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PREDICTIVE CONTROL OF A PROCESS WITH VARIABLE DEAD-TIME

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Abstract: This paper deals with the control of variable-delay processes, where the delay depends on the value of the manipulated variable, which results in a non-linear system difficult to control. As a reference process, the case of a heated tank where the controlled variable is the liquid temperature and the placement of the sensor introduce a transport delay in the control loop, has been considered. This challenging problem is approached from the perspective of predictive control, using the non-linear EPSAC controller.

Keywords: Variable dead-time, Predictive control, Nonlinear control, Delay systems

1. INTRODUCTION

Time delays in the feedback loop appear frequently in industrial processes, both from the process itself and caused by the controllers. The presence of dead-time in a feedback control loop has a significant impact on the performance of the control loop. Depending on the magnitude of the delay, performance of the feedback loop can be severely limited. A difficult and challenging problem is to control a process without instantaneous measurement of the state variables, or via delayed actuators (Richard, 2003).

Dead-time compensating controllers (DTC) have received attention in the literature. The most widely used method in industry is the Smith predictor, (Smith, 1957). In Hägglund (1996), a PI controller with model-based prediction (PPI) was presented. Another dead-time compensation in the controller design is proposed in Normey-Rico *et al*. (1997). A simple modified Smith predictor controller was also related in Chien *et al*. (2002). Dead-time compensators seems to be a good option for improving the system' response without increasing too much the complexity of the controllers. But all of these dead-time compensators consider stable or integrating processes with a constant dead-time, identified like:

$$P_n(s) = \frac{K_n}{T_n s + 1} e^{-ds} \quad \text{or} \quad P_n(s) = \frac{K_n}{s} e^{-ds}$$

In the process industry is not uncommon to find transport delays due to the placement of measuring elements far away from the process being controlled. These transport delays introduce a pure significant dead time that changes with the value of the flow, which, in turn, is often a manipulated variable. This means that the difficulty of the control problem increases significantly as there is a relationship between the dead-time and the manipulated variable. The model then becomes:

$$P_n(s) = G(s) e^{-d(u)s}$$

which is non-linear. The paper tries to contribute to the solution of this problem that is still an open one in spite of the many efforts developed in the past. (Richard, 2003). The approach follows the NMPC ideas, taking advantage of the non-linear EPSAC (De Keyser, 1998) implementation to gain in adequacy to the problem as well as in computation time. In this sense, it does not present a new approach, but shows the advantages of the proposed method when dealing with variable time delays due to its dependence of the manipulated variable. Other NMPC linearization methods has been proposed in the literature (Kouvaritakis, 1999), (Lee, 2002), (Alvarez, 2000) but none is focused to the variable delay problem, nor offers clear advantages over the non-linear EPSAC.

The paper content is split in five sections. After the introduction, section 2 describes the process to be controlled. In section 3 a brief review of the basic ideas of the non linear predictive control is presented followed by the nonlinear EPSAC algorithm. Section 4 is devoted to experimental results concerning the controlled process and finally, some concluding remarks are provided.

2. PROCESS DESCRIPTION

With the purpose of illustrating the ideas behind the proposed controller, a simple, but challenging problem has been chosen. The process considered in our case study consists of a stirred tank (Fig. 1) with inlet volumetric flow q. The liquid is heated by a constant amount of heat Q supplied by a submerged electrical heating element and flows out of the tank by overflow. The objective is to keep the tank temperature T at a desired value manipulating the inlet flow q whose temperature is T_i. The temperature is measured, not in the tank, but in the effluent pipe in a place located at a distance L from the tank. This distance L introduces a transportation delay to the control loop that depends on the flow, which is the manipulated variable.

The mathematical model that describes how the outlet temperature $T(t)$ responds to changes in inlet flow $q(t)$ is given by an energy balance:

$$Ah\frac{dT(t)}{dt} = q(t)\left(T_i - T(t)\right) + \frac{Q}{\rho c_p} \qquad (1)$$

where the constant parameters ρ, A, c_p , density, cross section and specific heat, are characteristic of the tank and the heat losses to the surroundings are assumed to be negligible.

The measured variable is denoted as T_d and its response will be the same as $T(t)$ except that it will be delayed by a transportation time $d(t)$:

$$T_d(t) = T(t - d(t)) \qquad (2)$$

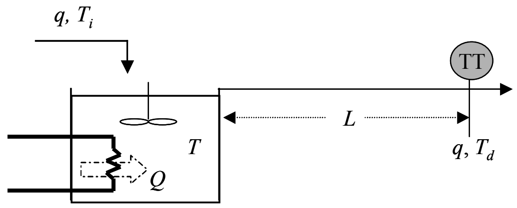

Fig. 1

The time delay $d(t)$ is inversely proportional to manipulated variable:

$$d(t) = \frac{\text{distance}}{\text{velocity}} = \frac{L}{\frac{q(t)}{S}} = \frac{LS}{q(t)} \qquad (3)$$

where S is the cross section of the effluent pipe. The model (1) is non-linear, but if the measured variable were T, it could be fairly well controlled using standard controllers. Nevertheless, the strong relationship between the dead-time and the output of the controller, provides a variable delay that increases the difficulty of the problem and either force to "detune" the controller in order to gain stability margin or oblige us to consider explicitly the dynamic non linear model of the process for the development and implementation of the control system.

3. NONLINEAR PREDICTIVE CONTROLLER

Nonlinear predictive control (NMPC) is a natural extension of the linear MPC technique. Most of the algorithms are based on the use of an internal non-linear plant model, which captures the main process characteristics, followed by a dynamic optimization that provides the optimal manipulated variables. In our case, the physical model (1)-(3) was used trying to reflect the dynamics of the process. In this section, the NMPC and EPSAC approaches will be presented briefly.

3.1 NMPC Controller

The objective of the non-linear model predictive control (NMPC) is finding the future optimal manipulated variable sequence in order to minimize a function based on a desired output trajectory over a prediction horizon. The cost function is the integral over the squares of the residuals between the model predicted outputs y_{pred} and the set point values r over the prediction interval $N_2\tau$ (where N_2 is the prediction horizon and τ is the sampling time). A typical formulation is

$$\min_{u(k/k),\ldots,u(k+N_u-1/k)} J = \int_{t_k}^{t_k+N_2\cdot\tau} \gamma\left[\left(y_{pred}(t) - r(t)\right)\right]^2 dt + \sum_{j=0}^{N_u-1} \beta\left[\Delta u(k+j|t)\right]^2 \qquad (4)$$

The change in the manipulated variable u is also included in the minimization. Other formulations include penalty terms or additional constraints in order to guarantee certain stability properties. In order to perform the optimization, it is necessary to parameterise the manipulated variable; otherwise an infinite number of decision variables would appear in the problem. An usual possibility is the discretization of the u along the control horizon (N_u) where the input remains constant over the sampling period τ.

$u(t) = u(k), \quad k\tau \leq t < (k+1)\tau$
$u(k) = u(N_u\text{-}1) \quad\quad k > N_u\text{-}1$

The minimization (4) is done under the constraints of the continuous model equations and the typical ones applied on the manipulated and controlled variables:

$$\begin{aligned} u_{\min} &\leq u(k) \leq u_{\max} \\ \Delta u_{\min} &\leq \Delta u(k) \leq \Delta u_{\max} \\ y_{\min} &\leq y(k) \leq y_{\max} \end{aligned} \tag{5}$$

Only the first component of the N_u moves optimal control sequence, is implemented, and the whole procedure is repeated every sampling period.

The controller law solution leads to a dynamic non-linear programming problem, which could be formulated generically as a real time minimization of a non-linear function with constraints, where the index J can be computed by simulation each time the optimization algorithm needs it.

3.2 Nonlinear iterative EPSAC formulation

The key idea of this formulation is to approximate the non-linear predictions by iterative linearizations around future trajectories, so that they converge to the true non-linear optimal solution. For this purpose, the future sequence of manipulated variables is considered as the sum of a basic future control scenario, called $u_{base}(t+k/t), k \geq 0$ and optimizing future control actions $\delta u(t+k/t), 0 \leq k \leq N_u - 1$:

$$u(t+k/t) = u_{base}(t+k/t) + \delta u(t+k/t) \tag{6}$$

In this way, see Fig.2, the output predictions can be considered in a first approximation as being the cumulative results of two effects:

$$y(t+k/t) \approx y_{base}(t+k/t) + y_{optimize}(t+k/t) \tag{7}$$

$k=1,..N_2$

The component $y_{base}(t+k/t)$ is calculated as the output of the non-linear model using the known (postulated) sequence $u_{base}(t+k/t)$ as the model input. The other component, $y_{optimize}(t+k/t)$, is the response to the $\delta u(t+k\ /\ t)$ component of the input. Notice that (7) is only an approximation based on the use of the superposition principle. We will come back on this point later on.

Assuming that the perturbations $\delta u(t+k\ /\ t)$ are small enough, its is possible to derive an expression for this term as the one that corresponds to a linearised model along $u_{base}(t+k/t)$. As the terms $\delta u(t+j\ /\ t)$ are impulses at times $t+k$, except the last one at time N_u-1, that is a step, and the effect at time t+k of an impulse at time $t+j$ is h_{k-j}, then the term $y_{optimize}(t+k/t)$ is the acumulative effect of a series

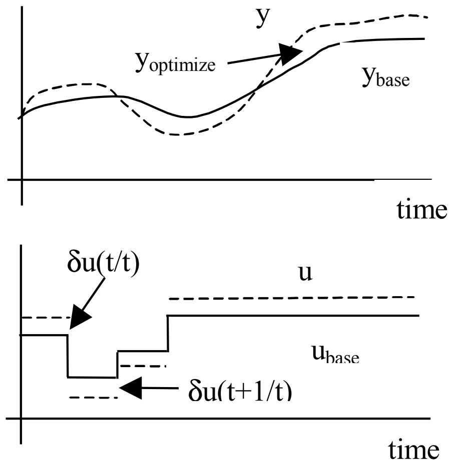

Fig.2

of impulse inputs and a step input (De Keyser, 1998):

$$\begin{aligned} y_{optimize}(t+k/t) &= h_k \delta u(t/t) + h_{k-1}\delta u(t+1/t) + \\ &... + g_{k-N_u+1}\delta u(t+N_u-1/t) \end{aligned} \tag{8}$$

where the parameters $h_1, h_2,.., h_k,...h_{N_2}$ are the coefficients of the unit impulse response of the system at the current operating point, whereas the values g_k refer to the unit step response coefficients. Notice that the value of the parameters reflects the effect of the variable delay around this specific control actions.

Using matrix notation, the prediction equation then becomes

$$\mathbf{Y} = \overline{\mathbf{Y}} + \mathbf{GU} \tag{9}$$

where

$$\begin{aligned} \overline{\mathbf{Y}} &= [y_{base}(t+N_1/t) \quad ... \quad y_{base}(t+N_2/t)]^T \\ \mathbf{U} &= [\delta u(t/t) \quad ... \quad \delta u(t+N_u-1/t)]^T \\ \mathbf{G} &= \begin{bmatrix} h_{N_1} & h_{N_1-1} & h_{N_1-2} & ... & g_{N_1-N_u+1} \\ h_{N_1+1} & h_{N_1} & h_{N_1-1} & ... & g_{N_1-N_u+2} \\ ... & ... & ... & ... & ... \\ h_{N2} & h_{N_2-1} & h_{N_2-2} & ... & g_{N_2-N_u+1} \end{bmatrix} \end{aligned} \tag{10}$$

A simple relationship exists between the control actions Δu and δu:

$$\begin{bmatrix} \Delta u(t/t) \\ \Delta u(t+1/t) \\ ... \\ \Delta u(t+N_u-1/t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} \delta u(t/t) \\ \delta u(t+1/t) \\ ... \\ \delta u(t+N_u-1/t) \end{bmatrix} + \mathbf{b} \tag{11}$$

with the matrix **A** and the vector **b** given by:

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 1 & 0 & 0 & ... & 0 \\ -1 & 1 & 0 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & ... & -1 & 1 \end{bmatrix}, \\ \mathbf{b} &= \begin{bmatrix} u_{base}(t/t) - u(t-1) \\ u_{base}(t+1/t) - u_{base}(t/t) \\ ... \\ u_{base}(t+N_u-1/t) - u_{base}(t+N_u-2/t) \end{bmatrix} \end{aligned} \tag{12}$$

In this description, the coefficients of the matrix **G** are computed using the linearized model about the current trajectory at each sampling instant. In case of a complex dynamic model, obtaining a linearized model is not an easy task. To avoid it, a possible alternative is to use the non-linear model to calculate the coefficients h_k and g_k from perturbations in the model using a simulation procedure. Taking into account the relationship (11), and recalling that $h_k = g_k - g_{k-1}$, a new formula is obtained for $y_{optimize}(t+k/t)$:

$$y_{optimize}(t+k/t) = g_k\left[u(t-1)+\Delta u(t)-u_{base}(t)\right] + \sum_{i=1}^{N_u-1} g_{k-i}\left[\Delta u(t+i)+u_{base}(t+i-1)-u_{base}(t+i)\right] \tag{13}$$

The step response coefficients can be calculated each sampling instant simulating the nonlinear model of the system with a particular future sequence $u^*(t+k/t)$ taking as initial conditions the current process state and evaluating the predictions $y^*(t+k/t)$.

A simple choice for $u^*(t+k/t)$ could be $u(t-1)+\Delta u^*(t)$. With this option and considering that

$$y^*(t+k/t) = y_{base}(t+k/t) + y^*_{optimize}(t+k/t) \tag{14}$$

the coefficients g_k verify the formula

$$c_0 g_k = y^*(t+k/t) - y_{base}(t+k/t) - g_{k-1}c_1 - g_{k-2}c_2 - \ldots - g_{k-N_u+1}c_{N_u-1} \tag{15}$$

where

$$\begin{aligned} g_0 &= 0 \\ c_0 &= u(t-1)+\Delta u^*(t)-u_{base}(t) \\ c_j &= u_{base}(t+j-1)-u_{base}(t+j),\ j\geq 1 \end{aligned} \tag{16}$$

Based on (9)...(12), the cost function is a quadratic form in **U**

$$J = \gamma\sum_{k=N_1}^{N_2}\left[y(t+k/t)-r(t+k/t)\right]^2 + \beta\sum_{k=0}^{N_u-1}\left[\Delta u(t+k/t)\right]^2 = \\ = \gamma\left(\mathbf{R}-\overline{\mathbf{Y}}-\mathbf{GU}\right)^T\left(\mathbf{R}-\overline{\mathbf{Y}}-\mathbf{GU}\right) + \beta\left(\mathbf{AU}+\mathbf{b}\right)^T\left(\mathbf{AU}+\mathbf{b}\right) \tag{17}$$

and the optimisation problem, the minimization of J subject to the constraints (5), can be solved with simple quadratic programming techniques (QP).

As mentioned before, as (7) is only an approximation because the superposition principle does not hold, the controls $u(t+k/t) = u_{base}(t+k/t) + \delta u(t+k/t)$, based on the solution $\delta u(t+k/t)$ of (17), are suboptimal. However if the approach is repeated iteratively in the same sampling instant by redefining $u_{base}(t+k/t) \equiv u(t+k/t)$ and recalculating $\delta u(t+k/t)$ and $u(t+k/t)$ until $\delta u(t+k/t) \approx 0$, then, as $\delta u(t+k/t) \approx 0$, the term $y_{optimize}(t+k/t)$ is practically zero and then (7) holds. So, the solution obtained in this iteration, converge to the optimal non-linear solution. In this way, the non-linear optimization problem is replaced by several QP problems.

To reduce the number of iterations, which is crucial for the efficiency of the algorithm, the initial value of $u_{base}(t+k/t)$ is important. A simple and effective choice, (De Keyser, 1998), is to start with the optimal control policy derived at the previous sample $u_{base}(t+k/t) \equiv u(t+k/t-1)$. In this paper, this strategy has been used, leading to a very small number of iterations per sampling time, usually three or four.

3.3 State estimation

On the other hand, the current state is needed as an initial condition at each iteration to predict the future behaviour and select an optimal control sequence. Since the state T of the nonlinear system is not directly measurable at time t, but with a delay, an estimation technique was implemented in order to reconstruct the current state of the system.

The technique chosen was a receding horizon one. The model state is computed based on the process model and past values of manipulated variable u. The present measured output $T_d(t)$ is used as initial condition to simulate the process model starting at past time $t - d$, and applying the known past discrete control sequence u(t-*NE*), u(*t*-*NE*+1),…,u(*t*-1). The receding horizon *NE* depends on the dead time: $NE=d/\tau$. So, the value $T(t)$ is the one obtained integrating the model from $TIME=t-NE*\tau$ to $TIME=t$ when the past controls are applied. (Fig.3)

4. SIMULATION RESULTS

Several experiments have been carried out in a simulated environment in order to test the nonlinear EPSAC controller on the process previously described and to compare it with other approaches. In all of them the sampling period is 20 seconds

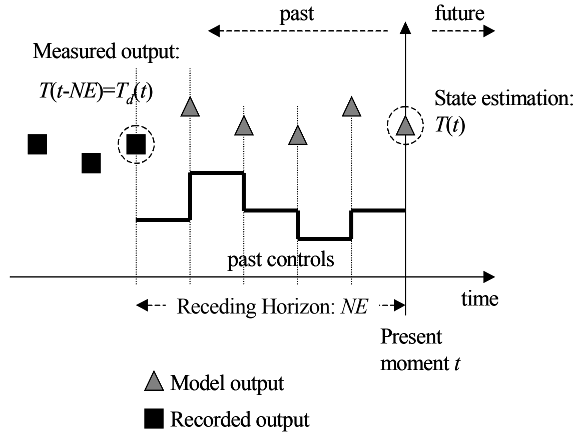

Fig.3 State estimation

whereas the other parameters, prediction horizon, control horizon and control weight, are N_2=15, N_u=1, γ=1, β=0 respectively. For the manipulated variables, the constraints were fixed to $u_{\min} = 25$, $u_{\max} = 100$ and their changes were limited to $\Delta u_{\min} = -15$; $\Delta u_{\max} = 15$.The controlled variables are constrained by $y_{\min} = 15$ and $y_{\max} = 30$.

The simulations have been performed using the simulation language EcosimPro with the process being represented by equations (1)..(3), where the parameters had the following values: the area of the tank was A=500 cm^2, the length of the pipe L=150 m, the initial level h=15 cm, the input temperature of the liquid T_i=15 ºC and the initial value of the inlet flow q=50 cm^3/s.

4.1 Setpoint tracking

Fig. 4 shows the evolution of the tank temperature with the proposed non-linear EPSAC controller during 10000 seconds where several set point step changes were performed, shown in dotted line. The estimation of the temperature in the tank is also shown. The controller was started with a reference value of 22.18 ºC (the steady-state value). The temperature T follows fairly well the set point changes. Notice that no delay appears in the graph because we have represented the temperature T in the tank and not the measured temperature T_d. Also notice that the graphs of T and its estimation are superposed , so that no clear difference can be seen between them.

The manipulated variable is represented in Fig. 5 and the time varying delay can be seen in Fig.6.

In order to compare these results with the ones obtained with a linear predictive controller with fixed delay time and to keep the same methodology, the linear EPSAC was applied with the following CARIMA model:

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \frac{C(q^{-1})}{1-q^{-1}}e(t) \qquad (18)$$

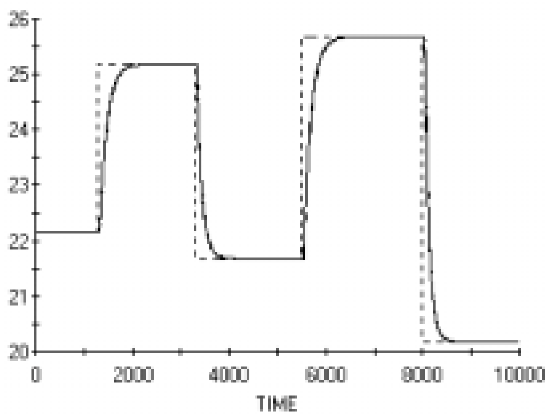

Fig.4 Setpoint tracking of the controlled variable

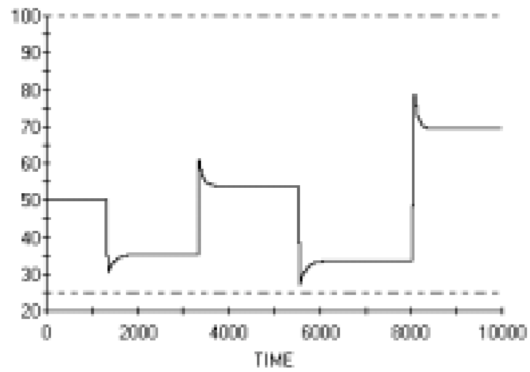

Fig. 5 The manipulated variable (non linear controller)

where

$$A(q^{-1}) = 1 - 0.8752q^{-1}$$
$$B(q^{-1}) = (-0.01793q^{-1})q^{-k_b}$$
$$C(q^{-1}) = 1$$

The open-loop time constant is $\tau = \frac{Ah}{q} = 150$ seconds.

Using the same parameters and constraints as the non linear controller and taking into account an identical simulation sequence, three cases was considered, with different values for the pure time delay k_b taking into account the range represented in Fig. 6.

In the first case, the constant model delay was underestimated. Fig. 7 illustrates the evolution of the temperature considering a constant small value for the dead-time: d=100 seconds < τ (k_b=5). The response has a very big overshoot and it is too oscillatory.

In the second experiment, shown in Fig. 8, an overestimated fix value of: d= 300=2τ (k_b=15) is considered. The response improves, but is oscillatory and worst than the one of Fig.4. Notice that overstating a delay is not a solution for the stabilisation of the system as it is shown in Fig.9, which corresponds to the same linear controller operating with a model with a fixed dead-time of d=440 ≈3τ (k_b=22). As we can see, the response is too oscillatory and clearly unacceptable.

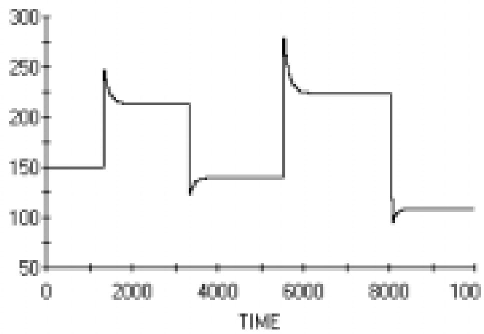

Fig. 6 The dead-time

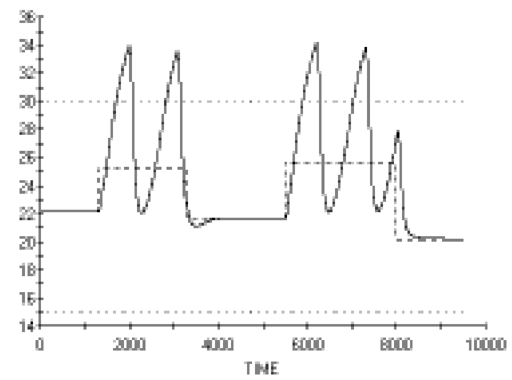

Fig.7 The controlled variable (using linear controller with d=100 seconds)

As a final test, the full NMPC controller was implemented and the experiments repeated. Results were identical to Fig. 4 but the computation time was twice the one needed for the non-linear EPSAC to obtain the same result.

5. CONCLUSIONS

In this paper an alternative approach to the solution of NMPC problems have been presented, applied to a difficult control problem due to the variable delay involved. One important point to mention is that the way in which the internal model is used in non-linear EPSAC, gives through the u_{base} the gross effect of the variable delay, particularly when $N_u > 1$, and provides an automatic fitting to the change of this effect via the step and impulse coefficients, improving its response. Another advantage is the reduction in computation time over a pure NMPC approach.

Notice that cases of variable, but computable, delay, like the one considered here, in which the main non-linearity is the variable delay, can be represented by the model

$$P_n(s) = G(s)e^{-d(u)s}$$

for which, the proposed non-linear EPSAC approach provides an attractive and simple solution.

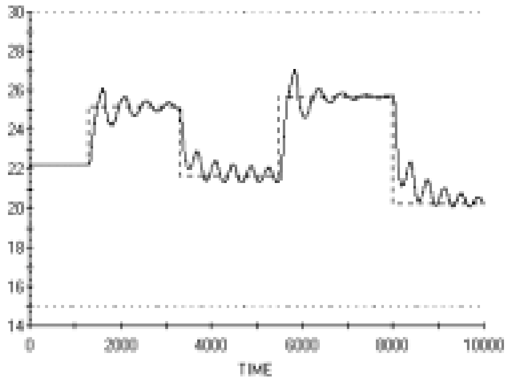

Fig.8 The controlled variable (using linear controller with dead-time d=300 seconds)

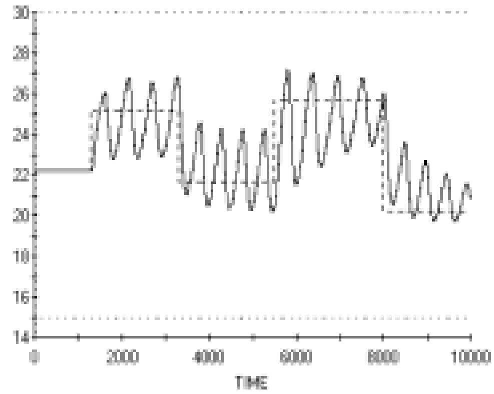

Fig.9 The controlled variable (using linear controller with dead-time d=440 seconds)

6. AKNOWLEDGMENTS

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