BER OF MRC FOR M-QAM WITH IMPERFECT CHANNEL ESTIMATION OVER CORRELATED NAKAGAMI-M FADING

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ABSTRACT

In this contribution, we provide an exact BER analysis for M-QAM transmission over arbitrarily correlated Nakagami-m fading channels with maximal-ratio combining (MRC) and imperfect channel estimation at the receiver. Assuming an arbitrary joint fading distribution and a generic pilot-based channel estimation method, we derive an exact BER expression that involves an expectation over (at most) 4 variables, irrespective of the number of receive antennas. The resulting BER expression includes well-known PDFs and the PDF of only the norm of the channel vector. In order to obtain the latter PDF for arbitrarily correlated Nakagami-m fading, several approaches from the literature are discussed. For identically distributed and arbitrarily correlated Nakagami-m channels with integer m, we present several BER performance results, which are obtained from numerical evaluation and confirmed by straightforward computer simulations. The numerical evaluation of the exact BER expression turns out to be much less time-consuming than the computer simulations.

1. INTRODUCTION

Diversity combining techniques are efficient means of mitigating the destructive effects of multipath fading on the performance of wireless communication systems [1, Chapt. 9]. When the channel state information (CSI) is known at the receiver, maximal-ratio combining (MRC) is the optimal way to combine the multiple received signals into a single signal with improved signal-to-noise ratio (SNR) [2]. In many practical applications, due to closely spaced diversity antennas, there exists correlated fading between the received diversity signals that results in a degradation of the diversity gain obtained [1]. Numerous papers deal with MRC performance analysis in the presence of arbitrarily correlated Nakagami-m fading channels, e.g., see [3–10] and references therein. The Nakagami-m distribution includes the Rayleigh distribution ($m = 1$) and the one-sided Gaussian distribution ($m = 1/2$) and is considered as a versatile statistical distribution that accurately models a variety of fading environments [11].

In practice, however, the CSI is not a priori available and the receiver has to estimate the diversity channels. The effect of estimation errors on the SNR of MRC in Rayleigh fading channels was examined in [12] and [13]. The symbol error probability (SEP) of antenna subset diversity (ASD), including the case of MRC, was studied in [14] for M-ary quadrature amplitude modulation (M-QAM) and phase-shift keying (M-PSK) on Rayleigh fading channels with imperfect channel estimation (ICE). The exact bit error rate (BER) for square/rectangular QAM with MRC and ICE in non-identical Rayleigh fading channels was given in [15]. A similar analysis for Rician fading channels was provided in [16]. In [17], approximate BER expressions were given for M-QAM with both MRC and equal-gain combining (EGC) in Nakagami fading channels with ICE. In [18], the exact bit error probability (BEP) for MRC diversity systems utilizing binary phase-shift keying (BPSK) was derived for arbitrary fading channels with ICE. The resulting BEP expression requires the evaluation of a single finite-range integral provided that one can obtain the moment generating function (MGF) of the norm square of the channel vector. However, since the analysis in [18] is based on the result of [19, Appendix B], it cannot be extended to non-binary signaling constellations.

In this paper, we provide an exact BER analysis for M-QAM signals with ICE and MRC at the receiver over arbitrarily correlated fading channels. In Section 2 we describe the observation model which includes the L arbitrary fading channels and a generic pilot-based linear channel estimation method. In Section 3, the exact BER is expressed as an expectation over (at most) 4 variables that includes known probability density functions (PDFs) and the PDF of the norm of the channel vector. In order to obtain the latter PDF for arbitrarily correlated Nakagami-m fading, several approaches from the literature are discussed in Section 4. Numerical and computer simulation performance evaluation results are presented and discussed in Section 5, assuming arbitrarily correlated and identically distributed (i.d.) Nakagami-m channels with integer m. The conclusions of the paper can be found in Section 6.

Throughout this paper, the superscript H represents the vector (matrix) conjugate transpose, while $\mathrm{diag}\{\mathbf{x}\}$, $\det(\mathbf{X})$ and $\mathbb{E}[x]$ denote the diagonal matrix with the elements of the vector $\mathbf{x}$ in the main diagonal, the determinant of matrix $\mathbf{X}$, and the expected value of x, respectively. Furthermore, unless otherwise indicated, the indexes k, ℓ, and n take values from the alphabet $\{1, 2, \ldots, L\}$.

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2. SYSTEM MODEL

Let us consider a wireless single-input multiple-output (SIMO) communication system with 1 transmit and L receive antennas, operating over a slow frequency non-selective arbitrarily correlated fading channel. Transmission is organized in frames consisting of K_p known pilot symbols and K uncoded data symbols. The pilot symbols are used by the receiver to estimate the channel, which is assumed to be constant within one frame of $N_\mathrm{fr} = K + K_\mathrm{p}$ symbols and changes independently from one frame to another (block fading). The $L \times N_\mathrm{fr}$ received signal matrix $\mathbf{R}_\mathrm{tot}$ is given by

$$\mathbf{R}_\mathrm{tot} = \left[\mathbf{R}_\mathrm{p}\,\mathbf{R}\right] = \mathbf{h}\left[\sqrt{E_\mathrm{p}}\mathbf{a}_\mathrm{p}\ \sqrt{E_\mathrm{s}}\mathbf{a}\right] + \left[\mathbf{W}_\mathrm{p}\,\mathbf{W}\right], \qquad (1)$$

where $\mathbf{h}$ is the $L \times 1$ complex channel vector with elements $h_\ell = r_\ell \exp(-j\,\phi_\ell)$ where r_ℓ is the fading envelope, $j^2 = -1$, and ϕ_ℓ is the random phase that is assumed to be uniformly distributed over the range $[0, 2\pi)$. The $1 \times K_\mathrm{p}$ pilot vector $\mathbf{a}_\mathrm{p}$ and the $1 \times K$ data vector $\mathbf{a} = [a_1, a_2, \ldots, a_K]$ consist of the pilot symbols and the information symbols, respectively. The $L \times N_\mathrm{fr}$ matrix $\left[\mathbf{W}_\mathrm{p}\,\mathbf{W}\right]$ describes additive spatially and temporally white noise and consists of independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables (RVs) with variance N_0. We assume a normalized M-QAM constellation for the information symbols ($\mathbb{E}\left[|a_i|^2\right] = 1$, $1 \le i \le K$), such that their average transmitted energy is E_s. Similarly, the energy of the transmitted pilot symbols is E_p.

In order to estimate $\mathbf{h}$ from the known pilot vector $\mathbf{a}_\mathrm{p}$ and the corresponding received signal matrix $\mathbf{R}_\mathrm{p}$, the receiver uses a linear channel estimator of the form

$$\hat{\mathbf{h}} = \frac{\alpha}{K_\mathrm{p}\sqrt{E_\mathrm{p}}}\mathbf{R}_\mathrm{p}\mathbf{a}_\mathrm{p}^H, \qquad (2)$$

with $\alpha \in \mathbb{R}$, such that $\hat{\mathbf{h}}$ can be decomposed into the sum of two statistically independent contributions as

$$\hat{\mathbf{h}} = \alpha\mathbf{h} + \mathbf{n}, \qquad (3)$$

where the entries of $\mathbf{n} = \left(\alpha/(K_\mathrm{p}\sqrt{E_\mathrm{p}})\right)\mathbf{W}_\mathrm{p}\mathbf{a}_\mathrm{p}^H$ are ZMCSCG RVs; the real and imaginary parts of the entries of $\mathbf{n}$ have a variance $\sigma_\mathbf{n}^2 = \alpha^2 N_0/(2K_\mathrm{p}E_\mathrm{p})$. Hence, when conditioned on $\mathbf{h}$, the channel estimate $\hat{\mathbf{h}}$ is a complex Gaussian RV with mean $\alpha\mathbf{h}$ and diagonal covariance matrix with diagonal elements $2\sigma_\mathbf{n}^2$. Both least-squares and linear MMSE estimation satisfy (2) with $\alpha = 1$ and $\alpha = K_\mathrm{p}E_\mathrm{p}/(K_\mathrm{p}E_\mathrm{p} + N_0)$, respectively [20].

Allocating a large total energy $K_\mathrm{p}E_\mathrm{p}$ to pilot symbols yields an accurate channel estimate, but on the other hand gives rise to a reduction of the symbol energy E_s. When E_b denotes the energy per information bit and $\gamma \triangleq E_\mathrm{p}/E_\mathrm{s}$ is the ratio of the pilot energy to the data energy, we have

$$E_\mathrm{s} = \frac{K}{K + \gamma K_\mathrm{p}}\log_2(M)E_\mathrm{b}, \qquad (4)$$

where M denotes the number of constellation points. Hence, E_s decreases when the number of pilot symbols K_p is increased.

3. BER FOR M-QAM OF MRC WITH ICE

We consider a mismatched L-branch MRC receiver that uses the estimated channel vector $\hat{\mathbf{h}}$ in the same way an MRC receiver with perfect channel knowledge (PCK) would use the actual channel vector $\mathbf{h}$. In this way, the detection algorithm reduces to symbol-by-symbol detection:

$$\hat{a}_i = \arg\min_{\tilde{a}} |u_i - \tilde{a}|, \; 1 \le i \le K. \qquad (5)$$

where the MRC decision variables u_i are given by

$$u_i = \frac{\hat{\mathbf{h}}^H\mathbf{r}_k}{\sqrt{E_\mathrm{s}}\left|\hat{\mathbf{h}}\right|^2}, \qquad (6)$$

with $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_K]$.

In this contribution, square M-QAM transmission with Gray mapping is considered, which is equivalent to $\sqrt{M}$-PAM transmission for both the in-phase and quadrature information bits. Because of the rotational symmetry of the M-QAM constellation, it is readily verified that the BERs related to the in-phase and quadrature bits are equal. Hence, it is sufficient to carry out the BER analysis for the in-phase bits only. Also, since the conditional BER is the same for all data symbols a_i, irrespective of i, we may drop the index i in the BER analysis. In this way, the BER for the mismatched MRC receiver is obtained by averaging the conditional BER for the in-phase bits (conditioned on the channel $\mathbf{h}$, the channel estimate $\hat{\mathbf{h}}$ and the transmitted symbol a). It is shown in [21, eq. (23)] that the resulting BER expression can be reduced to

$$\mathrm{BER} = \int \mathrm{BER}_\mathrm{R}(x_1, x_2, z, u) \; p(x_1, x_2, z \big| |\mathbf{h}| = u)\, p(u)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}z\mathrm{d}u \qquad (7)$$

where $|\mathbf{h}|$ is the norm of $\mathbf{h}$, $p(u)$ is the PDF of $|\mathbf{h}|$ and $\mathrm{BER}_\mathrm{R}(x_1, x_2, z, u)$ is derived in [21, eq. (10)–(22)]. When conditioned on $|\mathbf{h}|$, x_1, x_2, z are independent variables given by [21, eq. (20)–(22)], which satisfy the following properties:

- x_1 is a Gaussian RV with mean $\alpha|\mathbf{h}|$ and variance $\sigma_\mathbf{n}^2$.
- x_2 is a Gaussian RV with zero-mean and variance $\sigma_\mathbf{n}^2$.
- $z/\sigma_\mathbf{n}$ is a chi-distributed RV with $2L - 2$ degrees of freedom [22].

It is important to note that instead of the joint distribution $p(\mathbf{h})$ of the L complex-valued fading gains, we need only the distribution of the norm $|\mathbf{h}|$. Hence, the BER for M-QAM involves an expectation over only 4 random variables. The expectation (7) is evaluated numerically by approximating the 4-fold integral by a 4-fold sum, running over discretized versions of the continuous variables x_1, x_2, z and u. Considering the PDFs of the independent variables x_1, x_2 and z (conditioned on $|\mathbf{h}|$), the integrand in (7) is, apart from $p(u)$, the product of well-known analytical functions. Since the numerical evaluation of (7) requires the variable u to be discretized, the PDF $p(u)$ of $|\mathbf{h}|$ can be available either in analytical form or as a (discrete) histogram obtained from experiments.

The number of random variables to be considered in the expectation (7) can be further reduced for PAM and BPSK constellations and/or real-valued channels [21].

4. PDF OF THE NORM OF THE CHANNEL VECTOR IN CORRELATED NAKAGAMI-*M* FADING

Let r_ℓ's (with $\ell \in \{1,\ldots,L\}$ and $L \geq 2$) be arbitrarily correlated and not necessarily i.d. Nakagami-*m* distributed RVs with marginal PDFs given by [11, eq. (22)]

$$p_{r_\ell}(r) = \frac{2\, r^{2m_\ell - 1}}{\Gamma(m_\ell)\, \Omega_\ell^{m_\ell}} \exp\left(-\frac{r^2}{\Omega_\ell}\right) U(r), \tag{8}$$

with $\Gamma(\cdot)$ being the Gamma function [23, eq. (8.310/1)], $U(r)$ being the unit step function, $\Omega_\ell = \mathbb{E}\left[r_\ell^2\right]/m_\ell$ being a parameter related to the average fading power, and $m_\ell \geq 1/2$ being the fading parameters. The (k,n)-th element of the power correlation matrix, $\boldsymbol{\Sigma}$, of r_ℓ's is given by $\boldsymbol{\Sigma}^{k,n} = 1$ for $k = n$ and $\boldsymbol{\Sigma}^{k,n} = \boldsymbol{\Sigma}^{n,k} = \rho_{k,n}$ for $k \neq n$ with $0 \leq \rho_{k,n} < 1$. The $\rho_{k,n}$'s are the Nakagami-*m* power correlation coefficients, i.e., the correlation between r_k^2 and r_n^2 [1, eq. (9.195)].

Different approaches have been presented for deriving analytical expressions for the distribution of $|\mathbf{h}|^2$ when assuming arbitrarily correlated Nakagami-*m* fading channels, e.g., see [3, 5–10] and references therein. These approaches obtain arbitrarily correlated Nakagami-*m* RVs either from Gamma RVs [7–9] or from Gaussian RVs for integer values of m_ℓ [3, 5, 6, 10]. Analytical expressions for the moment generating function (MGF) of $|\mathbf{h}|^2$ have been derived for integer $m_\ell = m$, $\forall \ell$ [3, 5, 10], integer m_ℓ [6] and arbitrary m_ℓ [9]. Although, the obtained expressions in [3,5,10] can be straightforwardly used for the derivation of $p(u)$, this seems complicated using the MGF expression presented in [6] and rather difficult with that in [9]. On the other hand, the PDF-based approach has been used for deriving the distribution of $|\mathbf{h}|^2$ for arbitrary $m_\ell = m$, $\forall \ell$ [7] and for integer m_ℓ with the restriction that $\Omega_1 \neq \Omega_2 \neq \cdots \neq \Omega_L$ [8].

By using the vast majority of the aforementioned approaches, the PDF of $|\mathbf{h}|^2$, $f_{|\mathbf{h}|^2}(x)$, is given by either a finite or infinite sum of terms of the form

$$A x^{\xi - 1} \exp(-Bx)\, U(x), \tag{9}$$

where the parameters A, ξ and B depend on the Nakagami-*m* parameters m_ℓ, Ω_ℓ, and on the power correlation matrix $\boldsymbol{\Sigma}$.

For integer $m_\ell = m$, $\forall \ell$, the pdf of $|\mathbf{h}|^2$ is given by [3, 5]

$$f_{|\mathbf{h}|^2}(x) = \sum_{i=1}^{\kappa} \sum_{q=1}^{c_i m} \frac{C_{i,q}}{\lambda_i^q\, (q-1)!} x^{q-1} \exp\left(-\frac{x}{\lambda_i}\right) U(x). \tag{10}$$

Here, λ_i's, $i = 1, 2, \cdots, \kappa$, are the distinct eigenvalues of $\boldsymbol{\Omega}^{1/2}\, \boldsymbol{\Sigma}_{\mathbf{G}}\, \boldsymbol{\Omega}^{1/2}$, with corresponding algebraic multiplicities c_i, whereby the (k,n)-th element of the matrix $\boldsymbol{\Sigma}_{\mathbf{G}}$ is defined as $\Sigma_{\mathbf{G}}^{k,n} = \sqrt{\rho_{k,n}}$, and $\boldsymbol{\Omega} = \mathrm{diag}\{\Omega_1, \Omega_2, \ldots, \Omega_L\}$. The parameters $C_{i,q}$ are given by

$$C_{i,q} = \frac{\lambda_i^{q - c_i m}}{(c_i m - q)!} \left[\frac{\mathrm{d}^{c_i m - q}}{\mathrm{d}s^{c_i m - q}} \Psi_i(s)\right]\Big|_{s = -\frac{1}{\lambda_i}}, \tag{11}$$

where

$$\Psi_i(s) = \prod_{\substack{j=1 \\ j \neq i}}^{\kappa} (1 + s\lambda_j)^{-c_j m}. \tag{12}$$

Alternatively, by applying a tridiagonal decomposition to $\mathbf{W} = \boldsymbol{\Sigma}_{\mathbf{G}}{}^{-1}$ for integer $m_\ell = m$ and $\Omega_\ell = \Omega$, $\forall \ell$, $f_{|\mathbf{h}|^2}(x)$ can be expressed by fast convergent infinite summations [10] as

$$\begin{aligned} f_{|\mathbf{h}|^2}(x) = & \frac{\det(\mathbf{W})^m\, \Omega^{Lm}}{(m-1)!} \sum_{k_1, k_2, \ldots, k_{L-1} = 0}^{\infty} \left[\prod_{i=1}^{L-1} \left(\frac{p_{i,i+1}}{\Omega}\right)^{2k_i}\right] \\ & \times \left[\frac{\prod_{\ell=1}^{L} (b_\ell - 1)!}{\prod_{i=1}^{L-1} k_i!\, (k_i + m - 1)!}\right] \\ & \times \sum_{i=1}^{L} \sum_{t=1}^{b_i} \frac{D_{i,t}}{(t-1)!} x^{t-1} \exp\left(-\frac{p_{i,i}}{\Omega} x\right) U(x), \end{aligned} \tag{13}$$

where $p_{i,j}$ are the elements of the tridiagonal form of $\mathbf{W}$ [10, eq. (4)], $b_1 = k_1 + m$, $b_L = k_{L-1} + m$, $b_j = k_{j-1} + k_j + m$, $\forall j = 2, 3, \ldots, L-1$, and

$$D_{i,t} = \frac{1}{(b_i - t)!} \left[\frac{\mathrm{d}^{b_i - t}}{\mathrm{d}s^{b_i - t}} \Phi_i(s)\right]\Big|_{s = -\frac{p_{i,i}}{\Omega}} \tag{14}$$

with

$$\Phi_i(s) = \prod_{\substack{j=1 \\ j \neq i}}^{L} \left(s + \frac{p_{j,j}}{\Omega}\right)^{-b_j}. \tag{15}$$

For arbitrary $m_\ell = m$, $\forall \ell$, the pdf of $|\mathbf{h}|^2$ can be easily obtained from [7, eq. 5], and includes an infinite summation of terms of the form (9).

By using (10), (13) or the PDF resulting from [7, eq. 5], and a standard RVs transformation, $p(u)$ needed in (7) can be easily obtained for arbitrarily correlated Nakagami-*m* fading channels.

5. RESULTS AND DISCUSSION

In this section, we illustrate the exact BER analysis of MRC over arbitrarily correlated Nakagami-*m* fading channels by numerically evaluating (7). Assuming i.d. channels with integer $m_\ell = m$, $\forall \ell$, the PDF of the norm of the channel vector is easily obtained from (10). Channel state information is either known by the receiver or obtained through linear MMSE channel estimation ($E_p = E_s$). All numerical results have been obtained for QPSK transmission. For the correlation matrix (CM), $\boldsymbol{\Sigma}_{\mathbf{G}}$, the following correlations have been considered:

- *i*) Uncorrelated CM (U-CM), where $\boldsymbol{\Sigma}_{\mathbf{G}} = \mathbf{I}_L$, with $\mathbf{I}_L$ being the $L \times L$ identity matrix,
- *ii*) Linear CM (L-CM), with $\boldsymbol{\Sigma}_{\mathbf{G}}$ given by [5, eq. (38)],
- *iii*) Triangular CM (T-CM), with $\boldsymbol{\Sigma}_{\mathbf{G}}$ given by [5, eq. (37)],
- *iv*) Constant CM (C-CM), with $\Sigma_{\mathbf{G}}^{k,n} = 0.6$ for $k \neq n$,
- *v*) Arbitrary CM (A_1-CM), with $\boldsymbol{\Sigma}_{\mathbf{G}}$ given by [10, eq. (14)]), and
- *vi*) Arbitrary CM (A_2-CM), with $\boldsymbol{\Sigma}_{\mathbf{G}}$ given by [10, eq. (34)]).

Fig. 1 shows the BER curves for a three-branch MRC receiver ($L = 3$). The results are shown for both a PCK receiver and a mismatched receiver using MMSE channel estimation (with $K = 100$ and $K_p = 10$), for $m \in \{1, 4\}$, and for different correlation models. These correlation models include uncorrelated fading, with correlation matrix U-CM according to *i*), and a linear and a triangular antenna array, with

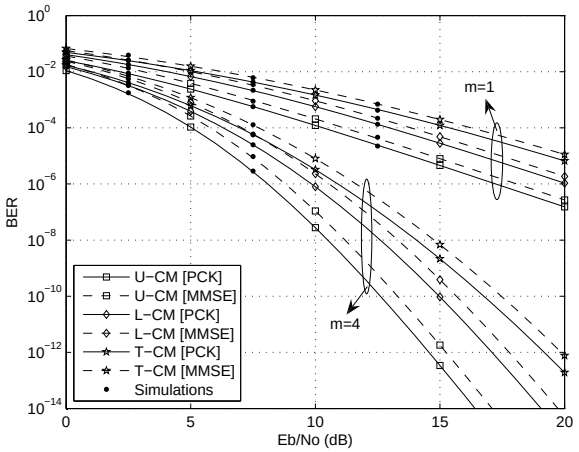

Figure 1: Three-branch MRC ($L = 3$) over correlated Nakagami-m channels, with uncorrelated (U-CM), linear (L-CM) and triangular (T-CM) correlation matrices. Computer simulations confirm the analytical curves obtained from numerical evaluation.

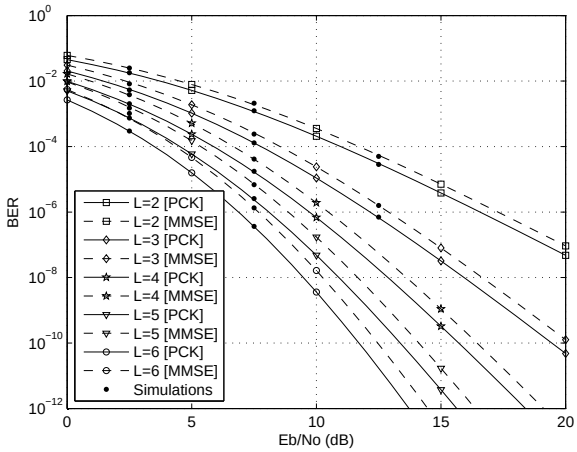

Figure 2: MRC over correlated Nakagami-m channels (QPSK, $m = 2$, constant correlation matrix (C-CM)). Results are shown for several values of the number of receive antennas L. Simulations match the analytical result from numerical evaluation.

correlation matrices L-CM and T-CM according to *ii*) and *iii*), respectively. Also shown in the figure are computer simulations that perfectly match the analytical curves obtained from numerical evaluation. From fig. 1, we can see how the fading severity parameter m, imperfect channel estimation and branch correlation affect the BER of MRC reception. A larger m indicates less severe fading and causes improved BER performance through a larger diversity gain [1, p. 797]. Both imperfect channel estimation and branch correlation degrade the BER through a horizontal shift of the BER curve for large SNR, but without affecting the diversity gain. For MRC on arbitrarily correlated Nakagami-m fading channels, the diversity gain equals mL [24]. Note that for highly correlated channels (e.g., the triangular correlation model T-CM), the BER degradation due to branch correlation is much larger than the degradation due to imperfect channel estimation.

Fig. 2 displays the BER curves for L-branch MRC reception over correlated Nakagami-m channels with $m = 2$, for several values of L. We assume a constant correlation model [4, eq. (11)] with correlation matrix C-CM according to *iv*). The results are shown for PCK and MMSE channel estimation with $K = 100$ and $K_p = 10$. Again, computer simulations confirm the result from numerical evaluation. From fig. 2, it is clear that the number of receive antennas L has a significant impact on the BER, because the application of multiple antennas benefits not only from an increased diversity order, but also from an array gain caused by combining the energy received by each of the antennas.

Fig. 3 shows the BER curves for MRC reception over arbitrarily correlated Nakagami-m channels. The results are shown for a PCK receiver and a mismatched receiver using MMSE channel estimation with $K = 100$ and several values of the number of pilot symbols (K_p), and for $m \in \{1,3\}$. Both four-branch ($L = 4$) and six-branch ($L = 6$) MRC are considered with correlation matrices A_1-CM and A_2-CM according to *v*) and *vi*), respectively. In order not to overload the figure, computer simulations are not shown in the figure.

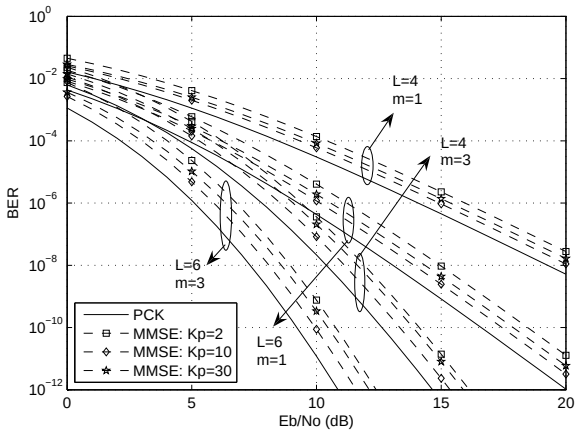

Figure 3: MRC over correlated Nakagami-m channels (QPSK, $m \in \{1,3\}$, $L = 4$ and $L = 6$ with corresponding arbitrary correlation matrices A_1-CM and A_2-CM). Results are shown for several values of the number of pilot symbols K_p.

6. CONCLUSIONS AND REMARKS

In this contribution, we derived an exact BER expression for M-QAM transmission over arbitrarily distributed (correlated) fading channels with maximal-ratio combining (MRC) and imperfect channel estimation (ICE) at the receiver. Channel state information was assumed to be obtained by a generic pilot-based linear channel estimation method, which includes the well-known least-squares estimation and linear minimum mean-square error (MMSE) estimation as special cases. The resulting BER expression involves an expectation over (at most) 4 variables, irrespective of the number of receive antennas, and includes known probability density functions (PDFs) and the PDF of the norm of the channel vector. Several existing approaches to obtain the latter PDF for arbitrarily correlated Nakagami-m fading were discussed.

Assuming arbitrarily correlated and i.d. Nakagami-m channels with integer m, numerical and computer simulation performance evaluation results were presented and discussed. Comparing the computing times resulting from numerical averaging and from straightforward simulation, it turns out that the numerical evaluation is to be preferred.

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