Stochastic modelling of energy harvesting for low power sensor nodes

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ABSTRACT

Battery lifetime is a key impediment to long-lasting low power sensor nodes. Energy or power harvesting mitigates the dependency on battery power, by converting ambient energy into electrical energy. This energy can then be used by the device for data collection and transmission. This paper proposes and analyses a queueing model to assess performance of such an energy harvesting sensor node. Accounting for energy harvesting, data collection and data transmission opportunities, the sensor node is modelled as a paired queueing system. The system has two queues, one representing accumulated energy and the other being the data queue. By means of some numerical examples, we investigate the energy-information trade-off.

1. INTRODUCTION

The problem of battery replacement and disposal is a key impediment to ubiquitous use of wireless sensors networks. Sensor networks are formed by a collection of intercommunicating sensor nodes, collecting spatially distributed data (temperature, humidity, movement, noise, ...). Sensors networks can be used in a large range of applications, including military, environmental, home and health applications [1]. Despite vast improvements on power consumption and ongoing developments in power management, the lifetime of wireless sensors is largely determined by the energy of onboard batteries [13]. To overcome dependency on batteries, current research effort focusses on wireless devices that extract the necessary energy from their environment [7]. Possible power sources include electromagnetic radiation, thermal energy as well as mechanical energy [8].

The specific dynamics of energy harvesting has also drawn the attention of the modelling community. Sensors being autonomous in deciding which information will be transmitted as well as when to transmit, various authors propose game theoretic models; see e.g. [11] for power control games in wireless networks. Accounting for energy harvesting, Tsuo et al [15] consider a Bayesian game where each node knows its local energy state. An evolutionary hawk and dove game with harvesting nodes transmitting either at high or low power is studied in [2, 5]. Specifically focussing solar power, optimal energy management for a sensor node that uses a sleep and wakeup strategy for energy conservation is studied by a bargaining game in [12].

Neither of these game-theoretic models assume that acquired data can be temporarily stored at the sensor node. To study data buffering at the sensor node, queueing theoretic modelling applies. [14] is a recent contribution on such a queueing theoretic approach. These authors analytically study stochastic stability of an energy harvesting node with data buffering and rely on simulation to assess its performance. Also the present contribution investigates a queueing model for a harvesting sensor node. In particular, we assess the performance of an energy harvesting sensor node accounting for uncertainty in data acquisition, in energy harvesting and in transmission opportunities. To this end, we investigate a queueing system with two queues: one queue represents the data buffer and one queue represents the available energy. Maximising versatility of the model at hand while keeping the analysis numerically tractable, we model data acquisition, energy harvesting and transmission by means of Markovian arrival processes; an "arrival" representing some acquired data, some harvested energy and a transmission opportunity (an encounter with another node or a base station) respectively.

Such two-buffer queueing problems are sometimes termed paired queues — pairing refers to the coupling between the queues, service is only possible if both queues are non-empty — and have been studied in various contexts including leakybucket access control [16, 17], kitting processes [4] in assembly and decoupling buffers in production systems [3].

Leaky-bucket access control in asynchronous transfer mode, introduces a virtual buffer (a bucket) at sender nodes. The virtual buffer is filled with tokens according to some well behaved process. For every transmission, a token is taken from the bucket and transmission is only allowed if there are tokens present. Hence, the data buffer and leaky bucket constitute a paired queueing system. Kitting is a particular strategy for supplying materials to an assembly line. Instead of delivering parts in containers of equal parts, kitting collects the necessary parts for a given end product into a specific container, called a kit, prior to arriving at an assembly unit. As kits can only be completed if all parts are present, the

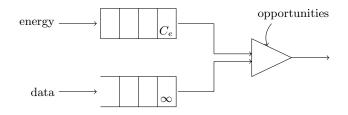


Figure 1: Stochastic model of energy harvesting for low power sensor nodes.

part buffers and the kitting operation constitutes a paired queueing system. Finally, decoupling buffers are used to reduce lead times in production systems by buffering semifinished products at some point in the production process. When there is demand, semi-finished products are taken out of the decoupling buffer and finished according to the demand. Again, paired queueing applies as the second production stage only starts if there are semi-finished products and demand.

Finally, paired queues have also been studied in a more abstract setting. Considering a system with two paired queues, Harrison shows that it is necessary to impose a restriction on the size of the buffer to ensure stability in the operations of a kitting process [6]. Similar observations where made by Latouche [9] who studied the difference of the queue lengths in such a paired queueing system.

The remainder of this paper is organised as follows. The paired queueing model under investigation and the notationally conventions are introduced in the next section. In section 3, the system is analysed as a quasi-birth-and-death process (QBD). Also, the numerical solution methodology is discussed and relevant performance measures are determined. To illustrate our approach, section 4 considers some numerical examples. Finally, conclusions are drawn in section 5.

2. MODEL DESCRIPTION

The energy harvesting sensor node is modelled as a queueing system with two queues, as depicted in figure 1. The energy queue has finite capacity C_e and stores energy extracted from the environment. The data queue keeps track of not yet transmitted data packets and has infinite capacity.

The amount of stored energy is discretised for modelling convenience. We make abstraction of the specifics of energy harvesting apart from the assumption that there is a continuous chance to come by some 'chunks' of energy. Therefore, we assume that energy arrives in accordance with a Markovian arrival process with state space \mathcal{K}_E . Let Ω_E^0 and Ω_E^1 denote the generator matrices of this arrival process, governing the state transitions when there are no arrivals and when there is an arrival, respectively. Analogously, the sensor picks up data in accordance with a Markovian arrival process with state space \mathcal{K}_A : whenever it picks up data, there is an arrival in the data queue. Let Ω_A^0 and Ω_A^1 denote the generator matrices of this arrival process, governing the state transitions when there are no arrivals and when there is an arrival, respectively.

Data can only be transmitted during transmission opportunities. Moreover, the two queues are paired, meaning that data can only be transmitted if the energy buffer is nonempty. Whenever a transmission occurs, a data packet departs but the level of the energy buffer may or may not decrease (this assumption allows for modelling the dynamics of the battery with fewer states). The arrivals of transmission opportunities being exogenous to the state of the sensor node, the departure process is a marked Markov process with state space \mathcal{K}_D . The generator matrices Ω_D^0 , Ω_D^1 and Ω_D^2 govern the state transitions of the departure process without transmission opportunities, with a transmission opportunity that leads to a decrease of the energy buffer and with a transmission opportunity that does not lead to such a decrease. Note that for the matrices Ω_E^0 , Ω_A^0 and Ω_D^0 , diagonal elements are assumed to be zero.

3. ANALYSIS

Modulating Markov chain. For ease of modelling, we first consider the Markov chain with state space $\mathcal{K} = \mathcal{K}_E \times \mathcal{K}_A \times \mathcal{K}_D$ that jointly describes the (marked) state changes of energy, arrival and departure processes. In the remainder, let \mathbf{I}_E , \mathbf{I}_A and \mathbf{I}_D denote identity matrices with size $|\mathcal{K}_E|$, $|\mathcal{K}_A|$ and $|\mathcal{K}_D|$, respectively. Note that the symbol \otimes denotes the Kronecker's product.

- The matrix **A** governs the transitions, when there are neither arrivals nor departures:
 - $\mathbf{A} = \Omega_E^0 \otimes \mathbf{I}_A \otimes \mathbf{I}_D + \mathbf{I}_E \otimes \Omega_A^0 \otimes \mathbf{I}_D + \mathbf{I}_E \otimes \mathbf{I}_A \otimes \Omega_D^0.$
- The matrix \mathbf{B}_E governs the transitions when there is an arrival in the energy buffer:

$$\mathbf{B}_E = \Omega_E^1 \otimes \mathbf{I}_A \otimes \mathbf{I}_D \,.$$

• The matrix **B**_A governs the transitions when there is an arrival in the data buffer:

$$\mathbf{B}_A = \mathbf{I}_E \otimes \Omega_A^1 \otimes \mathbf{I}_D \,.$$

• The matrices **C**₁ and **C**₂ govern the transitions when there is an arrival that drains the energy buffer and that does not drain this buffer, respectively:

$$\mathbf{C}_1 = \mathbf{I}_E \otimes \mathbf{I}_A \otimes \Omega_D^1$$
, $\mathbf{C}_2 = \mathbf{I}_E \otimes \mathbf{I}_A \otimes \Omega_D^2$.

REMARK 1. The matrices **A** till C_2 above are defined in terms of the characteristics of the different arrival processes. In the remainder, all results will be expressed in terms of the matrices as defined above. Hence, these results remain valid in the case that the different arrival processes are intercorrelated as well. In that case there is a single marked Markov process, with marks for data arrivals, energy arrivals and transmission opportunities.

Quasi-birth-death process. Having defined these transition matrices, we now focus on the queueing model at hand. To be more precise, the energy harvesting sensor node system is a continuous-time Markov chain with infinite state space $\mathbb{N} \times \{1, 2, \dots, C_e\} \times \mathcal{K}, \mathcal{K} = \{0, 1, \dots, K\}$. At any time, the state of the system is described by the triplet [n, m, i], n being the number of data packets available, m being the energy level and i being the state of the modulating chain.

The studied Markov process is a homogeneous quasi-birthand-death process (QBD), see [10]. In the present setting, the *level* or block-row index, indicates the data packets available while the phase, i.e. the index within a block element, indicates both the energy level and the state of the Markovian environment. The one-step transitions are restricted to states in the same level (from state (n, *, *) to state (n, *, *)) or in two adjacent levels (from state (n, *, *) to state (n + 1, *, *) or state (n - 1, *, *)).

We then find that the generator matrix of the Markov chain has the following block matrix representation,

$$\mathbf{Q} = \begin{bmatrix} \mathcal{B}_{0} & \mathcal{A}_{2} & 0 & 0 & \cdots \\ \mathcal{A}_{0} & \mathcal{A}_{1} & \mathcal{A}_{2} & 0 & \cdots \\ 0 & \mathcal{A}_{0} & \mathcal{A}_{1} & \mathcal{A}_{2} & \cdots \\ 0 & 0 & \mathcal{A}_{0} & \mathcal{A}_{1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} .$$
(1)

The blocks are given by,

$$\mathcal{B}_{0} = \begin{bmatrix} \mathbf{\underline{D}} & \mathbf{B}_{E} & 0 & \cdots & 0\\ 0 & \mathbf{\underline{D}} & \mathbf{B}_{E} & \cdots & 0\\ 0 & 0 & \mathbf{\underline{D}} & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & \mathbf{\underline{D}} \end{bmatrix}$$
(2)

$$\mathcal{A}_{2} = \begin{bmatrix} \mathbf{B}_{A} & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{B}_{A} & 0 & \cdots & 0 \\ 0 & 0 & \mathbf{B}_{A} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \mathbf{B}_{A} \end{bmatrix}$$
(3)
$$\mathcal{A}_{0} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \mathbf{C}_{1} & \mathbf{C}_{2} & \cdots & 0 & 0 \\ 0 & \mathbf{C}_{1} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{C}_{1} & \mathbf{C}_{2} \end{bmatrix}$$
(4)

$$\mathcal{A}_{1} = \begin{bmatrix} \mathbf{\underline{D}} & \mathbf{B}_{E} & 0 & \cdots & 0\\ 0 & \mathbf{D} & \mathbf{B}_{E} & \cdots & 0\\ 0 & 0 & \mathbf{D} & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & \mathbf{D} \end{bmatrix} .$$
 (5)

with $\mathbf{D} = \mathbf{A} - \partial \mathbf{A} - \partial \mathbf{C}_1 - \partial \mathbf{C}_2 - \partial \mathbf{B}_A - \partial \mathbf{B}_E$ and $\mathbf{\underline{D}} = \mathbf{D} + \mathbf{C}_1 + \mathbf{C}_2$, where the notation $\partial \mathbf{X}$ represents a diagonal matrix with diagonal elements equal to the row sums of \mathbf{X} .

Numerical solution. Having defined the different blocks of the QBD process, we now focus on its solution. Recall

that the state of the Markov chain was described by the triplet [n, m, i]; n is the size of the data buffer, m is the size of the energy buffer and i is the state of the modulating chain. Let $\pi(n, m, i)$ be the steady state probability to be in state [n, m, i]. A well-known method for finding the stationary distribution of QBD processes is the matrix-geometric method. Using the vector notation $\pi_k = (\pi(k, 0, 0), \pi(k, 0, 1), \ldots, \pi(k, C_e, K))$, the probability vectors can be expressed as,

$$\boldsymbol{\pi}_k = \boldsymbol{\pi}_0 \mathbf{R}^k. \tag{6}$$

where the so-called rate matrix ${\bf R}$ is the minimal non-negative solution of the non-linear matrix equation

$$\mathbf{R}^2 \mathcal{A}_0 + \mathbf{R} \mathcal{A}_1 + \mathcal{A}_2 = \mathbf{0} \,.$$

We compute the rate matrix by implementing the efficient iterative algorithm of [10], chapter 8.

Performance measures. Once the steady state probabilities have been determined numerically, we can calculate a number of interesting performance measures for the harvesting energy sensor node. For ease of notation, we introduce the marginal probability mass functions of the energy and the data queue content: $\pi^{(e)}(m) = \sum_{i \in \mathcal{K}} \sum_{n=0}^{\infty} \pi(n, m, i)$ and $\pi^{(d)}(n) = \sum_{i \in \mathcal{K}} \sum_{m=0}^{C_e} \pi(n, m, i)$.

Note that as the data queue is infinite, the throughput of the sensor node system η equals the data arrival rate λ_d . In addition, we have the following performance measures.

• The mean energy queue and the mean data queue: $E Q_e$ and $E Q_d$ respectively,

$$E Q_e = \sum_{m}^{C_e} \pi^{(e)}(m)m, \quad E Q_d = \sum_{n}^{\infty} \pi^{(d)}(n)n.$$

• The variance of the energy queue and the data queue: Var Q_e and Var Q_d respectively,

$$\operatorname{Var} Q_{e} = \sum_{m}^{C_{e}} \pi^{(e)}(m)m^{2} - (\operatorname{E} Q_{e})^{2},$$
$$\operatorname{Var} Q_{d} = \sum_{n}^{\infty} \pi^{(d)}(n)n^{2} - (\operatorname{E} Q_{d})^{2}.$$

• The mean delay *L* (calculated based on Little's theorem) is the average amount of time between the arrival of a data packet its transmission:

$$L = \frac{\mathrm{E}\,Q_d}{\lambda_d}$$

• As the energy queue has finite capacity, energy harvesting may be blocked. This happens when energy is captured but the queue is full. Hence, blocking corresponds to the loss probability in the energy queue. The loss probability is most easily expressed in terms of the throughput. We have,

$$b_e = \frac{\lambda_e - \eta}{\lambda_e} = \frac{\lambda_e - \lambda_d}{\lambda_e}$$

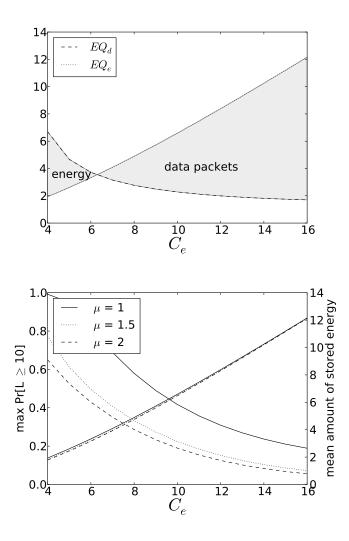


Figure 2: There is a trade-off between the mean amount of stored energy and stored data and between the delay.

4. NUMERICAL RESULTS

We now illustrate our approach by means of some numerical examples.

Poisson arrivals and exponential data transmission opportunities. As a first example, the difference between the mean energy queue and the mean data queue versus the capacity C_e is depicted in figure 2(a). We assume that energy units and data units arrive according to a Poisson process with parameter $\lambda_e = 0.6$ and $\lambda_d = 0.6$, respectively. The probability to use one unit of energy for data transmission p equals 0.8 and the data transmission opportunities are exponentially distributed with service rate μ equal to 1. As the figure shows, the buffer capacity of 4 results on average in no data and energy in the buffer. Under and above the level, energy and data are on average backlogged, respectively. Obviously, there is on average more amount of energy and less buffer of data as the capacity increases.

Figure 2(b) represents the trade-off between the maximum probability to have a delay higher or equal to 10 (left side) and the mean amount of stored energy (right side). Note that we calculated the delay distribution by using the onesided Chebyshev's inequality. Under the same parameter assumptions of figure 2(a), the maximum probability to have a delay higher or equal to 10 decreases and the mean amount of stored energy increases as the energy capacity increases for each service rate. Indeed, if more buffer capacity is available, it will be used — the energy queue increases such that there is on average less time required to transmit one data unit. Furthermore, we observe a slightly decrease of the amount of energy as the service rate μ increases. Indeed, the more data is transmitted per time unit, the higher the mean amount of energy used to transmit data. Finally, the maximum probability to have a delay equal or higher than 10 decreases as the service rate increases, as expected.

Markovian arrival process for energy. We also quantity the impact of irregular capture of energy. To this end we compare both buffers with Poisson arrivals to corresponding system with interrupted Poisson arrivals for the energy and Poisson arrivals for the data. The arrival interruptions account for inefficiency in the energy harvesting process.

The interrupted Poisson process considered here is a twostate Markovian process. In the active state, generated energy arrive in accordance with a Poisson process with rate λ_e whereas no new energy arrive in the inactive state. Let α and β denote the rate from the active to the inactive state and vice versa, respectively. We then use the following parameters to characterise the interrupted Poisson process (IPP),

$$\sigma = \frac{\beta}{\alpha + \beta}, \quad \kappa = \frac{1}{\alpha} + \frac{1}{\beta}, \quad \lambda_e^* = \lambda_e \sigma.$$

Note that σ is the fraction of time that the interrupted Poisson process is active, the absolute time parameter κ is the average duration of an active and an inactive period, and λ_e^* is the arrival load of energy.

Figure 3 shows the mean number of stored data packets versus the arrival load of energy with buffer capacity C_e equal to 5 and 10 for Poisson arrivals as well as for interrupted Poisson arrivals of energy. The probability to use one unit of energy for data transmission equals 0.8 and transmission times are exponentially distributed with service rate μ equal to 1. In addition, we set $\sigma = 0.8$ and $\kappa = 10$ for the interrupted Poisson process (e.g. $\lambda_e = 0.8$ for Poisson arrivals and $\lambda_e = 1.0$ for interrupted Poisson arrivals). The data arrival rate λ_d equals 0.6. As expected, the mean number of stored data packets decreases as the arrival rate of energy increases. Furthermore, the impact of the buffer capacity decreases as the arrival rate of energy λ_e increases. Finally, comparing interrupted Poisson and Poisson processes, burstiness in the energy harvesting process has a negative impact on performance — there is on average more time required to transmit one data unit. Figure 4 confirms the previous results. Indeed, the probability to have an empty energy queue decreases as the buffer capacity of energy decreases and the probability is higher for interrupted Poisson than for Poisson arrivals.

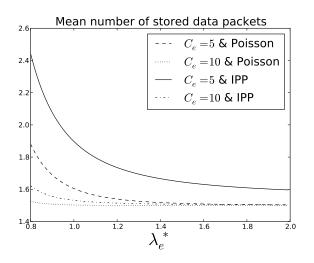


Figure 3: Irregular capture of energy results in a higher mean number of stored data packets.

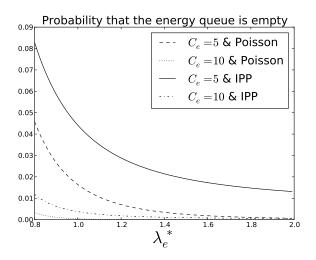


Figure 4: Irregular capture of energy and a small buffer capacity result in a higher probability that the energy queue is full.

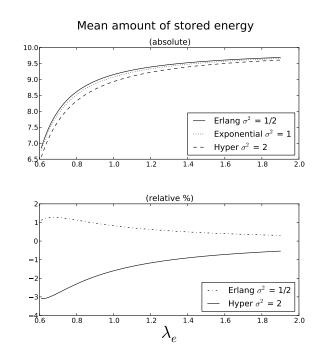


Figure 5: The shape of the data transmission opportunity distribution has no significant impact on the mean amount of stored energy.

Phase-type distributed data transmission opportunities. The last numerical example quantifies the impact of the distribution of the data transmission opportunity on the sensor node system. Figure 5 and 6 depict the mean amount of energy in the queue and the mean delay of the sensor node system. In both figures, the energy arrival rate λ_e is varied and different values of the variance of the data transmission opportunity distribution are assumed as indicated. The probability to use one unit of energy for data transmission p equals 0.8 and the mean service time equals 1 for all curves. We consider a two-phase hyper-exponential distribution (in which each phase has the same probability to occur) and a two-phase Erlang distribution. Note that two corner cases coincide both with an exponential distribution: a hyper-exponential distribution with unit variance and an Erlang distribution with one phase. Furthermore, the data arrival rate λ_d equals 0.6 and the energy capacity C_e equals 10. Clearly, figure 5 and 6 show respectively that the energy buffer content converges to maximum capacity and the mean delay decreases to a certain value as the energy arrival rate increases. The second plot shows values relative to the exponential distribution. Concerning the mean amount of stored energy, we observe that the data transmission opportunity distribution has no significant effect on this performance measure. However, the difference between σ^2 equal to 1/2, 1 and 2 for the mean delay remains constant and is significant. Finally, the mean amount of stored energy and the mean delay show respectively a slight decrease and increase as the variance of the data transmission opportunity distribution σ^2 increases.

5. CONCLUSION

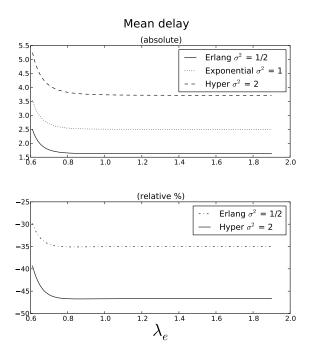


Figure 6: The shape of the data transmission opportunity distribution has significant impact on the mean delay.

In this paper, we analyse the performance of different energy harvesting sensor nodes. In particular, we investigate the impact of irregular capture of energy in the environment as well as the data transmission opportunity distribution on the performance of sensor node systems. In the studied system, both accumulated energy and data needs to be available for transmission. Furthermore, we assume that there is a probability that one unit of energy whether or not will be used to transmit one unit of data. Therefore, the studied sensor node system is modelled as a homogeneous quasi-birthand-death process (QBD) and solved with matrix-analytic methods.

As our numerical examples show, there is trade-off to be made between the storage cost of energy and the service level of the sensor node, as expected — e.g. a higher capacity causes on average a higher storage of energy and a smaller time between data availability and data transmission. Furthermore, irregular capture of energy has a negative effect on the performance of the sensor node system. However, system performance is partially insensitive to variation in the data transmission opportunity distribution. Future work will focus on determining the total cost of the studied sensor node system.

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