UNIVERSITEIT GENT

FACULTEIT **INGENIEURSWETENSCHAPPEN EN ARCHITECTUUR**

Dimensional exploration techniques for photonics

Philippe A.J.G. CHEVALIER

Ghent University

Who is this physicist?

Who is this physicist?

James Clerk Maxwell (*1831-†1879) at the age of 40 years.

Outline

J.C. Maxwell (*1831-†1879)

In 1869 Maxwell made the following remarks on the mathematical classification of physical quantities:

J.C. Maxwell (*1831-†1879)

In 1869 Maxwell made the following remarks on the mathematical classification of physical quantities:

> *"The first part of the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend.*

J.C. Maxwell (*1831-†1879)

In 1869 Maxwell made the following remarks on the mathematical classification of physical quantities:

> *"The first part of the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend. The next stage is the discovery of the mathematical form of the relations between these quantities.*

J.C. Maxwell (*1831-†1879)

In 1869 Maxwell made the following remarks on the mathematical classification of physical quantities:

> *"The first part of the growth of a physical science consists in the discovery of a system of quantities on which its phenomena may be conceived to depend. The next stage is the discovery of the mathematical form of the relations between these quantities. After this, the science may be treated as a mathematical science, and the verification of the laws is effected by a theoretical investigation of the conditions under which certain quantities can be most accurately measured, followed by an experimental realisation of these conditions, and actual measurement of the quantities."*

.

What is the language of physics ?

What is the language of physics ?

$$E = mc_0^2$$

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$$

$$\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c_0^2}{\hbar^2}\psi = 0$$

$$i\hbar\gamma^\mu\partial_\mu\psi - mc_0\psi = 0$$

$$\langle\mathbf{p}\rangle = \langle\psi|\frac{\hbar}{i}\nabla|\psi\rangle = \hbar\mathbf{k}$$

$$\langle E\rangle = \langle\psi|\frac{\hbar}{i}\frac{\partial}{\partial t}|\psi\rangle = \hbar\omega$$

$$G^{\mu\nu} \doteq R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = -\frac{8\pi G}{c_0^4}T^{\mu\nu}$$

$$\frac{\mathrm{d}^2x^\delta}{\mathrm{d}\tau^2} + \Gamma^\delta_{\beta\gamma}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}\frac{\mathrm{d}x^\gamma}{\mathrm{d}\tau} = 0$$

What are the research questions ?

What are the research questions ?

- Question 1: which cluster of related quantities fixed the value of the quantity being investigated, and did not under or over-determine it?

What are the research questions ?

- Question 1: which cluster of related quantities fixed the value of the quantity being investigated, and did not under or over-determine it?
- Question 2: which variables, material constants and universal constants should appear multiplied together in a hypothetical law of physics?

What are the research questions ?

- Question 1: which cluster of related quantities fixed the value of the quantity being investigated, and did not under or over-determine it?
- Question 2: which variables, material constants and universal constants should appear multiplied together in a hypothetical law of physics?
- Question 3: which quantities should be excluded?

What are the research questions ?

- Question 1: which cluster of related quantities fixed the value of the quantity being investigated, and did not under or over-determine it?
- Question 2: which variables, material constants and universal constants should appear multiplied together in a hypothetical law of physics?
- Question 3: which quantities should be excluded?

These research questions remain unanswered and block the progress in mathematical modeling of physical processes.

What are the base quantities of SI_{2018} ?

SI_{2018} **base quantity**	SI_{2018} **quantity symbol**	SI_{2018} **dimension symbol**
frequency	ν	F
action	h	A
velocity	c	V
electric charge	e	C
heat capacity	k	H
amount of substance	N_A	N
luminous efficacy	K_{cd}	K

Which dimensions for a physical quantity ?

Definition

The SI_{2018} dimension of a physical quantity q is expressed as a dimensional product:

$$\dim(q) = \mathrm{F}^{\alpha}\mathrm{A}^{\beta}\mathrm{V}^{\gamma}\mathrm{C}^{\delta}\mathrm{H}^{\epsilon}\mathrm{N}^{\zeta}\mathrm{K}^{\eta}\,;$$

where the exponents $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta \in \mathbb{Z}$ are called dimensional exponents.

Which dimensions for a physical quantity ?

Definition

The SI_{2018} dimension of a physical quantity q is expressed as a dimensional product:

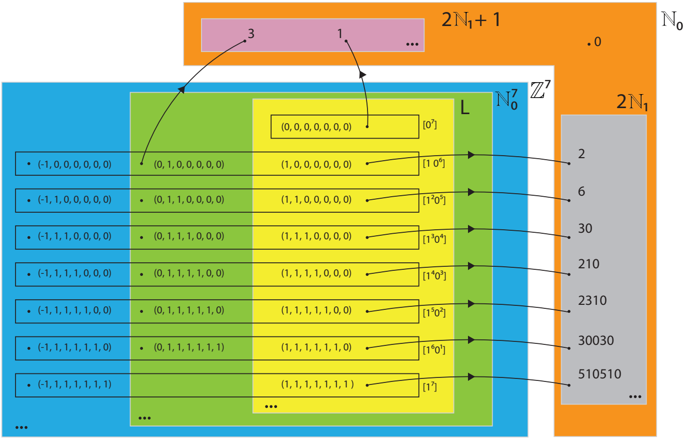
$$\dim(q) = \mathrm{F}^{\alpha}\mathrm{A}^{\beta}\mathrm{V}^{\gamma}\mathrm{C}^{\delta}\mathrm{H}^{\epsilon}\mathrm{N}^{\zeta}\mathrm{K}^{\eta}\,;$$

where the exponents $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta \in \mathbb{Z}$ are called dimensional exponents.

Example

$$\dim(\mathit{Energy}) = \mathrm{F}^1\mathrm{A}^1\mathrm{V}^0\mathrm{C}^0\mathrm{H}^0\mathrm{N}^0\mathrm{K}^0$$
$$\mathrm{dex}\left([\mathit{Energy}]\right) = (1, 1, 0, 0, 0, 0, 0)$$

How to codify physical quantities ?



How many equivalence classes exist ?

The number of equivalence classes that can be formed in a d-dimensional hypercube P_d^s is:

Number of equivalence classes

$$\#\left(P_d^s\right) = \binom{d+s-1}{s}$$

when the infinity norm $\ell_\infty = s$ and $s \in \mathbb{N}$.

How many equivalence classes exist ?

The number of equivalence classes that can be formed in a d-dimensional hypercube P_d^s is:

Number of equivalence classes

$$\#\left(P_d^s\right) = \binom{d+s-1}{s}$$

when the infinity norm $\ell_\infty = s$ and $s \in \mathbb{N}$.
OEIS A000579: $\#\left(P_7^s\right) = 1, 7, 28, 84, 210, 462, 924, 1\,716, 3\,003, 5\,005, 8\,008, 12\,376, 18\,564, 27\,132, 38\,760, 54\,264, 74\,613, 100\,947, 134\,596, 177\,100, 230\,230, 296\,010, 376\,740, 475\,020, 593\,775, 736\,281, 2\,760\,681 \ldots$

What is a Gödel code ?

Definition

$$\phi_d(f_1, \ldots, f_d) = \prod_{i=1}^{d} p_i^{f_i} \ ,$$

where p_i is the i-th prime number, $\mathbf{f} = (f_1, \ldots, f_d)$ and $f_i \in \mathbb{Z}_+$.

What is a Gödel code ?

Definition

$$\phi_d(f_1, \ldots, f_d) = \prod_{i=1}^{d} p_i^{f_i} \; ,$$

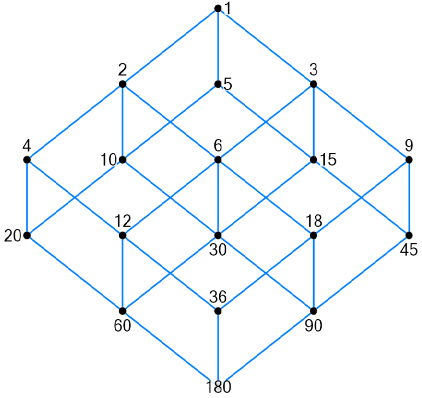
where p_i is the i-th prime number, $\mathbf{f} = (f_1, \ldots, f_d)$ and $f_i \in \mathbb{Z}_+$.

Example

$\phi_7(2, 2, 1, 0, 0, 0, 0) = 2^2 \cdot 3^2 \cdot 5^1 \cdot 7^0 \cdot 11^0 \cdot 13^0 \cdot 17^0 = 180$

What is the divisibility relation $n \mid m$?

What is the divisibility relation $n \mid m$?



Factorization of leader class $[(2^2, 1, 0^4)]$ based on the divisibility relation $n \mid m$ between the natural numbers n and m induces a lattice structure !

What is the cardinality of a leader class ?

Cardinality of a leader class

$$\#\left([w]\right) = 2^{7-d_0} \frac{7!}{d_0! d_1! d_2! \ldots d_s!} \; .$$

What is the cardinality of a leader class ?

Cardinality of a leader class

$$\#\left([w]\right) = 2^{7-d_0}\frac{7!}{d_0!d_1!d_2!\ldots d_s!} \ .$$

Example

$$\#\left([(2,2,1,0,0,0,0)]\right) = 2^{(7-4)}\frac{7!}{4!1!2!} = 840 \ .$$

What is the cardinality of a leader class ?

Cardinality of a leader class

$$\#\left([w]\right) = 2^{7-d_0}\frac{7!}{d_0!d_1!d_2!\ldots d_s!}\ .$$

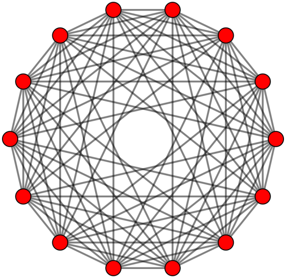
Example

$$\#\left([(2,2,1,0,0,0,0)]\right) = 2^{(7-4)}\frac{7!}{4!1!2!} = 840\ .$$

The set of the cardinalities of the leader classes is finite and counts 30 distinct elements.

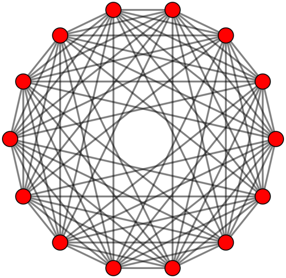
What are the symmetries of leader classes ?

What are the symmetries of leader classes ?

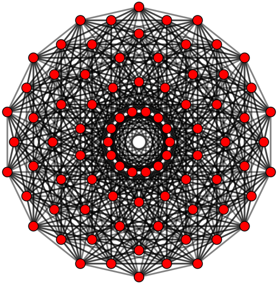


Symmetry of leader class $[(1, 0^6)]$ with $\#\left([(1, 0^6)]\right) = 14$ representing [frequency].

What are the symmetries of leader classes ?



Symmetry of leader class $[(1, 0^6)]$ with $\#\left([(1, 0^6)]\right) = 14$ representing [frequency].



Symmetry of leader class $[(1^2, 0^5)]$ with $\#\left([(1^2, 0^5)]\right) = 84$ representing [energy].

Parallelograms everywhere ?

Theorem (4-point theorem of physical binary form equations)

The binary form equation $[z] = [f(\Pi)][x][y]$ *is physically valid, with* $[f(\Pi)], [x], [y], [z]$ *distinct classes of physical quantities obeying the properties:*

$$\mathrm{dex}^{-1}([z]) \circ \mathrm{dex}([z]) = [z], \quad \mathrm{dex}^{-1}([f(\Pi)]) \circ \mathrm{dex}([f(\Pi)]) = [f(\Pi)],$$
$$\mathrm{dex}^{-1}([x]) \circ \mathrm{dex}([x]) = [x], \quad \mathrm{dex}^{-1}([y]) \circ \mathrm{dex}([y]) = [y],$$

if and only if, the 4-cycle **oyzxo** *is a parallelogram in the integer lattice* $\mathbb{Z}^7$ *and* $\mathrm{dex}([x]) = \mathbf{x}$, $\mathrm{dex}([y]) = \mathbf{y}$, $\mathrm{dex}([z]) = \mathbf{z}$, $\mathrm{dex}([f(\Pi)]) = \mathbf{o}$ *are distinct integer lattice points with* **o** *being the origin of the integer lattice* $\mathbb{Z}^7$ *.*

Two colors physics ?

Theorem (Bicoloring of binary form equations)

Any binary form equation $[z] = [f(\Pi)][x][y]$ *between distinct physical quantities* $[f(\Pi)], [x], [y], [z]$ *represents a distinct ordered coloring pattern* $(\mathrm{psc}(\mathbf{o}), \mathrm{psc}(\mathbf{x}), \mathrm{psc}(\mathbf{y}), \mathrm{psc}(\mathbf{z}))$ *that is an element of the set of ordered coloring patterns* $\{(0,0,0,0),(0,0,1,1),(0,1,0,1),(0,1,1,0)\}$.

where $\mathrm{psc}(\mathbf{x})$ represents the parity of the sum of the absolute value of the coordinates of the lattice point $\mathbf{x}$.

A fundamental principle ?

The decomposition of a vertex $\mathbf{z}$ in pairwise orthogonal vertices $\mathbf{x}$ and $\mathbf{y}$ assumes the existence of a system of Diophantine equations:

$$\mathbf{x} + \mathbf{y} - \mathbf{z} = \mathbf{0},$$
$$\mathbf{x} \cdot \mathbf{y} = 0,$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{Z}^7$.

A fundamental principle ?

The decomposition of a vertex $\mathbf{z}$ in pairwise orthogonal vertices $\mathbf{x}$ and $\mathbf{y}$ assumes the existence of a system of Diophantine equations:

$$\mathbf{x} + \mathbf{y} - \mathbf{z} = \mathbf{0},$$
$$\mathbf{x} \cdot \mathbf{y} = 0,$$

where $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{Z}^7$.

7D-hypersphere

$$(\mathbf{x} - \frac{\mathbf{z}}{2})^2 = (\frac{\mathbf{z}}{2})^2 \ ,$$

with center at $\frac{\mathbf{z}}{2}$ and radius $\|\frac{\mathbf{z}}{2}\|_2$.

How are rectangles distributed in $\mathbb{Z}_+^7$?

We determine the distribution of non-degenerated unique rectangles formed by 4 lattice points $\mathbf{o}, \mathbf{x}, \mathbf{y}, \mathbf{z}$ in $\mathbb{Z}_+^7$ as function of the infinity norm $\|\mathbf{z}\|_\infty = s$ where $\mathbf{z} = \mathbf{x} + \mathbf{y}$.

We define a sample space Ω consisting of 7D-hyperspheres with infinity norm $\|\mathbf{z}\|_\infty = s$ and search for the event of a unique perimeter p .

We find in $\mathbb{Z}_+^7$ for $\|\mathbf{z}\|_\infty \leq 10$, a total of 7 747 unique rectangles out of 6 510 466 998 rectangles.

The unique rectangles represent unique realizable binary form equations of the type $[z] = f(\Pi)[x][y]$ for the selected physical quantity $[z]$.

How are rectangles distributed in $\mathbb{Z}_+^7$?

infinity norm $\|\mathbf{z}\|_\infty = s$	L= # leader classes	UR = # unique rectangles	R = # rectangles	UR/R ratio
0	1	0	0	1
1	7	1	120	8.33E-03
2	28	7	7 196	9.73E-04
3	84	26	162 554	1.60E-04
4	210	79	1 341 957	5.89E-05
5	462	182	9 255 603	1.97E-05
6	924	333	40 532 530	8.22E-06
7	1716	693	168 302 117	4.12E-06
8	3003	1 180	523 421 602	2.25E-06
9	5005	1 999	1 637 895 896	1.22E-06
10	8008	3 247	4 129 547 423	7.86E-07
Total	19447	7 747	6 510 466 998	1.19E-06

The integer sequence R received the OEIS A240934 number .
The integer sequence UR received the OEIS A247557 number .

How to apply the methods to photonics?

We apply the 7D-hypersphere method to $[\mathbf{H}], [\mathbf{B}], [\mathbf{E}], [\mathbf{D}]$.

Magnetic field strength: $\mathrm{dex}([\mathbf{H}]) = (2, 0, -1, 1, 0, 0, 0)$; $class = [(2, 1^2, 0^4)]$

Magnetic induction: $\mathrm{dex}([\mathbf{B}]) = (2, 1, -2, -1, 0, 0, 0)$; $class = [(2^2, 1^2, 0^3)]$

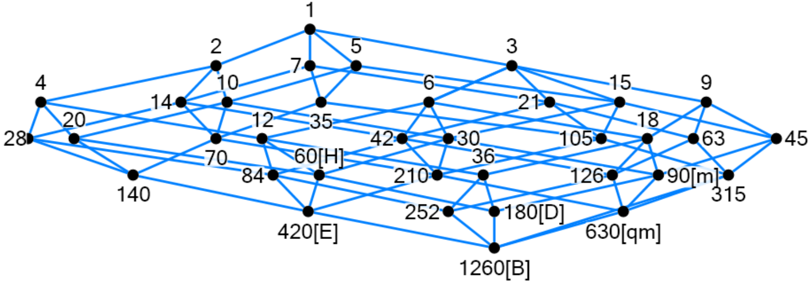
Electric field: $\mathrm{dex}([\mathbf{E}]) = (2, 1, -1, -1, 0, 0, 0)$; $class = [(2, 1^3, 0^3)]$

Electrical displacement: $\mathrm{dex}([\mathbf{D}]) = (2, 0, -2, 1, 0, 0, 0)$; $class = [(2^2, 1, 0^4)]$

Infinity norm of $[\mathbf{H}], [\mathbf{B}], [\mathbf{E}], [\mathbf{D}]$ is $\ell_\infty = 2$.
Gödel numbers are:
$\phi_7([\mathbf{H}]) = 60$, $\phi_7([\mathbf{D}]) = 180$, $\phi_7([\mathbf{E}]) = 420$, $\phi_7([\mathbf{B}]) = 1260$.

How to factorize the leader class of $\mathbf{B}$?

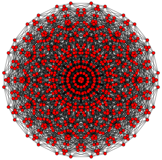
How to factorize the leader class of **B**?



Factorization of leader class $[(2^2, 1^2, 0^3)]$ based on the divisibility relation $n \mid m$.

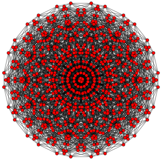
What symmetries for [$\mathbf{H}$], [$\mathbf{B}$], [$\mathbf{E}$], [$\mathbf{D}$] ?

What symmetries for [**H**], [**B**], [**E**], [**D**] ?

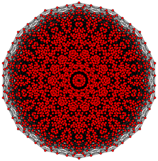


[**H**], $\# \left([(2,1^2,0^4)]\right) = 840$.

What symmetries for [**H**], [**B**], [**E**], [**D**] ?

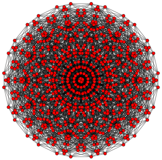


[**H**], $\#\left([(2,1^2,0^4)]\right) = 840$.

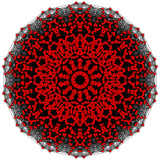


[**B**], $\#\left([(2^2,1^2,0^3)]\right) = 3360$.

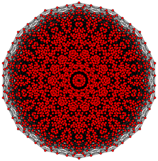
What symmetries for [**H**], [**B**], [**E**], [**D**] ?



[**H**], $\#\left([(2,1^2,0^4)]\right) = 840$.

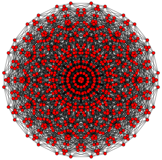


[**E**], $\#\left([(2,1^3,0^3)]\right) = 2240$.

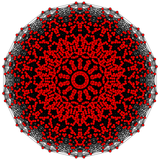


[**B**], $\#\left([(2^2,1^2,0^3)]\right) = 3360$.

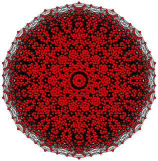
What symmetries for [**H**], [**B**], [**E**], [**D**] ?



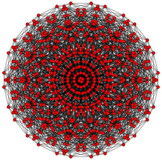
[**H**], $\#\left([(2,1^2,0^4)]\right) = 840$.



[**E**], $\#\left([(2,1^3,0^3)]\right) = 2240$.



[**B**], $\#\left([(2^2,1^2,0^3)]\right) = 3360$.



[**D**], $\#\left([(2^2,1,0^4)]\right) = 840$.

Integral representation of Maxwell's laws

$$\oint_{L(S)} \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot \mathrm{d}\mathbf{S} \qquad \oint_{L(S)} \mathbf{H} \cdot \mathrm{d}\mathbf{s} = \iint_S \mathbf{J} \cdot \mathrm{d}\mathbf{S} + \frac{\partial}{\partial t} \iint_S \mathbf{D} \cdot \mathrm{d}\mathbf{S}$$

$$\oiint_{S(V)} \mathbf{D} \cdot \mathrm{d}\mathbf{S} = \iiint_V \rho_f \, \mathrm{d}V \qquad \oiint_{S(V)} \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$$

Constitutive laws:

$$\mathbf{D} \cdot \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon \mathbf{E} \qquad \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

Electrical displacement [**D**]

The canonical factorization of $\phi_7([\mathbf{D}]) = 180$ results in 8 binary forms, 8 ternary forms, 1 quaternary form and 1 quinternary form.

Table: Distribution of rectangles in the 7D-hypersphere of $[(2^2, 1, 0^4)]$.

$[q]$	Perimeter p	Frequency f
$[(2^2, 1, 0^4)]$	6	1
$[(2^2, 1, 0^4)]$	7.6569	1
$[(2^2, 1, 0^4)]$	8.1199	16
$[(2^2, 1, 0^4)]$	8.3631	17
$[(2^2, 1, 0^4)]$	8.4721	26

Electrical displacement [**D**]

We select the non-degenerated rectangle of $[(2^2, 1, 0^4)]$ having a unique perimeter $p = 7.6569$.
We find $\mathbf{x} = (2,2,0,0,0,0,0)$ and $\mathbf{y} = (0,0,1,0,0,0,0)$.
The leader class representative $\mathbf{z} = (2,2,1,0,0,0,0)$ is mapped on $\mathbf{w} = (2,0,-2,1,0,0,0)$ by the signed permutation matrix P:

$$\mathrm{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Electrical displacement [D]

Multiplication of the matrix P with the lattice points $\mathbf{x} = (2, 2, 0, 0, 0, 0, 0)$ and $\mathbf{y} = (0, 0, 1, 0, 0, 0, 0)$ results in the new lattice points $\mathbf{u} = (2, 0, -2, 0, 0, 0, 0)$ and $\mathbf{v} = (0, 0, 0, 1, 0, 0, 0)$. The realizable binary form equation becomes $[\mathbf{w}] = f(\Pi)[\mathbf{u}][\mathbf{v}]$.

$$D = f(\Pi)(\frac{1}{S})q; \quad DS = f(\Pi)q$$

$$D_x S_x = f_x(\Pi)q; \quad D_y S_y = f_y(\Pi)q; \quad D_z S_z = f_z(\Pi)q$$

$$\oint\!\!\oint_{S(V)} (D_x \, \mathrm{d}S_x + D_y \, \mathrm{d}S_y + D_z \, \mathrm{d}S_z) = (f_x(\Pi) + f_y(\Pi) + f_z(\Pi))q$$

$$\oint\!\!\oint_{S(V)} \mathbf{D} \cdot \mathrm{d}\mathbf{S} = f(\Pi) \iiint_V \rho_f \, \mathrm{d}V = \iiint_V \rho_f \, \mathrm{d}V$$

Research question 1: What are the clusters of physical quantities?

Research question 1: What are the clusters of physical quantities?

1. We show that dimensionally equivalent physical SI_{2018} quantities are mapped to integer lattice points of $\mathbb{Z}^7$.

Research question 1: What are the clusters of physical quantities?

1. We show that dimensionally equivalent physical SI_{2018} quantities are mapped to integer lattice points of $\mathbb{Z}^7$.
2. We show that integer lattice points are partitioned, using signed permutations as equivalence relation, in leader classes with representative lattice point in $\mathbb{Z}_+^7$.

Research question 1: What are the clusters of physical quantities?

1. We show that dimensionally equivalent physical SI_{2018} quantities are mapped to integer lattice points of $\mathbb{Z}^7$.
2. We show that integer lattice points are partitioned, using signed permutations as equivalence relation, in leader classes with representative lattice point in $\mathbb{Z}^7_+$.
3. The cardinality of each leader class is an element of a finite set of 30 distinct cardinalities representative for the symmetries of the leader class.

Research question 1: What are the clusters of physical quantities?

1. We show that dimensionally equivalent physical SI_{2018} quantities are mapped to integer lattice points of $\mathbb{Z}^7$.
2. We show that integer lattice points are partitioned, using signed permutations as equivalence relation, in leader classes with representative lattice point in $\mathbb{Z}^7_+$.
3. The cardinality of each leader class is an element of a finite set of 30 distinct cardinalities representative for the symmetries of the leader class.
4. We show that each leader class has a unique Gödel number that generates a partial order between the physical quantities.

Research question 2: Which variables should appear in a hypothetical law of physics?

1. We show that each leader class has a unique 7D-hypersphere defining rectangles formed by 4 lattice points $\mathbf{o}, \mathbf{x}, \mathbf{y}, \mathbf{z}$ in $\mathbb{Z}_+^7$ where $\mathbf{z} = \mathbf{x} + \mathbf{y}$. The resulting rectangles are the geometric representation of the realizable binary form equations $[z] = f(\Pi)[x][y]$ for the selected physical quantity $[z]$.

Research question 2: Which variables should appear in a hypothetical law of physics?

1. We show that each leader class has a unique 7D-hypersphere defining rectangles formed by 4 lattice points $\mathbf{o}, \mathbf{x}, \mathbf{y}, \mathbf{z}$ in $\mathbb{Z}_+^7$ where $\mathbf{z} = \mathbf{x} + \mathbf{y}$. The resulting rectangles are the geometric representation of the realizable binary form equations $[z] = f(\Pi)[x][y]$ for the selected physical quantity $[z]$.
2. We show that the divisibility relation $n \mid m$ applied on the Gödel numbers creates the core lattice of the physical quantities up to a signed permutation.

Research question 3: Which quantities should be excluded?

1. We find in $\mathbb{Z}_+^7$ where $\|\mathbf{z}\|_\infty \leq 10$, a total of 7 747 unique rectangles out of 6 510 466 998 rectangles.

Future work

Future work

1. Generate the table of the elements of physics up to $\|\mathbf{z}\|_\infty \leq 26$.

Future work

1. Generate the table of the elements of physics up to $\|\mathbf{z}\|_\infty \leq 26$.
2. Expand the atlas of the discovered 7 747 unique rectangles up to $\|\mathbf{z}\|_\infty \leq 26$.

Future work

1. Generate the table of the elements of physics up to $\|\mathbf{z}\|_\infty \leq 26$.
2. Expand the atlas of the discovered 7 747 unique rectangles up to $\|\mathbf{z}\|_\infty \leq 26$.
3. Use the 7D-hypersphere method to solve hot topics in physics and engineering.

Future work

1. Generate the table of the elements of physics up to $\|\mathbf{z}\|_\infty \leq 26$.
2. Expand the atlas of the discovered 7 747 unique rectangles up to $\|\mathbf{z}\|_\infty \leq 26$.
3. Use the 7D-hypersphere method to solve hot topics in physics and engineering.
4. Generate a sequential orthogonal decomposition of the leader class $[(7,6,5,4,3,2,1)]$ with $\#([(7,6,5,4,3,2,1)]) = 645\,120$ and $\phi_7([(7,6,5,4,3,2,1)]) = 2\,677\,277\,333\,530\,800\,000$.

Future work

1. Generate the table of the elements of physics up to $\|\mathbf{z}\|_\infty \leq 26$.
2. Expand the atlas of the discovered 7 747 unique rectangles up to $\|\mathbf{z}\|_\infty \leq 26$.
3. Use the 7D-hypersphere method to solve hot topics in physics and engineering.
4. Generate a sequential orthogonal decomposition of the leader class $[(7,6,5,4,3,2,1)]$ with $\#([(7,6,5,4,3,2,1)]) = 645\,120$ and $\phi_7([(7,6,5,4,3,2,1)]) = 2\,677\,277\,333\,530\,800\,000$.
5. Generate the network of spherical codes of physical quantities on the unit $7D$ hypersphere.

Future work

1. Generate the table of the elements of physics up to $\|\mathbf{z}\|_\infty \leq 26$.
2. Expand the atlas of the discovered 7 747 unique rectangles up to $\|\mathbf{z}\|_\infty \leq 26$.
3. Use the 7D-hypersphere method to solve hot topics in physics and engineering.
4. Generate a sequential orthogonal decomposition of the leader class $[(7,6,5,4,3,2,1)]$ with $\#([(7,6,5,4,3,2,1)]) = 645\,120$ and $\phi_7([(7,6,5,4,3,2,1)]) = 2\,677\,277\,333\,530\,800\,000$.
5. Generate the network of spherical codes of physical quantities on the unit $7D$ hypersphere.
6. Derive the properties of the 7D-wavefunctions of $\mathbb{Z}^7$ satisfying 7D-Helmholtz equation and 7D-Schrödinger equation.

How to convert from SI_{2008} to SI_{2018} ?

The conversion matrix SI_{2018} is:

$$\mathrm{SI}_{2018} = \begin{bmatrix} -1 & 1 & -1 & 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

How to convert from SI_{2008} to SI_{2018} ?

SI_{2008} **base quantity**	SI_{2018} **lattice point**
length	$(-1, 0, 1, 0, 0, 0, 0)$
mass	$(1, 1, -2, 0, 0, 0, 0)$
time	$(-1, 0, 0, 0, 0, 0, 0)$
electric current	$(1, 0, 0, 1, 0, 0, 0)$
thermodynamic temperature	$(1, 1, 0, 0, -1, 0, 0)$
amount of substance	$(0, 0, 0, 0, 0, -1, 0)$
luminous intensity	$(2, 1, 0, 0, 0, 0, 1)$