CONVERGENCE OF THE BACKWARD EULER METHOD FOR NONLINEAR MAXWELL EQUATIONS.

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Abstract. Let Ω be an open and bounded domain in $\mathbb{R}^3$, with Lipschitz continuous boundary $\partial\Omega$. This domain is occupied with a nonlinear conducting material with electric conductivity $\sigma(\mathbf{x}, |\mathbf{E}|)$, which is assumed to be a monotone function of the form $|\mathbf{E}|^{\alpha-1}$. Possible current sources in the domain are gathered in the source term $\mathbf{F}$. Electromagnetic fields in this domain are described by the Maxwell equations, yielding the following hyperbolic PDE

$$
\begin{cases} \partial_{tt}\mathbf{E} + \partial_t\left(\sigma(\mathbf{x}, |\mathbf{E}|)\mathbf{E}\right) + \nabla\times\nabla\times\mathbf{E} = \mathbf{F}, & (\mathbf{x}, t) \in \Omega\times(0,T], \\ \mathbf{E}(\mathbf{x}, 0) = \mathbf{E}_0, \partial_t\mathbf{E}(\mathbf{x}, 0) = \mathbf{E}_0', & \mathbf{x}\in\Omega, \\ \mathbf{n}\times\mathbf{E}(\mathbf{x}, t) = 0, & (\mathbf{x}, t)\in\partial\Omega\times(0,T]. \end{cases} \tag{1}
$$

A power law for the conductivity occurs in various physical models such as the constitutive law for type-II superconductors [1] and the modelling of the nonlinear conductivity of the charge-density-wave state of $NbSe_3$ [2]. Similar nonlinearities have been studied in the context of quasi-static Maxwell equations, resulting in a nonlinear, parabolic PDE [3, 4].

We will study the convergence of a semi-discrete approximation scheme for (1), based on backward Euler's method. The major difficulty of the hyperbolic system, compared to the parabolic system resulting from quasi-static Maxwell equations, is the abscense of estimates on the second time derivative. However, we are able to prove convergence based on the boundedness of the sum of the first two terms in equation (1) in the sense of the dual space. The monotonicity of the function σ plays a vital role, since it allows to use the Minty-Browder technique [5] to pass weak convergence through a nonlinear term.

After the proof of convergence we will derive the corresponding error estimates, yielding also in the uniqueness of the problem in appropriate function spaces. Finally, the numerical scheme is tested using a finite element model using curl-conforming edge elements in 3D [6].

Keywords: nonlinear Maxwell's equations, semi-discrete scheme, error estimates

Mathematics Subject Classifications (2000): 65N12, 65N15,78M10

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