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INTRODUCING A NOVEL AUTO-TUNER AS AN EDUCATIONAL TOOL

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Abstract: This paper presents 2 student projects regarding a recently developed extension of the widely used relay-feedback PID auto-tuner. The proposed method consists of two steps: process identification and controller design. First, a non-iterative procedure is suggested for identification of two points on the process Nyquist curve. A second-order-plus-dead-time model is obtained and then used for PID controller design based on the internal model principle (IMC). For the identification of the two points on the Nyquist curve a pure relay in the feedback loop (as used in standard auto-tuning) and a relay which operates on the integral of the error are used. The method is illustrated on two applications: a continuous stirred tank reactor and a boost DC-DC converter. These examples are relevant for the students following the basic course in control engineering. The influence of changing a design parameter – the desired closed loop behaviour – is shown. The several tuning options lead to changes in the performance, making the method a useful tool in control education.

Keywords: auto-tuning, PID controller, relay experiment, Nyquist plot, chemical industry, power circuits, teaching.

1. INTRODUCTION

Despite the development of more advanced control strategies, the PID still holds the majority number of controllers used in industrial instrumentation. Their popularity is justified by the following advantages: they have a simple structure, their principle is well understood by instrumentation engineers, a good process model is not required and their control capabilities have proven to be adequate for most control loops. Moreover, due to process uncertainties, a more sophisticated control scheme is not necessarily more efficient in real-life applications than a well-tuned PID controller. However, it is common that PID controllers are poorly tuned in practice because the choice of controller parameters requires professional – specialized knowledge by the user. To simplify this task, PID controllers can incorporate *auto-tuning* capabilities, which may reduce dramatically the start-up period. The auto-tuners are equipped with a mechanism capable of automatically computing a reasonable good set of parameters when the regulator is connected to the process (Åström, *et al*., 1984; Åström, *et al*., 1992; Åström, *et al*., 1995; Leva, *et al*., 2002). Auto-tuning is a very desirable feature and almost every industrial PID controller provides it nowadays. These features provide easy-to-use controller tuning and have proven to be well accepted among process engineers.

For the automatic tuning of the PID controllers, several methods have been proposed. Some of these methods are based on identification of one point of the process frequency response, while others are based on the knowledge of some characteristic parameters of the open-loop process step response. The identification of a point of the process frequency response can be performed either using a proportional regulator, which brings the closed-loop system to the stability boundary, or by a relay forcing the process output to oscillate. Åström and Hägglund (Åström, *et al*., 1984; Åström, *et al*., 1992; Schei, 1994; Åström, *et al*., 1995; Leva, et al., 2002) report an important and interesting approach. Their method is based on the Ziegler-Nichols frequency domain design formula. A relay connected in a feedback loop with the process is used in order to determine the critical point.

This contribution briefly describes the development of an auto-tuning method based on the identification of *two* points on the process Nyquist curve (Nascu and De Keyser, 2003): the intersection with the real negative axis and that with the imaginary negative axis. The use of two relay feedback experiments provides information to determine a four parameters second-order-plus-dead-time (SOPDT) model. The use of this auto-tuning technique is suitable for illustrating the effect of design parameter upon the closed-loop behaviour. Its simple and effective approach could make a good educational tool for students following the basic course in control engineering.

The paper is organized as follows: the experimental setup is described in the next section, and the tuning method in the third section. Two examples are provided in the fourth section and a conclusion section summarizes the main ideas.

2. THE EXPERIMENTAL SETUP

A standard PID control system with single input single output, as shown in figure 1, is considered.

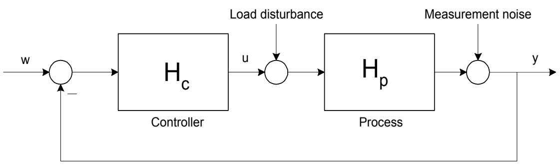

Fig. 1. Control system structure.

The process H_p is assumed to be linear, stable and proper. The PID controller has a non-interacting structure cascaded with a first order filter:

$$H_c(s) = K_c\left(1 + \frac{1}{sT_i} + sT_d\right)\frac{1}{T_f s + 1} \qquad (1)$$

For the purpose of PID-controller design, higher order models are of limited utility even if the process dynamics are theoretically of high order. From the control point of view, complex process models lead to complex controllers. Several papers on PID control are based on the idea of using second order with time delay models. In this paper it is assumed that the process dynamics can be described with reasonable accuracy by a second-order-plus-dead-time (SOPDT) model as:

$$H_p(s) = \frac{k_p\, e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}, \quad \theta \geq 0 \text{ and } \tau, \zeta > 0 \qquad (2)$$

In order to tune the PID controller we will approximate $e^{-\theta s}$ by *(1-θs)* and using the well-known IMC-PID design method (Morari and Zafiriou, 1989; Rivera *et al.*, 1986) we then obtain:

$$K_c = \frac{2\zeta\tau}{k_p(2T+\theta)}, \quad T_i = 2\zeta\tau$$
$$T_d = \frac{\tau^2}{2\zeta\tau}, \qquad T_f = \frac{T^2}{2T+\theta} \qquad (3)$$

thus resulting in the closed-loop transfer function:

$$H_{CL}(s) = \frac{H_p(s)\cdot H_c(s)}{1 + H_p(s)\cdot H_c(s)} = \frac{e^{-\theta s}}{(Ts+1)^2} \qquad (4)$$

The parameter T is the closed-loop time constant and is a design parameter which can be selected by the user in order to tune the controller *aggressiveness*.

The parameters of the process transfer function (2) are assumed to be unknown and - in order to identify them - 2 points of the frequency response have to be estimated using two relay feedback experiments: the first experiment uses an integrating relay IR (= a relay without hysteresis which has as input the integral of the error) and the second experiment uses a pure relay (without hysteresis). After performing these two experiments the values of the process frequency response at two different frequencies ω_1 and ω_2 are obtained:

$$H_p(j\omega_1) = a_1 + jb_1 \text{ and } H_p(j\omega_2) = a_2 + jb_2 \qquad (5)$$

The procedure how to perform these experiments and how to obtain the values for (a_i,b_i; i=1,2) is further explained in section 3.

3. THE TUNING METHOD

Note from (2) that $H_p(j\omega)$ can be written as follows:

$$H_p(j\omega) = H'_p(j\omega)\, H''_p(j\omega) \qquad (6)$$

where

$$H'_p(j\omega) = \frac{k_p}{1 - \tau^2\omega^2 + 2\zeta\tau\, j\omega}, \quad H''_p(j\omega) = (1 - \theta\, j\omega)$$

If we define - for a certain point on the Nyquist plot - the magnitude, the phase angle and the real and imaginary components of $H_p(j\omega)$, $H'_p(j\omega)$ and $H''_p(j\omega)$ to be M, α, a, b, respectively M', α', a' ,b' and M'', α'', a'', b'', then (Ogata, 1990):

$$M^2 = a^2 + b^2 = M'^2 M''^2 = (a'^2 + b'^2)(a''^2 + b''^2)$$
$$\alpha = \alpha' + \alpha'' \qquad (7)$$

$$tg(\alpha') = \frac{b'}{a'} = tg(\alpha - \alpha'') = \frac{tg(\alpha) - tg(\alpha'')}{1 + tg(\alpha)tg(\alpha'')} = $$
$$= \frac{\frac{b}{a} - \frac{b''}{a''}}{1 + \frac{b}{a}\frac{b''}{a''}} = \frac{a''b - ab''}{aa'' + bb''} \quad (8)$$

$$b' = a'\frac{a''b - ab''}{aa'' + bb''} \qquad a' = b'\frac{aa'' + bb''}{a''b - ab''} \quad (9)$$

From (9) - and afterwards (7) - it can be written:

$$(a'^2 + b'^2) = a'^2\left[1 + \left(\frac{a''b - ab''}{aa'' + bb''}\right)^2\right] =$$
$$= a'^2\frac{(a''^2 + b''^2)(a^2 + b^2)}{(aa'' + bb'')^2} = \quad (10)$$
$$= a'^2\frac{(a''^2 + b''^2)^2(a'^2 + b'^2)}{(aa'' + bb'')^2}$$

thus leading to:

$$aa'' + bb'' = a'(a''^2 + b''^2) = a'M''^2 \quad (11)$$

and using (9) again also to:

$$a''b - ab'' = b'M''^2 \quad (12)$$

Thus for the two points obtained from the relay feedback experiments we have:

$$a_1a_1'' + b_1b_1'' = a_1'M_1''^2 \qquad a_1''b_1 - a_1b_1'' = b_1'M_1''^2 \quad (13a)$$
$$a_2a_2'' + b_2b_2'' = a_2'M_2''^2 \qquad a_2''b_2 - a_2b_2'' = b_2'M_2''^2 \quad (13b)$$

The parameters a_j and b_j (j=1,2) can be obtained using the two relay feedback experiments. In order to identify a point on the process Nyquist curve, a relay connected in a feedback loop with the process is used, forcing the process output to oscillate. Therefore assume that d_j is the relay amplitude and ε_j is the relay hysteresis width. For the given values d_j and ε_j we will obtain oscillation with amplitude h_j and period T_j in the process output.

The first experiment uses an integrating relay ($\varepsilon_1=0$), identifying the point given by the intersection of the process Nyquist curve and the negative imaginary axis, $P_i(a_1,jb_1)$.

$$\omega_1 = \frac{2\pi}{T_1},\ a_1 = 0,\ b_1 = -\frac{\pi h_1}{4d_1},\ M_1^2 = b_1^2 \quad (14)$$

The second experiment uses a pure relay ($\varepsilon_2=0$) and the point given by the intersection of the process Nyquist curve and the negative real axis, $P_p(a_2,jb_2)$, is identified:

$$\omega_2 = \frac{2\pi}{T_2},\ a_2 = \frac{-\pi h_2}{4d_2},\ b_2 = 0,\ M_2^2 = a_2^2 \quad (15)$$

The real and imaginary components of $H_p'(j\omega_i)$ and $H_p''(j\omega_i)$ can be written as:

$$a_i' = \mathrm{Re}\left(H_p'(j\omega_i)\right) = \frac{k_p\left(1-\tau^2\omega_i^2\right)}{\left(1-\tau^2\omega_i^2\right)^2 + (2\zeta\tau\omega_i)^2} = \frac{\left(1-\tau^2\omega_i^2\right)M_i'^2}{k_p}$$
$$b_i' = \mathrm{Im}\left(H_p'(j\omega_i)\right) = \frac{k_p(-2\zeta\tau\omega_i)}{\left(1-\tau^2\omega_i^2\right)^2 + (2\zeta\tau\omega_i)^2} = \frac{-2\zeta\tau\omega_i M_i'^2}{k_p} \quad (16)$$
$$a_i'' = \mathrm{Re}\left(H_p''(j\omega_i)\right) = 1 \quad b_i'' = \mathrm{Im}\left(H_p''(j\omega_i)\right) = -\theta\omega_i$$

Inserting in (13) the relations (16) for $a_i',\ b_i',\ a_i'',\ b_i'';\ i = 1,2$ gives four equations with four unknown parameters: k_p, θ, τ^2 and $2\zeta\tau$:

$$a_1 - b_1\theta\omega_1 = \frac{\left(1-\tau^2\omega_1^2\right)M_1^2}{k_p} \quad (17a)$$

$$b_1 + a_1\theta\omega_1 = \frac{-2\zeta\tau\omega_1M_1^2}{k_p} \quad (17b)$$

$$a_2 - b_2\theta\omega_2 = \frac{\left(1-\tau^2\omega_2^2\right)M_2^2}{k_p} \quad (17c)$$

$$b_2 + a_2\theta\omega_2 = \frac{-2\zeta\tau\omega_2M_2^2}{k_p} \quad (17d)$$

We obtain the following parameters which can then be used in (3) to tune the PID:

$$\theta = \frac{a_2}{\omega_1b_1}, \quad k_p = \frac{a_2b_1^2\left(\omega_1^2 - \omega_2^2\right)}{b_1\omega_1(a_2\theta\omega_2^2 + b_1\omega_1)}$$
$$\tau^2 = \frac{1}{\omega_1^2}\left(1 + \frac{k_pa_2}{b_1^2}\right), \quad 2\zeta\tau = -\frac{k_p}{\omega_1b_1} \quad (18)$$

This auto-tuning technique will be illustrated with two examples in the following section.

4. EXAMPLES

Usually, the auto-tuning technique is presented for students opting for a specialization in control engineering – automation. However, classic control techniques (PID) are taught in earlier years, for all students, before choosing their specialization, as part of the basic course in control engineering.

Since the auto-tuner can be derived without the fully specialized knowledge of a control engineer, it would be useful that they are taught as well within the basic course of control. The need for simple but effective control techniques is justified by the fact that only a part of the students will specialize in control and automation, while the majority will need just basic knowledge. Therefore, the choice of applicative examples plays a crucial role in the good understanding and the motivation of the students.

The two applications described in this section are practical and describe real-life systems by use of (approximated) linear transfer functions. A continuous stirred tank reactor and a boost DC-DC converter make a suitable pair of illustrative examples as they are applications for chemical and respectively, electrical engineering students. As a result, the students not only learn to apply the auto-tuning control technique, but they are also motivated by the choice of examples in their (future) field of interest. For both examples, a suitable controller will be derived and their results will be discussed further in this section.

4.1 First example: a continuous stirred tank reactor (CSTR)

Such systems as a CSTR (see figure 2) are usually highly nonlinear, multivariable, complex and with limited stability. By linearization around a working (equilibrium) point, linear transfer functions can be obtained for each input-output relationship.

In this example, a simple case of a first order, irreversible chemical reaction $A \xrightarrow{k} B$ was considered. Schematically, the input-output system can be depicted as in figure 3. By linearization, a working set of 8 linear transfer functions is obtained, describing the relationship between each I/O variable. Only one of the transfer functions will be used in this section and it denotes the relationship between the cooling temperature T_c of the reactor's jacket and the concentration c_o of the output product:

$$H_1(s) = \frac{497.8e^{-0.5s}}{s^2 + 1.806s + 35.95} \qquad (19)$$

Fig. 2. Continuous stirred tank reactors (CSTR).

The poles of the transfer function are at $-0.903 \pm 5.927j$ and therefore close to the value of dead-time, making it difficult to control.

Recalling from section two, the identified model of form:

$$H_p(s) = \frac{k_p e^{-0.5}}{\tau^2 s^2 + 2\zeta\tau s + 1} \qquad (20)$$

and the closed-loop transfer function:

$$H_{CL}(s) = \frac{e^{-0.5}}{(Ts+1)^2} \qquad (21)$$

The closed loop responses are obtained using the IR method described in section three. In figure 4 are presented the step responses (left) and the load disturbance responses (right). Unit amplitude for set-point and load disturbance is applied. In the upper plots T is fixed at $2\zeta\tau/0.1$ and in the lower plots T is fixed at $2\zeta\tau/0.2$. For reasons concerning graphical clarity the open loop behaviour (dotted line) is scaled to unit.

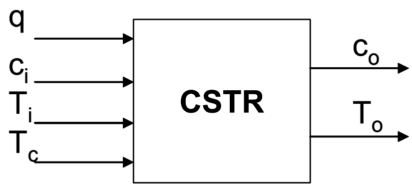

Fig. 3. Input-Output schematic of a continuous stirred tank reactor considering one single, irreversible, chemical reaction. The following notations apply: q – input flow of product A, c_i – the concentration of product A in the in-flow, T_i – the temperature of in-flow, T_c – the cooling temperature of the cooling jacket, c_o – the concentration of product A in the out-flow, T_o – the output temperature.

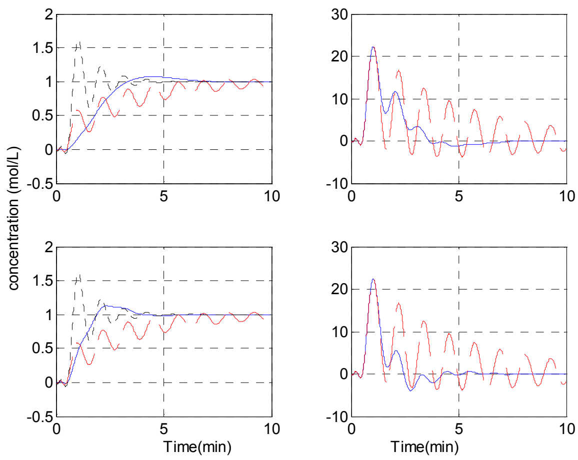

Fig. 4. The step responses (left) and the disturbance rejection (right) of the CSTR. Performance comparison between: open loop (dotted line), IR method (continuous line) and Ziegler-Nichols method (dashed line).

From figure 4 can be concluded that the IR auto-tuning method gives better performance than the classical Ziegler-Nichols auto-tuning method. The change of the user-defined parameter *T* is influencing the closed-loop behaviour of the controller with the process (upper plots vs. lower plots). The students can then decide themselves upon the suitable specifications for the process dynamics and characteristics. The open loop behaviour is significantly improved for the IR auto-tuning method (see step responses) by reducing the overshoot and the settling time. The difference between the classical Ziegler-Nichols (ZN) and the proposed IR method is more visible in the case of disturbance rejection (upper-right and lower-right plots). However, it can be observed that a sub-optimal tuning of the IR auto-tuner (by choosing the right value for the design-parameter *T*), leads to a similar performance as the ZN auto-tuner (that has no design parameter).

4.2 Second example: a DC-DC boost converter

The second example is taken from the topic of electrical engineering students. The real-life system is usually nonlinear and accuracy is a crucial specification. Disturbance rejection is another important requirement of power supply systems (De Keyser *et al.*, 2004).

The (switched) boost converter considered throughout this paper is represented in figure 5 and the differential equations describing the circuit are given by:

$$\begin{aligned} \dot{x}_1 &= -(1-q(t))\frac{1}{L}x_2(t) + \frac{1}{L}V_{in} \\ \dot{x}_2 &= (1-q(t))\frac{1}{C}x_1(t) - \frac{1}{RC}x_2(t) \end{aligned} \qquad (22)$$

with x_1 and x_2 the input inductor current (I), respectively the output capacitor voltage (V); V_{in} the nominal value of the external voltage source; R the resistance of the load; q denotes the switch position function and acts as the control input, taking values in the discrete set $\{0,1\}$.

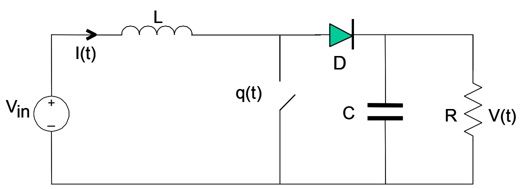

Fig. 5. Electrical scheme of a DC-DC boost converter.

When the switch is closed (q=1) the inductor current increases and the capacitor *C* discharges over the resistor *R* according to the relations:

$$L\frac{dx_1}{dt} = V_{in};\ C\frac{dx_2}{dt} = -\frac{x_2(t)}{R} \qquad (23)$$

Alternatively, when the switch is open (q=0), the variations of the inductor current and of the output capacitor voltage are described respectively by:

$$\begin{aligned} L\frac{dx_1}{dt} &= V_{in} - x_2(t) \\ C\frac{dx_2}{dt} &= x_1(t) - \frac{x_2(t)}{R} \end{aligned} \qquad (24)$$

Notice that in this situation, the current is decreasing continuously, because for a boost converter in nominal operation the output voltage $V(=x_2)$ is higher than the input voltage V_{in}. If the current possibly becomes zero, it will remain there until q=1, because the presence of the diode in the circuit prevents that the current flows in the opposite direction. The output to be controlled is x_2 and the objective is to bring it at a desired voltage V^*, which is higher than V_{in}.

The manipulated variable is the duty ratio (DR) defined as the time for which the switch is open (i.e. for q=1) over the total switching period. Linearization around a working point (e.g. for a duty-cycle of 0.5) leads to:

$$H_p(s) = \frac{968 \cdot 1371^2}{s^2 + 2 \cdot 0.0155 \cdot 1371 \cdot s + 1371^2} \qquad (25)$$

Figure 6 shows the tuning for IR method where in the upper plots *T* is fixed at *2ζτ/1* and in the lower plots *T* is fixed at *2ζτ/0.1.* The controller developed with the IR auto-tuning method gives good performance for step response (fast dynamics, no overshoot and minimum settling time). However, it does not perform a very good disturbance rejection but it is better than ZN method if the values for the design parameter *T* are not too large.

Another tuning set is made for the design parameter *T* settled at *2ζτ/0.1* in the upper plots and in the lower plots *T* is fixed at *2ζτ/0.01.* This tuning set is depicted in figure 7 on the same time scale. It can be observed that the specification for the design parameter *T* is again crucial for obtaining an (sub)optimal solution.

The corresponding controller parameters for the two examples are listed in the table below.

Table 1 Controller Parameters (see equation (3)). Lines 1-2 correspond to figure 4, lines 3-4 correspond to figure 6 and lines 5-6 correspond to figure 7.

K_c	T_i	T_d	T_f
0.00542	0.1252	0.406	0.5127
0.00918	0.1252	0.406	0.217
0.00024	0.0000253	0.02114	0.00000584
0.00004627	0.0000253	0.02114	0.00011326
0.00004627	0.0000253	0.02114	0.00011326
0.0000051	0.0000253	0.02114	0.0012497

5. CONCLUSIONS

The purpose of this contribution was to describe a recently developed auto-tuning method and its applicability within control engineering education system. The method is simple and as other auto-tuning techniques, does not require an advanced knowledge of control engineering theory.

It has been shown that the IR auto-tuning method yields good PID parameters for a specific class of processes. The method has however some limitations: it cannot be applied to processes with integrator – the open loop step response is continuously increasing, thus not stable; it cannot be used on processes with model approximation of order bigger than 2, since the identification model is assumed to be SOPDT. These limitations make the IR auto-tuning method not a general methodology for arbitrary process models.

In this contribution, both optimal and sub-optimal tuning parameters sets have been illustrated and the effect of several design values of T. Due to the fact that the students have the possibility to choose the closed-loop performance design, the method is suitable for use in basic course of control engineering. On the other hand, it also offers the possibility to learn and study a basic method for tuning controllers.

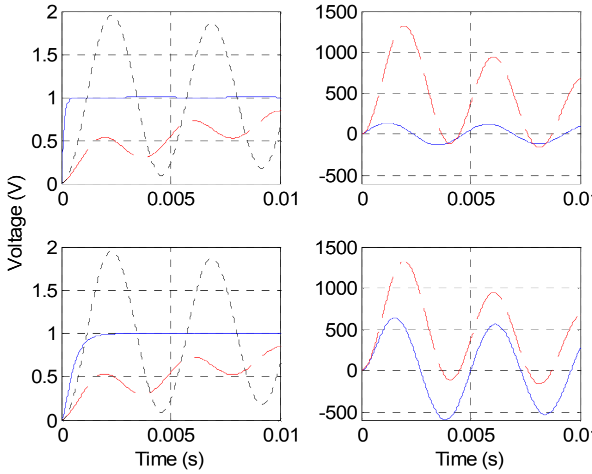

Fig. 6. Tuning set #1: the step responses (left) and the disturbance rejection (right) of the DC-DC boost converter. Performance comparison between: open loop (dotted line), IR method (continuous line) and Ziegler-Nichols method (dashed line).

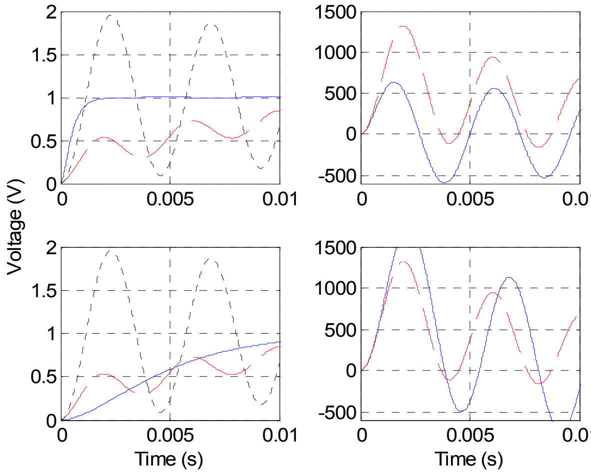

Fig. 7. Tuning set #2: the step responses (left) and the disturbance rejection (right) of the DC-DC boost converter. Performance comparison between: open loop (dotted line), IR method (continuous line) and Ziegler-Nichols method (dashed line).

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