a supplier. This kind of situation is encountered whenever a buyer owns the brand and/or designs the product, but outsources the production to one or more suppliers. From the customer's point of view, the ultimate goal is to get the right quantity of products in the highest possible quality at the right time. Capital investment to shorten the lead time and/or to improve product quality are considered effective ways to reach this goal. This paper investigates the trade-off between these two possible investments options under different conditions.

The proposed basic model is an integrated inventory model i.e. the goal is to minimize the joint total expected cost (sum of buyer's costs and supplier's costs) per time unit. The integrated system is triggered by the market demand. As this demand is considered stochastic, the buyer holds a certain level of stock. Whenever his stock level drops below the reorder point, the buyer places an order with the supplier, who thereupon immediately starts with the production. Each order is delivered to the buyer in a certain number of deliveries. During production it is possible that the production process gets out-of-control and thus produces defective items. These defective items can be repaired at the expense of an additional cost. The chance of the production process getting out-of-control can be influenced by investments in production quality.

There is a certain lead time before the buyer can get hold of his ordered products. Because of this lead time and the stochastic nature of the market demand, it is possible that the buyer experiences stock-outs. In this case it is assumed that a known proportion is backordered while the other part are lost sales. It is assumed that lead time can be shortened at the expense of an additional cost. Shortening the lead time implies that safety stock can be lower, stock-out losses can be reduced, and ultimately, service levels can be improved.

Our basic model is based on the work of Ouyang et al. (2006) and has 5 decision variables: the order size (Q), the reorder point (r), the lead time (L), the number of deliveries for each order (m) and the probability that the production process goes out-of-control each time another unit is produced (θ). Based on these five variables, Ouyang theoretically calculated a total expected cost for the buyer (TEC_b), consisting of inventory cost, ordering cost, delivery cost, backordering cost and crash cost, and for the vendor (TEC_v), consisting of inventory cost, set-up cost, rework cost and capital cost. The cost efficiency of the vendor-buyer system was analyzed through an evaluation of the joint total expected cost ($JTEC$), defined as:

$$JTEC = TEC_b + TEC_v \tag{1}$$

with

$$TEC_b(Q, r, L, m) = \tag{2}$$
$$\frac{D}{Q}\left\{A_b + mF + m\left[\pi + \pi_0(1-\beta)\right]E(X-r)^+\right\}$$
$$+h_b\left[\frac{Q}{2m} + r - DL + (1-\beta)E(x-r)^+\right] + \frac{mD}{Q}R(L)$$

and

$$TEC_v(Q, \theta, m) = \tag{3}$$
$$\frac{\alpha}{\delta}\ln\left(\frac{\theta_0}{\theta}\right) + \frac{A_v D}{Q} + \frac{sDQ\theta}{2} + \frac{h_v DQ}{m}\left(\frac{1}{P} + \frac{m-1}{2D} - \frac{m}{2P}\right)$$

Table 1: The basic model's decision variables

Q	Buyer's order quantity
r	Buyer's reorder point
L	Buyer's lead time
m	Number of deliveries per order
θ	Probability of the supplier's production process to go out-of-control ($\theta \leq \theta_0$)

Table 2: Ouyang's model parameters

D	average demand per period
σ	standard deviation on D
P	Production speed of the vendor (P > D)
A_b	Order cost for the buyer per order
A_v	Set-up for the vendor per set-up
F	Transportation cost per delivery
h_b	Buyer's holding cost per unit of time
h_v	Vendor's holding cost per unit of time
s	repair cost for the vendor per defect
π	Unit cost per late delivery
π_0	Unit marginal profit for the buyer
β	fraction of backorders ($0 \leq \beta \leq 1$)
θ_0	Original out-of-control probability
α	Vendor's capital cost per unit of time
δ	Percentage reduction of θ per invested dollar
X	unknown lead time distribution with mean: DL, stdev: $\sigma\sqrt{L}$

THE SIMULATION MODEL

Verification

We developed a discrete event simulation program in Matlab to analyze the dynamics of this model. To implement the program we had to step away from the original distribution-free approach and therefore could not directly use Ouyang's results to verify our simulation model. To overcome this problem, we also adapted Ouyang's equations and solved them using AMPL/CPLEX/MINOS.

When we compared the results from our program and the AMPL-model we found that there was a difference in interpretation for the lead time L. Ouyang consideres L fixed, so the buyer always receives its delivery L time units later than he ordered it. In certain cases however, it is possible that the production time is longer than L, rendering a delivery after exactly L time units infeasible. In Ouyang's theoretical model this possibility is not accounted for; but it could, however, disrupt our simulation model. Therefore we adapted Ouyang's equations again; these changes made the AMPL-results match the results from our simulations.

Table 3: Best results for the original model

Q	r	L	m	θ $\times 10^{-4}$	TECb	TECv	JTEC	stdev
500	220	42	2	0.1	1225	1288	2513	22.82
500	220	42	2	0.01	1221	1353	2573	5.36
500	170	28	2	0.1	1268	1313	2581	58.71
700	220	42	3	0.1	1249	1350	2599	7.55
700	170	28	3	0.1	1272	1333	2605	1.56
700	220	42	3	0.01	1245	1376	2621	1.57
500	220	42	3	0.1	1216	1416	2632	2.26
500	170	42	3	0.1	1264	1371	2636	4.32
500	170	28	3	0.1	1259	1389	2647	5.17
500	170	28	2	0.01	1280	1373	2652	3.66

Simulation Results

Using the simulation program we performed a 3^5 full-factorial experiment over the five variables. We chose three values for each variable and performed two simulations for each of the 243 possible combinations of the variables. We analyzed the results (Table 3), looking for relations between the variables and the total cost.
The driving factor of the total cost seemed to be the cost occurring for the buyer when his inventory is empty: lost profit due to lost sales and additional cost of backorders. This cost is due to the stochastic nature of the demand. To minimize this cost, it is necessary to reduce the probability of an empty stock. Therefore we need to balance the size of a delivery (defined by Q/m), the reorder point r and the lead time L. It is not easy to identify one global optimum for this situation, but there are several combinations of these variables that all give satisfying results.
Although the 4 variables Q, r, L and m will be responsible for the order of magnitude of the total cost, the variable θ is not unimportant. By choosing a good value for θ it is still possible to further reduce the costs, in most cases even by more than 10%.

MODIFICATIONS TO THE FAILURE MECHANISM

In the original model there is a probability θ that the production process goes out-of-control each time another unit is produced. Once the process is out of control, all remaining products of that production run will be damaged and they will need rework to be fixed. This failure mechanism is simple but not always very realistic. Freimer et al. (2006) describes three other failure mechanisms (see Figure 1) that are frequently used in other models. We will implement these three mechanisms in our simulation program and analyze the (cost) effects on the vendor-buyer system.

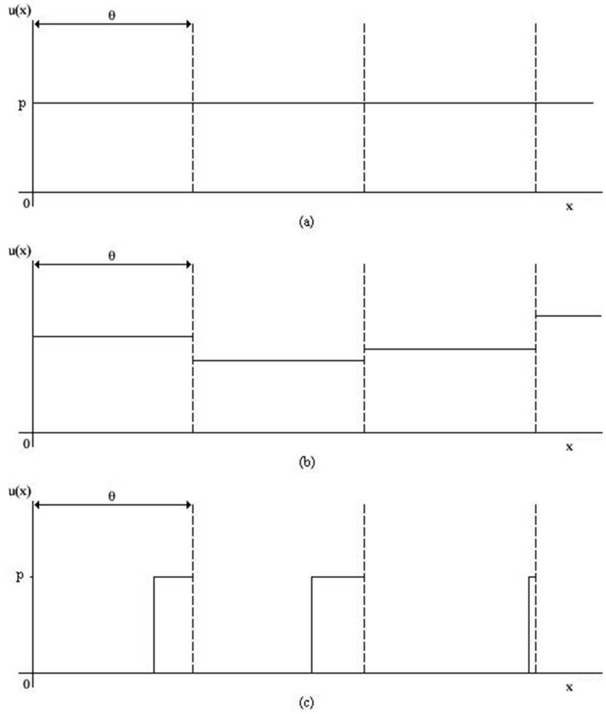

Figure 1: Freimer's different failure mechanisms:(a) Bernouilli failure, (b) Variable Bernouilli failure, (c) time to failure geometrically dictributed

(a) Bernoulli failure mechanism

For the Bernoulli-mechanism there is no longer a special out-of-control state for the production process. Every unit that is produced has the same probability p to be defective. The main implication is that the fraction of damaged products in one production run is no longer dependent on the length of the production run, because the expected fraction of defect goods will always be equal to p.

(b) Variable Bernoulli failure mechanism

The variable Bernoulli failure mechanism is very similar to the normal Bernoulli failure mechanism. The only difference is that the fraction of defective goods, p, is not the same for every production run. At the beginning of each production run the value for p is selected using a stochastic process based on a normal distribution with mean $\bar{p}$. When we consider multiple production runs the expected fraction of damaged goods will then be $\bar{p}$, but the variance of this fraction will be higher than in the previous case.

(c) Geometrically distributed time until failure

This last failure mechanism is a hybrid form between the original mechanism and the Bernoulli mechanism.

Table 4: Best results for the model with Bernoulli-failure

Q	r	L	m	p	TECb	TECv	JTEC	stddev
500	220	42	2	0.002675	1225	1302	2528	4.73
500	220	42	2	0.0005	1223	1339	2562	0.84
700	220	42	3	0.002675	1246	1322	2568	3.19
500	170	28	2	0.002675	1275	1304	2579	13.32
700	170	28	3	0.002675	1271	1326	2597	1.00
500	220	42	3	0.002675	1217	1382	2598	0.39
700	220	42	3	0.0005	1248	1361	2609	0.85
500	170	28	2	0.0005	1280	1338	2619	3.67
500	220	42	3	0.0005	1216	1420	2636	1.97
700	170	28	3	0.0005	1280	1360	2640	4.42

Table 5: Best results for the model with variable p Bernoulli-failure

Q	r	L	m	p	TECb	TECv	JTEC	stddev
500	220	42	2	0.002675	1230	1299	2529	1.08
500	220	42	2	0.0005	1229	1338	2568	2.67
700	220	42	3	0.002675	1248	1320	2568	1.99
500	170	28	2	0.002675	1273	1302	2575	4.76
500	220	42	3	0.002675	1216	1382	2598	0.77
700	170	28	3	0.002675	1279	1323	2601	8.67
500	170	28	2	0.0005	1267	1338	2605	3.20
700	220	42	3	0.0005	1246	1360	2605	1.81
700	170	28	3	0.0005	1272	1361	2633	1.96
500	220	42	3	0.0005	1215	1421	2636	0.79

Before the production process goes out-of-control, none of the products will be defective and after the process goes out-of-control a fraction p of the products will be defective. The probability of the process going out-of-control each time another unit is produced is θ'.

Simulation Results

We also performed a full-factorial experiment for each of these three new failure mechanisms. We chose the values for the new variables (p, $\bar{p}$ and θ') in such a way that the expected total number of defect units over time would be comparable to the original model.
The results (Tables 4, 5 and 6) show that the behavior of the system does not change. The best results are still obtained for the combinations of the variables that are corresponding to the best results for the original model. The only difference we observe is the variance of the total cost. The variance is lowest for the Bernoulli processes, higher for the hybrid mechanism and highest for the original mechanism.

MODIFICATIONS TO THE INVESTMENT MODEL

In the original model a continuous logarithmic capital investment function, $I(\theta) = \delta^{-1} \ln(\theta_0/\theta)$, is used to model the quality improvement on the vendor's side. This function is a good choice from a theoretical point of view, but the logarithmic function has only two parameters: θ_0, defined by the original quality of the production process and δ, which is essentially a scale parameter.

Table 6: Best results for the model with geometrically distributed time until failure

Q	r	L	m	θ' $\times 10^{-3}$	TECb	TECv	JTEC	stddev
500	220	42	2	0.0175	1226	1296	2522	18.3656
700	220	42	3	0.0175	1247	1293	2540	10.2122
500	170	28	2	0.0175	1289	1288	2578	22.8749
500	220	42	2	0.0018	1230	1355	2585	4.8385
700	170	28	3	0.0175	1276	1313	2589	11.4437
500	220	42	3	0.0175	1216	1389	2606	14.9262
700	220	42	3	0.0018	1250	1375	2626	0.6856
500	170	28	2	0.0018	1277	1354	2631	11.8734
500	170	28	3	0.0175	1259	1375	2634	10.1493
700	170	28	3	0.0018	1271	1378	2649	4.7341

Therefore the fitting capabilities are very limited in a real situation. To overcome this we introduce two new investment functions in our simulation program.

Piecewise linear function

If we use a piecewise linear function, we can divide the interval of θ in different parts and use a linear approximation in each part. The more pieces are used, the better the fit will be. In figure 2, we let θ vary between 10^{-4} and 10^{-5} in steps of 10^{-5}.

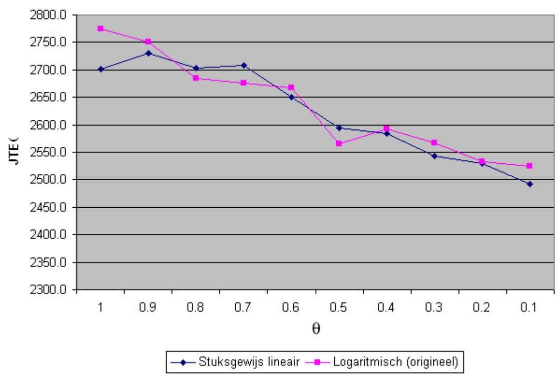

Figure 2: Comparison of the results for the logarithmic and piecewise linear investment function

Discrete function

In a real situation it could be very difficult to deduce a continuous investment function. In many cases quality improvement is done on a project basis. A certain project has a certain price and results in a corresponding quality improvement. This is no longer a continuous function but a discrete one. For the comparison in Figure 3, eight different discrete values of θ were chosen, ranging from 1.7 10^{-4} to 6 10^{-6}.

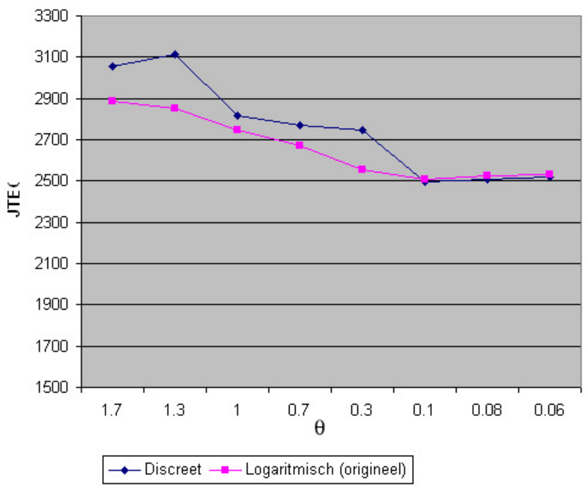

Figure 3: Comparison of the results for the logarithmic and discrete investment function

Simulation Results

We also did experiments with these new investment functions. We let θ change and kept the other variables constant. In both cases we saw the expected differences in total cost, but these were very small (Figure 2, Figure 3).

CONCLUSION

We have introduced several modifications to the original integrated inventory model. From our experiments we can conclude that the total cost is influenced mainly by the buyer in case he can not supply timely to the end customer. From the results it can be seen that it is necessary to carefully balance the buyer's order quantity Q, the buyer's reorder point r, the buyer's lead time L and the number of deliveries per order m in order to keep the total costs low. The experiments with the different failure mechanism and investment functions showed the model capable of providing meaningful results and therefore its useability in practical situations, where e.g. piecewise linear investment functions will be more readily available from historical data than the logarithmic function from the original theoretical model.

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