# BER Performance of OSTBCs on Correlated Fading Channels With Imperfect Channel Estimation 

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#### Abstract

: In this contribution, we examine the bit error rate (BER) performance of orthogonal space-time block codes (OSTBCs) under flat-fading channels with imperfect channel state information. We present new exact BER expressions for PAM and QAM signal constellations, regardless of the fading distribution. Furthermore, we show how these expressions can be efficiently and accurately evaluated for the case of arbitrarily correlated Nakagami- $m$ fading channels. As the high diversity order resulting from the application of OSTBCs gives rise to small BER values, the numerical evaluation of the presented BER expressions is much faster than straightforward computer simulations.


Keywords: Error analysis, OSTBC, channel estimation, correlated Nakagami fading

## 1. Introduction

Orthogonal space-time block codes (OSTBCs) are a remarkably beneficial transmit diversity technique, since they provide full diversity gain and result in a very simple maximum-likelihood (ML) detection algorithm, based only on linear processing at the receiver $[1,2]$. When imperfect channel estimation (ICE) is taken into account, the performance of OSTBC systems is usually studied considering independent and identically distributed (i.i.d.) Rayleigh fading. In [3], a single-integral BER expression was given for Alamouti's code with pilot-symbol assisted modulation (PSAM). An exact closed-form BER expression for square OSTBCs was derived in [4] for pilot-based minimum mean-square error (MMSE) channel estimation and pulse amplitude modulation (PAM) or quadrature amplitude modulation (QAM) constellations. Using an approach based on characteristic functions, an exact closed-form expression for the pairwise error probability (PEP) of both orthogonal and non-orthogonal space-time codes with least-squares (LS) channel estimation was obtained in [5]. In this paper, we provide an exact analytical BER analysis for OSTBCs with PAM or QAM constellations and ICE on flat-fading channels with an arbitrary joint pdf. Also, we show how these exact BER expressions can be efficiently and accurately evaluated in the particular case of arbitrarily correlated Nakagami- $m$ fading channels.

## 2. System model

We consider a multiple-input multiple-output (MIMO) wireless communication system with $L_{\mathrm{t}}$ transmit and $L_{\mathrm{r}}$ receive antennas. The transmitted data symbols at each transmit antenna are assumed to be coded according to an $L_{\mathrm{t}} \times K_{\mathrm{c}}$ ( $K_{\mathrm{c}}$ denotes the block length) OSTBC matrix $\mathbf{C}$, the entries of which are linear combinations of $N_{\mathrm{s}}$ information symbols $s_{i}=s_{i, \mathrm{R}}+j s_{i, \mathrm{I}}, 1 \leq i \leq N_{\mathrm{s}}$, and their complex conjugate $s_{i}^{*}$

$$
\begin{equation*}
\mathbf{C}=\sum_{i=1}^{N_{\mathrm{s}}}\left(\mathbf{C}_{i} s_{i}+\mathbf{C}_{i}^{\prime} s_{i}^{*}\right) . \tag{1}
\end{equation*}
$$

In (1), the $L_{\mathrm{t}} \times K_{\mathrm{c}}$ matrices $\mathbf{C}_{i}$ and $\mathbf{C}_{i}^{\prime}$ comprise the coefficients of $s_{i}$ and $s_{i}^{*}$, respectively. Since scaling of $\mathbf{C}$ does not affect its orthogonality, we assume that $\mathbf{C}$ is scaled in such way that

$$
\begin{equation*}
\mathbf{C C}^{H}=\left(\lambda \sum_{i=1}^{N_{\mathrm{s}}}\left|s_{i}\right|^{2}\right) \mathbf{I}_{L_{\mathrm{t}}}, \tag{2}
\end{equation*}
$$

where $\lambda \triangleq K_{\mathrm{c}} / N_{\mathrm{s}}$ and $\mathbf{I}_{L_{\mathrm{t}}}$ is the $L_{\mathrm{t}} \times L_{\mathrm{t}}$ identity matrix. For square OSTBCs, i.e., $L_{\mathrm{t}}=K_{\mathrm{c}}$, it is readily verified that $\mathbf{C}^{H} \mathbf{C}=\mathbf{C} \mathbf{C}^{H}$.

We organize the data transmission in frames consisting of $K_{\mathrm{p}}$ known pilot symbols used for channel estimation and $K / K_{\mathrm{c}}$ coded symbol matrices $\mathbf{C}(k)$, with $K$ being a multiple of $K_{\mathrm{c}}$ and $k$ denoting the block index. With $L \triangleq L_{\mathrm{t}} L_{\mathrm{r}}$ denoting the diversity order, the $L$ complex channel coefficients are distributed according to an arbitrary joint pdf that characterizes the fading. The $L_{\mathrm{r}} \times L_{\mathrm{t}}$ complex random matrix $\mathbf{H}$, containing all channel coefficients, is assumed to remain constant during the length of one frame of $K+K_{\mathrm{p}}$ symbols, such that the receiver separately observes the $L_{\mathrm{r}} \times K_{\mathrm{c}}$ matrices

$$
\begin{equation*}
\mathbf{R}(k)=\sqrt{E_{\mathbf{s}}} \mathbf{H C}(k)+\mathbf{W}(k), \tag{3}
\end{equation*}
$$

with $1 \leq k \leq K / K_{\mathrm{c}}$, and the $L_{\mathrm{r}} \times K_{\mathrm{p}}$ matrix

$$
\begin{equation*}
\mathbf{R}_{\mathrm{p}}=\sqrt{E_{\mathrm{p}}} \mathbf{H C} \mathbf{C}_{\mathrm{p}}+\mathbf{W}_{\mathrm{p}} \tag{4}
\end{equation*}
$$

where the noise matrices $\mathbf{W}(k)$ and $\mathbf{W}_{\mathrm{p}}$ contain i.i.d. zero-mean (ZM) circularly symmetric complex Gaussian (CSCG) random variables (RVs) with variance $N_{0}$. In the remainder of this paper, we will drop the block index $k$ for notational simplicity.

In order to estimate the channel, we assume orthogonal pilot sequences, i.e., $\mathbf{C}_{\mathrm{p}} \mathbf{C}_{\mathrm{p}}^{H}=$ $K_{\mathrm{p}} \mathbf{I}_{L_{\mathrm{t}}}$, and LS channel estimation [6]

$$
\begin{equation*}
\hat{\mathbf{H}}=\frac{1}{K_{\mathrm{p}} \sqrt{E_{\mathrm{p}}}} \mathbf{R}_{\mathrm{p}} \mathbf{C}_{\mathrm{p}}^{H} \tag{5}
\end{equation*}
$$

such that the estimated channel $\hat{\mathbf{H}}$ can be decomposed into the sum of two statistically independent contributions, i.e., $\hat{\mathbf{H}}=\mathbf{H}+\mathbf{N}$. Note that the $L_{\mathrm{r}} \times L_{\mathrm{t}}$ estimation noise matrix $\mathbf{N} \triangleq\left[1 /\left(K_{\mathrm{p}} \sqrt{E_{\mathrm{p}}}\right)\right] \mathbf{W}_{\mathrm{p}} \mathbf{C}_{\mathrm{p}}^{H}$ consists of ZM CSCG RVs, the real and imaginary parts of which have variance $\sigma_{\mathbf{N}}^{2}=N_{0} /\left(2 K_{\mathrm{p}} E_{\mathrm{p}}\right)$. Hence, when conditioned on $\mathbf{H}$, the estimated channel coefficients $\hat{\mathbf{H}}_{\ell, k}\left(1 \leq \ell \leq L_{\mathrm{r}}, 1 \leq k \leq L_{\mathrm{t}}\right)$ are CSCG RVs with mean $\mathbf{H}_{\ell, k}$ and variance $2 \sigma_{\mathrm{N}}^{2}$. Increasing the total energy $K_{\mathrm{p}} E_{\mathrm{p}}$ allocated to pilot symbols
improves the channel estimate, but also reduces the symbol energy $E_{s}$ available for data transmission. Indeed, with $E_{\mathrm{b}}, \gamma=E_{\mathrm{p}} / E_{\mathrm{s}}, M$, and $\rho=N_{\mathrm{s}} /\left(L_{\mathrm{t}} K_{\mathrm{c}}\right)$ denoting the energy per information bit, the ratio of the pilot energy $E_{\mathrm{p}}$ to the symbol energy $E_{\mathrm{s}}$, the number of constellation points and the code rate, respectively, we have $E_{\mathrm{s}}=\frac{K}{K+\gamma K_{\mathrm{p}}} \rho \log _{2}(M) E_{\mathrm{b}}$, which is a decreasing function of $K_{\mathrm{p}}$.

We consider a mismatched ML receiver that uses the estimated channel $\hat{\mathbf{H}}$ in the same way as an ML receiver would apply $\mathbf{H}$. In this way, the detection algorithm for the information symbols $s_{i}$ can be shown to reduce to symbol-by-symbol detection

$$
\begin{equation*}
\hat{s}_{i}=\arg \min _{\tilde{s}}\left|u_{i}-\tilde{s}\right|, \quad 1 \leq i \leq N_{\mathrm{s}} \tag{6}
\end{equation*}
$$

where the minimization is over the symbols $\tilde{s}$ belonging to the considered signaling constellation, and the decision variables $u_{i}=u_{i, \mathrm{R}}+j u_{i, \mathrm{I}}$ are given by

$$
\begin{equation*}
u_{i}=\frac{\operatorname{tr}\left(\mathbf{C}_{i}^{H} \hat{\mathbf{H}}^{H} \mathbf{R}+\mathbf{R}^{H} \hat{\mathbf{H}} \mathbf{C}_{i}^{\prime}\right)}{\lambda \sqrt{E_{\mathrm{s}}}\|\hat{\mathbf{H}}\|_{F}^{2}} \tag{7}
\end{equation*}
$$

with $\operatorname{tr}(\cdot)$ and $\|\cdot\|_{F}$ denoting the trace and the Frobenius norm of a matrix, respectively.

## 3. BER analysis

For square $M$-QAM transmission with Gray mapping, the BER can be obtained by averaging the BERs related to the in-phase and quadrature-phase bits of all information symbols $s_{i}$ in the code matrix

$$
\begin{equation*}
\operatorname{BER}=\frac{1}{2 N_{\mathrm{s}}} \sum_{i=1}^{N_{\mathrm{s}}} \mathbb{E}\left[\operatorname{BER}_{i, \mathrm{R}}(\mathbf{s}, \mathbf{H}, \hat{\mathbf{H}})+\operatorname{BER}_{i, \mathrm{I}}(\mathbf{s}, \mathbf{H}, \hat{\mathbf{H}})\right] \tag{8}
\end{equation*}
$$

where $\mathrm{BER}_{i, \mathrm{R}}(\mathbf{s}, \mathbf{H}, \hat{\mathbf{H}})$ and $\mathrm{BER}_{i \mathrm{I}}(\mathbf{s}, \mathbf{H}, \hat{\mathbf{H}})$ denote the conditional BERs related to the in-phase and quadrature-phase information bits of $s_{i}$, respectively, conditioned on $\mathbf{s}$, $\mathbf{H}$, and $\hat{\mathbf{H}}$. Denoting by $\Psi$ the $M$-QAM constellation, $\mathbf{s}$ is uniformly distributed over $\Psi^{N_{\mathrm{s}}}$. Furthermore, $\Psi_{\mathrm{R}}$ and $\Psi_{\mathrm{I}}$ are the sets containing the real and imaginary parts of the constellation points, respectively. Referring to the projections of the decision area of a QAM symbol $b=b_{\mathrm{R}}+j b_{\mathrm{I}}$ on the real and imaginary axis as the decision regions of $b_{\mathrm{R}}$ and $b_{\mathrm{I}}$, respectively, we can write $\mathrm{BER}_{i, \mathrm{q}}(\mathbf{s}, \mathbf{H}, \hat{\mathbf{H}})$, with $\mathrm{q}=\mathrm{R}$ or $\mathrm{q}=\mathrm{I}$, as follows

$$
\begin{equation*}
\mathrm{BER}_{i, \mathrm{q}}(\mathbf{s}, \mathbf{H}, \hat{\mathbf{H}})=2 \sum_{b_{\mathrm{q}} \in \Psi_{\mathrm{q}}} \frac{N\left(s_{i, \mathrm{q}}, b_{\mathrm{q}}\right)}{\log _{2} M} P_{i, \mathrm{q}}\left(\mathbf{s}, b_{\mathrm{q}}, \mathbf{H}, \hat{\mathbf{H}}\right), \tag{9}
\end{equation*}
$$

where $N\left(s_{i, \mathrm{q}}, b_{\mathrm{q}}\right)$ represents the Hamming distance between the bits allocated to $s_{i, \mathrm{q}}$ and $b_{\mathrm{q}}$, and $P_{i, \mathrm{q}}\left(\mathbf{s}, b_{\mathrm{q}}, \mathbf{H}, \hat{\mathbf{H}}\right)$ is the probability that $u_{i, \mathrm{q}}$ is located inside the decision area of $b_{\mathrm{q}}$, when $\mathbf{s}, \mathbf{H}$, and $\hat{\mathbf{H}}$ are known. Taking (1) and (3) into account, expanding the decision variable (7) yields $u_{i}=u_{i}^{\prime}+n_{i}$, where $u_{i}^{\prime}$ is a function of $\mathbf{s}, \mathbf{H}$, and $\hat{\mathbf{H}}$, and $n_{i, \mathrm{R}}$ represents ZM CSCG noise with variance $N_{0} /\left(\lambda E_{\mathrm{s}}\|\hat{\mathbf{H}}\|_{F}^{2}\right)$. When $d_{1}\left(b_{\mathrm{q}}\right)$ and $d_{2}\left(b_{\mathrm{q}}\right)$ denote the boundaries of the decision area of $b_{\mathrm{q}}$, with $d_{1}\left(b_{\mathrm{q}}\right)<d_{2}\left(b_{\mathrm{q}}\right), P_{i, \mathrm{q}}\left(\mathbf{s}, b_{\mathrm{q}}, \mathbf{H}, \hat{\mathbf{H}}\right)$
is given by

$$
\begin{align*}
& P_{i, \mathrm{q}}\left(\mathbf{s}, b_{\mathrm{q}}, \mathbf{H}, \hat{\mathbf{H}}\right)=Q\left\{\sqrt{2 \lambda \frac{E_{\mathrm{s}}}{N_{0}}}\|\hat{\mathbf{H}}\|_{F}\left[d_{1}\left(b_{\mathrm{q}}\right)-u_{\mathrm{i}, \mathrm{q}}^{\prime}\right]\right\} \\
&-Q\left\{\sqrt{2 \lambda \frac{E_{\mathrm{s}}}{N_{0}}}\|\hat{\mathbf{H}}\|_{F}\left[d_{2}\left(b_{\mathrm{q}}\right)-u_{\mathrm{i}, \mathrm{q}}^{\prime}\right]\right\} \tag{10}
\end{align*}
$$

where $Q($.$) denotes the Gaussian Q$-function [7, Eq. (4.1)]. Note from (8)-(10) that the evaluation of the BER requires averaging over $4 L$ real-valued continuous RVs, i.e., the real and imaginary parts of the elements of $\mathbf{H}$ and $\hat{\mathbf{H}}$, and over $N_{\mathrm{s}}$ discrete RVs, i.e., the $N_{\mathrm{s}}$ symbols in s. By using a carefully selected coordinate transformation, however, we will decrease the number of RVs involved in the expectation (8) and, accordingly, the computational complexity related to its numerical evaluation. Let us introduce the $2 L$ dimensional real-valued column vectors $\hat{\mathbf{h}}$ and $\mathbf{h}$, consisting of the real and imaginary parts of all elements of $\hat{\mathbf{H}}$ and $\mathbf{H}$, respectively:

$$
\left\{\begin{array}{l}
\hat{\mathbf{h}}=\left[\hat{\mathbf{h}}_{1, \mathrm{R}}^{T}, \hat{\mathbf{h}}_{1, \mathrm{I}}^{T}, \cdots, \hat{\mathbf{h}}_{N_{\mathrm{s}}, \mathrm{R}}^{T}, \hat{\mathbf{h}}_{N_{\mathrm{s}, \mathrm{I}}}^{T}\right]^{T}  \tag{11}\\
\mathbf{h}=\left[\mathbf{h}_{1, \mathrm{R}}^{T}, \mathbf{h}_{1, \mathrm{I}}^{T}, \cdots, \mathbf{h}_{N_{\mathrm{s}}, \mathrm{R}}^{T}, \mathbf{h}_{N_{\mathrm{s}}, \mathrm{I}}^{T}\right]^{T},
\end{array}\right.
$$

with $\hat{\mathbf{h}}_{i, \mathrm{R}}+j \hat{\mathbf{h}}_{i, \mathrm{I}}$ and $\mathbf{h}_{i, \mathrm{R}}+j \mathbf{h}_{i, \mathrm{I}}$ denoting the $i$-th column of $\hat{\mathbf{H}}$ and $\mathbf{H}$, respectively. It follows from (11) that $|\hat{\mathbf{h}}|=\|\hat{\mathbf{H}}\|_{F}$ and $|\mathbf{h}|=\|\mathbf{H}\|_{F}$, with $|\cdot|$ denoting the norm of a vector. Using (11), it can be shown that $u_{i, \mathrm{q}}^{\prime}$ in (10) can be written as

$$
\begin{equation*}
u_{i, \mathrm{q}}^{\prime}=\frac{1}{\lambda\left(x_{1}^{2}+x_{2}^{2}+z^{2}\right)}\left(\lambda x_{1} s_{i, \mathrm{q}}|\mathbf{h}|+x_{2}\left|\mathbf{M}_{i, \mathrm{q}} \mathbf{h}\right|\right) \tag{12}
\end{equation*}
$$

where the $2 L \times 2 L$ matrix $\mathbf{M}_{i, \mathrm{q}}$ is a function of the transmitted symbol vector $\mathbf{s}$ and the coefficient matrices $\mathbf{C}_{j}$ and $\mathbf{C}_{j}^{\prime}, 1 \leq j \leq N_{\mathrm{s}}$. Because of space limitations, $\mathbf{M}_{i, \mathrm{q}}$ is not given in explicit form. Also, it can be shown that $x_{1}, x_{2}$, and $z$, when conditioned on $\mathbf{h}$, are independent RVs satisfying the following properties: $x_{1}$ and $x_{2}$ are Gaussian RVs with mean $|\mathbf{h}|$ and zero, respectively, and variance $\sigma_{\mathbf{N}}^{2} ; z / \sigma_{\mathbf{n}}$ follows the chi-distribution with $2 L-2$ degrees of freedom. Hence, considering that $|\hat{\mathbf{h}}|^{2}=x_{1}^{2}+x_{2}^{2}+z^{2}$, it is easily seen that the conditional BERs given by (9) can be rewritten as functions that depend on $\mathbf{H}$ through the random vector $\mathbf{h}$, and on $\hat{\mathbf{H}}$ through only 3 RVs : $x_{1}, x_{2}$, and $z$; we denote these functions by $\mathrm{B}_{i, \mathrm{q}, 1}\left(\mathbf{s}, \mathbf{h}, x_{1}, x_{2}, z\right)$. Note that the dependence on $\mathbf{h}$ is only through $|\mathbf{h}|$ and $\left|\mathbf{M}_{i, \mathrm{q}} \mathbf{h}\right|$, with $\mathbf{M}_{i, \mathrm{q}}$ depending on $\mathbf{s}$. Because of this substitution, the BER reduces to an expectation over $2 L+3$ continuous RVs, i.e., the $2 L$ components of $\mathbf{h}, x_{1}, x_{2}$, and $z$, and $N_{\mathrm{s}}$ discrete RVs, i.e., the components of $\mathbf{s}$ :

$$
\begin{equation*}
\mathrm{BER}=\frac{1}{2 N_{\mathrm{s}}} \sum_{i=1}^{N_{\mathrm{s}}} \mathbb{E}\left[\mathrm{~B}_{i, \mathrm{R}, 1}\left(\mathbf{s}, \mathbf{h}, x_{1}, x_{2}, z\right)+\mathrm{B}_{i, \mathrm{I}, 1}\left(\mathbf{s}, \mathbf{h}, x_{1}, x_{2}, z\right)\right] \tag{13}
\end{equation*}
$$

In the case of square OSTBCs, the BER expression (13) can be considerably simplified. For these OSTBCs, it can be shown that the magnitude of $\mathbf{M}_{i, \mathrm{q}} \mathbf{h}$ used in (12) is given by $\left|\mathbf{M}_{i, \mathrm{q}} \mathbf{h}\right|=\lambda|\mathbf{h}| \sqrt{|\mathbf{s}|^{2}-s_{i, \mathrm{q}}^{2}}$. Hence, $\mathrm{B}_{i, \mathrm{q}, 1}\left(\mathbf{s}, \mathbf{h}, x_{1}, x_{2}, z\right)$ in (13), with $\mathrm{q}=\mathrm{R}$ or $\mathrm{q}=\mathrm{I}$ can be rewritten as a function that depends on $\mathbf{h}$ through only the norm $|\mathbf{h}|$
of the channel vector; we denote this function by $\mathrm{B}_{i, \mathrm{q}, 2}\left(\mathbf{s},|\mathbf{h}|, x_{1}, x_{2}, z\right)$. Moreover, since the statistical properties of $\mathbf{s}_{i, \mathrm{q}}$ are independent of $i$ and q , it is easily seen that the BER for square OSTBCs reduces to

$$
\begin{equation*}
\mathrm{BER}=\mathbb{E}\left[\mathrm{B}_{i, \mathrm{q}, 2}\left(\mathbf{s},|\mathbf{h}|, x_{1}, x_{2}, z\right)\right], \tag{14}
\end{equation*}
$$

which involves the expectation over only 4 continuous RVs, i.e, the norm $|\mathbf{h}|$ of the channel vector $\mathbf{h}, x_{1}, x_{2}$ and $z$, and $N_{\mathrm{s}}$ discrete RVs, i.e., the components of $\mathbf{s}$.

By numerically evaluating (13) and (14), the BER for OSTBCs can be efficiently and accurately obtained. In order to minimize the associated computation time, a proper combination of numerical integration techniques, such as the quadrature rule [8, Sec. 4.1], in which case a $J$-fold integral is approximated by a $J$-fold sum, and Monte-Carlo integration $[8$, Sec. 7.7] with importance sampling $[8$, Sec. 7.9.1] can be used. For instance, the expectation over $x_{1}, x_{2}$, and $z$ can be evaluated by means of the quadrature rule, while the expectation over $\mathbf{s}$ and $\mathbf{h}$ can be evaluated through Monte-Carlo integration.

## 4. Numerical results

In this section, BER results are presented for correlated Nakagami- $m$ fading channels with uniformly distributed phases, assuming that $E_{\mathrm{p}}=E_{\mathrm{s}}$. The Nakagami- $m$ distribution is considered as a versatile statistical distribution that accurately models a variety of fading environments [7,9-11]. Denoting by $\alpha$ the magnitude of a complex-valued channel coefficient, its pdf in the case of Nakagami- $m$ fading is given by [7, Eq. (2.20)]

$$
\begin{equation*}
p_{\alpha}(r)=\frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^{m} r^{(2 m-1)} \exp \left(-\frac{m}{\Omega} r^{2}\right), \quad r \geq 0 \tag{15}
\end{equation*}
$$

with $\Gamma(\cdot)$ being the Gamma function [12, Eq. (8.310/1)], $\Omega=\mathbb{E}\left[\alpha^{2}\right]$ being the average fading power, and $m \geq 1 / 2$ being the fading parameter. In case of a correlated Nakagami fading MIMO channel, the marginal pdf of the magnitude $\alpha_{i, j}$ of the channel coefficient $\mathbf{H}_{i, j}$ follows (15), with parameters $m_{i, j}$ and $\Omega_{i, j}$ possibly depending on $i$ and $j$. Stacking the $L=L_{\mathrm{t}} L_{\mathrm{r}}$ fading magnitudes $\alpha_{i, j}$ into an $L$-dimensional column vector $\boldsymbol{\alpha}$, such that $\boldsymbol{\alpha}_{i+(j-1) L_{\mathrm{r}}}=\alpha_{i, j}$, the elements of the $L \times L$ power correlation matrix $\boldsymbol{\Sigma}$ are defined as [7, Eq. (9.195)]. Also, we use the MIMO channel model proposed in [13], such that $\boldsymbol{\Sigma}=\boldsymbol{\Sigma}_{\mathrm{t}} \otimes \boldsymbol{\Sigma}_{\mathrm{r}}$, where $\boldsymbol{\Sigma}_{\mathrm{t}}$ and $\boldsymbol{\Sigma}_{\mathrm{r}}$ are the $L_{\mathrm{t}} \times L_{\mathrm{t}}$ transmit and $L_{\mathrm{r}} \times L_{\mathrm{r}}$ receive power correlation matrices, respectively, and $\otimes$ denotes the Kronecker product.

Let us consider Alamouti's code ( $L_{\mathrm{t}}=K_{\mathrm{c}}=N_{\mathrm{s}}=2$ ), which is given by [1, Eq. (32)]. Fig. 1 displays the BER curves for Alamouti's code along with 4-QAM signaling, operating over correlated identically distributed (i.d.) Nakagami- $m$ channels with $\Omega=1$ and $L_{\mathrm{r}}=3$. In order to obtain the BER, we evaluate the expectation over $|\mathbf{h}|, x_{1}$, $x_{2}$, and $z$ in (14) by means of the quadrature rule, while the expectation over $\mathbf{s}$ is exactly obtained by a finite summation over all possible symbol vectors. The pdf of $|\mathbf{h}|$ can be easily obtained from [9] in the case of identical and integer $m$. The BER results are shown for both a receiver with perfect channel knowledge (PCK) and a mismatched receiver using (5) with $K=200$, and $K_{\mathrm{p}}=20$. Further, we assume that $m=\{1,3\}$, that no antenna correlation occurs at the transmitter side, and that the receive antennas are either uncorrelated (unc) or constructed as a linear array (lin), the configuration of which is depicted in [10, Fig. 4(b)], with power correlation matrix $\boldsymbol{\Sigma}_{\mathrm{r}}$ given by [10, Eq. (38)]. From Fig. 1, we can see how the fading parameter m, ICE,


Figure 1: BER for Alamouti's code with 4-QAM signaling operating over correlated i.d. Nakagami- $m$ channels, with $m=\{1,3\}$ and $\Omega=1$. Computer simulations confirm the analytical results obtained from numerically evaluating (14).
and antenna correlation affect the BER of maximal-ratio diversity (MRD) reception. A larger $m$ indicates less severe fading, such that the BER performance is improved. Both imperfect channel estimation and antenna correlation degrade the BER through a horizontal shift of the BER curve for large $E_{\mathrm{b}} / N_{0}$. Interestingly, the BER degradation due to antenna correlation is much larger than the degradation due to ICE.

Fig. 2 shows the BER versus the number of pilot symbols $K_{\mathrm{p}}$ for the $3 \times 4$ OSTBC given by [1, Eq. (39)], operating over correlated i.d. Nakagami- $m$ fading channels with $m=2$ and $\Omega=1$, under the assumption that $E_{\mathrm{b}} / N_{0}=10 \mathrm{~dB}$. To obtain the BER, the expectation over $\mathbf{s}, \mathbf{h}, x_{1}, x_{2}$, and $z$ in (13) has been evaluated by means of MonteCarlo integration with importance sampling. Clearly, the expectation over $\mathbf{h}$ requires the generation of $L$ correlated Nakagami- $m$ RVs; in [11], it is shown how these RVs can be obtained from $2 m$ i.i.d. $L$-dimensional ZM Gaussian random vectors, in case of identical and integer $m$. The power correlation matrix $\boldsymbol{\Sigma}_{\mathrm{r}}$ of the mismatched dualantenna receiver $\left(L_{\mathrm{r}}=2\right)$ is given by

$$
\boldsymbol{\Sigma}_{\mathrm{r}}=\left(\begin{array}{cc}
1 & 0.2  \tag{16}\\
0.2 & 1
\end{array}\right)
$$

and the transmitted information symbols belong to a $64-\mathrm{QAM}$ constellation. The results are shown for $K=\{48,200,800\}$ information symbols and for the power correlation


Figure 2: BER versus $K_{\mathrm{p}}$ for the $3 \times 4$ OSTBC given by [1, Eq. (39)] with 64-QAM.
matrix $\boldsymbol{\Sigma}_{\mathrm{t}}$ at the transmitter side being given by either [10, Eq. (38)] (lin) or the identity matrix (unc). From Fig. 2, we notice that the optimal number of pilot symbols grows with the number of information symbols $K$, and that antenna correlation does not affect this optimal number. For large $K$, obtaining the optimal number of pilot symbols is not very critical, as the BER grows only slowly when more pilot symbols are added.

## 5. Conclusions and remarks

In this contribution, we investigated the effect of ICE on the BER performance of OSTBCs operating over correlated flat-fading channels. For both non-square and square OSTBCs, we proposed new exact BER expressions which can be efficiently and accurately evaluated by a combination of numerical integration techniques. Our numerical results illustrate how the proposed analysis can be applied to investigate the effect of channel estimation and fading correlation on the BER performance of OSTBC systems in the case of Nakagami- $m$ fading.

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