Logistics and spatial planning

Case studies on optimizing the supply chain and reorganizing the urban freight distribution

Ir. Thomas Dubois

Ghent University

Logistics and spatial planning

Case studies on optimizing the supply chain and reorganizing the urban freight distribution

Logistiek en ruimtelijke planning

Casestudy's omtrent het optimaliseren van de supply chain en de reorganisatie van de stedelijke goederendistributie

Ir. Thomas Dubois

Doctoral dissertation

Ghent University
2015

Supervisor:
Prof. dr. Frank Witlox
Department of Geography
Faculty of Sciences
Ghent University

Co-supervisor:
Prof. em. dr. Georges Allaert
Centre for Mobility and Spatial Planning
Faculty of Engineering and Architecture
Ghent University

Examination committee
Prof. dr. Frank Witlox (supervisor), Ghent University
Prof. em. dr. Georges Allaert (co-supervisor), Ghent University
Dr. Levent Aksoy, Maltepe University
Prof. dr. Mats Johnsson, Lund University
Dr. Veronique Van Acker, University of Amsterdam, Ghent University
Prof. dr. Mehmet Tanyas, Maltepe University
Prof. dr. Philippe De Maeyer, Ghent University
Prof. dr. Ben Derudder (chair), Ghent University

Contents

1
Introduction

In this dissertation we study urban freight distribution and the optimization of different aspects of the supply chain regarding transportation costs and inventory holding costs. More specifically we search for locations of urban facilities that minimize transportation costs, we search for inventory policies that minimize the inventory holding costs under fill rate constraints and we study the merger of distribution centres to minimize transportation costs. In these optimization studies we consider joint transportation and study how optima and costs change if the transportation is organized more jointly e.g. by making round trips with larger vehicles and with more deliveries per round trip. The dissertation contains a case study on the Istanbul fruit and vegetable wholesale market and a case study on a possible road transshipment centre in Istanbul. Also included in the dissertation is research in which optimization topics are studied more generally and in which theorems regarding optimization problems are proved for simplified conditions.

Based on [1] we consider a supply chain to contain all parties involved, directly or indirectly, in satisfying customer demand. This includes suppliers, manufacturers, distributors, retailers and customers. Some aspects of supply chains are the facilities (e.g. shops, distribution centres and plants), the products (e.g. inventories in facilities and products carried on vehicles) and the transportation of products between facilities (by e.g. semi-trailer trucks, pickup trucks, trains or ships). Other aspects of supply chains include information flows, fund flows and pricing. In this chapter we will introduce subjects relevant to the other chapters and we will study relevant literature.

1.1. Urban freight distribution

Cities are typical areas with large populations and with high population density. Because of the large number of people living in cities, a large amount of goods is needed in cities. Much of these goods are transported from areas outside the city to the city and inside the city. Because of the high population density the environmental aspect of urban freight distribution is important. Typical for urban transportation are also traffic jams and small streets. In [2] urban freight distribution is discussed for large cities in Europe. We will study two types of urban freight distribution in this dissertation: individual transport distribution and milk run distribution. We consider a distribution centre with customers in the city. In the case of individual transport distribution vehicles make round trips during which goods are delivered to only one customer per round trip and round trips start at customer locations or at the distribution centre. In the case of milk run distribution vehicles make round trips during which goods are delivered to different customers per round trip and round trips start at the distribution centre. Typically larger vehicles are used in the case of milk run distribution. In chapter 5 of the dissertation we will study the change in fuel costs if the distribution method is changed from individual transport distribution to milk run distribution for the Istanbul fruit

and vegetable wholesale market. In [3] urban milk run distribution is discussed including environmental considerations.

Milk run distribution is also studied in the field of operations research. Let us consider a distribution centre and customers. The distribution centre and the customers are considered the nodes of a network with links between all nodes. The costs of transportation between two nodes are given for all links. Based on [4], [5] and [6] we consider a vehicle routing problem to be a problem in which vehicle routes need to be determined such that (i) vehicle routes start and end at the distribution centre, (ii) every customer is visited once, (iii) the costs are minimal and (iii) given side constraints are satisfied. Possible side constraints are: (i) cargo mass restrictions on the vehicles (with the demand of each customer given), (ii) restrictions on the number of deliveries per vehicle route, (iii) restrictions on the length of vehicle routes (with the route length between two nodes given for all links), (iv) restrictions on the duration of vehicle routes (with the transport time between two nodes given for all links and the delivery duration given for each customer) (v) restrictions on the start moments and end moments of the deliveries (with the transport time between two nodes given for all links and the delivery duration given for each customer), (vi) restrictions on the number of vehicle routes. In this dissertation we will not study vehicle routing problems because in the case studies we consider (chapters 3, 4, 5 and 9) data on the customer locations is lacking except for estimations of the numbers of customers in different districts.

1.2. Economic geography and location

The location of industries is one of the topics that is studied in the field of economic geography. An early contribution in the study of industrial location is [7] in which the location of an industrial facility is searched that minimizes the sum of the costs of transportation between two supplier areas and the facility and the facility and one market area. Other contributions are [8] and [9]. In the following paragraphs (based on [10] and [11]) we give an introduction to the research on industrial location by using a simplified example in which we consider an industry and study the dependence of the costs on the plant location for plants in this industry.

We consider an industry with the following characteristics. (i) The cost of the land needed to operate a plant is independent of the plant location. (ii) The raw materials needed for manufacturing products are located at location A. (iii) The workers needed for operating a plant live around location B. (iv) The centre of the area where the manufactured products are sold (the market) is at location C. (v) The part of the costs that is independent of the plant location is 100 dollar per manufactured product. (vi) The cost of transporting raw materials from A to the plant location is proportional to the distance between A and the plant location and the cost is 1 dollar per km per manufactured product. (vii) The labour costs increase with 1 dollar per manufactured product if the distance between B and the plant location increases with 1 km. (viii) The cost of transporting the manufactured products from the plant to the market is proportional to the distance between the plant location and the market centre C and the cost is 1 dollar per km per manufactured product. (ix) Manufactured products are sold at the market at a price of 165 dollar per manufactured product.

Figure 1 shows the locations A, B and C. We determined the location O for which the costs are minimal. Methods to find this location are discussed in the next section. We also

determined the cost contour lines of 155, 160, 165, ... and 200 dollar per manufactured product. The location O and these cost contour lines are shown in figure 1. Because the price at the market is 165 dollar per product, the 165 dollar cost contour line is the border of the area in which plants of the considered industry make profit. This cost contour line is called the margin to profitability ([10]). Figure 2 shows the costs per manufactured product and the price per manufactured product as functions of the plant location. The price per product is constant: 165 dollar per product. Plants of the considered industry make profit at locations for which the costs per product are lower than the price per product. Figure 3 shows the costs per manufactured product and price per manufactured product for plant locations for which the y-coordinate is zero (we use the coordinate system of figure 1). The curve in figure 3 that corresponds to the costs per product is called the space cost curve ([10]).

In chapters 3 and 4 of this dissertation we will study the dependence of the transportation costs of the customers and suppliers of a wholesale market on the location of this wholesale market. In this study we will not take into account three points (A, B and C) but fifteen points (fourteen district centres and the suppliers entry point). Figure 4 of chapter 4 shows transportation fuel cost contour lines and figure 3 of chapter 4 shows the transportation fuel costs as a function of the location of the wholesale market.
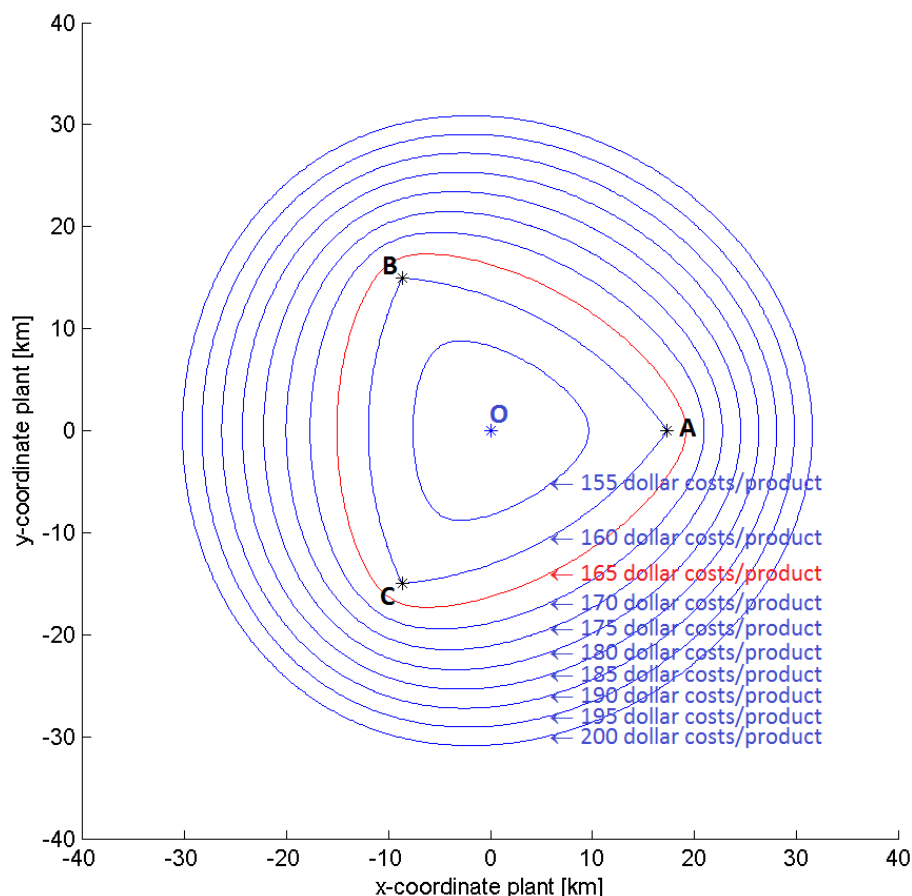
Figure 1.1. Points A, B and C, the plant location O that minimizes the cost and different cost contour lines including the margin to profitability (red).
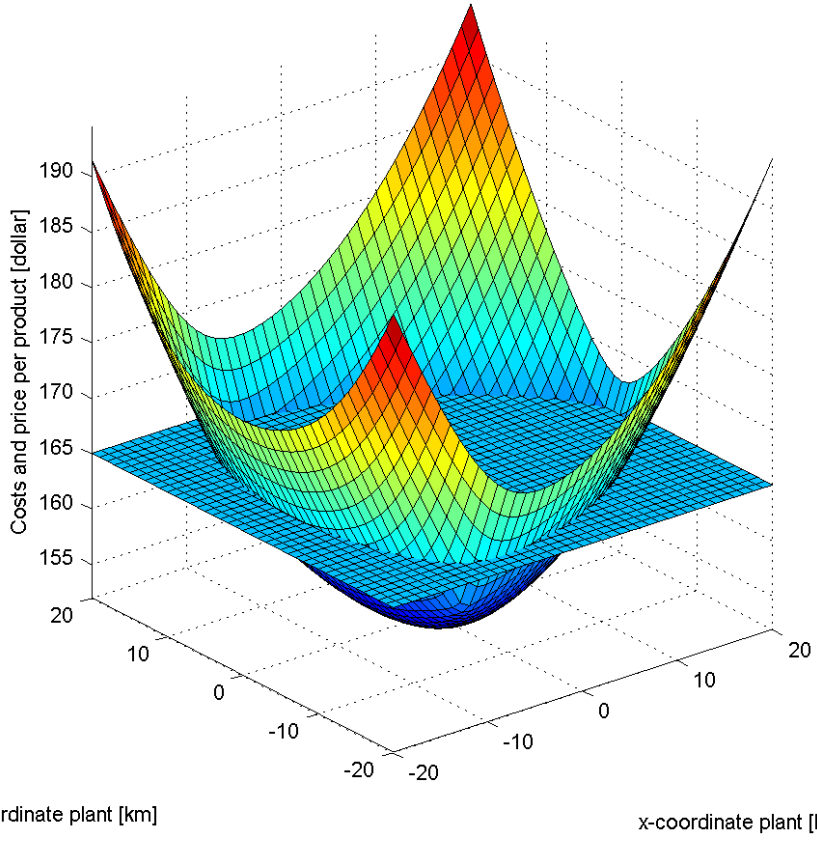
Figure 1.2. Costs per product and price per product as functions of the plant location.
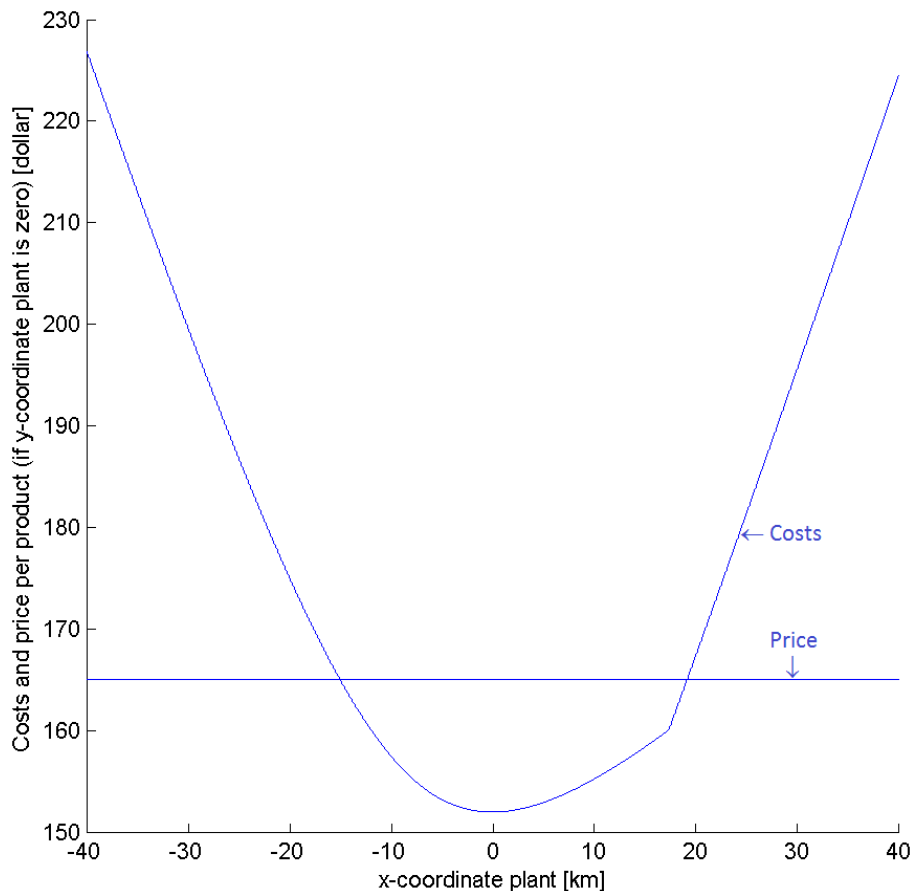
Figure 1.3. Costs per product and price per product as functions of the *x*-coordinate of the plant location if the *y*-coordinate of the plant location is zero.

Aspects that are not taken into account in this simplified example are e.g. entrepreneurial skills, subsidy and the advantages of different plants together in an area. If the costs are reduced by the entrepreneurial skills of the plant management, the margin of profitability changes for this plant and the area of profitability for this plant becomes larger. Subsidy that is dependent on the plant location changes the costs and possibly changes the area of profitability. Different plants together in an area possibly reduces plant costs in that area by e.g. allowing joint organization. Also not taken into account in the simplified example are changes over time in the price and the costs.

Besides the approach of [7], [8] and [9], called the (neo)classical approach ([12], [13]), which focuses on the minimization of costs or the maximization of profit, also other approaches have been proposed in the field of economic geography. A disadvantage of the (neo)classical approach is that different aspects of location decisions other than profit maximization are not taken into account such as personal preferences of the decision maker, social aspects and influences of institutions. Another disadvantage is that often the decision maker has not enough data to make accurate profit maximization calculations. In the behavioural approach the personal characteristics of the decision maker are taken into account. An example of this approach is found in [14] ([12]). Important for this approach are contributions of H.A. Simon such as [15], [16] and [17]. In the institutional approach the roles of different institutions are taken into account. Economic behaviour is studied in relation with institutional frameworks and it is assumed that agents are guided by institutional rules ([13]). Examples of institutional differences are organizational routines, business cultures and legal frameworks ([13]).

Another approach, the evolutionary approach, is discussed in [13] and [18]. In this approach the locations of economic activities are studied with taking into account their histories.

In this dissertation we estimate transportation costs and inventory holding costs, but we see the results only as a first step. We advise decision makers to also take into account other aspects, such as environmental aspects, social aspects, cultural aspects, political aspects, legal aspects, historical aspects and psychological aspects and to use also input from different approaches. In this dissertation we also discuss environmental aspects and an idea for further research is to study more profoundly the environmental aspects.

1.3. Operations research and location: facility location analysis

Facility location analysis is the study of facility location problems. Based on [19] and [20] we consider a facility location problem to be a problem in which optimal locations of facilities need to be determined according to a given objective. Often there are locations given which are being served by the facilities, these locations are called customer locations and it is said that demand occurs at these given locations. In [21] and [22] facility location problems and corresponding models are categorized in four main categories. In the first category, the analytic facility location problems, the demand is assumed to be distributed with a given density function (often uniform demand) over an area and the possible facility locations are all points of that area. Typically, a large number of simplifying assumptions are made in this first category. In the second category, the continuous facility location problems, demand occurs at discrete points and the set of possible facility locations is the set of all points of an area. In network facility location problems, demand occurs only on parts of a network and the possible facility locations are all nodes and all points on links of the network. In the last category, the discrete facility location problems, the set of points where demand occurs and the set of possible facility locations are finite. Table 1 shows the differences between these facility location problems.

Table 1.1. Facility location problems.

	Customer locations are given by	Facility locations are restricted to
	a density function over an area or	
Analytic problems	a density function over an area and a finite number of points	an area
Continuous problems	a finite number of points	an area
	a density function over a network or	
	a finite number of points on a network or	
Network problems	a density function over a network and a finite number of points on this network	a network
Discrete problems	a finite number of points	a finite number of points

Other articles, such as [23] and [24], distinguish facility location problems according to the number of facilities, the used objective, the used distance or the type of demand. In some facility location problems the number of facilities is a given and in other facility location problems the optimal number needs to be determined. Often used objectives are minisum (minimizing the sum of all transportation costs) and minimax (minimizing the maximum transportation cost between a facility and a customer). Often used distances are the Euclidean distance, the rectilinear distance and, in a network context, shortest path length. There are facility location problems with discrete demand (demand occurs only at a finite number of points) and with continuous demand (with a density function over an area).

In the following four subsections we will discuss the four types of facility location problems from table 1 and we will give urban examples of these facility location problems.

1.3.1. Analytic facility location problems

In analytic facility location problems customer locations are given by a density function over an area. Let us consider a city in which a facility needs to be located. In [25], [26], [27] urban population densities are studied and the urban population density in a location x in the city is approximated by:

$$D_p(x) = D_{p0}e^{-\gamma d(x,c)} \tag{1}$$

with γ and D_{p0} constants, c the location of the city centre and $d(a,b)$ the distance between location a and b for all locations a and b. If the density of customer locations is proportional to the population density, we get for the customer location density in a location x in the city:

$$D_c(x) = D_{c0}e^{-\gamma d(x,c)} \tag{2}$$

with D_{c0} a constant.

Figure 4 illustrates this customer location density for a city with a radius of 20 km, $\gamma=0.1$ and D_{c0} such that the integral of D_c over the city equals 1000 (D_{c0} is approximately 2.7). As distance we used the Euclidean distance. Customers are only located in the city, outside the city the customer location density is zero.
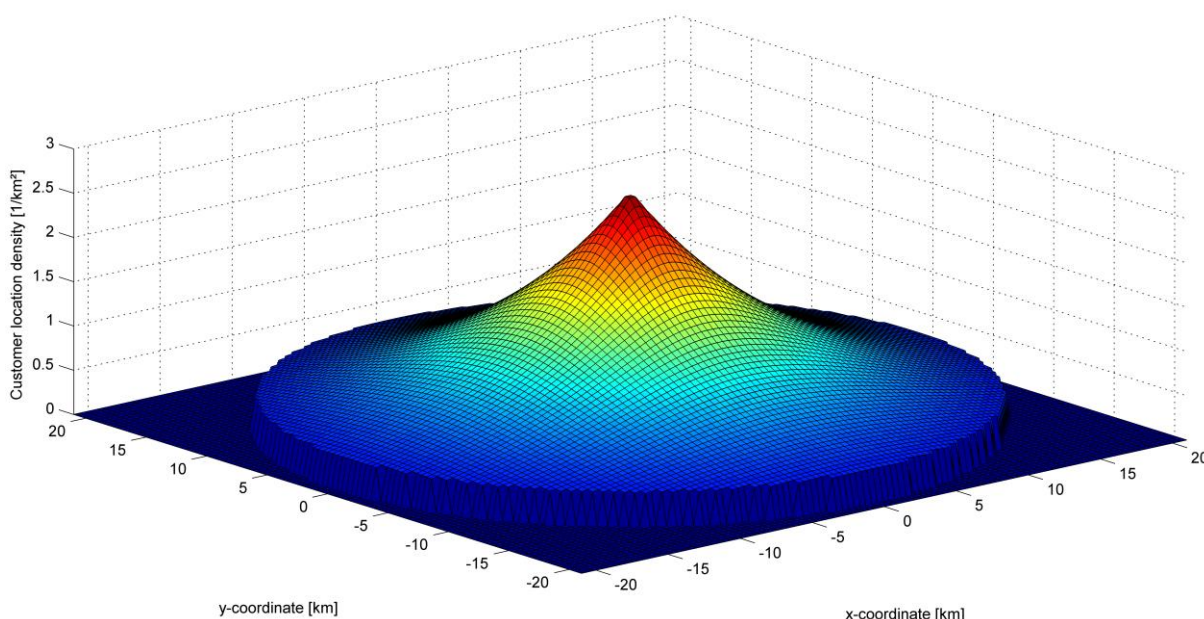
Figure 1.4. Illustration of an urban customer location density function according to (2) for a disc-shaped city with a radius of 20 km. The city centre has coordinates (0,0) and the customer location density outside the city is zero.

An example of an analytic facility location problem is the problem of determining the location of a facility in the city such that the total transportation cost is minimal. We consider transportation between the urban facility and the customer locations in the city and transportation between the urban facility and a supplier location outside the city. This example is further studied in chapter 2. Other examples of analytic facility location problems

include [28], [29] and [30]. In [28] a single facility location problem is studied with continuous uniform demand in a disc and the objective is to minimize the quintile share ratio of the distances between the facility and the customer locations. In [29] a single facility location problem is studied with continuous uniform demand in a rectangle and with the minisum objective, the used distance is the rectilinear distance and a high-speed road is taken into account. In [30] a location problem is studied with a given number of facilities and with continuous demand in an area, the minimax objective is used and the used distance is the Euclidean distance.

1.3.2. Continuous facility location problems

Let us consider a continuous facility location problem with one facility for which the optimal location needs to be determined, a number of given customer locations, the minisum objective and the Euclidean distance. This is the continuous 1-median problem with Euclidean distance. Location problems with the minisum objective and p facilities for which optimal locations need to be determined are called p-median problems and location problems with the minimax objective and p facilities for which optimal locations need to be determined are called p-centre problems ([31] and [32]). A method for finding the optimal facility location for the continuous 1-median problem with Euclidean distance is the Weiszfeld algorithm ([33], pp. 14-15), according to [20] and [34] named after [35]. [36] contains a translation of [35] and an introduction to [35]. If the facility coordinates are (x,y), the customer locations coordinates are (x_1,y_1), (x_2,y_2), ..., (x_n,y_n) and the cost for transportation between (x,y) and (x_i,y_i) over a time period T is $w_i\sqrt{(x-x_i)^2+(y-y_i)^2}$ (w_1, w_2, ..., w_n are called the weights of the customers), then the total transportation cost over time period T is:

$$TC = \sum_{i=1}^{n} w_i \sqrt{(x-x_i)^2+(y-y_i)^2} . \tag{3}$$

For determining the optimal facility location we apply partial differentiation with respect to x and y and we equate the results with zero:

$$\sum_{i=1}^{n} \frac{w_i(x-x_i)}{\sqrt{(x-x_i)^2+(y-y_i)^2}} = 0 \tag{4}$$

$$\sum_{i=1}^{n} \frac{w_i(y-y_i)}{\sqrt{(x-x_i)^2+(y-y_i)^2}} = 0 . \tag{5}$$

By manipulating the equations we get:

$$x = \frac{\displaystyle\sum_{i=1}^{n} \frac{w_i x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}{\displaystyle\sum_{i=1}^{n} \frac{w_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}} \tag{6}$$

$$y = \frac{\displaystyle\sum_{i=1}^{n} \frac{w_i y_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}}{\displaystyle\sum_{i=1}^{n} \frac{w_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}}} \tag{7}$$

If we use equation (6) and (7) iteratively, we get the Weiszfeld algorithm. We start with a first guess, e.g.:

$$x^{(0)} = \frac{\sum\limits_{i=1}^{n} w_i x_i}{\sum\limits_{i=1}^{n} w_i} \qquad (8)$$

$$y^{(0)} = \frac{\sum\limits_{i=1}^{n} w_i y_i}{\sum\limits_{i=1}^{n} w_i}\ . \qquad (9)$$

This location is called the centre-of-gravity location ([33], p. 15).
Then, based on (6) and (7), we use

$$x^{(l+1)} = \frac{\sum\limits_{i=1}^{n} \dfrac{w_i x_i}{\sqrt{(x^{(l)} - x_i)^2 + (y^{(l)} - y_i)^2}}}{\sum\limits_{i=1}^{n} \dfrac{w_i}{\sqrt{(x^{(l)} - x_i)^2 + (y^{(l)} - y_i)^2}}} \qquad (10)$$

$$y^{(l+1)} = \frac{\sum\limits_{i=1}^{n} \dfrac{w_i y_i}{\sqrt{(x^{(l)} - x_i)^2 + (y^{(l)} - y_i)^2}}}{\sum\limits_{i=1}^{n} \dfrac{w_i}{\sqrt{(x^{(l)} - x_i)^2 + (y^{(l)} - y_i)^2}}} \qquad (11)$$

for generating $(x^{(1)}, y^{(1)})$, $(x^{(2)}, y^{(2)})$, $(x^{(3)}, y^{(3)})$, $(x^{(4)}, y^{(4)})$, ... According to p. 15 of [33] this iterative procedure converges to an optimum if none of the iteration points coincides with a customer location and none of the customer locations is optimal.

An example to illustrate this iterative procedure is the following. We consider one customer location in every district of the Asian side of Istanbul (there are fourteen districts on the Asian side of Istanbul) plus a customer location at the boundary of Istanbul. In this example, for the fourteen customers in Istanbul, we assume the weight of a customer to be proportional to the population of the district in which the customer is located. The customer at the boundary has a weight greater than any of the customers in Istanbul. Figure 5 shows the locations and weights of the fifteen customers. Every day vehicles drive from each customer location to the facility and back. The total cost of transportation per day between customer location (x_i, y_i) and the facility location (x, y) is $w_i \sqrt{(x - x_i)^2 + (y - y_i)^2}$. We search for the optimal facility location by using the Weiszfeld algorithm. The starting point (centre-of-gravity location) and the resulting points of the first ten iterations are shown in figure 5 but are more clearly visible in figure 6, which shows an enlarged part of figure 5. The resulting point after 10000 iterations is also shown in figure 5 and 6. A second example is shown in Figure 7 and 8. This example is similar to the first example but with the customer at the Istanbul boundary omitted. In this second example the distance between the starting point and the point after 10000 iterations is larger than in the first example. In the first example the starting point and the points after the first ten iterations are all located at a distance greater than 1 km from the point after 10000 iterations. In the second example the starting point and the point after one iteration are at a distance greater than 1 km from the point after 10000 iterations while the points after two,

three, ..., ten iterations are all located at a distance less than 1 km from the point after 10000 iterations.
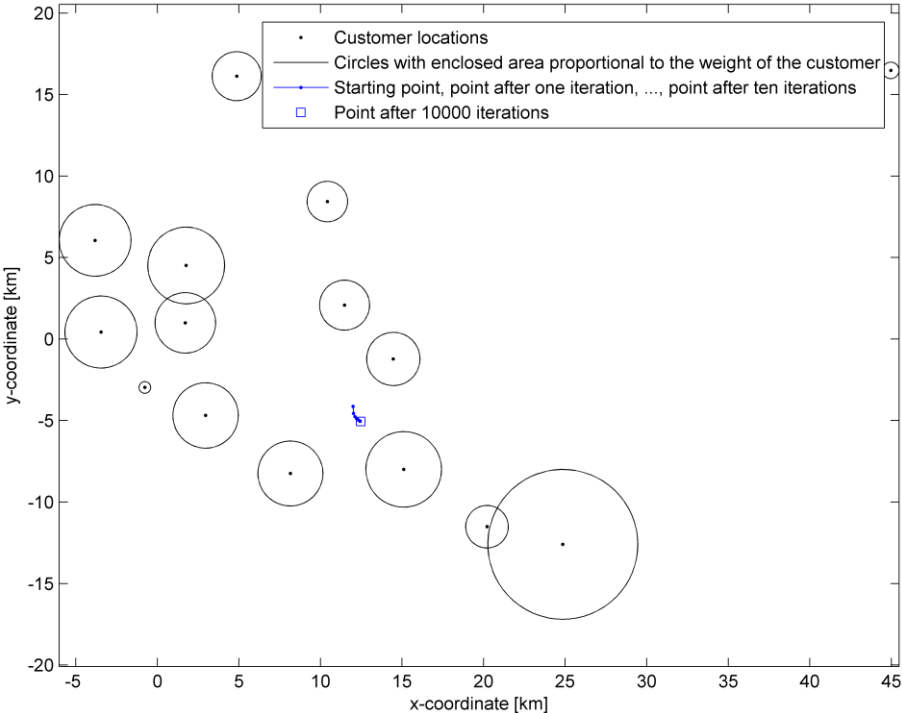
Figure 1.5. Illustration of the Weiszfeld algorithm with fifteen customer locations.
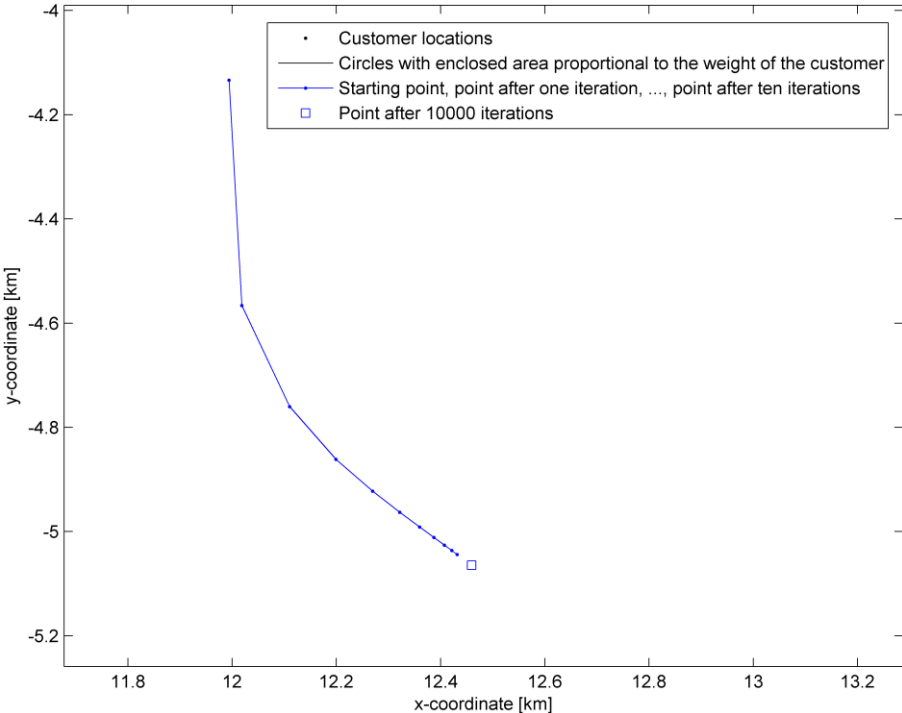
Figure 1.6. Illustration of the Weiszfeld algorithm with fifteen customer locations (enlarged).
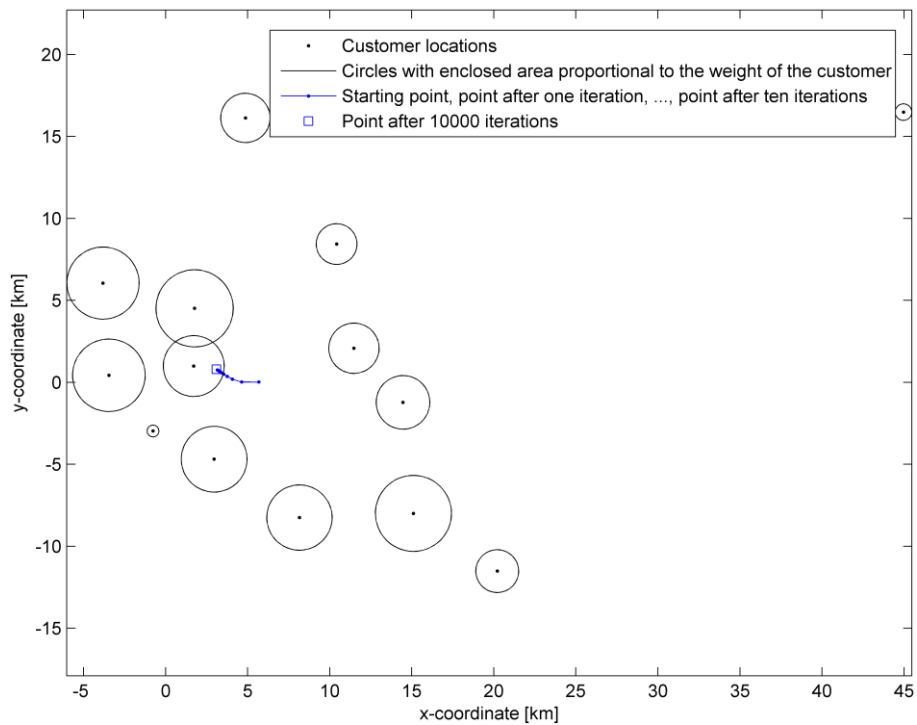
Figure 1.7. Illustration of the Weiszfeld algorithm with fourteen customer locations.
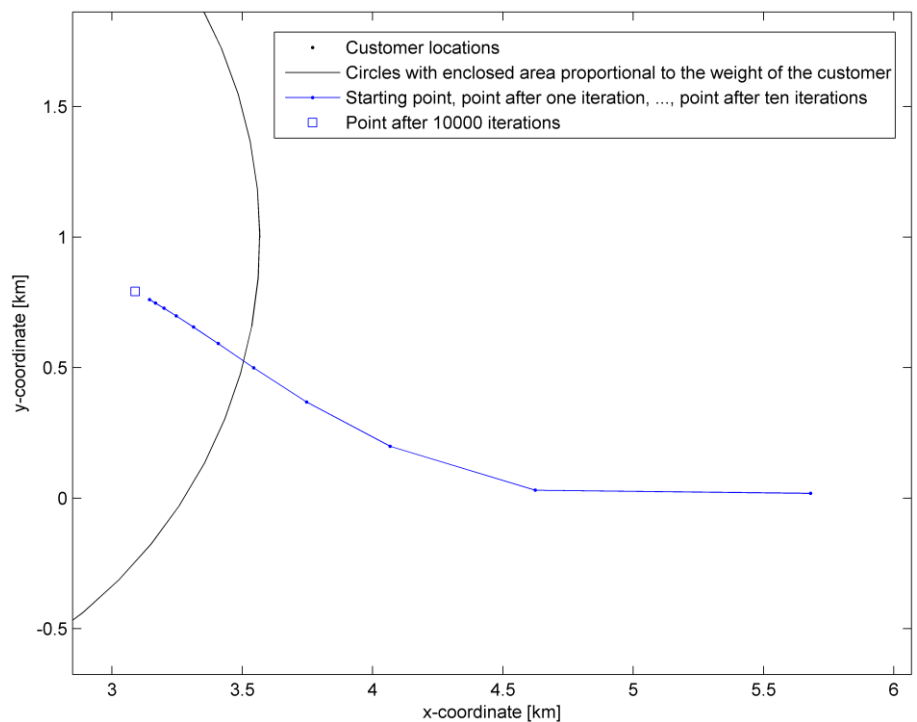
Figure 1.8. Illustration of the Weiszfeld algorithm with fourteen customer locations (enlarged).

Another method to solve single facility problems is the exhaustive search method. In this method the area to which the optimal facility location is restricted is divided into polygons (e.g. squares) with dimensions smaller than the required precision of the optimal facility location and the total cost is calculated for one point in every polygon. The point with the

lowest cost is proposed as approximation of the optimal facility location. A disadvantage of the Weiszfeld algorithm is the problem of determining how many iterations are enough for a given precision. The exhaustive search method does not have this problem. The generated results of exhaustive search method other than the optimal location, i.e. the cost at different locations, is often also useful regarding the location problem. An example of the exhaustive search method is included in section 4 of chapter 4.

In [37] the continuous 1-median problem with Euclidean distance is also studied. More specifically, [37] contains a search for the facility location that is the most robust regarding changes in customer locations and the conclusion of this article is that this location coincides with the optimal location regarding cost minimization.

In [38] a continuous single facility location problem with the minisum objective and rectilinear distance is studied. In this location problem not only the optimal facility location needs to be determined, also the optimal positioning of a highway needs to be determined. This problem is called the freeway and facility location problem. The article presents an algorithm to solve the problem and discusses the solutions.

If the optimal location of more than one facility needs to be determined, there is an extra aspect of the problem: the assignment (allocation) of customers to facilities. Therefore this problem is sometimes called the location-allocation problem [39]. Often every customer is assigned to the closest facility [23]. In [39] a generalization of the Weiszfeld algorithm for multiple facilities is presented.

In [40] the continuous p-median problem with Euclidean distance is studied. In [40] an algorithm is presented to solve this problem. Every iteration in this algorithm contains two steps: one step using the original continuous location problem and one step using a discretization of the original continuous location problem.

1.3.3. Network facility location problems

Let us consider a network with given customer locations on the network (on the nodes or links). Every customer location i has a given weight w_i and every link j of the network has a given length l_j. We define the network travel distance between two points of the network as the minimal length of a path that is part of the network and that connects these two points (shortest path length). We assume that for every customer location on a link, the length of the two link parts on both sides of the customer location are given (and they add up to the length of the link). It is assumed that the cost of transportation between a customer location i and a facility equals w_i times the network travel distance between the customer location i and the facility.

The network 1-median problem is the network facility location problem in which a location of a facility on a network needs to be determined such that the total transportation cost is minimal. The network p-median problem is the network facility location problem in which locations of p facilities on the network need to be determined such that the total transportation cost is minimal, assuming that customers are served by the closest facility (a facility for which the network travel distance to the customer is minimal) ([41], [24]). The network p-centre problem is the network facility location problem in which the locations of p facilities on the

network need to be determined such that the maximum cost of transportation between a customer location and the closest facility to this customer location is minimal ([41], [24]).

An example of a network 1-median problem is illustrated in figure 9. The network models a part of a city with the links modelling streets. A possible choice for the length of a link is the length of the corresponding street. Another possible choice for the length of a link, sometimes made for urban applications where travel time is very important, is the estimated average time to drive through the corresponding street at a relevant time of the day. Figure 9 shows the network and the given customer locations on the network. The facility location on the network that minimizes the total transportation cost needs to be determined.
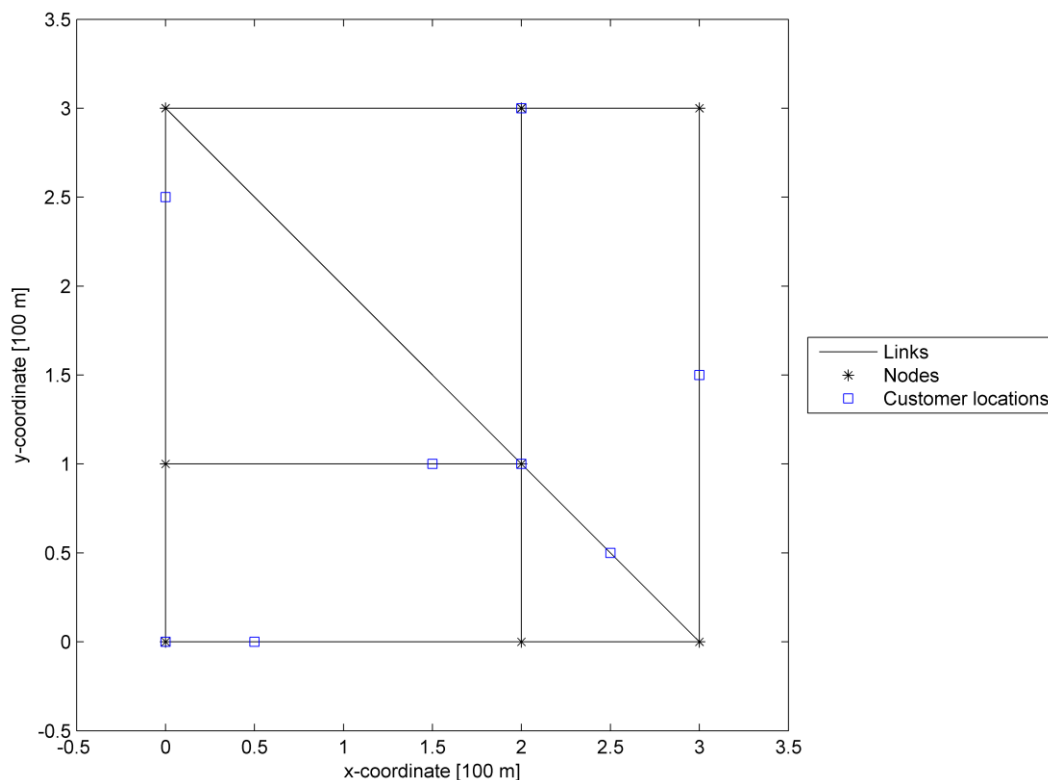
Figure 1.9. Illustration of a network facility location problem: a network (with eight nodes and twelve links) containing eight customer locations is given, an optimal facility location on the network needs to be determined.

In [42] it is shown for the network 1-median problem that if the demand locations are only at nodes, at least one optimal facility location coincides with a node. In [43] it is shown for the network p-median problem that if the demand locations are only at nodes, at least one optimal set of facility locations contains only nodes. If there is also a finite number of customer locations on the links of the network, at least one optimal set of facility locations contains only points that are nodes or customer locations. This is because for the network obtained by considering the customer locations as nodes, at least one optimal set of facility locations contains only nodes. If we apply this to our example of figure 9, we get that one of the thirteen points that are nodes or customer locations is optimal. The problem of determining such a point is a discrete facility location problem. [41] is a review article on the network p-median and p-centre problem and in [44] network facility location problems involving tree network structures are studied.

1.3.4. Discrete facility location problems

In discrete facility location problems, facility locations are restricted to a given finite set of points, the candidate locations. We consider n given customer locations p_1, p_2, ..., p_n and m given candidate locations q_1, q_2, ... q_m. Given is also the total cost per year c_{ij} of transportation between customer location p_i and candidate location q_j if the customer at p_i is only served by a facility located at q_j, for all $i \in \{1,2,...,n\}$ and $j \in \{1,2,...,m\}$. The discrete p-median problem is the problem in which p facility locations need to be selected from the set of candidate locations such that the total transportation cost is minimal ([45], [21]). The problem is formulated as a zero-one linear program as follows ([23], [21]):

minimize

$$\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} x_{ij} \tag{12}$$

such that

$$\sum_{j=1}^{m} x_{ij} = 1, \text{ for all } i \in \{1,2,...,n\} \tag{13}$$

$$x_{ij} \leq y_j, \text{ for all } i \in \{1,2,...,n\} \text{ and for all } j \in \{1,2,...,m\} \tag{14}$$

$$\sum_{j=1}^{m} y_j = p \tag{15}$$

$$x_{ij} \in \{0,1\}, \text{ for all } i \in \{1,2,...,n\} \text{ and for all } j \in \{1,2,...,m\} \tag{16}$$

$$y_j \in \{0,1\}, \text{ for all } j \in \{1,2,...,m\}. \tag{17}$$

With x_{ij} and y_j ($i \in \{1,2,...,n\}$, $j \in \{1,2,...,m\}$) the $n{\times}m{+}m$ variables to be determined. x_{ij} is 1 if the customer location p_i is served by a facility at candidate location q_j and 0 otherwise. y_j is 1 if there is a facility at q_j and 0 otherwise. (12) is called the objective function and the equalities, inequalities and zero-one restrictions of (13), (14), (15), (16) and (17) are called the constraints ([46],[21]).

In the previous subsection we found that for the facility location problem of figure 9 (a 1-median problem) the candidate locations are the customer locations and the nodes of the network (in total 13 locations). Therefore in the zero-one linear program formulation of the problem of figure 9 obtained by application of (12)-(17), n is 8, m is 13, the number of variables is $n{\times}m{+}m{=}117$, the number of terms in (12) is $n{\times}m{=}104$ and the number of equalities or inequalities in (13), (14) and (15) is $n{+}n{\times}m{+}1{=}113$. If instead of one facility location, three facility locations need to be determined such that the total transportation cost is minimal, the number of equalities or inequalities stays the same, the only difference in the zero-one program is that (15) changes from $\sum_{j=1}^{13} y_j = 1$ to $\sum_{j=1}^{13} y_j = 3$.

If the number of facilities and the number of candidate solutions are small, the exhaustive search method, i.e. calculating the cost for every candidate solution and then selecting the one with the lowest cost, is one of the solution methods, if the number of facilities and the number

of candidate solutions are larger and exhaustive search takes too much time, other solution methods such as heuristic algorithms are more suitable. In [46] the discrete p-median problem is studied using the zero-one linear programming formulation. In [21], [23] and [45] overviews are given of solution methods for the discrete p-median problem. More specifically, [45] gives an extensive overview of heuristic algorithms.

Another discrete facility location problem is the discrete simple plant location problem. In this problem costs for opening facilities are also considered and the number of facilities need to be determined together with the location of the facilities ([47], [23]). We consider n given customer locations $p_1, p_2, ..., p_n$ and m given candidate locations $q_1, q_2, ... q_m$. Given is also the total cost per year c_{ij} of the customer at p_i being served only by a facility at q_j, for all $i \in \{1,2,...,n\}$ and $j \in \{1,2,...,m\}$. Given is also the cost per year f_j which is the cost of opening a facility at q_j divided by the number of years the facilities are planned to be active plus the cost per year of keeping a facility at q_j open, for all $j \in \{1,2,...,m\}$. In the discrete simple plant location problem the number of facilities needs to be determined and the locations of the facilities need to be selected from the set of candidate locations such that the total cost is minimal. The problem is formulated as a zero-one linear program as follows (similarly as in [32], [48] and [22]):

minimize

$$\sum_{j=1}^{m} f_j y_j + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \tag{18}$$

such that

$$\sum_{j=1}^{m} x_{ij} = 1, \text{ for all } i \in \{1,2,...,n\} \tag{19}$$

$$x_{ij} \leq y_j, \text{ for all } i \in \{1,2,...,n\} \text{ and for all } j \in \{1,2,...,m\} \tag{20}$$

$$x_{ij} \in \{0,1\}, \text{ for all } i \in \{1,2,...,n\} \text{ and for all } j \in \{1,2,...,m\} \tag{21}$$

$$y_j \in \{0,1\}, \text{ for all } j \in \{1,2,...,m\}. \tag{22}$$

With x_{ij} and y_j ($i \in \{1,2,...,n\}$, $j \in \{1,2,...,m\}$) the $n \times m + m$ variables to be determined. x_{ij} is 1 if the customer location p_i is served by a facility at candidate location q_j and 0 otherwise. y_j is 1 if there is a facility at q_j and 0 otherwise.

In [47] the discrete simple plant location problem is studied and an extensive overview of solution methods is given. In [48] the discrete simple plant location problem and other related discrete facility location problems are studied including solution methods for these problems. [23] and [22] both contain a brief study and overview of the discrete simple plant location problem including references to other articles on the subject.

1.3.5. Urban facility location and the reorganization of urban freight distribution

In the last four subsections we discussed four types of facility location models. In different chapters of this dissertation we will study practical facility location problems by using one of the model types: analytic, continuous, network or discrete facility location models. The model decision depends on the data available regarding customer locations and candidate facility

locations, the urban environment, the required precision and the amount of time available to search for the solution. If there are for example only two candidate facilities, a discrete model and the exhaustive search method are suitable. If we have data about the number of customers per city district, if all locations in the city are candidate facility locations and if the required facility location precision is km, a continuous model is a suitable approximation in some cases and the exhaustive search method over every square kilometre of the city is then a suitable solution method.

One of the research topics of this dissertation is the relation between reorganization of urban freight distribution, changes in costs and changes in the optimal facility location. We will study this in the dissertation for a practical urban case: we will make an extensive case study of a practical urban single facility location problem with customers in the city and suppliers outside the city. In this case study we will study the actual urban freight distribution situation and make an estimation of the transportation costs. Then we will try to optimize the urban freight distribution method and the facility location, taking into account the sum of the customers costs and the suppliers costs and also taking into account the environmental situation. The following example illustrates that reorganization of the transportation sometimes changes the optimal facility location. Let us consider five customer locations c_1, c_2, ..., c_5 in the city with weights w_1, w_2, ... , w_5 such that the daily transportation cost of the goods from facility location f to customer location c_i is w_i times the distance from f to c_i for all $i \in \{1,2,3,4,5\}$ and let us also consider supplier locations south of the city with given weights. By replacing the actual internal combustion vehicles of the customers by electric vehicles the cost per km will possibly decrease according to [49] and [50] and therefore also the weights of the customers will decrease. If the weights of the customers in the city decrease and the weights of the suppliers south of the city remain constant, we expect the optimal facility location to move to the south.

1.4. Inventory management

In this section we discuss inventory management, more specifically in subsection 1.4.1 we present different inventory policies and discuss them and in subsection 1.4.2 we consider inventories with joint capacitated replenishment.

1.4.1. Inventory policies

Let us consider a facility which stores products, e.g. a retail shop. By delivering products to customers, the stock on hand, i.e. the number of products in the facility, decreases and by ordering products from a supplier the stock on hand increases. We assume that the time between the placement of an order by the retailer and the arrival of the ordered products at the retail shop, which is called the lead time ([1]), is L. If the number of products on hand is zero and demand occurs we distinguish two possibilities. (i) Backorders: the unsatisfied demand of a customer will be satisfied immediately after replenishment. (ii) Lost sales: unsatisfied demand of a customer is lost, e.g. the customer buys the product at another retail shop, buys a similar product or does not buy a product if the stock on hand is zero. We define the inventory position as the number of products on hand plus the number of products on order minus the number of products backordered. Regarding the customers demand, some problems consider

deterministic customers demand, i.e. the moment and quantity of every demand occurrence is given, e.g. one product is sold every hour, other problems consider stochastic customer demand, i.e. probabilities concerning the customer demand are given, e.g. it is given that the demands during different weeks are independently and identically distributed and the probability mass function of the demand during a week is given.

A policy concerning when to place an order and how much to order is called an inventory or replenishment policy ([1]). Two categories of inventory policies are (i) continuous review, where the inventory position is tracked continuously, and (ii) periodic review, where the inventory position is checked (reviewed) at equidistant points in time ([1]). The time between two consecutive reviews is called the review period ([51]).

An example of a continuous review inventory policy is the continuous review reorder level order quantity inventory policy (or the continuous review (R,Q) inventory policy). In this policy Q products (the order quantity) are ordered if the inventory position decreases to a fixed level, the reorder level (or reorder point) R ([52]). This inventory policy with backorders is illustrated in figure 10, which shows the stock on hand as a function of time and the inventory position as a function of time. Figure 11 also shows the stock on hand as a function of time and the inventory position as a function of time for a continuous review reorder level order quantity inventory policy but with lost sales. Demand occurs at the same moments in both figures. A difference between both figures is that in the third week during the time that there is no stock on hand the inventory position decreases in figure 10 but remains constant in figure 11. An example of a periodic review inventory policy is the periodic review order-up-to level inventory policy (or periodic review base stock inventory policy). In this policy every T weeks the inventory position is checked and an order is placed to raise the inventory level to a fixed level, the order-up-to level (or base stock level) S ([53]). This inventory policy with backorders is illustrated in figure 12, which shows the stock on hand as a function of time and the inventory position as a function of time. Figure 13 also shows the stock on hand as a function of time and the inventory position as a function of time for a periodic review order-up-to level inventory policy but with lost sales. Demand occurs at the same moments in both figures. A difference between figure 12 and figure 13 is that the inventory position is nonnegative in figure 13 but in figure 12 the inventory position has also negative values (in the fifth week). Another example of a periodic review inventory policy is the periodic review reorder level order-up-to level inventory policy. In this policy every T weeks the inventory position is checked and an order is placed to raise the inventory level to a fixed level, the order-up-to level S, if the inventory position is equal to or less than R, the reorder level ([54]).
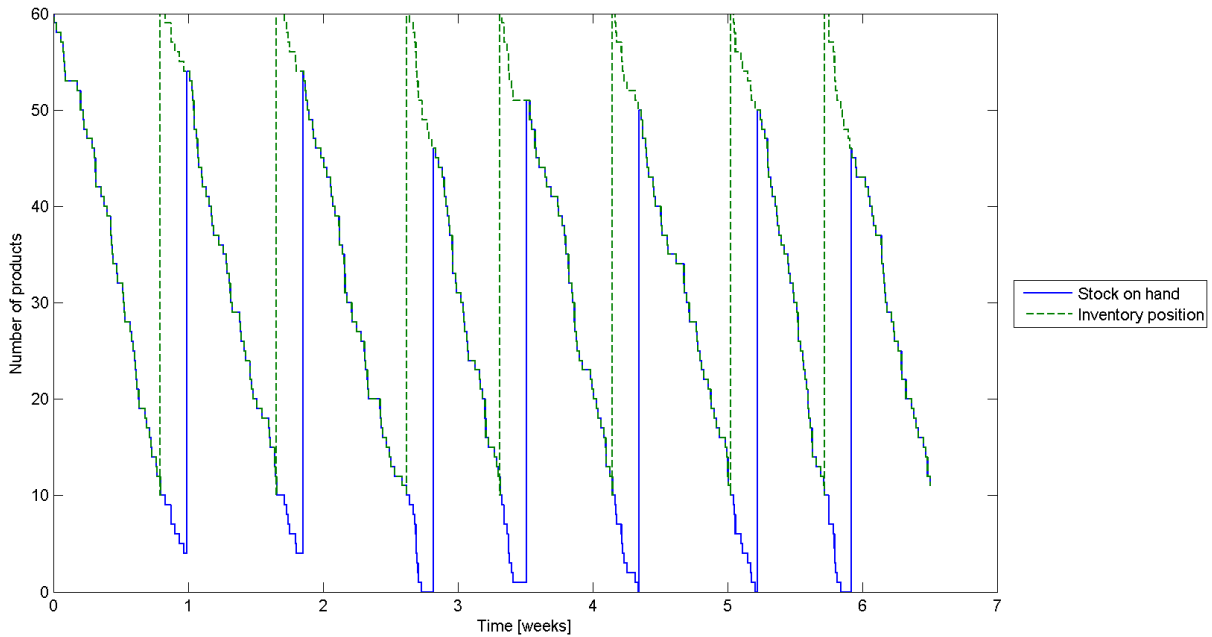
Figure 1.10. Stock on hand as a function of time and the inventory position (stock on hand plus the number of products on order minus the number of products backordered) as a function of time for a continuous review reorder level order quantity inventory policy with backorders. The reorder level is 10 products, the order quantity is 50 products and the lead time is 0.2 weeks. The demand is simulated as a Poisson process with the time between two consecutive demand occurrences being exponentially distributed with a mean of 3 hours.
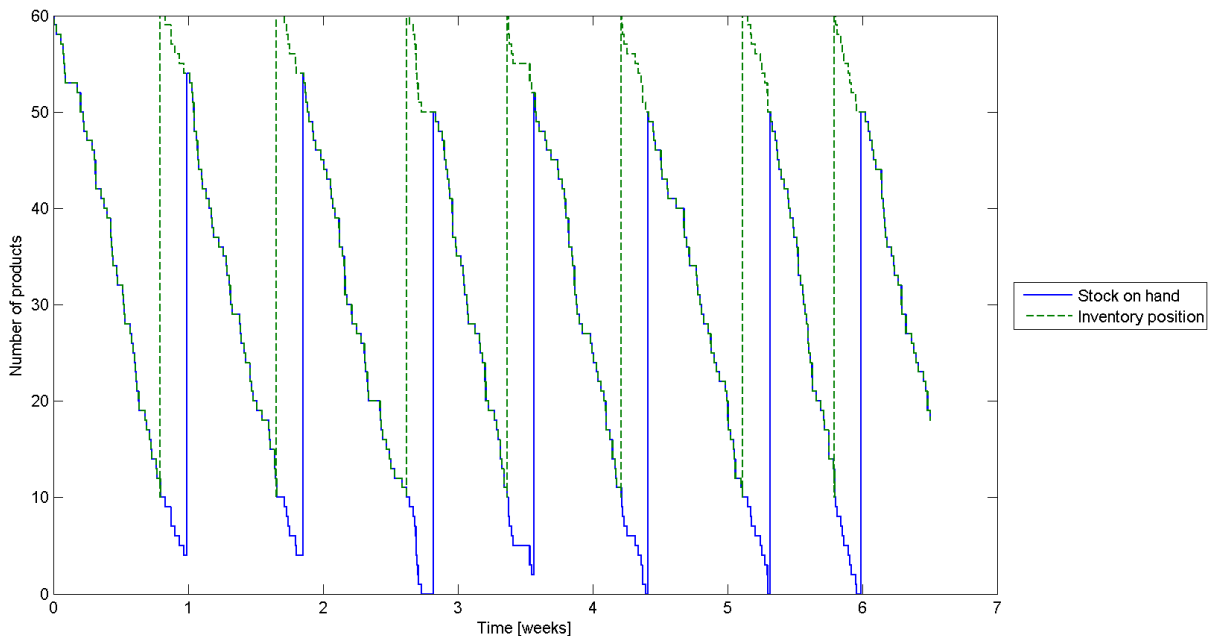
Figure 1.11. Stock on hand as a function of time and the inventory position as a function of time for a continuous review reorder level order quantity inventory policy with lost sales. The reorder level is 10 products, the order quantity is 50 products and the lead time is 0.2 weeks. The same demand occurrences are used as in figure 10.
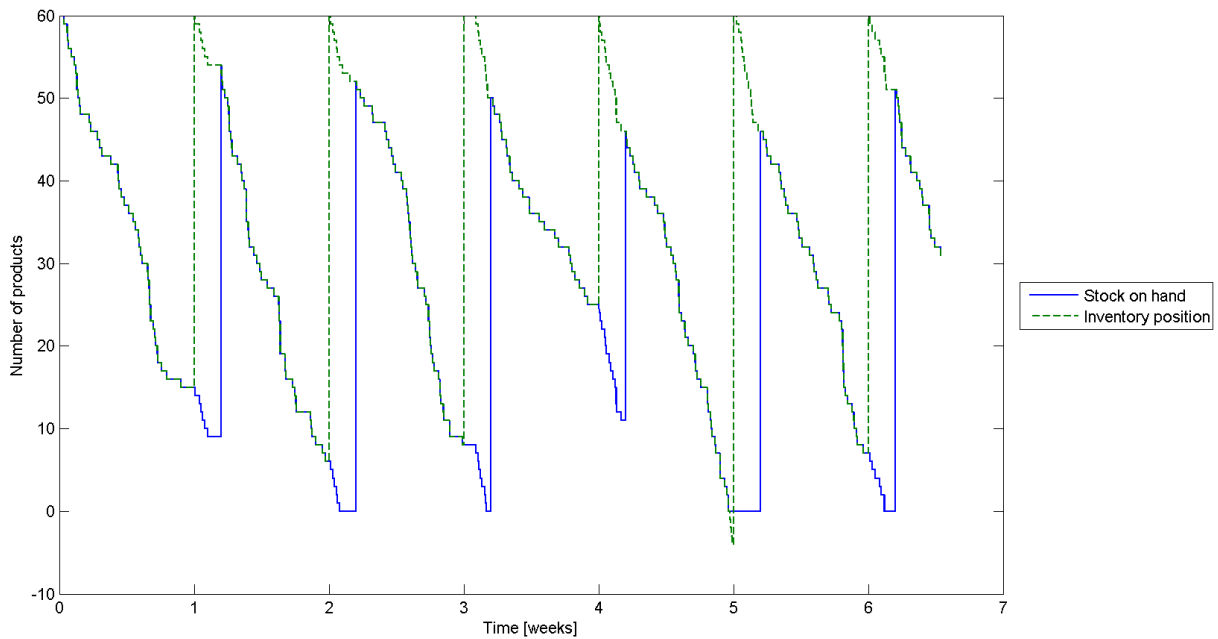
Figure 1.12. Stock on hand as a function of time and the inventory position as a function of time for a periodic review order-up-to level inventory policy with backorders. The review period is 1 week, the order-up-to level is 60 products and the lead time is 0.2 weeks. The demand is simulated as a Poisson process with the time between two consecutive demand occurrences being exponentially distributed with a mean of 3 hours.
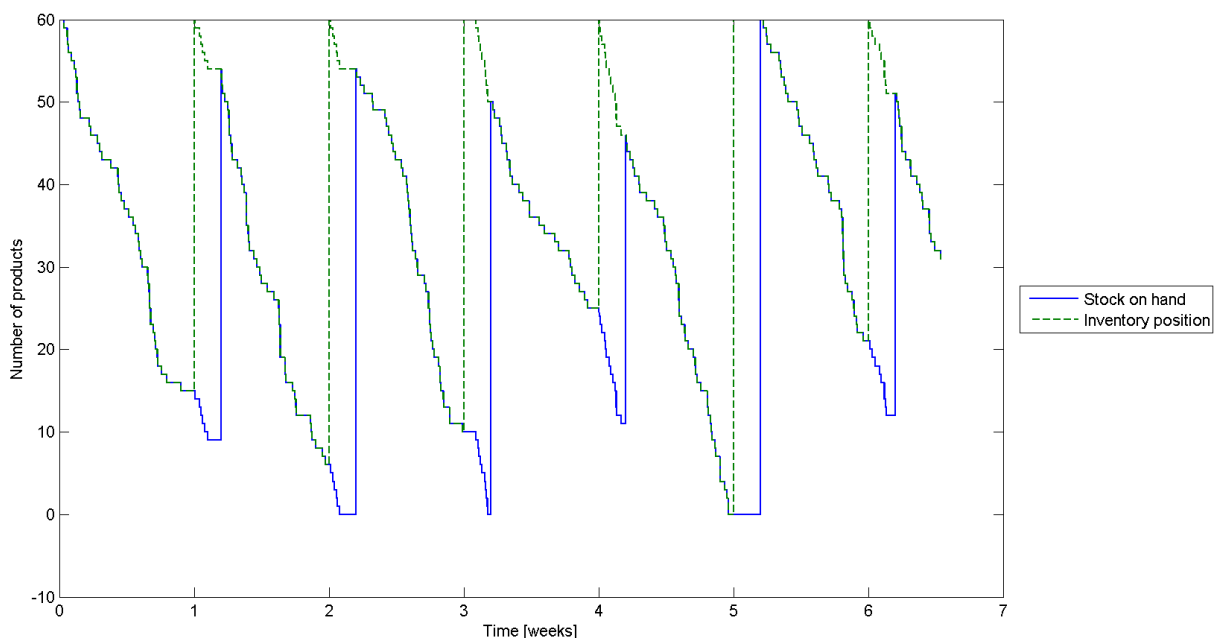
Figure 1.13. Stock on hand as a function of time and the inventory position as a function of time for a periodic review order-up-to level inventory policy with lost sales. The review period is 1 week, the order-up-to level is 60 products and the lead time is 0.2 weeks. The same demand occurrences are used as in figure 12.

In [55] the fill rate of an inventory policy combined with a given lead time and given customer demand is defined as the proportion of the expected satisfied demand to the expected demand. Another definition used in literature for the fill rate (e.g. in [56] and [57]) is the expectation of the proportion of the satisfied demand to the demand. According to [58] and [57], both definitions agree if an infinite horizon is considered, i.e. the limits of both expressions agree as the time considered approaches infinity. The fill rate is often considered as a measure of customers satisfaction. If the fill rate increases, the customer demand is more often satisfied directly from inventory (i.e. without backorder or without the customer going to another retail shop).

Concerning the costs related with inventories, costs often considered include the following costs ([1],[59],[52]): holding cost, order cost, review cost, purchase cost and shortage cost. Holding cost is the cost related with storing the products in the facility. This cost depends on the amount of time the products are stored and the number of products that are stored. The order cost is the cost related with the placement of an order and is dependent on or independent of the number of products ordered depending on the situation. The review cost is the cost of checking the inventory position, the purchase cost is the cost of purchasing products from the supplier and the shortage cost is the cost related with not being able to satisfy customers demand. Selling products to customers is a source of revenue for the retailer. Examples of inventory problems are: (i) the problem of determining parameters of inventory policies such that the considered costs are minimal, (ii) the problem of determining parameters of inventory policies such that the profit (revenues minus costs) is maximal, (iii) the problem of determining parameters of inventory policies such that the fill rate minus a given value is nonnegative and minimal and (iv) the problem of determining parameters of inventory policies such that the considered costs are minimal and the fill rate is greater than a given value.

In [52] a continuous review reorder level order quantity inventory policy with backorders is considered and the demand during the lead time is assumed to be normally distributed. The reorder level and order quantity for which the order cost plus holding cost is minimal and the fill rate is greater than or equal to a given value are searched for. In the paper a technique is presented for finding an approximation of the optimal reorder level and order quantity. In [53] a formula for the fill rate is obtained for a periodic review order-up-to level inventory policy under normally distributed demand and with backorders and a positive fixed lead time. [60] is a review article on inventory management with lost sales. It contains a literature study on inventory management with lost sales and a discussion of continuous review policies with lost sales and of periodic review policies with lost sales. In chapter 6 and 7 we study a periodic review order-up-to level inventory policy with lost sales and zero lead time and we consider the replenishment to be capacitated, i.e. the number of products delivered is less than a fixed given number (the capacity). We determine the fill rate in case the order-up-to level, the probability mass function of the demand during a review period and the capacity are given. Also determined is the order-up-to level that minimizes the holding cost such that the fill rate is greater than or equal to a given value, for a given probability mass function of the demand during a review period and a given capacity.

1.4.2. Joint capacitated replenishment of different inventories

Because of the size and the maximum gross vehicle mass, we consider vehicles to be capacitated, i.e. they deliver at most c products, with c called the capacity. Therefore the number of products delivered to a retail shop is equal to or less than the number of products ordered by the retailer if the supplier decides to use only one vehicle and to deliver only once per order. Therefore we expect the fill rate to be lower if replenishment is capacitated than if replenishment is uncapacitated (if the capacity is not very large). In chapter 6 we determine the fill rate for a periodic review inventory policy with capacitated replenishment. In chapter 7 we determine the order-up-to level that minimizes the expected inventory holding cost per unit of time such that the fill rate is greater than or equal to a given value for a periodic review inventory policy with capacitated replenishment.

Until now we considered individual inventories, but sometimes different inventories are replenished jointly. For example one capacitated vehicle replenishes once a week five retail shops by making a milk run (round trip). In this situation it is possible that one inventory influences another inventory. If one retailer orders a small number of products there is more place in the vehicle for the other four retailers than if this retailer orders a larger number of products. In chapter 8 we will study this situation for retailers who apply periodic review order-up-to level inventory policies with given minimal values for the fill rates. With joint capacitated replenishment the question arises how to divide the products if the sum of the number of products ordered by the different retailers is greater than the replenishment capacity. In [61] three allocation schemes are discussed for dividing scarce capacity: proportional, linear and uniform allocation. In proportional allocation the proportion of the quantity a retailer receives to the total capacity is equal to the order of this retailer to the sum of all orders, if the sum of all orders is greater than the capacity. In linear allocation every retailer receives the ordered quantity minus the quotient of the shortage and the number of retailers. If this quantity is smaller than zero for a retailer, this retailer receives nothing and the quantities for the other retailers are recalculated. In uniform allocation the capacity is divided equally among the retailers. If a retailer ordered less than this quantity, this retailer receives the ordered quantity and the remaining part of the capacity is divided equally among the remaining retailers. In chapter 8 we will compare n inventories being replenished individually with vehicles of capacity c and n inventories being replenished jointly with a vehicle of capacity $n{\times}c$.

1.5. Istanbul case studies

This dissertation contains case studies on the urban freight distribution and the location of a fruit and vegetable wholesale market in Istanbul and on the urban freight distribution of distribution centres and a possible road transshipment centre in Istanbul. Istanbul has a population of $14.2{\times}10^6$ people in 2013 with $9.2{\times}10^6$ people on the European side of Istanbul and $5.0{\times}10^6$ people on the Asian side of Istanbul ([62]). The area of Istanbul is divided into 39 districts with 25 districts on the European side and 14 districts on the Asian side. Figure 14 shows the different districts with each district in a different colour than the neighbouring districts. There are two main fruit and vegetable wholesale markets in Istanbul. One on the European side in the Kocatepe neighbourhood of the Bayrampasa district and one on the Asian side in the Içerenköy neighbourhood of the Atasehir district. The customers of the Kocatepe market are mainly located on the European side of Istanbul and the customers of the

Içerenköy market are mainly located on the Asian side of Istanbul. In chapters 3, 4 and 5 we will study the urban freight distribution and the location of the fruit and vegetable wholesale market on the Asian side of Istanbul (in this dissertation we call this market the Istanbul fruit and vegetable wholesale market). There are plans to move this market to the Aydinli neighbourhood of the Tuzla district. Figure 14 shows the location of the Kocatepe market, the Içerenköy market and the planned Aydinli market on the map of Istanbul. Most suppliers of the markets enter Istanbul via the E-80 road (indicated on figure 14) and a main road for the customers of the fruit and vegetable wholesale market on the Asian side of Istanbul is the D-100 road (also indicated on figure 14). In chapter 9 we will compare the urban freight distribution of six distribution centres in the Tuzla district with the urban freight distribution of one possible road transshipment centre also in the Tuzla district. Figure 14 shows the locations of the considered distribution centres and the possible road transshipment centre. The customers of the distribution centres and road transshipment centre are mainly located on the Asian side of Istanbul.

Figure 1.14. Map of Istanbul with the location of the fruit and vegetable wholesale market in Kocatepe, the locations of the fruit and vegetable wholesale market in Içerenköy and the possible fruit and vegetable wholesale market in Aydinli (studied in chapters 3, 4 and 5) and the locations of the six distribution centres (DCs, red) and the possible road transhipment centre (RTC, green) studied in chapter 9.
Source: Istanbul Metropolitan Municipality website [63].

1.6. Research questions and the structure of the dissertation

The main research questions of this dissertation are:
RQ1: How to estimate the transportation fuel costs for different urban freight distribution methods?

RQ2: How to estimate the optimal (optimal regarding transportation fuel costs) location of an urban facility with customers in the city and all suppliers entering the city through the same point for different urban freight distribution methods?

RQ3: How to estimate the optimal (optimal regarding inventory holding costs, under fill rate constraints) order-up-to levels of periodic review inventory policies applied by urban retailers for different urban freight distribution methods?

Before we deal with RQ2 and RQ3 we first deal with the following research questions:

RQ2': How to estimate the optimal (optimal regarding transportation fuel costs) location of an urban facility with customers in the city and all suppliers entering the city through the same point for a particular urban freight distribution method?

RQ3': How to estimate the fill rates or the optimal (optimal regarding inventory holding costs, under fill rate constraints) order-up-to levels of periodic review inventory policies applied by urban retailers for a particular urban freight distribution method?

Chapters 5 and 9 deal with RQ1, chapters 3 and 4 deal with RQ2', chapter 2 and section 10.1 deal with RQ2, chapters 6 and 7 deal with RQ3' and chapter 8 and section 10.3 deal with RQ3. Section 10.1 uses chapters 3 to 5 and section 10.3 uses chapters 6 to 8. Chapter 2 deals with RQ2 in the context of an urban facility in a city modelled as a disc with an exponential population density function. Chapters 3 to 5 and sections 10.1 and 10.2 deal with RQ1, RQ2' and RQ2 in the context of the Istanbul fruit and vegetable wholesale market. Chapters 6 to 8 and section 10.3 deal with RQ3' and RQ3 in the context of inventory systems with capacitated replenishments. Chapter 9 deals with RQ1 in the context of six distribution centres and one road transshipment centre in Istanbul. Figure 15 shows the different chapters with research questions and arranges them in three overlapping categories: transportation reorganization, location and inventory.
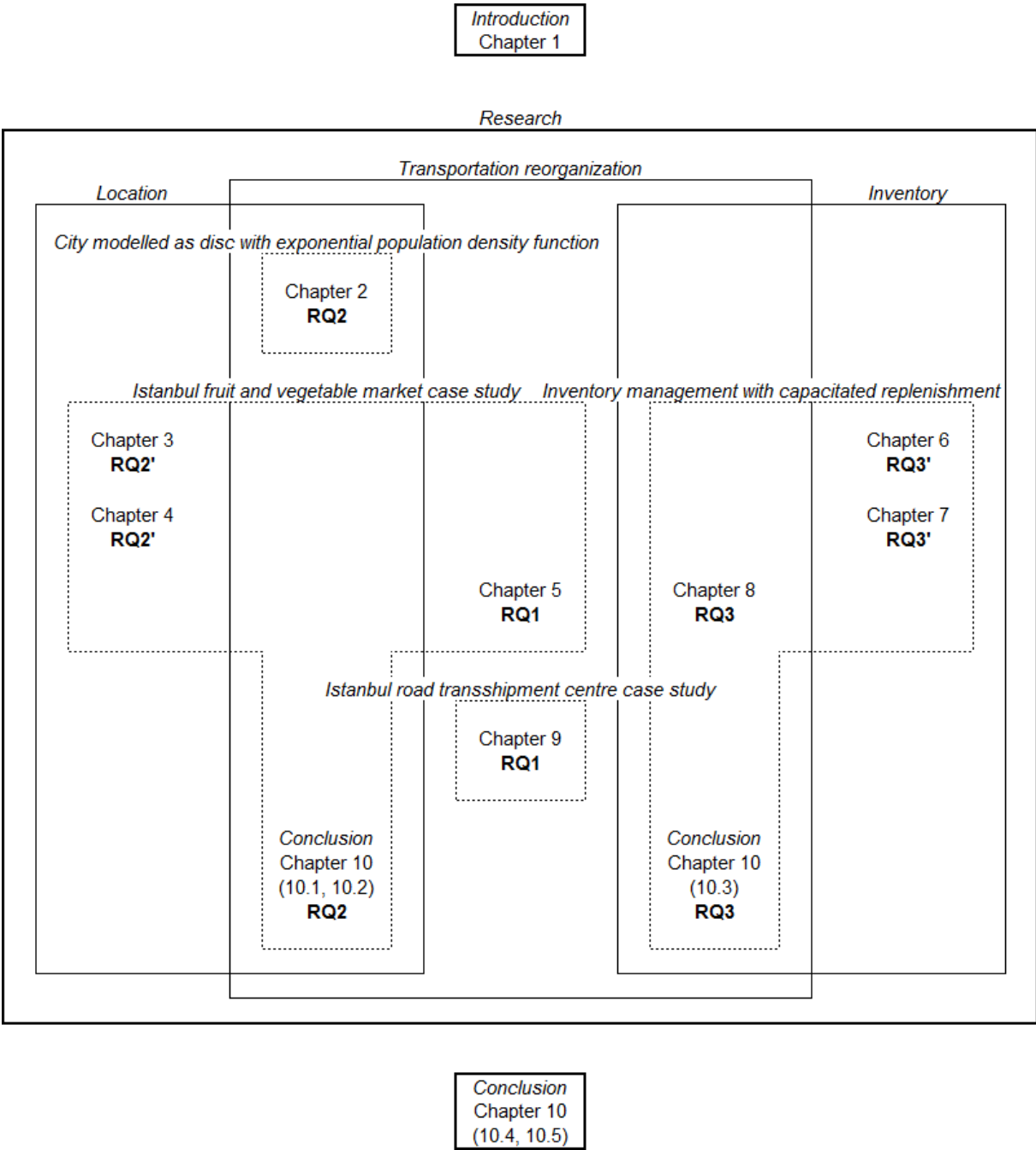
Figure 1.15. Structure of the dissertation.

1.7. Overview

Chapter 2 contains research on urban facility location optimization. We consider an analytic facility location problem: we search for the location of an urban facility that minimizes the total customers and suppliers transportation cost. We model the city as a disc with radius R and we assume that the urban population density and the customer location density at a distance r from the city centre is proportional to $\exp(-\gamma r)$ with γ a real number. It is also assumed that all supplier vehicles enter the city at the same point and that the transportation

routes between the suppliers entry point and the facility and between the facility and the customer locations go via straight lines. We reduce the location problem to the problem of finding the minimum of a function with only one variable over the interval [0,1]. This enables us to solve the problem numerically. In chapter 2 we also introduce the parameter α. The study in this chapter does not consider a specific facility and the results in this chapter are generally applicable if the assumptions are met. Because these assumptions are only roughly met for most urban facilities, the results of this chapter are considered to be first rough estimations for most urban facilities. In the next four chapters we will study a specific urban facility.

Chapter 3 is a case study of the Istanbul fruit and vegetable wholesale market. We consider a discrete facility location problem: we compare the actual location to the planned new location by estimating the total customers and suppliers fuel cost for both locations. For making these estimations we first need to study the location of the customers and suppliers and the fuel economy of the customer vehicles and the supplier vehicles. For the customer locations we assume the number of customers in a district to be proportional to the population of that district. The suppliers enter the Istanbul province via the E-80 road and the kilometres travelled from the supplier locations to the location where the suppliers enter the city is the same for the two market locations, therefore in our study we locate the suppliers close to where this road crosses the Istanbul border.

In chapter 4 we consider a continuous facility location problem: we search over the Asian side of Istanbul for the location of the Istanbul fruit and vegetable wholesale market that minimizes the total customers and suppliers transportation cost. We do this by using two methods. In the first method an exhaustive search is performed over every square kilometre of the Asian side of Istanbul and road network travel distances (driving distances) are estimated from the Euclidean distances between the suppliers entry location and the market location and between the market location and the customer locations. In the second method we simplify the continuous location problem to a discrete location problem with fifteen candidate locations: thirteen locations easy accessible from main roads, the actual market location and the planned new market location. In this method the distances of the supplier routes are determined by using a route planner.

In chapter 3 and 4 we consider individual transportation for the customers, which is the actual distribution method for the Istanbul fruit and vegetable wholesale market. Every costumer organizes the transportation from the market to the customer location independently of the other customers and mostly vans or pickup trucks are used in this distribution method. In chapter 5 we study the change in transportation cost if the distribution is reorganized: we study the transportation cost if goods are delivered to the customers by trucks making round trips (milk runs) with several deliveries per trip. We compare the transportation cost of individual transportation distribution to milk run distribution for the Istanbul fruit and vegetable wholesale market. Our calculations are made for the planned new location of the market in the Aydinli neighbourhood.

In chapter 6 we study a periodic review inventory system with capacitated replenishments. E.g. a retailer orders every week a number of products to raise the stock on hand to a fixed level (the order-up-to level) and products are delivered once a week by a truck shortly after the placement of the order. Because of the capacity of the truck (the maximum gross vehicle mass is reached if the truck contains c products) the maximum number of products delivered

is c. In this context we use the fill rate as a measure for the customer satisfaction. The fill rate is the expected satisfied demand during a large number of review periods over the expected demand during the same large number of review periods. Satisfied demand is the part of the demand that is satisfied directly from the inventory of the retailer (without backorders). In chapter 6 four theorems concerning the considered inventory system are proved. These theorems allow us to present a method for determining the fill rate for a given order-up-to level, capacity and probability mass function of the review period demand. After presenting this method for determining the fill rate, the inventory system is studied further by performing several computation experiments for Poisson distributed review period demand.

In chapter 7 we study the same inventory system as in chapter 6, but now we use a continuous probability function instead of a discrete probability mass function for the review period demand. After determining the fill rate for a given order-up-to level, capacity and probability function of the review period demand, we search for the optimal order-up-to level (the order-up-to level that minimizes the expected inventory holding cost per unit of time) for a given required degree of customer satisfaction (minimum fill rate), capacity and probability function of the review period demand. If the order-up-to level decreases, the expected average stock on hand during one review period does not increase and therefore the expected inventory holding cost per unit of time does not increase. Therefore we search the minimal order-up-to level such that the degree of customer satisfaction is greater than or equal to the required degree of customer satisfaction. An iterative procedure is used to find the optimal order-up-to level. The optimal order-up-to level in the capacitated case is greater than or equal to the optimal order-up-to level in the uncapacitated case because in the capacitated case the stock on hand just after replenishment is less than or equal to the order-up-to level.

In chapter 8 we consider a series of n inventory systems similar to the inventory systems in chapters 6 and 7 and we study how the optimal order-up-to levels change if the transportation is organized jointly. We compare two situations. In the first situation (individual transportation), every retailer is supplied by one vehicle with a capacity of c products. In the second situation (joint transportation), the n retailers are supplied by one vehicle with a capacity of $n \times c$ products and this vehicle makes one milk run every replenishment cycle with deliveries to the n retailers. We determine the optimal order-up-to levels (the minimal order-up-to levels such that each fill rate is greater than or equal to the required fill rate) of the inventory policies of the supplied retailers with individual transportation and with joint transportation. The optimal order-up-to levels of the n retailers with joint transportation are determined by an algorithm that simulates a large number of replenishment cycles.

In chapter 9 we consider six distribution centres in the Tuzla district of Istanbul, this district is close to the Istanbul province border. We study how the total transportation cost changes if these distribution centres merge into one road transshipment centre. The merger enables a reorganization of the distribution of the goods to the customers in the city. This type of urban freight distribution reorganization differs from the urban freight distribution reorganization in chapter 5 and 8: in chapter 9 different suppliers merge while in chapter 5 and 8 different trips from one facility to one customer and back are replaced by one round trip. We compare the situation before and after the merger by estimating the total customers and suppliers transportation fuel cost for both situations.

We conclude with chapter 10 where we put the results of previous chapters together. We focus on the results concerning joint transportation (joint transportation by integrating the

delivery to customers and joint transportation by integrating the suppliers) and how the transportation cost, the inventory holding cost, the optimal facility location and the optimal order-up-to levels change if the urban freight distribution is organized more jointly.

In section 1 of chapter 10 we put together the results of the different chapters on the Istanbul fruit and vegetable wholesale market (chapters 3, 4 and 5) and the results of chapter 2 on urban facility location optimization. We study how the optimal location of the Istanbul fruit and vegetable wholesale market (optimal regarding the total transportation fuel cost) changes if the urban freight distribution is reorganized from individual transportation to joint transportation. We also study the optimal market location in function of the parameter α (introduced in chapter 2) and take into account environmental considerations. We conclude by giving advice on the location and the urban freight distribution method of the Istanbul fruit and vegetable wholesale market, based on the results of the research in chapters 2, 3, 4 and 5. Section 2 of chapter 10 contains a sensitivity analysis.

In section 3 of chapter 10 we put together the results of the different chapters on inventory management (chapters 6, 7 and 8). We consider different retailers applying periodic review policies such that the order-up-to levels are minimal and the fill rates are greater than or equal to 99%. We compare uncapacitated replenishment, individual capacitated replenishment and joint capacitated replenishment of the different inventories by using the results of chapters 6, 7 and 8 and simulation. The order-up-to levels and the expected inventory holding costs per unit of time are calculated for the three cases.

We end the dissertation with a section in which we discuss the research results and possibilities for future research and a section in which the research presented in this dissertation is summarized.

References

[1] S. Chopra, P. Meindl, Supply chain management: strategy, planning and operation, Third edition, Pearson Prentice Hall, Upper Saddle River, New Jersey, 2007.

[2] L. Dablanc, Goods transport in large European cities: difficult to organize, difficult to modernize, Transportation Research Part A: Policy and Practice 41 (3) (2007) 280-285.

[3] N. Arvidsson, The milk run revisited: a load factor paradox with economic and environmental implications for urban freight transport, Transportation Research Part A 51 (2013) 56-62.

[4] G. Laporte, The vehicle routing problem: an overview of exact and approximate algorithms, European Journal of Operational Research 59 (3) (1992) 345-358.

[5] J.-F. Cordeau, M. Gendreau, G. Laporte, J.-Y. Potvin, F. Semet, A guide to vehicle routing heuristics, Journal of the Operational Research Society 53 (5) (2002) 512-522.

[6] G.B. Dantzig, J.H. Ramser, The truck dispatching problem, Management Science 6 (1) (1959) 80-91.

[7] A. Weber, C.J. Friedrich, Theory of the location of industries, University of Chicago Press, Chicago, Illinois, 1929.

[8] E.L. Greenhut, Plant location in theory and in practise: the economics of space, University of North Carolina Press, Chapel Hill, North Carolina, 1956.

[9] W. Isard, Location and space-economy: a general theory relating to industrial location, market areas, land use, trade, and urban structure, Technology Press of Massachusetts Institute of Technology, Cambridge, Massachusetts, 1956.

[10] D.M. Smith, Theoretical framework for geographical studies of industrial location, Economic Geography 42 (2) (1966) 95-113.

[11] D.M. Smith, Industrial location: an economic geographical analysis, Second edition, John Wiley & Sons, New York, 1981.

[12] F. Witlox, Towards a relational view on industrial location theory, Tijdschrift voor Economische en Sociale Geografie 91 (2) (2000) 135-146.

[13] R.A. Boschma, K. Frenken, Why is economic geography not an evolutionary science? Towards an evolutionary economic geography, Journal of Economic Geography 6 (3) (2006) 273-302.

[14] P. Dicken, Some aspects of the decision making behavior of business organizations, Economic Geography 47 (3) (1971) 426-437.

[15] H.A. Simon, A behavioral model of rational choice, Quarterly Journal of Economics 69 (1) (1955) 99-118.

[16] H.A. Simon, Rational choice and the structure of the environment, Psychological Review 63 (2) (1956) 129-138.

[17] H.A. Simon, Theories of Decision-Making in Economics and Behavioral Science, American Economic Review 49 (3) (1959) 253-283.

[18] R. Boschma and R. Martin, Editorial: constructing an evolutionary economic geography, Journal of Economic Geography 7 (2007) (5) 537-548.

[19] H.K. Smith, G. Laporte, P.R. Harper, Locational analysis: highlights of growth to maturity, Journal of the Operational Research Society 60 (Supplement 1) (2009) S140-S148.

[20] R.L. Francis, L.F. McGinnis, J.A. White, Locational analysis, European Journal of Operational Research 12 (3) (1983) 220-252.

[21] M.S. Daskin, What you should know about location modeling, Naval Research Logistics 55 (4) (2008) 283-294.

[22] C.S. ReVelle, H.A. Eiselt, M.S. Daskin, A bibliography for some fundamental problem categories in discrete location science, European Journal of Operational Research 184 (3) (2008) 817-848.

[23] C.S. ReVelle, H.A. Eiselt, Location analysis: A synthesis and survey, European Journal of Operational Research 165 (1) (2005) 1-19.

[24] M.L. Brandeau, S.S. Chiu, An overview of representative problems in location research, Management Science 35 (6) (1989) 645-674.

[25] C. Clark, Urban Population Densities, Journal of the Royal Statistical Society. Series A (General) 114 (4) (1951) 490-496.

[26] E.S. Mills, J.P. Tan, A comparison of urban population density functions in developed and developing countries, Urban Studies 17 (3) (1980) 313-321.

[27] J.F. McDonald, Econometric studies of urban population density: A survey, Journal of Urban Economics 26 (3) (1989) 361-385.

[28] T. Drezner, Z. Drezner, B. Hulliger, The Quintile Share Ratio in location analysis, European Journal of Operational Research 238 (1) (2014) 166-174.

[29] A.R. Odoni, G. Sadiq, Two planar facility location problems with high-speed corridors and continuous demand, Regional Science and Urban Economics 12 (4) (1982) 467-484.

[30] A. Suzuki, Z. Drezner, The p-center location problem in an area, Location Science 4 (1-2) (1996) 69-82.

[31] S.H. Owen, M.S. Daskin, Strategic facility location: a review, European Journal of Operational Research 111 (1998) 423-447.

[32] P.M. Dearing, Location problems, Operations Research Letters 4 (1985) 95-98.

[33] R.F. Love, J.G. Morris, G.O. Wesolowsky, Facilities location: models and methods, Elsevier Science, New York, 1988.

[34] T.S. Hale, C.R. Moberg, Location science research: a review, Annals of Operations Research 123 (2003) 21-35.

[35] E. Weiszfeld, Sur le point pour lequel la somme des distances de n points donnés est minimum, Tôhoku Mathematical Journal (First series) 43 (1937) 355-386.

[36] E. Weiszfeld, F. Plastria, On the point for which the sum of the distances to n given points is minimum, Annals of Operations Research 167 (2009) 7-41.

[37] M. Ciligot-Travain, S. Traoré, On a robustness property in single-facility location in continuous space, TOP 22 (2014) 321-330.

[38] J.M. Diaz-Banez, M. Korman, P. Pérez-Lantero, I. Ventura, Locating a single facility and a high-speed line, European Journal of Operational Research 236 (2014) 69-77.

[39] C. Iyigun, A. Ben-Israel, A generalized Weiszfeld method for the multi-facility location problem, Operations Research Letters 38 (2010) 207-214.

[40] J. Brimberg, Z. Drezner, N. Mladenovic, S. Salhi, A new local search for continuous location problems, European Journal of Operational Research 232 (2014) 256–265.

[41] B.C. Tansel, R.L. Francis, T.J. Lowe, Location on networks: a survey. Part I: the p-center and p-median problems, Management Science 29 (4) (1983) 482-497.

[42] S.L. Hakimi, Optimal locations of switching centers and the absolute centers and medians of a graph, Operations Research 12 (1964) 450-459.

[43] S.L. Hakimi, Optimal distribution of switching centers in a communication network and some related theoretic graph theoretic problems, Operations Research 13 (1965) 462-475.

[44] B.C. Tansel, R.L. Francis, T.J. Lowe, Location on networks: a survey. Part II: exploiting tree network structure, Management Science 29 (4) (1983) 498-511.

[45] N. Mladenovic, J. Brimberg, P. Hansen, J.A. Moreno-Pérez, The p-median problem: a survey of metaheuristic approaches, European Journal of Operational Research 179 (2007) 927-939.

[46] C.S. ReVelle, R.W. Swain, Central facilities location, Geographical Analysis 2 (1) (1970) 30-42.

[47] J. Krarup, P.M. Pruzan, The simple plant location problem: survey and synthesis, European Journal of Operational Research 12 (1983) 36-81.

[48] C.H. Aikens, Facility location models for distribution planning, European Journal of Operational Research 22 (1985) 263-279.

[49] B.A. Davis, M.A. Figliozzi, A methodology to evaluate the competitiveness of electric delivery trucks, Transportation Research Part E 49 (2013) 8-23.

[50] D.-Y. Lee, V.M. Thomas, M.A. Brown, Electric urban delivery trucks: energy use, greenhouse gas emissions, and cost-effectiveness, Environmental Science & Technology 47 (2013) 8022-8030.

[51] S. Axsäter, Inventory control, Kluwer Academic Publishers, Boston, Massachusetts, 2000.

[52] S. Axsäter, A simple procedure for determining order quantities under a fill rate constraint and normally distributed lead-time demand, European Journal of Operational Research 174 (2006) 480-491.

[53] E.A. Silver, D.P. Bischak, The exact fill rate in a periodic review base stock system under normally distributed demand, Omega International Journal of Management Science 39 (2011) 346-349.

[54] E.A. Silver, H. Naseraldin and D.P. Bischak, Determining the reorder point and order-up-to-level in a periodic review system so as to achieve a desired fill rate and a desired

average time between replenishments, Journal of the Operational Research Society 60 (2009) 1244-1253.

[55] R.H. Teunter, Note on the fill rate of single-stage general periodic review inventory systems, Operations Research Letters 37 (1) (2009) 67-68.

[56] M.J. Sobel, Fill rates of single-stage and multistage supply systems, Manufacturing and Service Operations Management 6 (1) (2004) 41-52.

[57] J. Zhang, J. Zhang, Fill rate of single-stage general periodic review inventory systems, Operations Research Letters 35 (4) (2007) 503-509.

[58] J. Chen, D.K.J. Lin, D.J. Thomas, On the single item fill rate for a finite horizon, Operations Research Letters 31 (2) (2003) 119-123.

[59] A.G. Lagodimos, I.T. Christou, K. Skouri, Optimal (r,nQ,T) batch ordering with quantized supplies, Computers & Operations Research 39 (2012) 259-268.

[60] M. Bijvank, I.F.A. Vis, Lost-sales inventory theory: a review, European Journal of Operational Research 215 (2011) 1-13.

[61] G.P. Cachon, M.A. Lariviere, An equilibrium analysis of linear, proportional and uniform allocation of scarce capacity, IIE Transactions 31 (9) (1999) 835-849.

[62] http://rapor.tuik.gov.tr/reports/rwservlet?adnksdb2&ENVID=adnksdb2Env&report=wa_turkiye_ilce_koy_sehir.RDF&p_il1=34&p_kod=1&p_yil=2013&p_dil=2&desformat=html (21.08.2014) (2013 Istanbul district population data on site of Turkish Statistical Institute).

[63] http://www.ibb.gov.tr/tr-TR/SiteImages/Haber/nisan2009/04052009idarisinirharitasi.jpg (22.08.2014).

2

Urban facility location and transportation cost

Abstract

In this paper we search for the location of a facility in a city that minimizes the total transportation cost. We consider goods to be transported from supplier locations outside the city to the facility and from the facility to customer locations in the city. We model the city as a disc with radius R and we assume the population density and customer location density at a distance r from the city centre to be proportional to exp($-\gamma r$) with γ a real number. Furthermore we assume that all supplier vehicles enter the city at the same point. We calculate the total cost of the customers and suppliers transportation in the city in function of the facility location and we prove three theorems about the optimal facility location. One of these theorems enables us to find the optimal location numerically. We also present graphs of the optimal location in function of an introduced parameter for different values of γR.

Keywords: facility location, urban facility, urban population density.

2.1. Introduction

Based on [1] and [2] we consider a location problem to be a problem in which optimal locations of facilities need to be determined according to a given objective. Often there are locations given which are being served by the facilities, these locations are called customer locations and it is said that demand occurs at these given locations. In [3] and [4] location problems are categorized in four main categories. In the first category, the analytic problems, the demand is assumed to be distributed with a given density function (often uniform demand) over an area and the possible facility locations are all points of that area. Typically, a large number of simplifying assumptions are made in this first category. In the second category, the continuous problems, demand occurs at discrete points and the set of possible facility locations is the set of all points of an area. In network problems, demand occurs only on parts of a network and the possible facility locations are all nodes and all points on links of the network. In the last category, the discrete problems, the set of points where demand occurs and the set of possible facility locations are finite. The location problem of this article belongs to the first category. Other articles, such as [5] and [6], distinguish location problems according to the number of facilities, the used objective, the used distance or the type of demand. In some location problems the number of facilities is a given and in other location

problems the optimal number needs to be determined. Often used objectives are minisum (minimizing the sum of all transportation costs) and minimax (minimizing the maximum transportation cost between a facility and a customer). Often used distances are the Euclidean distance, the rectilinear distance and, in a network context, shortest path length. There are location problems with discrete demand (demand occurs only at a finite number of points) and with continuous demand (with a density function over an area). In our article we consider a single facility location problem with the minisum objective, Euclidean distances and mixed (continuous and discrete) demand. Other articles that consider continuous demand are [7], [8] and [9]. In [7] a single facility location problem is studied with continuous uniform demand in a disc and the objective is to minimize the quintile share ratio of the distances between the facility and the customer locations. In [8] a single facility location problem is studied with continuous uniform demand in a rectangle and with the minisum objective, the used distance is the rectilinear distance and a high-speed road is taken into account. In [9] a location problem is studied with a given number of facilities and with continuous demand in an area, the minimax objective is used and the used distance is the Euclidean distance.

In this article we use an element of the field of urban studies: the urban population density function. We assume that the customer location density is proportional to the population density. The urban population density function is discussed in [10], [11] and [12]. In [10] exponential urban population density functions are proposed and the parameters are estimated for different cities and for different years. The exponential urban population density function is also discussed in [11] and parameter estimates are given for several cities. [12] is a review article on urban population densities.

2.2. Problem description

We consider a city in which a facility will be built and we search for the location that minimizes the total transportation cost. The facility, for example a wholesale market, is supplied by supplier vehicles coming from outside the city and the customers of the facility are located in the city. We model the city area as a disc with a radius of R (a positive real number) kilometre and the centre of this disc we call the city centre. We assume that all supplier vehicles enter the city at the same point, which we call the suppliers (entry) location. With every location in the city we associate a vector, element of $\mathbb{R}^2$, by using a Cartesian coordinate system with origin in the city centre, distances measured in kilometre and the suppliers location on the positive part of the X-axis. Figure 1 is an illustration of the location problem under study, it shows the suppliers location, some customer locations, the city border and centre and the coordinate system. In this illustration the city radius is 20 km. We define the scalar product of two vectors (x_1,y_1) and (x_2,y_2) as $x_1x_2+y_1y_2$ and denote it as $(x_1,y_1)\cdot(x_2,y_2)$. We define the (Euclidean) norm $\|x\|$ of a vector x as $\sqrt{x\cdot x}$ and we define the (Euclidean) distance $d(x,y)$ between two vectors x and y as $\|y-x\|$. We assume that every workday N_s supplier vehicles go from the suppliers location to the facility and back and every workday N_c customer vehicles go from N_c customer locations to the facility and back and that the transportation cost per kilometre is c_s for a supplier vehicle and c_c for a customer vehicle. The supplier routes in the city and the customer routes are assumed to go via straight lines between the facility and the suppliers location or customer locations. Figure 1 shows the supplier routes in the city and the customer routes for the showed customers.
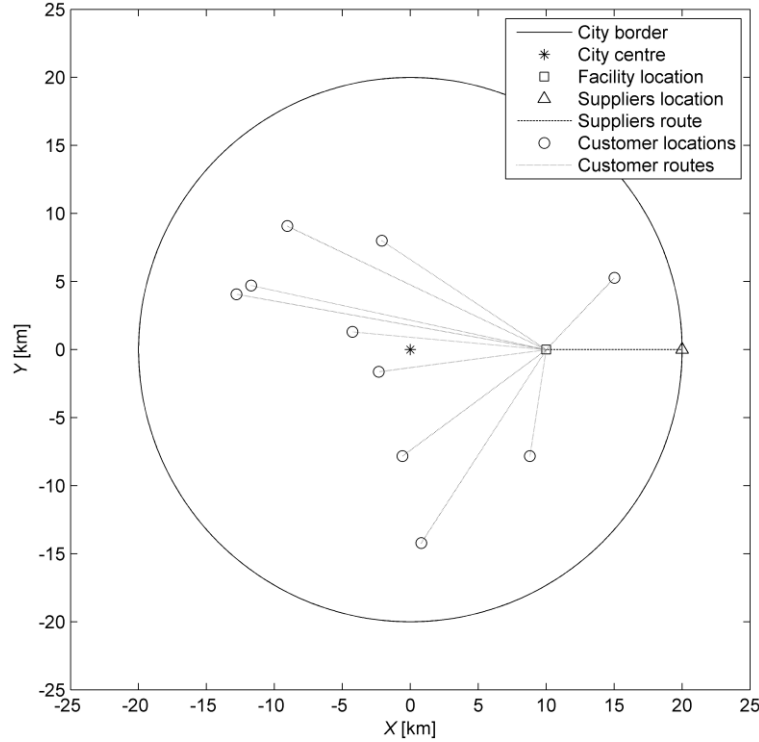
Figure 2.1. Illustration of the facility location, the suppliers location and suppliers urban route, some customer locations and customer routes, the city border and the city centre.

Regarding the location of the customers, we assume that the customer location density is proportional to the population density. And we assume that the population density in km^{-2} at a location with vector $\boldsymbol{x}$ is

$$D_p(\boldsymbol{x}) = D_{p0}e^{-\gamma\|\boldsymbol{x}\|} \tag{1}$$

with D_{p0} a positive real number and γ a real number. This urban population density function corresponds with formula 1 of [12]. The norm of the gradient of this function at a location with vector $\boldsymbol{x}$ in the city is $|\gamma|D_p(\boldsymbol{x})$, therefore the magnitude of γ is the norm of the density gradient over the density. The customer location density in km^{-2} at a location with vector $\boldsymbol{x}$ is then

$$D_c(\boldsymbol{x}) = D_{c0}e^{-\gamma\|\boldsymbol{x}\|} \tag{2}$$

with D_{c0} a positive real number. Because the total number of customer locations in the city is N_c every workday, the following holds:

$$\int_0^R dr\, 2\pi r D_{c0} e^{-\gamma r} = N_c . \tag{3}$$

After calculating the integral we find for D_{c0} the following expression:

$$D_{c0} = \frac{N_c\gamma^2}{2\pi\left(1-(1+\gamma R)e^{-\gamma R}\right)}, \text{ if } \gamma\neq 0 \tag{4}$$

and

$$D_{c0} = \frac{N_c}{\pi R^2}, \text{ if } \gamma=0. \tag{5}$$

Therefore we find for the customer location density:

$$D_c(\boldsymbol{x}) = \frac{N_c \gamma^2}{2\pi \left(1 - (1 + \gamma R)e^{-\gamma R}\right)} e^{-\gamma \|\boldsymbol{x}\|}, \text{ if } \gamma \neq 0 \tag{6}$$

and

$$D_c(\boldsymbol{x}) = \frac{N_c}{\pi R^2}, \text{ if } \gamma = 0. \tag{7}$$

We assume that if the urban facility is at a location with vector $\boldsymbol{f}$, the total transportation cost for supplying all customers during one workday (TC_c) is:

$$TC_c = 2c_c \int\limits_{disc} d\boldsymbol{x} D_c(\boldsymbol{x}) d(\boldsymbol{f}, \boldsymbol{x}) \tag{8}$$

with *disc* the set of vectors with a norm less than or equal to R.

2.3. Total urban transportation cost

In this section we search for an expression for the total urban transportation cost during one workday, this is the total transportation cost of customers and suppliers transportation in the city.

Theorem 1. The total urban transportation cost during one workday (TC) of the system under study with the urban facility at a location with coordinates ($r_f\cos(\theta_f), r_f\sin(\theta_f)$) is

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)} +$$

$$\frac{2N_c c_c \gamma^2}{\pi - \pi(1 + \gamma R)e^{-\gamma R}} \int\limits_0^R dr \int\limits_0^\pi d\theta r e^{-\gamma r} \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta)}, \text{ if } \gamma \neq 0 \tag{9}$$

and

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)} +$$

$$\frac{4N_c c_c}{\pi R^2} \int\limits_0^R dr \int\limits_0^\pi d\theta r \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta)}, \text{ if } \gamma = 0. \tag{10}$$

Proof. The total urban transportation cost during one workday (TC) is the suppliers urban transportation cost plus the customers transportation cost during one workday:

$$TC = 2\sum\nolimits_{i=1}^{N_s} c_s d(\boldsymbol{f}, \boldsymbol{s}) + TC_c \tag{11}$$

with $\boldsymbol{f}$ the facility location vector and $\boldsymbol{s}$ the suppliers location vector. By using (8), we get:

$$TC = 2N_s c_s d(\boldsymbol{f}, \boldsymbol{s}) + 2c_c \int\limits_{disc} d\boldsymbol{x} D_c(\boldsymbol{x}) d(\boldsymbol{f}, \boldsymbol{x}). \tag{12}$$

Because of the definition of distance and norm, we get:

$$TC = 2N_s c_s d(\boldsymbol{f}, \boldsymbol{s}) + 2c_c \int\limits_{disc} d\boldsymbol{x} D_c(\boldsymbol{x}) \sqrt{\boldsymbol{x} \cdot \boldsymbol{x} + \boldsymbol{f} \cdot \boldsymbol{f} - 2\boldsymbol{x} \cdot \boldsymbol{f}}. \tag{13}$$

If the suppliers location vector is ($R,0$) and if the facility location vector is ($r_f\cos(\theta_f), r_f\sin(\theta_f)$), then we get by using polar coordinates:

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)}$$

$$+ 2c_c \int\limits_0^R dr \int\limits_0^{2\pi} d\theta r D_c\left((r\cos(\theta), r\sin(\theta))\right) \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta - \theta_f)}. \tag{14}$$

By using (2) and manipulating the second term further, we get:

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)}$$
$$+ 2c_c D_{c0} \int_0^R dr \int_0^{2\pi} d\theta r e^{-\gamma r} \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta - \theta_f)} . \tag{15}$$

By using substitution, we get:

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)} + 2c_c D_{c0} \int_0^R dr \int_{-\theta_f}^{2\pi - \theta_f} d\theta r e^{-\gamma r} \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta')} . \tag{16}$$

Which is equal to

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)} + 2c_c D_{c0} \int_0^R dr \int_0^{2\pi} d\theta r e^{-\gamma r} \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta')} \tag{17}$$

because of the properties of the cosine function.
By manipulating the second term further, we get:

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)} + 4c_c D_{c0} \int_0^R dr \int_0^{\pi} d\theta r e^{-\gamma r} \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta')} . \tag{18}$$

By using (4) and (5), we get

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)} +$$
$$\frac{2N_c c_c \gamma^2}{\pi - \pi(1 + \gamma R)e^{-\gamma R}} \int_0^R dr \int_0^{\pi} d\theta r e^{-\gamma r} \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta')} , \text{ if } \gamma \neq 0 \tag{19}$$

and

$$TC = 2N_s c_s \sqrt{\left(R - r_f \cos(\theta_f)\right)^2 + r_f^2 \sin^2(\theta_f)} +$$
$$\frac{4N_c c_c}{\pi R^2} \int_0^R dr \int_0^{\pi} d\theta r \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta')} , \text{ if } \gamma = 0, \tag{20}$$

which completes the proof. $\square$

2.4. Optimal facility location

In this section we search for the location in the city that minimizes the total transportation cost.

Theorem 2. The total transportation cost is minimal only if the facility location lies on the nonnegative part of the *X*-axis.

Proof. The second term in (18) is an expression for the total customer transportation cost with $(r_f \cos(\theta_f), r_f \sin(\theta_f))$ the facility location vector:

$$4c_c D_{c0} \int_0^R dr \int_0^{\pi} d\theta r e^{-\gamma r} \sqrt{r^2 + r_f^2 - 2rr_f \cos(\theta')} . \tag{21}$$

This expression is independent of θ_f, therefore the total customer transportation cost is independent of θ_f.

Because of the first term in (12) the total suppliers urban transportation cost is lower if the distance between the facility location and the suppliers location is smaller.

Because of the two last sentences, we get that if the vector of the facility location is (x,y) with $y{\neq}0$, then the total urban transportation cost for a facility location with vector $\left(\sqrt{x^2+y^2},0\right)$ is less because the total customers transportation cost is equal and the total suppliers urban transportation cost is less. Therefore for every facility location not on the nonnegative part of the X-axis, there is a location on the nonnegative part of the X-axis with a lower total transportation cost. $\square$

Theorem 3. The total transportation cost is minimal if the urban facility location vector is $(x_{min},0)$ if and only if: the function f over the interval $[0,1]$ is minimal at x_{min}/R. With f the following function over the interval $[0,1]$:

$$f(x) = \frac{(\gamma R)^2}{\pi - \pi(1+\gamma R)e^{-\gamma R}} \int_0^1 dr \int_0^\pi d\theta r e^{-\gamma Rr} \sqrt{r^2 + x^2 - 2rx\cos(\theta)} - \frac{N_s c_s}{N_c c_c} x, \text{ if } \gamma{\neq}0 \tag{22}$$

and

$$f(x) = \frac{2}{\pi} \int_0^1 dr \int_0^\pi d\theta r \sqrt{r^2 + x^2 - 2rx\cos(\theta)} - \frac{N_s c_s}{N_c c_c} x, \text{ if } \gamma=0. \tag{23}$$

Proof. Because of theorem 2, if an urban facility location minimizes the total transportation cost, then this facility location lies on the nonnegative part of the X-axis. If the facility location has coordinates $(x,0)$ with $x{\leq}R$ and if we subtract $2N_s c_s R$ from both sides of the equation of (9) and then divide both sides by $2N_c c_c$, we get:

$$\frac{TC - 2N_s c_s R}{2N_c c_c} = \frac{\gamma^2}{\pi - \pi(1+\gamma R)e^{-\gamma R}} \int_0^R dr \int_0^\pi d\theta r e^{-\gamma r} \sqrt{r^2 + x^2 - 2rx\cos(\theta)} - \alpha x, \text{ if } \gamma{\neq}0 \tag{24}$$

with

$$\alpha = \frac{N_s c_s}{N_c c_c}. \tag{25}$$

By dividing both sides of the equation of (24) by R and using substitution, we get:

$$\frac{TC - 2N_s c_s R}{2N_c c_c R} = \frac{(\gamma R)^2}{\pi - \pi(1+\gamma R)e^{-\gamma R}} \int_0^1 dr_{rel} \int_0^\pi d\theta r_{rel} e^{-\gamma R r_{rel}} \sqrt{r_{rel}^2 + x_{rel}^2 - 2r_{rel} x_{rel} \cos(\theta)} - \alpha x_{rel}, \text{ if } \gamma{\neq}0 \tag{26}$$

with

$$x_{rel} = \frac{x}{R}. \tag{27}$$

In the case of $\gamma=0$, we get by using (10):

$$\frac{TC - 2N_s c_s R}{2N_c c_c R} = \frac{2}{\pi} \int_0^1 dr_{rel} \int_0^\pi d\theta r_{rel} \sqrt{r_{rel}^2 + x_{rel}^2 - 2r_{rel} x_{rel} \cos(\theta)} - \alpha x_{rel}, \text{ if } \gamma=0. \tag{28}$$

Because TC is minimal for a facility location if and only if $\dfrac{TC - 2N_s c_s R}{2N_c c_c R}$ is minimal for that facility location: $(x_{min},0)$ is a facility location in the city for which TC is minimal if and only if x_{min}/R is a value for which the function f is minimal over the interval $[0,1]$, with

$$f(x) = \frac{(\gamma R)^2}{\pi - \pi(1 + \gamma R)e^{-\gamma R}} \int_0^1 dr \int_0^\pi d\theta r e^{-\gamma R r} \sqrt{r^2 + x^2 - 2rx\cos(\theta)} - \alpha x, \text{ if } \gamma \neq 0 \tag{29}$$

and

$$f(x) = \frac{2}{\pi} \int_0^1 dr \int_0^\pi d\theta r \sqrt{r^2 + x^2 - 2rx\cos(\theta)} - \alpha x, \text{ if } \gamma = 0. \tag{30}$$

This completes the proof. □

Theorem 4. If $\dfrac{N_s c_s}{N_c c_c} \geq 1$, then the total transportation cost is minimal if the urban facility location coincides with the suppliers location.

Proof. We get the total urban transportation cost in case the facility location coincides with the suppliers location by replacing f by s in (12):

$$TC_{f=s} = 2N_c c_c \frac{\int_{disc} dx D_c(x)d(s,x)}{N_c}. \tag{31}$$

We will prove that if $\dfrac{N_s c_s}{N_c c_c} \geq 1$, for every urban facility location the total transportation cost is greater than or equal to the total transportation cost in case the facility location is the suppliers location. By manipulating (12), we get:

$$TC = 2N_c c_c \left(\frac{N_s c_s}{N_c c_c} d(f,s) + \frac{\int_{disc} dx D_c(x)d(f,x)}{N_c} \right). \tag{32}$$

By using (31), we get:

$$TC = TC_{f=s} + 2N_c c_c \left(\frac{N_s c_s}{N_c c_c} d(f,s) + \frac{\int_{disc} dx D_c(x)(d(f,x) - d(s,x))}{N_c} \right). \tag{33}$$

Because of the triangle inequality and because the customer location density is positive at every location in the city, we get:

$$TC \geq TC_{f=s} + 2N_c c_c \left(\frac{N_s c_s}{N_c c_c} d(f,s) - \frac{\int_{disc} dx D_c(x)d(f,s)}{N_c} \right). \tag{34}$$

Because of (3), we get:

$$TC \geq TC_{f=s} + 2N_c c_c \left(\frac{N_s c_s}{N_c c_c} d(f,s) - d(f,s) \right). \tag{35}$$

If $\dfrac{N_s c_s}{N_c c_c} \geq 1$, the second term in (35) is greater than or equal to zero, which completes the proof. □

Theorem 3 enables us to find the optimal facility location numerically by reducing the facility location problem under study to the problem of finding the minimum of a function with one variable. Figure 2 shows the *X*-coordinate of the optimal facility location over *R* in function of $\alpha = \dfrac{N_s c_s}{N_c c_c}$ for *γR* equal to 0, 1, 2, 3, 4 and 5. If *α*=1, the facility location that minimizes the total transportation cost is at the city border, which is in accordance with theorem 4. We notice that for the graphs shown in figure 2 if *γ* increases and if *α* and *R* remain constant, the optimal facility location moves towards the city centre and if *α* increases, e.g. by making the customer transportation more efficient, and if *γ* and *R* remain constant, the optimal facility location moves towards the city border.
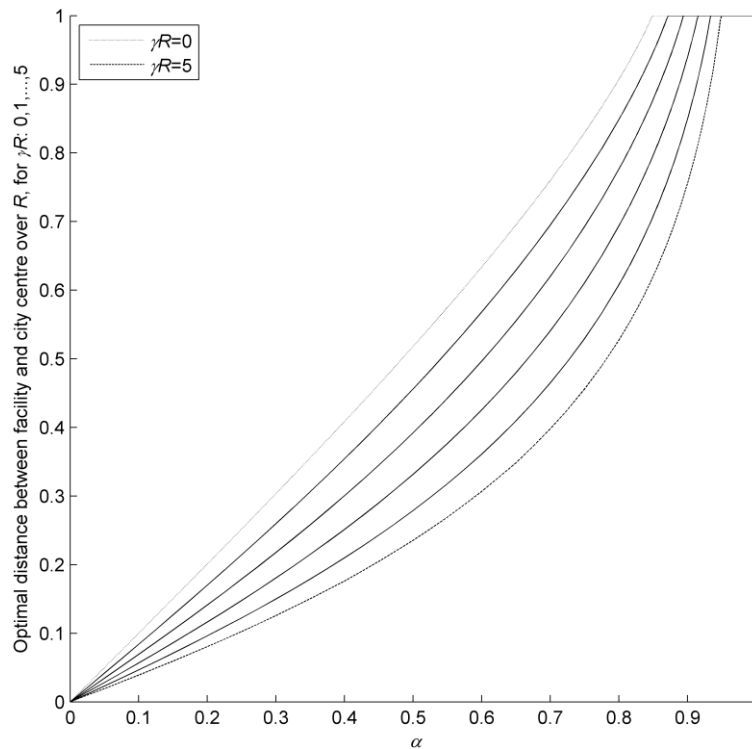
Figure 2.2. The optimal location of the urban facility (*X*-coordinate over *R*) in function of *α*, for *γR* equal to 0, 1, 2, 3, 4 and 5.

2.5. Conclusion

We found an expression for the total urban transportation cost of the system under study (theorem 1) and we proved three theorems (theorem 2, 3 and 4) about the optimal urban facility location. Theorem 2 restricts the solution of the location problem to the nonnegative part of the *X*-axis. Theorem 3 enables us to find the optimal facility location numerically and also shows that the coordinates of the optimal location over *R* only depend on the parameters *γR* and $\alpha = \dfrac{N_s c_s}{N_c c_c}$. Theorem 4 shows that the optimal facility location is the suppliers location if *α*≥1.

References

[1] H.K. Smith, G. Laporte, P.R. Harper, Locational analysis: highlights of growth to maturity, Journal of the Operational Research Society 60 (Supplement 1) (2009) S140-S148.

[2] R.L. Francis, L.F. McGinnis, J.A. White, Locational analysis, European Journal of Operational Research 12 (3) (1983) 220-252.

[3] M.S. Daskin, What you should know about location modeling, Naval Research Logistics 55 (4) (2008) 283-294.

[4] C.S. ReVelle, H.A. Eiselt, M.S. Daskin, A bibliography for some fundamental problem categories in discrete location science, European Journal of Operational Research 184 (3) (2008) 817-848.

[5] C.S. ReVelle, H.A. Eiselt, Location analysis: A synthesis and survey, European Journal of Operational Research 165 (1) (2005) 1-19.

[6] M.L. Brandeau, S.S. Chiu, An overview of representative problems in location research, Management Science 35 (6) (1989) 645-674.

[7] T. Drezner, Z. Drezner, B. Hulliger, The Quintile Share Ratio in location analysis, European Journal of Operational Research 238 (1) (2014) 166-174.

[8] A.R. Odoni, G. Sadiq, Two planar facility location problems with high-speed corridors and continuous demand, Regional Science and Urban Economics 12 (4) (1982) 467-484.

[9] A. Suzuki, Z. Drezner, The p-center location problem in an area, Location Science 4 (1-2) (1996) 69-82.

[10] C. Clark, Urban Population Densities, Journal of the Royal Statistical Society. Series A (General) 114 (4) (1951) 490-496.

[11] E.S. Mills, J.P. Tan, A comparison of urban population density functions in developed and developing countries, Urban Studies 17 (3) (1980) 313-321.

[12] J.F. McDonald, Econometric studies of urban population density: A survey, Journal of Urban Economics 26 (3) (1989) 361-385.

3

Istanbul fruit and vegetable wholesale market location and transportation cost: comparison of two locations

Abstract

There is one fruit and vegetable wholesale market on the Asian side of Istanbul. This market is in the İçerenköy neighbourhood in the district of Ataşehir. There are plans to move the market to the Aydınlı neighbourhood in the district of Tuzla. In this article we compare the supply chain fuel costs of both market locations. We estimate the average fuel cost per kilometre for the customer and the supplier vehicles. Then we present an approximate distribution of the customers over the Asian side of Istanbul. In the next step we use a map of Istanbul to find the driving distances in Istanbul for the customers and suppliers. Finally we conclude with the differences in fuel cost per day for the supply chain, the customers and the suppliers.

Keywords: city logistics, fruit and vegetable market, location analysis, spatial logistics.

3.1. Introduction

In the İçerenköy neighbourhood in the district of Ataşehir is a fruit and vegetable wholesale market. This is the only fruit and vegetable wholesale market on the Asian side of Istanbul and its customers are coming from all districts on the Asian side of Istanbul. There are plans to move this market to a new location in Aydınlı in the district of Tuzla, so that the market can enlarge. The expensive land in İçerenköy, which is more in the centre of the Asian side of Istanbul, can then be used for other purposes such as e.g. offices or a shopping centre. Figure 1 shows both market locations on a satellite/aerial photo of the Asian side of Istanbul. In this paper we will study the difference in fuel costs of the customer and supplier vehicles between the Aydınlı market case and the İçerenköy market case.

Next section we present some data about the actual fruit and vegetable market in İçerenköy. Section 3 deals with the fuel costs per kilometre for the customer vehicles and the supplier vehicles. In section 4 we will determine the average driving distances for the customer and the

supplier vehicles to both markets. We will use the data from section 2 to section 4 in section 5 where we will determine the difference in fuel costs between the two market locations, for the customers, the suppliers and the total supply chain. Section 6 contains the conclusion and ideas for further research.
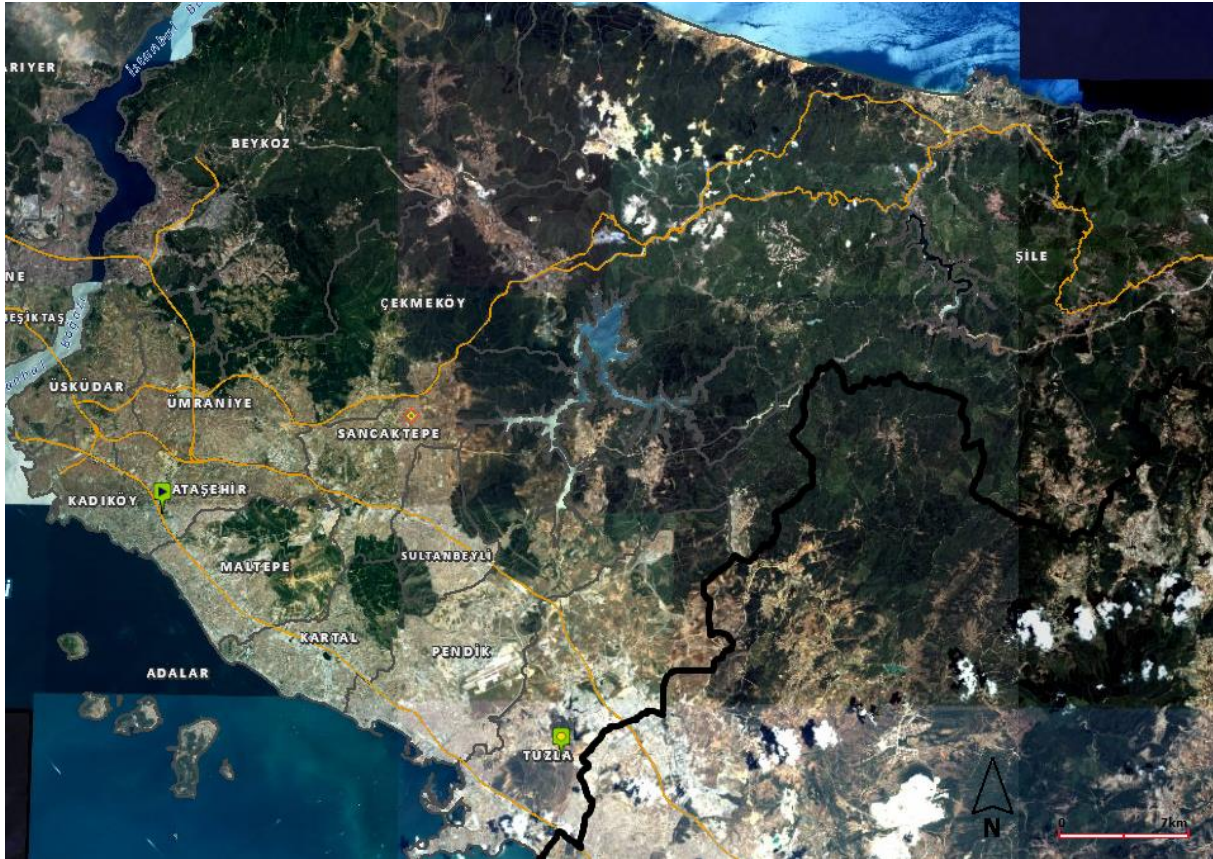
Figure 3.1. The location of the İçerenköy market (triangle) and the Aydınlı market (circle) on a satellite/aerial photo. Source: Istanbul Metropolitan Municipality website City map [1].

3.2. İçerenköy market data

In this section we present some data about the fruit and vegetable market in İçerenköy. We received these data from the management of the İçerenköy fruit and vegetable market. Approximate 20 ton trucks bring the fruit and vegetables from the producers (mostly farmers in Antalya, Ankara and Adana) to the fruit and vegetable market. The customers of the fruit and vegetable market mostly need the fruit and vegetables for street markets, greengroceries and other stores, restaurants and hotels. The customers are located on the Asian side of Istanbul (there is another fruit and vegetable market on the European side of Istanbul). The customer vehicles are 3.5 ton trucks or smaller. The supplier trucks arrive between 14:00 and 2:00 at the fruit and vegetable market and the fruit and vegetable market is open for customers between 2:00 and 9:00. Most supplier trucks arrive around 22:00 and 23:00, most customers come in the morning. There are more or less 540 supplier trucks and more or less 2200 customer vehicles per day. In this article we will use 2200 for the number of customer

vehicles per day and 540 for the number of supplier vehicles per day for both locations of the
fruit and vegetable market (İçerenköy and Aydınlı).

3.3. Fuel costs per kilometre for customer vehicles and supplier vehicles

We collected fuel consumption data for different truck classes from the Transport Energy
Data Book [2]. The customer vehicles belong to class 1 (less than 2 722 kg gross vehicle
weight) and class 2 (2 722 kg – 4 536 kg gross vehicle weight). The supplier vehicles belong
to class 8 (more than 14 969 kg gross vehicle weight). For the fuel prices (in TL/litre) we used
the prices from the Petrol Ofisi fuel distribution company website [3]. Most customer vehicles
use gasoline and the supplier vehicles use diesel. We assume that the fuel consumption of an
empty supplier truck is half the fuel consumption of a supplier truck with cargo. This
assumption is based on interviews with truck drivers at the İçerenköy market carried out by us
in January 2012. Table 1 shows the fuel consumption, fuel price and fuel cost per km for
truck classes 1, 2 and 8. This table also contains the average fuel cost per km of class 1 and 2,
and the average fuel cost per km of class 8 with and without cargo; which we use as an
estimation of the fuel cost per km for the costumer and supplier vehicles respectively.

Table 3.1. Fuel consumption, fuel price and fuel cost per km for different truck classes.

Gross vehicle weight class	Fuel economy*	Fuel consumption	Fuel price**	Gasoline or diesel	Fuel cost per km
	[miles per gallon]	[litre per 100 km]	[TL/litre]		[TL/km]
Class 1 (less than 2722 kg)	17,6	13,4	4,4	gasoline	0,59
Class 2 (2722-4536 kg)	14,3	16,4	4,4	gasoline	0,72
Class 8 (more than 14969 kg)	5,7	41,3	3,9	diesel	1,61
Average class 1 and class 2	16,0	**14,7**	4,4	gasoline	**0,65**
Average with cargo and empty class 8		**30,9**	3,9	diesel	**1,21**
*: data from Transport Energy Data Book (http://cta.ornl.gov/data/index.shtml), Chapter 5, from a research in 2002, USA					
**: data from Petrol Ofisi website (http://gm.poas.com.tr/pompafiyat/pompafiyatgrid.aspx), February 2012					

In the rest of the article we will use as fuel cost 0.6 TL/km for the customer vehicles and 1.2
TL/km for the supplier vehicles.

3.4. Driving distances

In this section we will estimate the average driving distance to the İçerenköy market and the
Aydınlı market for a customer vehicle and a supplier vehicle. For this we need the start
locations of the customer vehicles. We assume the number of customers in a district to be
proportional to the population of that district. Table 2 shows the population of the 14 districts
on the Asian side of Istanbul. The data come from the Turkish Statistical Institute
(TURKSTAT) website [4]. Table 2 also contains the number of customer vehicles per day for
each district.

Table 3.2. The districts on the Asian side of Istanbul, with the most populated districts at the top, the population of each district, the proportion of district population to the Asian side population, the number of customer vehicles per day for each district and the proportion of the district population to the total Istanbul population.

	District	Population*	Percent of population Asian side Istanbul	Number of customers per day	Percent of population Istanbul
1	Ümraniye	603431	12,88	283	4,55
2	Pendik	585196	12,49	275	4,41
3	Kadiköy	532835	11,37	250	4,02
4	Üsküdar	526947	11,25	247	3,98
5	Maltepe	438257	9,36	206	3,31
6	Kartal	432199	9,23	203	3,26
7	Atasehir	375208	8,01	176	2,83
8	Sultanbeyli	291063	6,21	137	2,20
9	Sancaktepe	256442	5,47	120	1,93
10	Beykoz	246136	5,25	116	1,86
11	Tuzla	185819	3,97	87	1,40
12	Çekmeköy	168438	3,60	79	1,27
13	Sile	28119	0,60	13	0,21
14	Adalar	14221	0,30	7	0,11
	Total Asian side Istanbul:	4684311	100,00	2200	35,34
	Total in Istanbul:	13255685			

*: data from Turkish Statistical Institute (TURKSTAT) website (http://www.turkstat.gov.tr), January 2012

We will assign one location "the centre" to every district, then we can determine the driving distances from all centres to both market locations using Google Maps (http://maps.google.com) or the Istanbul Metropolitan Municipality website City map [1]. Figure 2 shows the different districts on the map of the Asian side of Istanbul.

Figure 3.2. Map with the districts of the Asian side of Istanbul.

Source: Istanbul Metropolitan Municipality website City map [1].

We use the estimation of the geometric centre of the populated area of a district as the centre of that district. For the district Adalar (the islands) we locate the centre in Bostancı, which has a ferryboat connection with the islands. The map of the Istanbul Metropolitan Municipality website uses coordinates which we will call "ibb-coordinates". For this article we choose the İçerenköy market as origin of the coordinate system (İçerenköy-coordinates). Table 3 contains the coordinates of all district centres and the driving distance from each district centre to the İçerenköy market and the Aydınlı market. We used the route planner of the Istanbul Metropolitan Municipality website City map to determine the driving distances. The table also shows the weighted average of the customer vehicle driving distances with the district populations as weights.

We locate the supplier vehicles on the E-80 route just outside Istanbul. The driving distances from this location to both markets are included in table 3. Table 3 also contains the coordinates from this location and the coordinates from the markets.

Table 3.3. Customer and supplier vehicle starting locations and driving distances to both markets

		Percent of population Asian side**	ibb-x-coordinate [km]	ibb-y-coordinate [km]	İçerenköy-x-coordinate [km]	İçerenköy-y-coordinate [km]	Driving distance to İçerenköy market* [km]	Driving distance to Aydinli market* [km]
Costumers	Ümraniye	12,88	426,3	4543,3	1,8	4,5	9,7	37,4
	Pendik	12,49	439,6	4530,7	15,1	-8,0	21,2	15,3
	Kadiköy	11,37	421,1	4539,2	-3,5	0,4	4,4	35,0
	Üsküdar	11,25	420,7	4544,8	-3,8	6,1	9,6	36,7
	Maltepe	9,36	427,5	4534,1	3,0	-4,7	6,0	24,9
	Kartal	9,23	432,7	4530,5	8,2	-8,2	15,3	16,7
	Atasehir	8,01	426,2	4539,7	1,7	1,0	2,4	35,9
	Sultanbeyli	6,21	439,0	4537,5	14,5	-1,2	16,6	14,0
	Sancaktepe	5,47	436,0	4540,8	11,5	2,1	17,4	28,2
	Beykoz	5,25	429,4	4554,9	4,9	16,1	23,8	56,2
	Tuzla	3,97	444,7	4527,2	20,2	-11,5	29,0	4,6
	Çekmeköy	3,60	434,9	4547,2	10,4	8,4	20,1	36,0
	Sile	0,60	469,5	4555,2	45,0	16,5	67,7	76,4
	Adalar	0,30	423,8	4535,8	-0,8	-3,0	4,1	28,7
							Weighted average: 13,2	Weighted average: 29,0
Suppliers			449,4	4526,1	24,9	-12,6	33,0	7,1
Markets	Içerenköy		424,5	4538,7	0,0	0,0		
	Aydinli		445,3	4526,0	20,8	-12,8		

*: data from Istanbul Metropolitan Municipality website City map (http://sehirrehberi.ibb.gov.tr), February 2012
**: data from Turkish Statistical Institute (TURKSTAT) website (http://www.turkstat.gov.tr), January 2012

3.5. Difference in total fuel costs

With the data of previous sections we can calculate the difference in total fuel costs between both locations. For subsequent calculations we will use the following symbols and values for the respective quantities:
- number of customer vehicles per day: $N_c = 2200$

- number of supplier vehicles per day: $N_s = 540$
- fuel cost per kilometre for customer vehicle in TL/km: $C_c = 0.6$
- fuel cost per kilometre for supplier vehicle in TL/km: $C_s = 1.2$
- average driving distance from customer location to İçerenköy market in km: $D_{cI} = 13.2$
- average driving distance from customer location to Aydınlı market in km: $D_{cA} = 29.0$
- average driving distance from supplier location to İçerenköy market in km: $D_{sI} = 33.0$
- average driving distance from supplier location to Aydınlı market in km: $D_{sA} = 7.1$

For the total fuel costs per day C_I (in TL) in the case of the İçerenköy market we find:
$$C_I = 2D_{cI}N_cC_c + 2D_{sI}N_sC_s \tag{1}$$
$$C_I = 34\ 848 + 42\ 768 = 77\ 616 \tag{2}$$
So the total customer fuel costs are 34 848 TL per day in the case of the İçerenköy market.

For the total fuel costs per day C_A (in TL) in the case of the Aydınlı market we find:
$$C_A = 2D_{cA}N_cC_c + 2D_{sA}N_sC_s \tag{3}$$
$$C_A = 76\ 560 + 9\ 201.6 = 85\ 761.6 \tag{4}$$
So the total customer fuel costs are 76 560 TL per day in the case of the Aydınlı market.

The difference in total fuel costs per day between both market locations (in TL):
$$C_A - C_I = 8\ 145.6 \tag{5}$$

We see that for the used data the Aydınlı location is in total more or less 8 000 TL more expensive per day. This is more or less 10% of the total fuel costs per day for the İçerenköy location (with the starting point and end point of the supplier vehicles located at the boundary of Istanbul). The fuel costs of the customers will more or less double if the market moves to Aydınlı. For the suppliers the Aydınlı location will be an advantage with a total fuel cost decrease of almost 34 000 TL per day, this is more or less 62 TL per vehicle per trip to the market and back.

3.6. Conclusion and further research

With the used estimations we found that regarding fuel costs in total the Aydınlı location is more expensive, but only more or less 8 000 TL per day. For the customers the fuel costs will more or less double if the market moves to Aydınlı. For the suppliers the fuel costs will decrease with in total more or less 34 000 TL per day if the market moves to Aydınlı.

Further research can be done by taking into account also other costs and the environment. An other extension of this research is the study of the optimal location of the fruit and vegetable market (not restricted to İçerenköy or Aydınlı but anywhere on the Asian side of Istanbul) and the study of the possibility of an underground market.

References

[1] http://sehirrehberi.ibb.gov.tr (24.02.2012) (Istanbul Metropolitan Municipality website City map)

[2] http://cta.ornl.gov/data/tedb30/Edition30_Chapter05.pdf (06.02.2012) (Transport Energy
Data Book, Chapter 5 Heavy Vehicles and Characteristics)

[3] http://gm.poas.com.tr/pompafiyat/pompafiyatgrid.aspx (07.02.2012) (Gasoline and diesel
prices for Istanbul) (Petrol Ofisi website)

[4] http://rapor.tuik.gov.tr/reports/rwservlet?adnksdb2&ENVID=adnksdb2Env&report=wa_
turkiye_ilce_koy_sehir.RDF&p_il1=34&p_kod=1&p_yil=2010&p_dil=2&desformat=html
(23.01.2012) (Turkish Statistical Institute (TURKSTAT) website)

4

Istanbul fruit and vegetable wholesale market location and transportation cost: a search for the optimal location

Abstract

There is one fruit and vegetable market on the Asian side of Istanbul. This market is located in the İçerenköy neighbourhood in the district of Ataşehir and there are plans to move this market to a new location. In this article we search for the market location that minimizes the supply chain fuel costs. First we approach the problem by using Euclidean distances for estimating the driving distances of the customers and the suppliers. In a subsequent approach we analyse fifteen locations and calculate the fuels costs by using a route planner for determining the driving distances of the suppliers and Euclidean distances for estimating the driving distances of the customers. We conclude by calculating the difference in fuel costs with the İçerenköy market for the found optimal market place using a route planner for determining all driving distances.

Keywords: city logistics, fruit and vegetable market, location analysis, spatial logistics.

4.1. Introduction

In this article a facility location analysis is done for the fruit and vegetable wholesale market on the Asian side of Istanbul. This market, now in the İçerenköy neighbourhood in the district of Ataşehir, is one of the two large wholesale distribution centres for fruit and vegetables in Istanbul. The other wholesale market is on the European side of Istanbul, in the district of Bayrampaşa. The fruit and vegetable market on the Asian side, which is planned to move to a new location, was already subject of logistic studies: [1] contains a search for the optimal layout design of the fruit and vegetable market and in [2] the difference in supply chain fuel costs for the actual market location and the planned market location is estimated. Literature on optimal location case studies for other facilities include [3] and [4].

4.2. İçerenköy fruit and vegetable market

Through the management of this market we collected the following data about this market: the number of customer vehicles is more or less 2200 per day, the number of supplier vehicles is more or less 540 per day, the market is open all days except Sunday and the suppliers are coming mainly from Antalya, Ankara and Adana. In [2] the average fuel cost per kilometre for customer vehicles and supplier vehicles is estimated: 0.6 TL/km for the customer vehicles and 1.2 TL/km for the supplier vehicles (these values are the average of with and without cargo). These estimations in [2] are based on fuel consumption data in [5] and fuel prices in [6]. In İçerenköy there is no place for the market to extend and there are plans to move the market to Aydınlı in the district Tuzla of Istanbul. Information on fruit and vegetable wholesale markets in Turkey, including a case study on Antalya's fruit and vegetable wholesale market, can be found in [7].

4.3. Location of the customers and suppliers

Similar to [2] we estimate the number of customers in a district of the Asian side of Istanbul to be proportional to the population of that district. This results in the customer distribution shown in table 1. The population data come from the Turkish Statistical Institute (TURKSTAT) website [8].

For estimating the location of the customers we use the following method. In every district we designate a point, which we name "the district centre", where we will locate all customers in that district. We use the estimation of the geometric centre of the populated area of a district as the centre of that district. For the district Adalar (the islands) we locate the centre in Bostancı, which has a ferryboat connection with the islands.

The map of the Istanbul Metropolitan Municipality website [9] uses coordinates which we will call "ibb-coordinates". In this article we use also a coordinate system with the İçerenköy market as origin (İçerenköy-coordinates). Both coordinate systems have x-axes that point eastward. Table 1 contains the coordinates of all district centers.

Because the suppliers are coming mainly from Antalya, Ankara and Adana, they enter Istanbul from Gebze (Kocaeli). Most likely they come from the E80 route and in Gebze they can choose between staying on the E80 route or going to the D100 route. Because we are mostly interested in the difference in costs between the different market locations, we position the suppliers at the closest place to Istanbul where the E80 and D100 cross one another. The coordinates of this place in the district Gebze, are shown in table 1.

Table 4.1. Customers and suppliers locations and costumer distribution over the districts of the Asian side of Istanbul.

	District	District center ibb-x-coordinate [km]	District center ibb-y-coordinate [km]	District center İçerenköy-x-coordinate [km]	District center İçerenköy-y-coordinate [km]	District population*	Percent of population Asian side Istanbul	Number of customers per day
Costumers locations								
1	Ümraniye	426,3	4543,3	1,8	4,5	603431	12,88	283
2	Pendik	439,6	4530,7	15,1	-8,0	585196	12,49	275
3	Kadiköy	421,1	4539,2	-3,5	0,4	532835	11,37	250
4	Üsküdar	420,7	4544,8	-3,8	6,1	526947	11,25	247
5	Maltepe	427,5	4534,1	3,0	-4,7	438257	9,36	206
6	Kartal	432,7	4530,5	8,2	-8,2	432199	9,23	203
7	Atasehir	426,2	4539,7	1,7	1,0	375208	8,01	176
8	Sultanbeyli	439,0	4537,5	14,5	-1,2	291063	6,21	137
9	Sancaktepe	436,0	4540,8	11,5	2,1	256442	5,47	120
10	Beykoz	429,4	4554,9	4,9	16,1	246136	5,25	116
11	Tuzla	444,7	4527,2	20,2	-11,5	185819	3,97	87
12	Çekmeköy	434,9	4547,2	10,4	8,4	168438	3,60	79
13	Sile	469,5	4555,2	45,0	16,5	28119	0,60	13
14	Adalar	423,8	4535,8	-0,8	-3,0	14221	0,30	7
Suppliers location								
	District	ibb-x-coordinate [km]	ibb-y-coordinate [km]	İçerenköy-x-coordinate [km]	İçerenköy-y-coordinate [km]			
	Gebze (Kocaeli)	457,8	4519,0	33,3	-19,7			

*: data from Turkish Statistical Institute (TURKSTAT) website (http://www.turkstat.gov.tr), January 2012

4.4. Optimal location using Euclidean distances for estimating all driving distances

In this section we search for the location on the Asian side of Istanbul that is optimal regarding to the total supply chain fuel costs by using Euclidean distances for estimating the driving distances for customers and suppliers. By the Euclidean distance between a place with ibb-coordinates (x_1,y_1) and a place with ibb-coordinates (x_2,y_2) we mean $((x_2-x_1)^2+(y_2-y_1)^2)^{1/2}$. In Figure 1 we see the district centres and the suppliers location shown in the coordinate system with the İçerenköy market as origin, the x-axis pointing eastward and the y-axis pointing northward (the İçerenköy coordinate system).

Now we calculate the total cost for an array of points, one point per square kilometre. Figure 2 shows these points and figure 3 shows the total cost in function of the place of the market for all these points. For convenience the coordinate system is rotated over 35 degrees with the İçerenköy market as fixed point so that the x-axis is roughly parallel to the Marmara Sea coast of the Asian side of Istanbul (the rotated İçerenköy coordinate system).
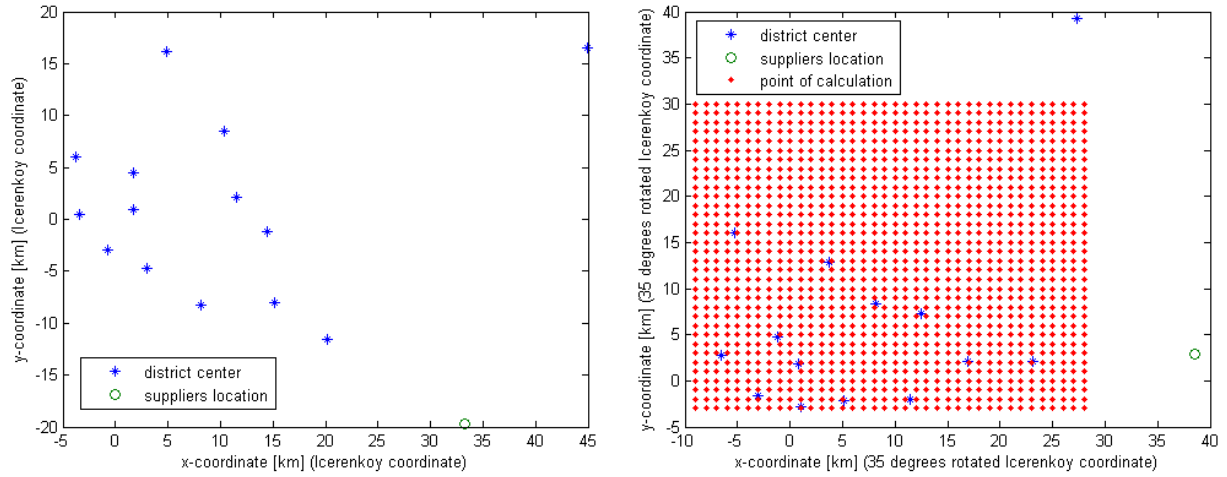
Figure 4.1 and 4.2. Figure 4.1 (left): The locations of the centres of the fourteen districts of the Asian side of Istanbul and the suppliers location. Figure 4.2 (right): The district centres locations, the suppliers location and the points where the total costs are calculated, in the rotated İçerenköy coordinate system.

In this section we use the following formula for calculating the total supply chain fuel costs for the market in the point with coordinates (x,y):

$$TC(x,y) = \left(\sum_{i=1}^{14} \left(2 \times \sqrt{(x_i - x)^2 + (y_i - y)^2} \times N_i \times C_c \right) + 2 \times \sqrt{(x_s - x)^2 + (y_s - y)^2} \times N_s \times C_s \right) \times \alpha$$

(1)

In this formula we used for the following quantities the respective symbols:
- coordinates of the centre of district i in km: (x_i, y_i)
- coordinates of the suppliers location in km: (x_s, y_s)
- number of customer vehicles per day from district i: N_i
- number of supplier vehicles per day: $N_s = 540$
- fuel cost per kilometre for customer vehicle in TL/km: $C_c = 0.6$
- fuel cost per kilometre for supplier vehicle in TL/km: $C_s = 1.2$
- average ratio of the driving distance to the Euclidean distance: $\alpha = 1.36$ (estimated in the appendix).

The calculation point with the lowest total costs has rotated İçerenköy coordinates (13 km,3 km), İçerenköy-coordinates (12 km,-5 km) and ibb-coordinates (437 km,4534 km). The total cost in this point is 87×10^3 TL per day. The location of this optimal point can be seen in figure 4 and 5. Figure 4 is a contour plot of the extra supply chain fuel cost per day in TL compared to the minimum as a function of the place of the market. In figure 5 the optimal market place is shown on the map of Istanbul together with the İçerenköy and Aydınlı market locations.
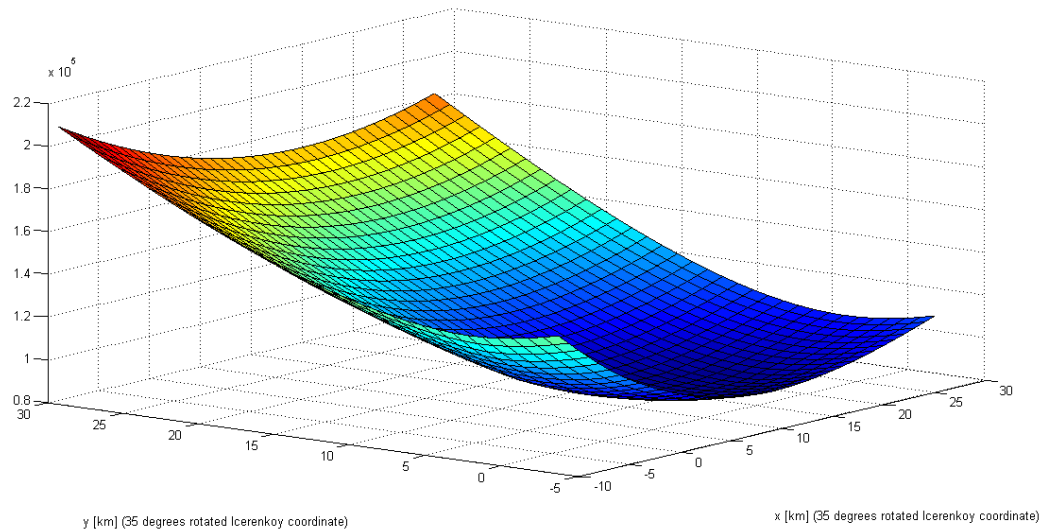
Figure 4.3. Total supply chain fuel cost per day in TL as a function of the place of the fruit and vegetable market using 35 degrees rotated İçerenköy coordinates.
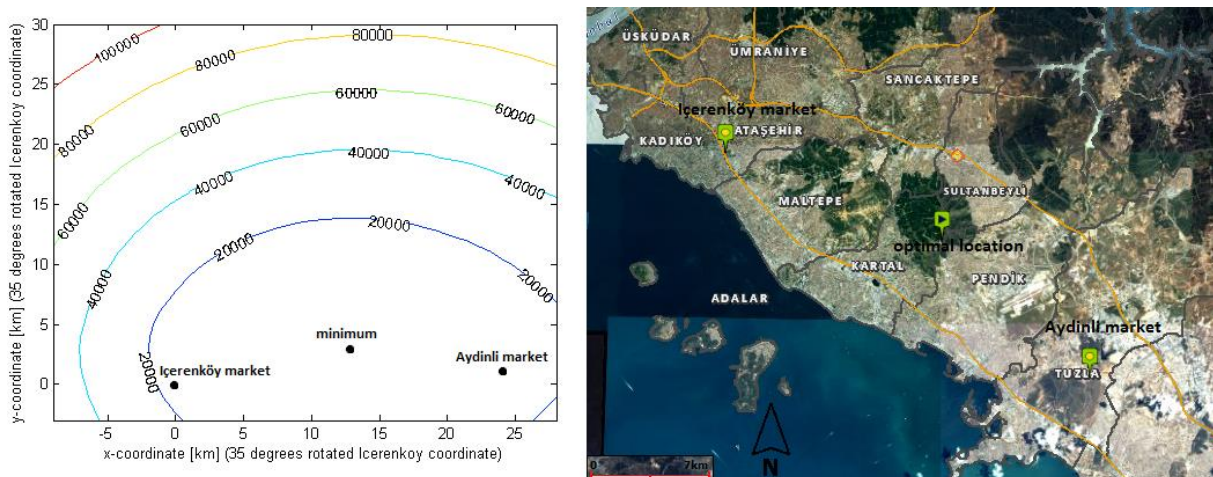
Figure 4.4 and 4.5. Figure 4.4 (left): Extra supply chain fuel costs per day in TL compared to the minimum costs as a function of the place of the fruit and vegetable market, using the 35 degrees rotated İçerenköy coordinate system. Figure 4.5 (right): The optimal market location together with the İçerenköy and Aydınlı market locations on the map of Istanbul.

4.5. Analysis of 15 locations using Euclidean distances for estimating the customers driving distances and a route planner for determining the suppliers driving distances

In previous section we found an optimal location using Euclidean distances for customers and suppliers: we determined the Euclidean distances and then multiplied it by 1.36 for estimating the driving distance. In this section we want to refine the solution method by using a route planner for determining the driving distances for the supplier vehicles, for the customers the same method as in previous section is used. We do this for thirteen locations easy accessible

from main roads and for the İçerenköy and Aydınlı market locations. Figure 6 shows these fifteen locations on the map of Istanbul.

We used the Google Maps route planner [10] for determining the suppliers driving distances. For location 12 and 15 we used also the route planner from the Istanbul Metropolitan Municipality website City map [9], because this route planner was able to find a shorter route. Table 2 contains the obtained supply chain fuel costs for every location, together with the customers and suppliers driving distances and fuel costs. The values in the column "Customers driving distance" are obtained by multiplying the customers Euclidean distances by 1.36. The used fuel costs per km are 0.6 TL/km and 1.2 TL/km for customers and suppliers, and the number of vehicles per day are 2200 and 540 for customers and suppliers.

In figure 6 for every location the difference in supply chain fuel cost per day with the İçerenköy location is shown. We see that the locations with lowest total costs are situated between İçerenköy and Aydınlı. We conclude from the figure that over the fifteen locations considered, the Samandıra junction (location 5) is the one with minimal supply chain fuel costs: 15×10^3 TL fuel costs per day less than the İçerenköy location, according to the used method.
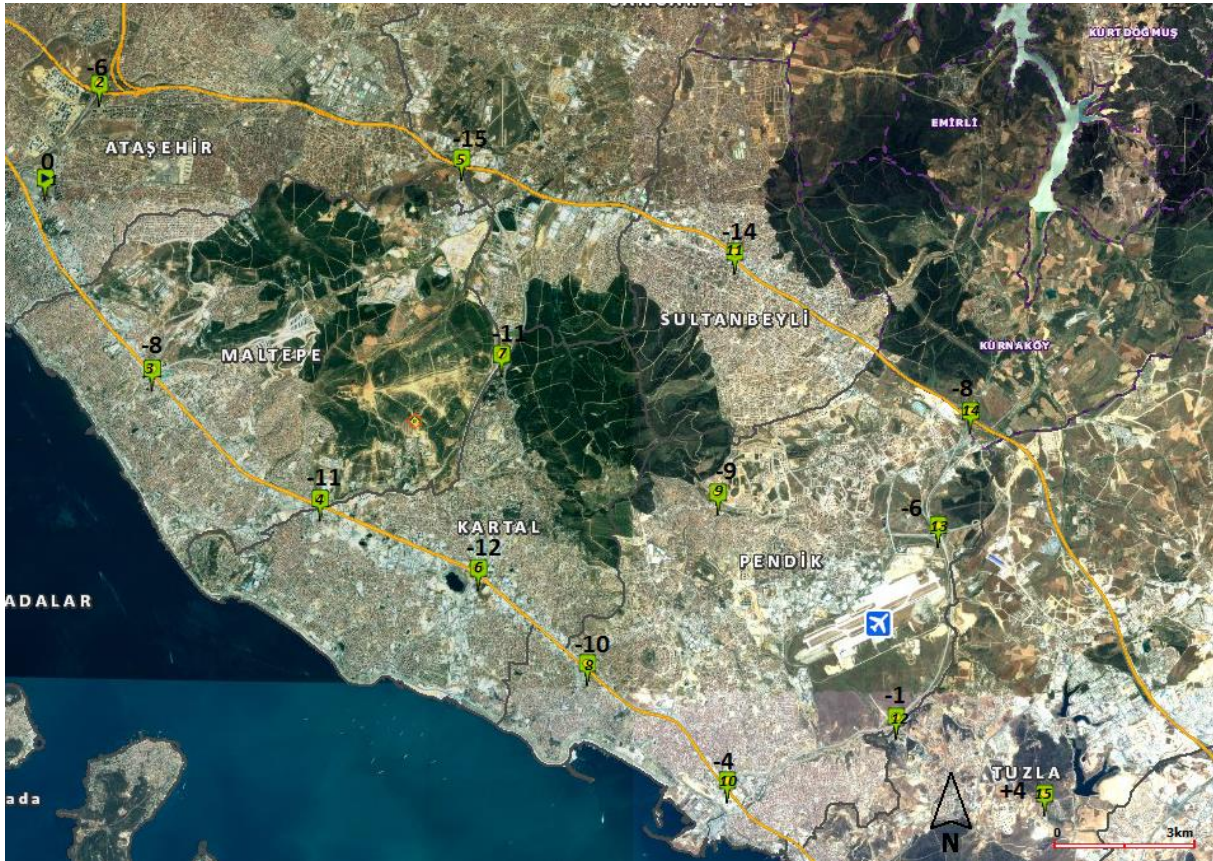
Figure 4.6. The fifteen market locations considered in this section, together with the supply chain fuel costs per day relative to the İçerenköy market location costs (in 10^3 TL).

Table 4.2. Table of the fifteen market locations considered in this section together with their customers and suppliers distances and fuel costs per day and the supply chain fuel costs per day.

	ibb x-coordinate [km]	ibb y-coordinate [km]	Description	Costumers Euclidean distance [10³ km]	Costumers driving distance [10³ km]	Costumers fuel costs [10³ TL]	Supplier driving distance to market* [km]	Suppliers driving distance [10³ km]	Suppliers fuel costs [10³ TL]	Suppliers road description	Supply chain fuel costs [10³ TL]	Supply chain fuel costs relative to İçerenköy [10³ TL]
15 market locations												
1	424,5	4538,7	İçerenköy Market	43,3	58,9	35,4	45,4	49,0	58,8	E80	94,2	0,0
2	425,7	4540,7	E80, Atasehir/Ümraniye	41,5	56,5	33,9	41,7	45,0	54,0	E80	87,9	-6,3
3	426,8	4534,8	D100, Maltepe	44,5	60,5	36,3	38,4	41,5	49,8	D100	86,1	-8,1
4	430,2	4532,1	D100, Maltepe/Kartal	48,3	65,6	39,4	33,9	36,6	43,9	D100	83,3	-10,9
5	433,2	4539,1	E80, Atasehir/Sancaktepe	43,9	59,7	35,8	33,8	36,5	43,8	E80	79,6	-14,6
6	433,6	4530,6	D100, Tas Ocagi, Kartal	52,7	71,6	43,0	30,3	32,7	39,3	D100	82,3	-11,9
7	434,0	4535,1	Maltepe/Kartal/Sancaktepe	46,2	62,8	37,7	35,1	37,9	45,5	D100	83,2	-11,0
8	435,8	4528,7	D100, Pendik	60,4	82,2	49,3	27,3	29,5	35,4	D100	84,7	-9,5
9	438,6	4532,2	Pendik	57,2	77,8	46,7	29,5	31,9	38,2	E80	84,9	-9,3
10	438,7	4526,2	D100, Pendik	72,8	99,1	59,4	23,4	25,3	30,3	D100	89,8	-4,4
11	438,9	4537,2	E80, Sultanbeyli	54,1	73,6	44,1	27,7	29,9	35,9	E80	80,0	-14,2
12	442,2	4527,5	Pendik/Tuzla	77,0	104,7	62,8	23,2	25,1	30,1	E80, Patlayici Maddeler Yolu	92,9	-1,3
13	443,1	4531,5	Pendik	71,4	97,1	58,3	23,3	25,2	30,2	E80	88,5	-5,7
14	443,8	4533,9	E80, Pendik	71,3	97,0	58,2	21,7	23,4	28,1	E80	86,3	-7,9
15	445,3	4526,0	Aydinli Market	90,2	122,7	73,6	19,0	20,5	24,6	E80, Patlayici Maddeler Yolu	98,3	4,1
Suppliers location												
	457,8	4519,0	intersection E80 and D100, near Gebze									

*: data from http://maps.google.com, for location 12 and 15 also http://sehirrehberi.ibb.gov.tr (Istanbul Metropolitan Municipality website City map)

4.6. Location analysis using a route planner for determining all driving distances

In this section we determine all driving distances with a route planner. We do this for the situation with the market in İçerenköy, Aydınlı and near to the Samandıra junction, this is the place were the D100-E80 connection road connects with the E80 and the place found in previous section as optimal regarding the supply chain fuel costs. Further we compare the results obtained by the three different methods (the method of section 4, 5 and 6) for the market locations İçerenköy, Aydınlı and the Samandıra junction.

Table 3 shows the customers driving distances from the different districts to the Samandıra junction, determined with the route planner of the Istanbul Metropolitan Municipality website City map [9]. The average driving distance for a customer to this market location is 13 km (the weighted average of the driving distances from the district centres to the Samandıra junction, with the district populations as weights). The supplier driving distance from the intersection of the E80 and D100 near Gebze to the Samandıra junction is 33.8 km. With these data we can calculate the customers and suppliers fuel costs per day, C_c and C_s. For obtaining the customers and suppliers fuel cost per day in TL we use the following formulas:

$$C_c = d_c \times 2 \times 2200 \times 0.6 \tag{2}$$

$$C_s = d_s \times 2 \times 540 \times 1.2 \tag{3}$$

With d_c and d_s the average costumer driving distance and the suppliers driving distance to the market in km. Table 4 shows the results. We see that considering these three locations for the customers fuel costs İçerenköy and the Samandıra junction are similar and optimal and for the suppliers fuel costs Aydınlı is optimal. For the total supply chain fuel costs the Samandıra junction is optimal (with a difference of more or less 15×10^3 TL per day with İçerenköy, which is a similar result as in section 5). Aydınlı is per day more are less 8×10^3 TL more expensive in supply chain fuel costs, which is a result similar to the result of [2] (the small difference with [2] is a result of the use of different route planners in this article and [2] for determining the suppliers driving distances).

Table 4.3. The driving distances to the Samandıra junction for the customers and the suppliers, determined with a route planner.

	Location	Percentage of population Asian side**	ibb-x-coordinate [km]	ibb-y-coordinate [km]	İçerenköy-x-coordinate [km]	İçerenköy-y-coordinate [km]	Driving distance to Samandira junction* [km]
Costumers	Ümraniye	12.88	426.3	4543.3	1.8	4.5	9.1
	Pendik	12.49	439.6	4530.7	15.1	-8.0	13.6
	Kadiköy	11.37	421.1	4539.2	-3.5	0.4	13.4
	Üsküdar	11.25	420.7	4544.8	-3.8	6.1	17.9
	Maltepe	9.36	427.5	4534.1	3.0	-4.7	10.6
	Kartal	9.23	432.7	4530.5	8.2	-8.2	12.4
	Atasehir	8.01	426.2	4539.7	1.7	1.0	8.2
	Sultanbeyli	6.21	439.0	4537.5	14.5	-1.2	6.3
	Sancaktepe	5.47	436.0	4540.8	11.5	2.1	4.0
	Beykoz	5.25	429.4	4554.9	4.9	16.1	28.3
	Tuzla	3.97	444.7	4527.2	20.2	-11.5	22.4
	Çekmeköy	3.60	434.9	4547.2	10.4	8.4	12.0
	Sile	0.60	469.5	4555.2	45.0	16.5	58.6
	Adalar	0.30	423.8	4535.8	-0.8	-3.0	13.2
						Weighted average***:	13.0
Suppliers	intersection E80 and D100, near Gebze		457.8	4519.0	33.3	-19.7	33.8
Market	Samandira junction		433.2	4539.1	8.7	0.4	

*: data from Istanbul Metropolitan Municipality website City map (http://sehirrehberi.ibb.gov.tr), August 2012
**: data from Turkish Statistical Institute (TURKSTAT) website (http://www.turkstat.gov.tr), January 2012
***: with the district populations as weights

Table 4.4. Average driving distances to the market and customers, suppliers and supply chain fuel costs for the market in İçerenköy and Aydınlı and near the Samandıra junction.

Market location	Costumers average driving distance to market [km]	Suppliers driving distance to market [km]	Customers fuel costs [10⁸ TL]	Suppliers fuel costs [10⁸ TL]	Supply chain fuel costs [10⁸ TL]	Supply chain fuel costs relative to İçerenköy supply chain fuel costs [10⁸ TL]
İçerenköy	13.2	45.4	34.8	58.8	93.7	0.0
Aydinli	29.0	19.0	76.6	24.6	101.2	7.5
Samandira junction	13.0	33.8	34.4	43.8	78.2	-15.5

Table 5 compares the results of the three different methods used in this article. We notice that in this study method 1 (from section 4) was very rough (with differences from method 3 up to 18%), but method 2 (from section 5) and method 3 (from this section) have more similar results (differences are less the 3%). This is consistent with the values of the ratio of the total driving distance to the total Euclidean distance, which vary more for the suppliers than the customers over the three locations and are more close to the used 1.36 for the customers than for the suppliers.

Table 4.5. Comparison of the results of the methods from section 4, 5 en 6.

Location	Customers fuel costs method 1, 2 [10⁸ TL]	Customers fuel costs method 3 [10⁸ TL]	Customers ratio total driving distance to total Euclidean distance	Suppliers fuel costs method 1 [10⁸ TL]	Suppliers fuel costs method 2, 3 [10⁸ TL]	Suppliers ratio total driving distance to total Euclidean distance	Supply chain fuel costs Method 1 [10⁸ TL]	Supply chain fuel costs Method 2 [10⁸ TL]	Supply chain fuel costs Method 3 [10⁸ TL]
İçerenköy	35.4	34.8	1.34	68.2	58.8	1.17	103.6	94.2	93.7
Aydinli	73.6	76.6	1.41	25.3	24.6	1.32	98.9	98.3	101.2
Samandira junction	35.8	34.4	1.30	56.0	43.8	1.06	91.9	79.6	78.2

Method 1: driving distance is estimated by the Euclidean distance times 1.36 for customers and suppliers
Method 2: driving distance is determined with a route planner for suppliers and estimated by the Euclidean distance times 1.36 for the customers
Method 3: all distances determined by route planner

4.7. Conclusion and further research

In this article we looked for the optimal new location for the fruit and vegetable wholesale market at the Asian side of Istanbul regarding the total supply chain fuel costs. We used three methods. The first method (section 4) uses Euclidean distances for calculating the customers and suppliers driving distances. The second method (section 5) uses Euclidean distances for calculating the customers driving distances and a route planner for determining the suppliers driving distances. The third method (section 6) uses a route planner for determining all driving distances. By using the third method (the most accurate method of the three methods) we found the Samandıra junction as optimal place for the fruit and vegetable wholesale market on the Asian side of Istanbul regarding the total supply chain fuel costs. Construction of new roads, for example to Aydınlı, or organizing the transport differently, for example replenishing the customers by large trucks making milk runs, will affect the results. It is important to mention that traffic considerations, environmental considerations and land costs are not taken into account in this study. Further research can be done by taking into account also these considerations.

Appendix: estimation of the average ratio of the driving distance to the Euclidean distance

In this appendix we calculate the ratio of the sum of all driving distances to the sum of all Euclidean distances met in [2]. In section 4 and 5 the driving distance between two points is estimated by multiplying the Euclidean distance between these two points by this ratio.

In table A.1 we show the driving and Euclidean distances to the İçerenköy and Aydınlı market for all customers and suppliers. In Table A.2 the calculation of the overall ratio is illustrated. The conclusion is that for the data met in [2] the overall ratio of the driving distance to the Euclidean distance is 1.36.

Table 4.A.1. Overview of all driving distances and Euclidean distances met in [2].

District	Number of vehicles per day**	ibb x-coordinate [km]	ibb y-coordinate [km]	x-coordinate [km]	y-coordinate [km]	Driving distance to Içerenköy market* [km]	Euclidean distance from Içerenköy market* [km]	Ratio driving distance to Euclidean distance	Driving distance to Aydinli market* [km]	Euclidean distance from Aydinli market* [km]	Ratio driving distance to Euclidean distance
Customers											
Ümraniye	283	426.3	4543.3	1.8	4.5	9.7	4.8	2.0	37.4	25.7	1.5
Pendik	275	439.6	4530.7	15.1	-8.0	21.2	17.1	1.2	15.3	7.5	2.1
Kadiköy	250	421.1	4539.2	-3.5	0.4	4.4	3.5	1.3	35.0	27.6	1.3
Üsküdar	247	420.7	4544.8	-3.8	6.1	9.6	7.2	1.3	36.7	31.0	1.2
Maltepe	206	427.5	4534.1	3.0	-4.7	6.0	5.5	1.1	24.9	19.6	1.3
Kartal	203	432.7	4530.5	8.2	-8.2	15.3	11.6	1.3	16.7	13.5	1.2
Atasehir	176	426.2	4539.7	1.7	1.0	2.4	2.0	1.2	35.9	23.6	1.5
Sultanbeyli	137	439.0	4537.5	14.5	-1.2	16.6	14.5	1.1	14.0	13.2	1.1
Sancaktepe	120	436.0	4540.8	11.5	2.1	17.4	11.7	1.5	28.2	17.6	1.6
Beykoz	116	429.4	4554.9	4.9	16.1	23.8	16.8	1.4	56.2	33.0	1.7
Tuzla	87	444.7	4527.2	20.2	-11.5	29.0	23.3	1.2	4.6	1.4	3.3
Çekmeköy	79	434.9	4547.2	10.4	8.4	20.1	13.4	1.5	36.0	23.6	1.5
Sile	13	469.5	4555.2	45.0	16.5	67.7	47.9	1.4	76.4	37.9	2.0
Adalar	7	423.8	4535.8	-0.8	-3.0	4.1	3.1	1.3	28.7	23.7	1.2
	Total: 2200	Weighted average (centre of gravity):	Weighted average (centre of gravity):	Weighted average (centre of gravity):	Weighted average (centre of gravity):	Weighted average: 13.2	Weighted average: 9.8	Ratio total driving distance to total Euclidean distance:	Weighted average: 29.0	Weighted average: 20.6	Ratio total driving distance to total Euclidean distance:
		430.2	4538.8	5.7	0.0	Total***: 28976.4	Total***: 21635.9	1.34	Total***: 63758.4	Total***: 45255.7	1.41
Suppliers											
Çayirova (in Kocaeli)	540	449.4	4526.1	24.9	-12.6	33.0	27.9	1.18	7.1	4.0	1.75
						Total****: 17797.5	Total****: 15046.1		Total****: 3814.3	Total****: 2182.0	
Markets											
Atasehir	(Içerenköy)	424.5	4538.7	0.0	0.0						
Tuzla	(Aydinli)	445.3	4526.0	20.8	-12.8						

*: data from Istanbul Metropolitan Municipality website City map (http://sehirrehberi.ibb.gov.tr), February 2012
**: we assume the number of customers in a particular district to be proportional to the population of that district,
we use the district population numbers from the Turkish Statistical Institute (TURKSTAT) website (http://www.turkstat.gov.tr) (January 2012)
***: for all 2200 customer vehicles to the market
****: for all 540 supplier vehicles to the market

Table 4.A.2. Calculation of the ratio of the total driving distance to the total Euclidean distance for the cases studied in [2].

	Driving distance [km]	Euclidean distance [km]	Ratio driving distance to Euclidean distance
All customer vehicles to Içerenköy market	28976	21639	1.34
All customer vehicles to Aydinli market	63758	45256	1.41
All supplier vehicles to Içerenköy market	17798	15046	1.18
All supplier vehicles to Aydinli market	3814	2182	1.75
total:	114347	84123	**1.36**

References

[1] M. Tanyaş, U.R. Tuzkaya, O.N Alp, N. Demirtaş, H. Baraçlı, Distribution center layout design selection by using axiomatic design: a specific case for fruits and vegetables wholesale market hall, 9th International Logistics and Supply Chain Congress 2011 Proceedings, 2011.

[2] T. Dubois, B. Bamyacı, M. Tanyaş,L. Aksoy,H. Vanlı, F. Witlox, G. Allaert, A location comparison based on fuel costs for one of the fruit and vegetable markets in Istanbul, 10th International Logistics and Supply Chain Congress 2012 Proceedings, 2012.

[3] G. Tuzkaya, S. Önüt, U.R. Tuzkaya, B. Gülsün, An analytic network process approach for locating undesirable facilities: An example from Istanbul, Turkey, Journal of Environmental Management 88 (2008) 970-983.

[4] F. Zhang, D.M. Johnson, J.W. Sutherland, A GIS-based method for identifying the optimal location for a facility to convert forest biomass to biofuel, Biomass and Bioenergy 35 (2011) 3951-3961.

[5] http://cta.ornl.gov/data/tedb30/Edition30_Chapter05.pdf (06.02.2012).

[6] http://gm.poas.com.tr/pompafiyat/pompafiyatgrid.aspx (07.02.2012).

[7] S. Yilmaz, I. Yilmaz, Evaluation of the wholesale market system for fresh fruits and vegetables in Turkey: A case study from Antalya Metropolitan Municipality, New Zealand Journal of Crop and Horticultural Science 36 (2008) 85-95.

[8] http://rapor.tuik.gov.tr/reports/rwservlet?adnksdb2&ENVID=adnksdb2Env&report=wa_turkiye_ilce_koy_sehir.RDF&p_il1=34&p_kod=1&p_yil=2010&p_dil=2&desformat=html (23.01.2012) (Turkish Statistical Institute (TURKSTAT) website).

[9] http://sehirrehberi.ibb.gov.tr (08.08.2012) (Istanbul Metropolitan Municipality website City map).

[10] http://maps.google.com (23.07.2012).

5

Istanbul fruit and vegetable wholesale market location and joint transportation

Based on: T. Dubois, M. Bamyaci, H. Vanli, L. Aksoy, M. Tanyas, G. Allaert, F. Witlox, Case study on different distribution methods for an urban wholesale market, in: C. Cebeci, G. Büyüközkan (eds.), 11th International Logistics and Supply Chain Congress 2013 Proceedings, 2013, pp. 313-319.

Abstract

In this paper we study different distribution methods for the fruit and vegetable wholesale market on the Asian side of Istanbul. Currently the distribution of the goods from the market to the customers is organised individually by the customers. In a different approach the distribution is organised by a logistics company and the goods are delivered to the customers, who place orders, by trucks making milk runs. We estimate for both distribution methods the total fuel cost, the total driving distance and the total costs. Also included in our research is the pricing of the delivery of the goods by the logistics company.

Keywords: distribution, milk run, city logistics, wholesale market.

5.1. Introduction

There are two fruit and vegetable wholesale markets in Istanbul. One is on the European side of Istanbul, in the Kocatepe neighbourhood of the Bayrampasa district, the other wholesale market is on the Asian side in the Içerenköy neighbourhood of the Atasehir district. There are plans to move the market that is now in Içerenköy to a new location in the Aydinli neighbourhood of the Tuzla district. Figure 1 shows the different market locations on a satellite/aerial photo of Istanbul. Large trucks bring the fruit and vegetables from the producers (mostly farmers in Antalya, Ankara and Adana) to the Istanbul fruit and vegetable markets. The customers of the fruit and vegetable markets mostly need the fruit and vegetables for street markets, greengroceries and other stores, restaurants and hotels. The facilities of customers of the Kocatepe market are located on the European side of Istanbul and the facilities of customers of the other market are located on the Asian side of Istanbul. The customers of the markets organise the transport of the goods from the wholesale market to their facilities usually with small trucks or cars. The supplier trucks arrive between 14:00 and 2:00 at the fruit and vegetable markets and the fruit and vegetable markets are open for customers between 2:00 and 9:00. Most supplier trucks arrive around 22:00 and 23:00, most customers come in the early morning. The markets are open every day except Sunday. There are on average more or less 540 supplier trucks and more or less 2200 customer vehicles per

day at the Içerenköy market. The data on the Istanbul fruit and vegetable wholesale markets in this section are provided by the Istanbul fruit and vegetable wholesale markets management. Information on the fruit and vegetable wholesale market system in Turkey and a case study on the Antalya fruit and vegetable wholesale market, can be found in [2].

In [3] and [4] a location analysis is made for the fruit and vegetable wholesale market on the Asian side of Istanbul: the total supply chain fuel cost is compared for different market locations. In this article, however, not the location but the method of distribution is studied for a fixed market location, Aydinli. We will compare two methods of distribution of the goods from the Aydinli wholesale market to the facilities of the customers.

Figure 5.1. Different fruit and vegetable wholesale market locations (Kocatepe/Bayrampasa, Içerenköy/Atasehir and Aydinli/Tuzla) on a satellite/aerial photo of Istanbul.
Source: Istanbul Metropolitan Municipality website city map [1].

5.2. Methods of distribution

The current distribution method of the Istanbul fruit and vegetable wholesale markets for transporting the goods from the market to the facilities of the customers is "individual transport": the customers organise the transport independent from each other. A different method of distribution is "distribution with milk runs": the customer orders the desired goods (e.g. via a website) and then the goods are delivered at the costumer's location by a logistics

company that organises the distribution of all customers by making rounds (milk runs) with trucks (several customers are supplied during one round).

For more information on milk run distribution we refer to [5] and [6]. In [5] an algorithm is presented to design milk runs in a setting with several suppliers, several customers and a distribution centre. Milk runs with in addition to deliveries at customer facilities also pickups at supplier facilities are considered in [5]. In [6] the relation between the load factor and costs and emission is studied in the context of urban milk runs.

5.3. Estimation of distribution costs

In this section we will estimate the customer costs for individual transport distribution and milk run distribution. We use Aydinli, the possible new location of the fruit and vegetable market on the Asian side, as the market location. First we want to locate the facilities of the customers. We assume these facilities are located at the Asian side of Istanbul. We follow [3] and [4] by assuming that the number of customer facilities in a district is proportional to the population of that district. District population data are available at the website of the Turkish Statistical Institute [7]: at [8] we find the 2010 population data of all districts of Istanbul. Given that the total number of customer vehicles is 2200 (see introduction), we assume that there are also 2200 customer facilities. Table 1 (third column) shows the number of customer facilities for every district on the Asian side of Istanbul.

For the driving distance from the Aydinli market to a location in a district, we use a single value for every district. We use the driving distance (determined by the route planner of the Istanbul Metropolitan Municipality website city map [1]) from the Aydinli market to the estimated geometric centre of the populated area of the district. Table 1 (second column) shows the driving distance from the Aydinli market to every district on the Asian side of Istanbul.

In [3] the fuel cost per km is estimated: 0.6 TL/km for the customer vehicles and 1.2 TL/km for the supplier vehicles. The following two subsections contain the average customer cost calculations for individual transport distribution and milk run distribution.

Table 5.1. Comparison of the daily driving distances and daily fuel costs for individual transport and distribution with milk runs.

District	Driving distance to Aydinli market [km]	Number of customers	Number of milk runs	Total driving distance with individual transport [km]	Total driving distance with milk runs [km]	Total fuel cost with individual transport [TL]	Total fuel cost with milk runs [TL]
Ümraniye	37,4	283	77	21199	6296	12719	7555
Pendik	15,3	275	74	8416	2500	5050	3000
Kadiköy	35,0	250	68	17523	5204	10514	6245
Üsküdar	36,7	247	67	18188	5402	10913	6482
Maltepe	24,9	206	56	10260	3047	6156	3657
Kartal	16,7	203	55	6763	2009	4058	2410
Atasehir	35,9	176	48	12651	3757	7590	4509
Sultanbeyli	14,0	137	37	3829	1137	2297	1365
Sancaktepe	28,2	120	33	6785	2015	4071	2418
Beykoz	56,2	116	31	12993	3859	7796	4631
Tuzla	4,6	87	24	807	240	484	288
Çekmeköy	36,0	79	21	5701	1693	3421	2032
Sile	76,4	13	4	2018	599	1211	719
Adalar	28,7	7	2	384	114	230	137
		Total:	Total:	Total:	Total:	Total:	Total:
		2200	594	1,28E+05	3,79E+04	7,65E+04	4,54E+04

5.3.1. Individual transport distribution

The customers cost contains four parts: the fuel cost, the labour cost (paying the drivers), the vehicle maintenance cost and the financial cost (cost related to the initial purchase of the vehicle). We make the following assumptions: (i) the sum of the labour cost and vehicle maintenance cost is equal to the fuel cost; (ii) the financial cost is 10% of the fuel cost. We assume a small customer financial cost because we assume that most customers need the vehicles also for other purposes.

We calculate the total daily customer driving distance (to and from the market) for individual transport distribution as follows:

total daily customer driving distance
$= \sum_{i=1}^{14}$ (number of customer facilities in district i × 2 × driving distance from market to district i) (1)

The formula contains 14 terms, each term related with one of the 14 districts of the Asian side of Istanbul. The result is 128×10^3 km. The total daily customer fuel cost is the total daily customer driving distance times 0.6 TL/km (see above), which is 76.5×10^3 TL. If we divide these results by 2200, we get 58 km for the average daily customer driving distance and 35 TL for the average daily customer fuel cost. Table 1 shows the total daily customer driving distance and total daily customer fuel cost for every district on the Asian side of Istanbul (fifth and seventh column).

By using assumption (i) and (ii) we get 35 TL for the sum of the average daily customer labour cost and average daily customer maintenance cost, and 3 TL for the average daily customer financial cost. Therefore the average daily total customer cost is 73 TL. (We rounded the numbers after making all calculations.)

5.3.2. Milk run distribution

The average customer cost is equal to the average price for delivery by the logistics company that supplies the customers. For estimating this cost we first estimate the logistics company cost, this cost contains four parts: the fuel cost, the labour cost (paying the drivers), the vehicle maintenance cost and the financial cost (cost related to the initial purchase of the trucks). We make the following assumptions: (i) the labour cost is 85% of the fuel cost; (ii) the sum of the financial cost and the vehicle maintenance cost is equal to the fuel cost.

For calculating the fuel cost, we first need to calculate the total driving distance. For this we estimate the average number of deliveries during a milk run and the driving distance of a milk run to a given district. In the introduction it is mentioned that the market on the Asian side of Istanbul is supplied by on average 540 trucks per day. We assume that the average cargo mass of a logistics company truck is similar to the average cargo mass that is delivered by a supplier truck. If we assume that for milk run distribution, the daily number of milk runs is 110% of the daily number of supplier trucks, then we get 594 for the number of daily milk runs. Because there are 2200 customer facilities, we get 3.7 for the average number of deliveries in one milk run. The driving distance of a milk run with all deliveries in a given district is estimated as 110% of twice the driving distance from the market to that district. Using these assumptions we calculate the total daily milk run driving distance as follows:

total daily milk run driving distance
$= \sum_{i=1}^{14}$ ((number of customer facilities in district i / (2200 / 594)) × 1.1 × 2 × driving distance from market to district i) (2)

The result is 37.9×10^3 km. This is 30% of the total daily driving distance of the customer vehicles in the case of individual transport. The total daily milk run fuel cost is the total daily milk run driving distance times 1.2 TL/km (see above), which is 45.4×10^3 TL. This is 59% of the total daily customer fuel cost in the case of individual transport. In Table 1 the total daily driving distance and the total daily fuel cost is shown for every district on the Asian side of Istanbul (sixth and eighth column).

By using assumption (i) and (ii), we get 38.6×10^3 TL for the daily labour cost and 45.4×10^3 TL for the sum of the daily financial cost and the daily vehicle maintenance cost. Therefore the daily total cost for the logistics company is 130×10^3 TL. The total cost per customer is then 59 TL. If the profit of the logistics company is 24% of the costs, then the average customer price for delivery is 0.24 × 59 TL + 59 TL = 73 TL or the average daily customer cost in the case of individual transport. The daily logistics company profit is then 31.1×10^3 TL. If the profit of the logistics company is 20% of the costs, then the average customer price is 0.2 × 59 TL + 59 TL = 71 TL, which is lower than the average customer cost in the case of individual transport. The daily logistics company profit is then 25.9×10^3 TL. If the profit of the logistics company is 10% of the costs, then the average customer price is 0.1 × 59 TL + 59 TL = 65 TL. The daily logistics company profit is then 13.0×10^3 TL. (We rounded the numbers after making all calculations.) Table 2 shows the results of these calculations and also contains the proportion of customer cost in the case of milk runs to customer cost in the case of individual transport for different values of the proportion of profit to cost for the logistics company.

The average daily customer cost in the case of milk run distribution is dependent on the pricing of the delivery by the logistics company. According to our calculations it is possible to do the pricing such that milk run distribution is cheaper than individual transport distribution for every customer and there is profit for the logistics company.

Table 5.2. The daily profit of the logistics company, the average price for delivery and the proportion of customer cost in the case of milk runs to customer cost in the case of individual transport for different values of the proportion of profit to cost for the logistics company.

Profit logistics company / cost logistics company	Daily profit logistics company [TL]	Average price for delivery [TL]	Average customer cost milk runs / average customer cost individual transport
0%	0,00E+00	59	81%
10%	1,30E+04	65	89%
20%	2,59E+04	71	97%
24%	3,11E+04	73	100%

5.4. Conclusion

We compared two methods of distribution of the goods of the Aydinli fruit and vegetable wholesale market to the customer facilities: individual transport organised by each customer and milk run distribution organised by a logistics company. For both methods we roughly estimated the costs for the customers. The result of this study is that according to our estimations it is possible to organise the milk run distribution such that the average cost for the customer is lower than the average cost for the customer in the case of individual transport distribution and the logistics company makes profit.

References

[1] http://sehirrehberi.ibb.gov.tr (31.07.2013) (Istanbul Metropolitan Municipality website city map).
[2] S. Yilmaz, I. Yilmaz, Evaluation of the wholesale market system for fresh fruits and vegetables in Turkey: A case study from Antalya Metropolitan Municipality, New Zealand Journal of Crop and Horticultural Science 36 (2008) 85-95.
[3] T. Dubois, M. Bamyaci, M. Tanyas, L. Aksoy, H. Vanli, F. Witlox, G. Allaert, A location comparison based on fuel costs for one of the fruit and vegetable markets in Istanbul, in: G. Gürsoy, M. Tanyas (eds.), 10th International Logistics and Supply Chain Congress 2012 Proceedings, Istanbul Kemerburgaz University & Logistics Association (LODER) & Ideal Kultur Publishing, Istanbul, 2012, pp. 179-184.
[4] T. Dubois, M. Tanyas, M. Bamyaci, H. Vanli, L. Aksoy, F. Witlox, G. Allaert, A search for the optimal location regarding the supply chain fuel costs for one of the fruit and vegetable markets in Istanbul, in: G. Gürsoy, M. Tanyas (eds.), 10th International Logistics and Supply Chain Congress 2012 Proceedings, Istanbul Kemerburgaz University & Logistics Association (LODER) & Ideal Kultur Publishing, Istanbul, 2012, pp. 544-552.
[5] T. Du, F.K.Wang, P. Lu, A real-time vehicle-dispatching system for consolidating milk runs, Transportation Research Part E 43 (2007) 565-577.
[6] N. Arvidsson, The milk run revisited: A load factor paradox with economic and environmental implications for urban freight transport, Transportation Research Part A 51 (2013) 56-62.

[7] http://www.turkstat.gov.tr (03.08.2013) (Turkish Statistical Institute website).
[8]   http://rapor.tuik.gov.tr/reports/rwservlet?adnksdb2&ENVID=adnksdb2Env&report=wa_
turkiye_ilce_koy_sehir.RDF&p_il1=34&p_kod=1&p_yil=2010&p_dil=2&desformat=html
(03.08.2013).

6

Inventory management, capacitated replenishment and customer satisfaction: determining the fill rate

Based on:
- T. Dubois, G. Allaert, F. Witlox, Determining the fill rate for a periodic review inventory policy with capacitated replenishments, lost sales and zero lead time, Operations Research Letters 41 (2013) 726-729.
- T. Dubois, F. Witlox, G. Allaert, M. Tanyaş, Computational experiments on the fill rate of a capacitated periodic review inventory system, in: I. Sabuncuoglu, B. Bidanda (eds.), International IIE (Institute of Industrial Engineers) Conference Proceedings, 2013, p. 289.

Abstract

In this chapter a method is presented to calculate the fill rate of a periodic review order-up-to inventory system with capacitated replenishments, lost sales and zero lead time. We consider discrete demand. The method is based on a number of theorems concerning the inventory system, which we prove. We show that the initial inventory positions of the different review periods form a Markov chain and we determine the transition matrix of this Markov chain. Furthermore we study for what probability mass functions of the review period demand the Markov chain has a unique stationary distribution. We conclude the first part of the study by presenting a method to determine the fill rate for any probability mass function of the review period demand.

In the second part of the chapter the inventory system is further studied by performing several computation experiments for Poisson distributed review period demand. We start with showing an inventory profile based on the simulation of the demand as a Poisson process. Furthermore we give an illustrated example of the application of the proposed method. We conclude by presenting a series of graphs in which the fill rate of the inventory system is studied as a function of three variables: the order-up-to level, the replenishment capacity and the expectation value of the review period demand.

Keywords: inventory, periodic review, fill rate, capacitated replenishment, lost sales.

6.1. Introduction

In several publications the fill rate is discussed for periodic review inventory systems with uncapacitated replenishment. For example Johnson et al. [1] study different fill rate

expressions for inventory systems with backorders and normally distributed demand and compare these expressions experimentally via simulation experiments. The fill rate of an uncapacitated periodic review inventory system with backorders and continuous period demand is also studied in [2], [3] and [4]. Sobel [2] discusses besides single-stage systems also multistage systems and similar as in [1] the lead time is assumed to be a multiple of the review period. This is not assumed in Zhang et al. [3] and Silver et al. [4]. In [2] and [3] general continuous demand and normal demand are considered, [1] and [4] focus on normal demand. Guijarro et al. [5] discuss fill rate definitions and expressions for uncapacitated periodic review inventory systems with lost sales and discrete demand. In this paper however, periodic review inventory systems with a limited replenishment capacity are studied. Unlike [1], [2], [3], [4] and [5], in this paper the lead time is assumed to be negligible. In a part of [2], capacity is also considered, but in the context of multistage systems with process limitations. In [6] and [7] finite horizon fill rates are considered and compared with the infinite horizon fill rate.

We consider a single-item inventory system that applies a periodic review order-up-to inventory policy with lost sales and zero lead time. Because of the lost sales assumption and the zero lead time assumption, the inventory position (number of products on hand minus number of products backlogged plus number of products on order) equals the stock level (number of products on hand). In such inventory policy the stock level is reviewed periodically and every review an order is placed to raise the stock level to a fixed level, the order-up-to level s (a positive integer). We assume the demand during one review period (period between two reviews) to be discrete with a given probability mass function. We consider a review period to begin when the order is placed and to end just before the next order is placed. Following characteristics are assumed for the inventory system under study: (i) the order is placed immediately after review; (ii) the lead time is zero, i.e. the order arrives immediately after the order is placed; (iii) the demands during different review periods are independently and identically distributed; (iv) the demand during a particular review period is independent of every stock level at the beginning of a review period that precedes that review period or coincides with that review period; (v) unsatisfied demands result in lost sales; and (vi) replenishment is capacitated with capacity c (a positive integer), i.e. if more than c products are ordered, only c are delivered.

In this paper we determine the fill rate of a periodic review inventory system with capacitated replenishments. A similar problem was already studied by Mapes [8], who determined the service level of a capacitated periodic review inventory system approximately by simulation. In this paper a new method to determine the fill rate is presented which is exact given the used fill rate definition and the above stated six assumptions. Similarly as in [9], we define the fill rate of a periodic review inventory system as the proportion of the expected satisfied demand to the expected demand (see (23) for the exact formula). Another definition used in literature for the fill rate (e.g. in [2] and [3]) is the expectation of the proportion of the satisfied demand to the demand. According to [6] and [3], both definitions agree if an infinite horizon is considered.

6.2. Determination of the fill rate

In this section we will determine the fill rate β of a periodic review order-up-to inventory system with order-up-to level s and replenishment capacity c. We assume $c<s$ because when c is greater than or equal to s replenishment is not capacitated. Let D_t be the random variable associated with the demand during the review period t, I_t the random variable associated with the stock level at the beginning of review period t, f_D the probability mass function of D_t (with the set of the integers as domain and value zero for negative integers) and f_{I_t} the probability mass function of I_t, for all $t\in\{1,2,...\}$. We assume the stock level at the beginning of the first review period to be c, $c+1$, ... or s. Because of the used inventory policy, the following holds:

$$I_t = \min\{s,\max\{I_{t-1}-D_{t-1},0\}+c\}, \text{ for all } t\in\{2,3,...\}. \tag{1}$$

We continue by first proving four theorems and then presenting a method to find the fill rate based on these theorems. For (finite state) Markov chain theory we refer to [10], chapter 4.

Theorem 1. $I_1,I_2,I_3,...$ is a Markov chain.

Proof. For proving theorem 1, we need to prove the following:

$$P(I_t = i_t|I_{t-1}=i_{t-1}\cap I_{t-2}=i_{t-2}\cap...\cap I_1=i_1) = P(I_t=i_t|I_{t-1}=i_{t-1}), \text{ for all } t\in\{2,3,...\} \text{ and}$$

for all $i_1,...,i_t\in\{c,c+1,...,s\}$ for which $P(I_{t-1}=i_{t-1}\cap...\cap I_1=i_1)\neq0$ and $P(I_{t-1}=i_{t-1})\neq0$ $\tag{2}$

We start with the definition of conditional probability and (1) and then use assumption (iv). For all $t\in\{2,3,...\}$ and for all $i_1,...,i_t\in\{c,c+1,...,s\}$ for which $P(I_{t-1}=i_{t-1}\cap...\cap I_1=i_1)\neq0$ and $P(I_{t-1}=i_{t-1})\neq0$:

$$P(I_t = i_t|I_{t-1}=i_{t-1}\cap I_{t-2}=i_{t-2}\cap...\cap I_1=i_1)$$

$$= \frac{P(\min\{s,\max\{I_{t-1}-D_{t-1},0\}+c\}=i_t\cap I_{t-1}=i_{t-1}\cap...\cap I_1=i_1)}{P(I_{t-1}=i_{t-1}\cap...\cap I_1=i_1)} \tag{3}$$

$$= \frac{P(\min\{s,\max\{i_{t-1}-D_{t-1},0\}+c\}=i_t\cap I_{t-1}=i_{t-1}\cap...\cap I_1=i_1)}{P(I_{t-1}=i_{t-1}\cap...\cap I_1=i_1)} \tag{4}$$

$$= P(\min\{s,\max\{i_{t-1}-D_{t-1},0\}+c\}=i_t). \tag{5}$$

Similarly, for all $t\in\{2,3,...\}$ and for all $i_1,...,i_t\in\{c,c+1,...,s\}$ for which $P(I_{t-1}=i_{t-1}\cap...\cap I_1=i_1)\neq0$ and $P(I_{t-1}=i_{t-1})\neq0$:

$$P(I_t = i_t|I_{t-1}=i_{t-1}) \quad = \frac{P(\min\{s,\max\{I_{t-1}-D_{t-1},0\}+c\}=i_t\cap I_{t-1}=i_{t-1})}{P(I_{t-1}=i_{t-1})} \tag{6}$$

$$= \frac{P(\min\{s,\max\{i_{t-1}-D_{t-1},0\}+c\}=i_t\cap I_{t-1}=i_{t-1})}{P(I_{t-1}=i_{t-1})} \tag{7}$$

$$= P(\min\{s,\max\{i_{t-1}-D_{t-1},0\}+c\}=i_t). \tag{8}$$

Combination of (5) and (8) yields (2), which completes the proof. $\square$

Theorem 2. The element at row i and column j of the transition matrix P of Markov chain $I_1,I_2,I_3,...$ with states $c,c+1,...,s$ is:

$$p_{ij} = \sum_{k=0}^{\infty} f_D(k)\delta(\min\{s,\max\{c-1+i-k,0\}+c\}-c+1-j), \text{ for all } i,j\in\{1,2,...,s\text{-}c+1\} \tag{9}$$

with $\delta(x)=1$ if $x=1$ and $\delta(x)=0$ if $x\neq1$ for every integer x.

Proof. For all $i,j \in \{1,2,...,s-c+1\}$ and for all $t \in \{2,3,...\}$ for which $P(I_{t-1}=c-1+i)\neq 0$:

$$p_{ij} = P(I_t = c - 1 + j \mid I_{t-1} = c - 1 + i) \tag{10}$$

$$= P(\min\{s, \max\{c-1+i-D_{t-1}, 0\} + c\} = c - 1 + j) \tag{11}$$

$$= \sum_{k=0}^{\infty} f_D(k)\delta\big(\min\{s, \max\{c-1+i-k, 0\} + c\} - c + 1 - j\big) \tag{12}$$

For getting (10) we applied the definition of transition matrix and for getting (11) we used (8). $\square$

Theorem 3.

- If $f_D(c)\neq 1$, then for the Markov chain $I_1, I_2, I_3, ...$ the following matrix equation in the variable $[f_I(c)\, f_I(c+1)\, ...\, f_I(s)]^T$, with $0 \leq f_I(c) \leq 1$, $0 \leq f_I(c+1) \leq 1$, ... and $0 \leq f_I(s) \leq 1$, has a unique solution

$$
\begin{bmatrix}
1 & 1 & 1 & ... & 1 \\
p_{12} & p_{22}-1 & p_{32} & ... & p_{s-c+1\,2} \\
p_{13} & p_{23} & p_{33}-1 & ... & p_{s-c+1\,3} \\
... & ... & ... & ... & ... \\
p_{1\,s-c+1} & p_{2\,s-c+1} & p_{3\,s-c+1} & ... & p_{s-c+1\,s-c+1}-1
\end{bmatrix}
\begin{bmatrix}
f_I(c) \\
f_I(c+1) \\
f_I(c+2) \\
... \\
f_I(s)
\end{bmatrix}
=
\begin{bmatrix}
1 \\
0 \\
0 \\
... \\
0
\end{bmatrix}
\tag{13}
$$

and for these $f_I(c)$, $f_I(c+1)$, ... and $f_I(s)$

$$\lim_{n \to \infty} f_{I_n}(i) = f_I(i), \text{ for all } i \in \{c,c+1,...,s\}. \tag{14}$$

- If $f_D(c)=1$, then $f_{I_t}(i)=f_{I_1}(i)$ for all $t \in \{1,2,...\}$ and for all $i \in \{c,c+1,...,s\}$.

Proof. The transition matrix of the Markov chain is

$$
P =
\begin{bmatrix}
p_{11} & p_{12} & ... & p_{1\,s-c+1} \\
p_{21} & p_{22} & ... & p_{2\,s-c+1} \\
... & ... & ... & ... \\
p_{s-c-1\,1} & p_{s-c-1\,2} & ... & p_{s-c+1\,s-c+1}
\end{bmatrix}.
\tag{15}
$$

Because of theorem 2, we get

$$
P =
\begin{bmatrix}
\sum_{k=c+m}^{\infty} f_D(k) + f_D(c+m-1) + ... + f_D(c) & f_D(c-1) & f_D(c-2) & ... \\
\sum_{k=c+m}^{\infty} f_D(k) + f_D(c+m-1) + ... + f_D(c+1) & f_D(c) & f_D(c-1) & ... \\
\sum_{k=c+m}^{\infty} f_D(k) + f_D(c+m-1) + ... + f_D(c+2) & f_D(c+1) & f_D(c) & ... \\
... & ... & ... & ... \\
\sum_{k=c+m}^{\infty} f_D(k) + f_D(c+m-1) & f_D(c+m-2) & f_D(c+m-3) & ... \\
\sum_{k=c+m}^{\infty} f_D(k) & f_D(c+m-1) & f_D(c+m-2) & ...
\end{bmatrix}
$$

$$\left[\begin{array}{cc} f_D(c-m+1) & \sum_{k=-\infty}^{c-m} f_D(k) \\[2ex] f_D(c-m+2) & f_D(c-m+1)+\sum_{k=-\infty}^{c-m} f_D(k) \\[2ex] f_D(c-m+3) & f_D(c-m+2)+f_D(c-m+1)+\sum_{k=-\infty}^{c-m} f_D(k) \\[2ex] ... & ... \\[2ex] f_D(c) & f_D(c-1)+...+f_D(c-m+1)+\sum_{k=-\infty}^{c-m} f_D(k) \\[2ex] f_D(c+1) & f_D(c)+...+f_D(c-m+1)+\sum_{k=-\infty}^{c-m} f_D(k) \end{array}\right] , \text{ with } m=s\text{-}c. \quad (16)$$

Case 1: $f_D(c) \neq 1$ and $f_D(x)=0$ for all $x \in \{0,1,...,c-1\}$.
By studying (16) we conclude that P is a lower triangular matrix and for every state the
probability to go to state c in a number of steps is positive and the probability to go from state
i to state j in a number of steps is zero if $i<j$. Therefore state c is recurrent and the other states
are transient.

Case 2: $f_D(c) \neq 1$ and $f_D(x)=0$ for all $x \in \{c+1,c+2,...\}$.
By studying (16) we conclude that P is an upper triangular matrix and for every state the
probability to go to state s in a number of steps is positive and the probability to go from state
i to state j in a number of steps is zero if $j<i$. Therefore state s is recurrent and the other states
are transient.

Case 3: if $f_D(c) \neq 1$ and $f_D(x) \neq 0$ for at least one integer x smaller than c and $f_D(x) \neq 0$ for at least
one integer x larger than c.
Similarly with case 1 and 2, for every state the probability to go to state c in a number of steps
is positive and for every state the probability to go to state s in a number of steps is positive.
Therefore s and c are in the same communication class and every state communicates with c
or is a transient state. We conclude that also in this case there is one recurrent communication
class and all other states are transient.

In case 1, 2 and 3 the recurrent communication class is aperiodic because in case 1 and 3 the
probability for going from state c to state c in one step is positive, and in case 2 and 3 the
probability for going from state s to state s in one step is positive. Application of theorem 6A
on page 118 of [10] yields the following two statements:
- there is a unique left probability eigenvector of P with eigenvalue 1   (17)
(this vector is called the stationary distribution of the Markov chain)
- let $\begin{bmatrix} f_I(c) & f_I(c+1) & ... & f_I(s) \end{bmatrix}$ be this vector, then

$$\lim_{n\to\infty} (P)^n = \begin{bmatrix} f_I(c) & f_I(c+1) & ... & f_I(s) \\ f_I(c) & f_I(c+1) & ... & f_I(s) \\ ... & ... & ... & ... \\ f_I(c) & f_I(c+1) & ... & f_I(s) \end{bmatrix}. \quad (18)$$

(18) allows us to calculate the following limit, what results in the proof of (14).

$$\lim_{n\to\infty} \begin{bmatrix} f_{In}(c) & f_{In}(c+1) & ... & f_{In}(s) \end{bmatrix} = \lim_{n\to\infty} \left(\begin{bmatrix} f_{I1}(c) & f_{I1}(c+1) & ... & f_{I1}(s) \end{bmatrix}(P)^n \right) \quad (19)$$

$$= \begin{bmatrix} f_I(c) & f_I(c+1) & ... & f_I(s) \end{bmatrix} \tag{20}$$

It is equivalent with (17) that $\begin{bmatrix} f_I(c) & f_I(c+1) & ... & f_I(s) \end{bmatrix}^T$ the unique right probability eigenvector is of $\boldsymbol{P}^T$ with eigenvalue 1. Therefore the following matrix equation has a unique probability vector solution.

$$\begin{bmatrix} p_{11}-1 & p_{21} & ... & p_{s-c+1\,1} \\ p_{12} & p_{22}-1 & ... & p_{s-c+1\,2} \\ ... & ... & ... & ... \\ p_{1\,s-c+1} & p_{2\,s-c+1} & ... & p_{s-c+1\,s-c+1}-1 \\ 1 & 1 & ... & 1 \end{bmatrix} \begin{bmatrix} f_I(c) \\ f_I(c+1) \\ ... \\ ... \\ f_I(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ ... \\ 0 \\ 1 \end{bmatrix} \tag{21}$$

Considered as a system of linear equations, (21) contains $s-c+2$ equations. The first equation in this system of linear equations equals the opposite of the sum of the other equations except the last one, because for all $i \in \{1,2,...,s-c+1\}$

$$\sum_{k=1}^{s-c+1} p_{ik} = 1. \tag{22}$$

Omitting the first equation in (21) therefore yields an equivalent matrix equation. With the first equation omitted and the last equation first we obtain matrix equation (13) which is equivalent with (21).

Case 4: if $f_D(c)=1$.
Then $\boldsymbol{P}$ is the $(s-c+1)\times(s-c+1)$ identity matrix. Therefore $f_{It}(i)=f_{I1}(i)$, for all $t \in \{1,2,...\}$ and for all $i \in \{c,c+1,...,s\}$. $\square$

Using the notation of this section the infinite horizon fill rate β of the periodic review inventory system under study is:

$$\beta = \lim_{n \to \infty} \frac{E(\min\{I_1,D_1\} + \min\{I_2,D_2\} + ... + \min\{I_n,D_n\})}{E(D_1 + D_2 + ... + D_n)}. \tag{23}$$

Theorem 4.

$$\lim_{n \to \infty} \frac{E(\min\{I_1,D_1\} + \min\{I_2,D_2\} + ... + \min\{I_n,D_n\})}{E(D_1 + D_2 + ... + D_n)} = 1 - \frac{\displaystyle\sum_{i=c}^{s} f_I(i) \sum_{j=i+1}^{\infty} (j-i)f_D(j)}{\displaystyle\sum_{j=1}^{\infty} j f_D(j)} \tag{24}$$

with $f_I(i) = \lim_{n \to \infty} f_{In}(i)$, for all $i \in \{c,c+1,...,s\}$.

Proof. $\displaystyle\lim_{n \to \infty} \frac{E(\min\{I_1,D_1\} + \min\{I_2,D_2\} + ... + \min\{I_n,D_n\})}{E(D_1 + D_2 + ... + D_n)}$

$$= \lim_{n \to \infty} \frac{E(\min\{I_1,D_1\}) + E(\min\{I_2,D_2\}) + ... + E(\min\{I_n,D_n\})}{nE(D_1)} \tag{25}$$

$$= \frac{\displaystyle\lim_{n \to \infty} E(\min\{I_n,D_n\})}{E(D_1)} \tag{26}$$

$$= \frac{\lim\limits_{n\to\infty} E(D_n - \max\{D_n - I_n, 0\})}{E(D_1)} \tag{27}$$

$$= 1 - \frac{\lim\limits_{n\to\infty} E(\max\{D_n - I_n, 0\})}{E(D_1)} \tag{28}$$

$$= 1 - \frac{\lim\limits_{n\to\infty} \sum\limits_{i=c}^{s} \sum\limits_{j=0}^{\infty} \max\{j-i, 0\} f_{In}(i) f_D(j)}{\sum\limits_{j=0}^{\infty} j f_D(j)} \tag{29}$$

$$= 1 - \frac{\sum\limits_{i=c}^{s} \sum\limits_{j=i+1}^{\infty} (j-i) f_I(i) f_D(j)}{\sum\limits_{j=1}^{\infty} j f_D(j)} \tag{30}$$

$$= 1 - \frac{\sum\limits_{i=c}^{s} f_I(i) \sum\limits_{j=i+1}^{\infty} (j-i) f_D(j)}{\sum\limits_{j=1}^{\infty} j f_D(j)} \tag{31}$$

For getting (26) we used:

$$\lim_{n\to\infty} \frac{f(1) + f(2) + \ldots + f(n)}{n} = \lim_{n\to\infty} f(n) \tag{32}$$

with f a function from the natural numbers to the real numbers for which the sequence $f(1), f(2), \ldots$ converges. Furthermore, we used assumption (iii) for getting (25), (28) and (29) and assumption (iv) for getting (29). $\square$

Theorems 1, 2, 3 and 4 put forward a method to determine the fill rate of the studied inventory system. First we construct the matrix $\boldsymbol{P}$ with the aid of theorem 2 or (16). Subsequently, if $f_D(c) \neq 1$, we solve matrix equation (13). According to theorem 3, this matrix equation has a unique probability vector solution and this solution gives us $\lim_{n\to\infty} f_{In}(c)$, $\lim_{n\to\infty} f_{In}(c+1)$, ... and $\lim_{n\to\infty} f_{In}(s)$. Notice that $\lim_{n\to\infty} f_{In}(c)$, $\lim_{n\to\infty} f_{In}(c+1)$, ... and $\lim_{n\to\infty} f_{In}(s)$ do not depend on the stock level at the beginning of the first review period, if $f_D(c) \neq 1$. If $f_D(c) = 1$ however, $\lim_{n\to\infty} f_{In}(i)$ equals $f_{I1}(i)$ for every state i. Finally we use these limits to calculate the fill rate of the inventory system with the aid of theorem 4.

6.3. Computational experiments

In this section we study the inventory system by performing several computation experiments for Poisson distributed review period demand.

6.3.1 Inventory profile for demand simulated as Poisson process
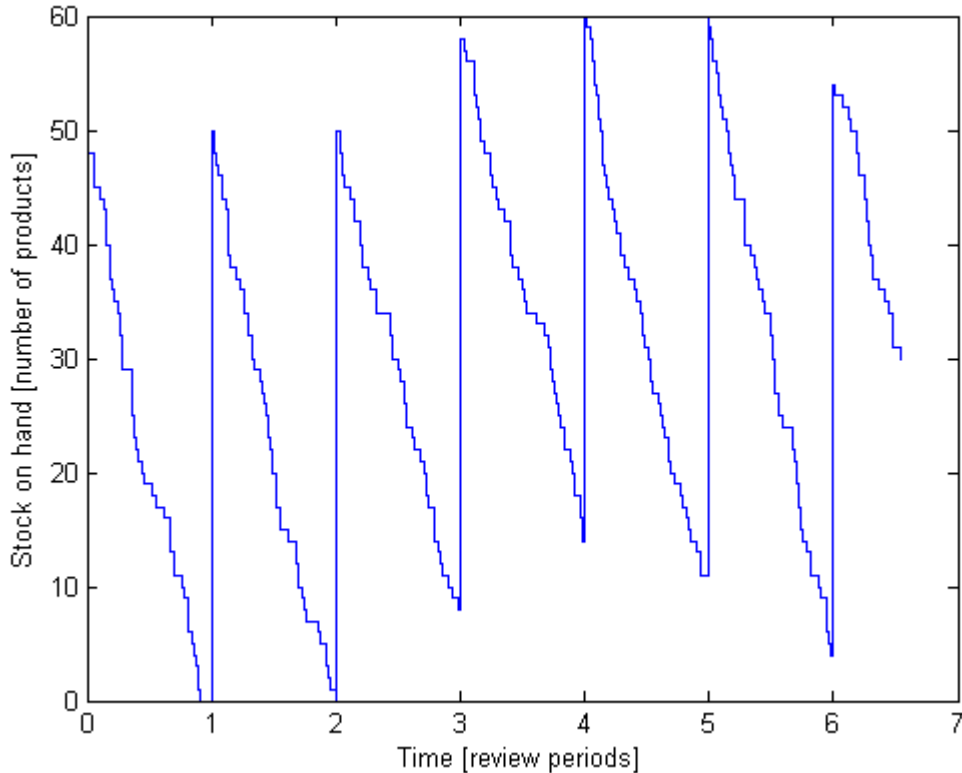
Figure 6.1. Stock on hand as a function of time.

Figure 1 shows the inventory profile of a capacitated periodic review inventory system with s=60, c=50 and Poisson distributed review period demand with mean μ=48. The graph is a result of a simulation of the demand as a Poisson process. The inventory position at the beginning of the first review period is 50. We see that just after replenishment the inventory level is 50, 51, ... or 60. If at the end of a review period the inventory level is zero, then at the beginning of the next review period the inventory level is 50 (the capacity c). If at the end of a review period the inventory level is 10 or larger, then at the beginning of the next review period the inventory level is 60 (the order-up-to level s).

6.3.2. Illustrated example of determining the fill rate for Poisson distributed review period demand

In this subsection we determine the fill rate for a capacitated periodic review inventory system with order-up-to level s=60, capacity c=50 and Poisson distributed review period demand with mean μ=48. Figure 2 shows the probability mass function of the review period demand.
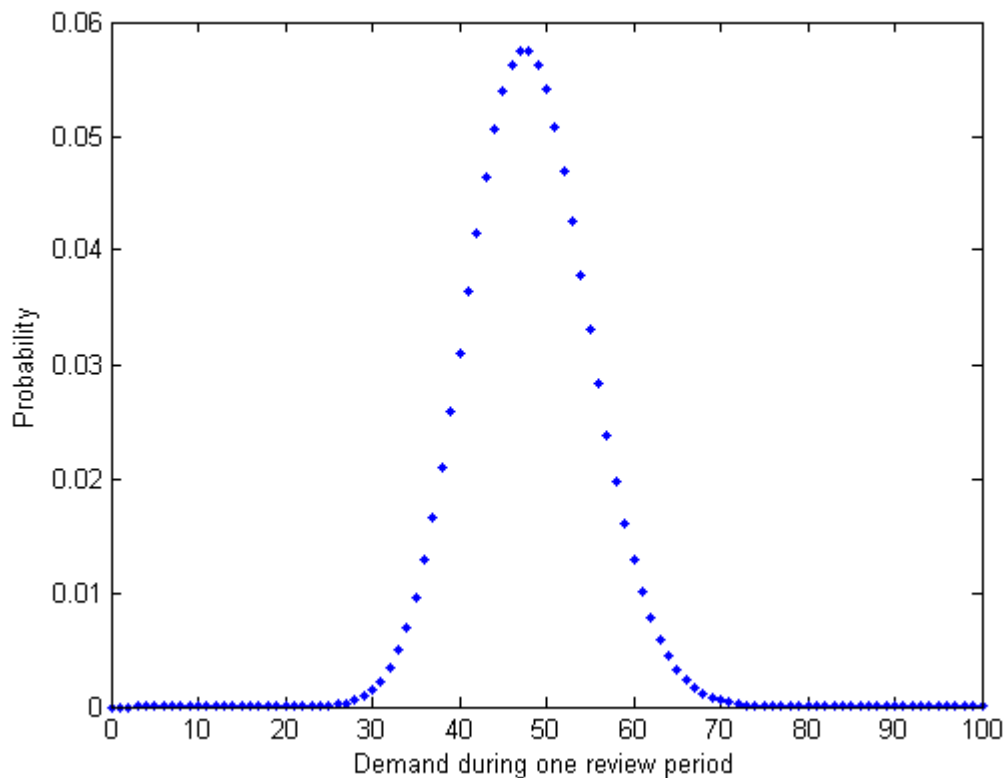
Figure 6.2. Probability mass function of the Poisson distributed (with mean 48) review period demand.

Application of theorem 2 gives us the transition matrix P, in this case an 11×11 matrix ($s-c+1=11$). Figure 3 shows this matrix (with the elements rounded to integer multiples of 0.0001.)

0.4054	0.0563	0.0575	0.0575	0.0563	0.0539	0.0506	0.0464	0.0415	0.0363	0.1383
0.3513	0.0541	0.0563	0.0575	0.0575	0.0563	0.0539	0.0506	0.0464	0.0415	0.1746
0.3005	0.0509	0.0541	0.0563	0.0575	0.0575	0.0563	0.0539	0.0506	0.0464	0.2162
0.2535	0.0470	0.0509	0.0541	0.0563	0.0575	0.0575	0.0563	0.0539	0.0506	0.2625
0.2110	0.0425	0.0470	0.0509	0.0541	0.0563	0.0575	0.0575	0.0563	0.0539	0.3131
0.1732	0.0378	0.0425	0.0470	0.0509	0.0541	0.0563	0.0575	0.0575	0.0563	0.3670
0.1402	0.0330	0.0378	0.0425	0.0470	0.0509	0.0541	0.0563	0.0575	0.0575	0.4233
0.1119	0.0283	0.0330	0.0378	0.0425	0.0470	0.0509	0.0541	0.0563	0.0575	0.4808
0.0881	0.0238	0.0283	0.0330	0.0378	0.0425	0.0470	0.0509	0.0541	0.0563	0.5383
0.0684	0.0197	0.0238	0.0283	0.0330	0.0378	0.0425	0.0470	0.0509	0.0541	0.5946
0.0523	0.0160	0.0197	0.0238	0.0283	0.0330	0.0378	0.0425	0.0470	0.0509	0.6487

Figure 6.3. Transition matrix P.

Solving the matrix equation of theorem 3 yields the function f_I, to which the sequence f_{I1}, f_{I2}, f_{I3}, ... converges pointwise. Figure 4 shows a graph of this function.
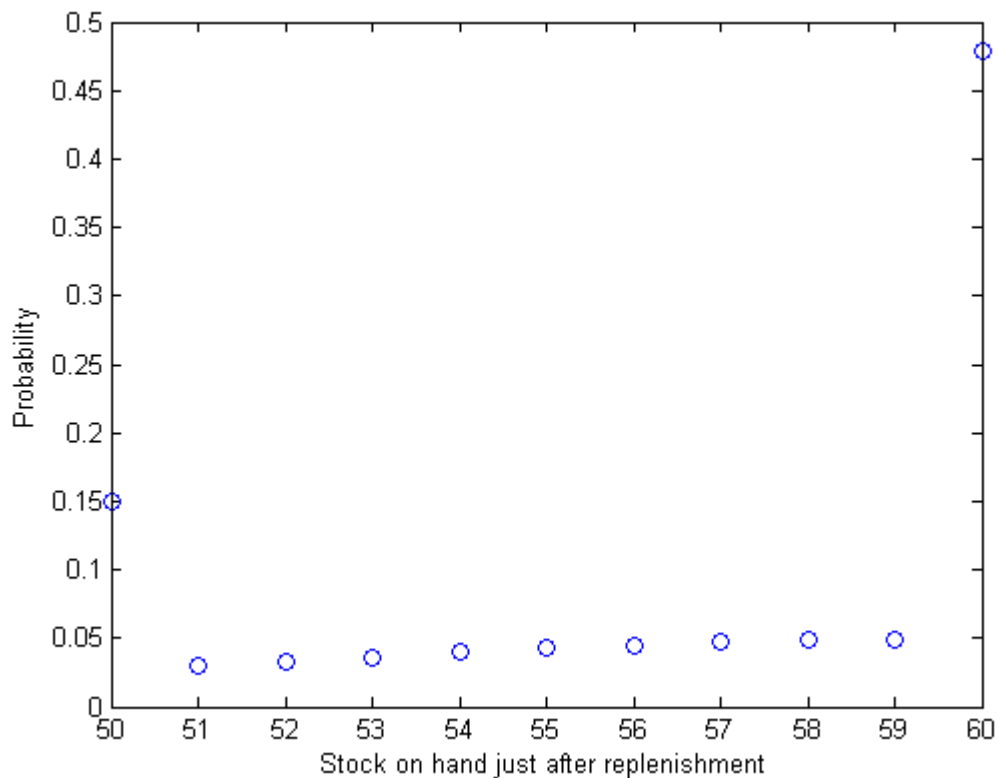
Figure 6.4. The stationary distribution f_I of the Markov chain $I_1, I_2, I_3, ...$

Application of theorem 4 gives us the fill rate: 0.9877 (rounded to an integer multiple of 0.0001). This is smaller than the fill rate for the uncapacitated periodic review inventory system with $s=60$ (0.9970), and greater than the uncapacitated periodic review inventory system with $s=50$ (0.9606).

6.3.3. The fill rate as a function of the expectation value of the review period demand, the replenishment capacity and the order-up-to level for Poisson distributed review period demand.
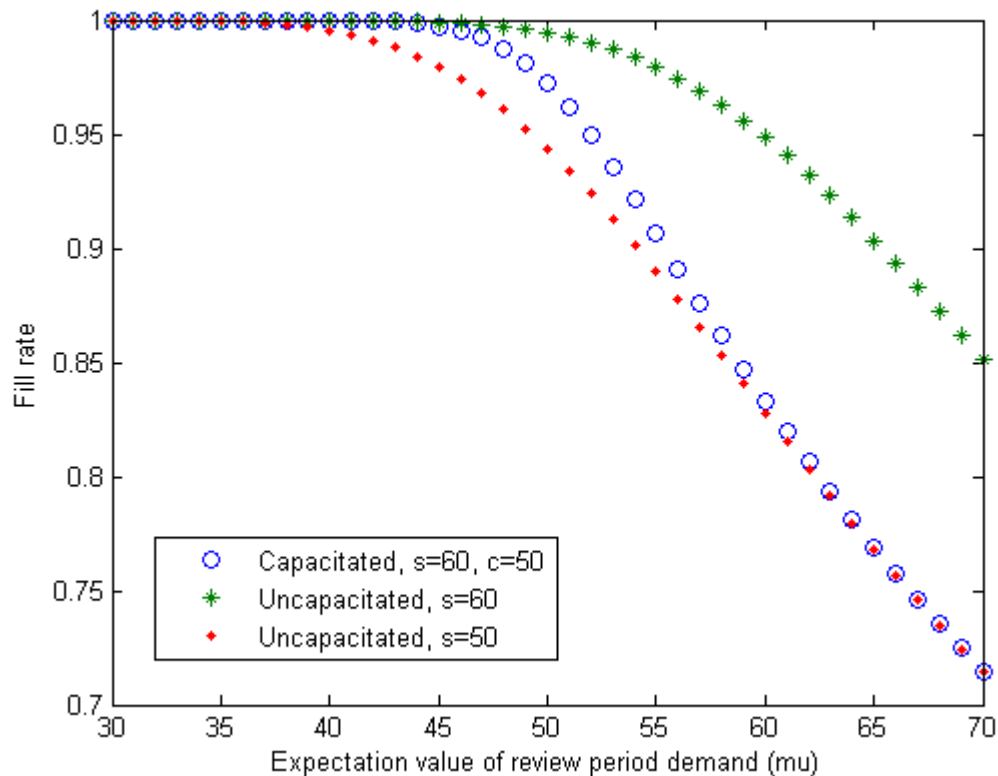
Figure 6.5. The fill rate as a function of the expectation value of the review period demand.

Figure 5 shows the fill rate of a capacitated periodic review inventory system as a function of the expectation value of the review period demand for Poisson distributed review period demand, for a constant order-up-to level (s=60) and a constant replenishment capacity (c=50). We compare this capacitated system with two uncapacitated systems: one with order-up-to level 60 and one with order-up-to level 50. For small values of the mean review period demand the fill rate of the capacitated system comes close to the fill rate of the first uncapacitated system in figure 5 and for large values of the mean review period demand the fill rate of the capacitated system comes close to the fill rate of the second uncapacitated system.
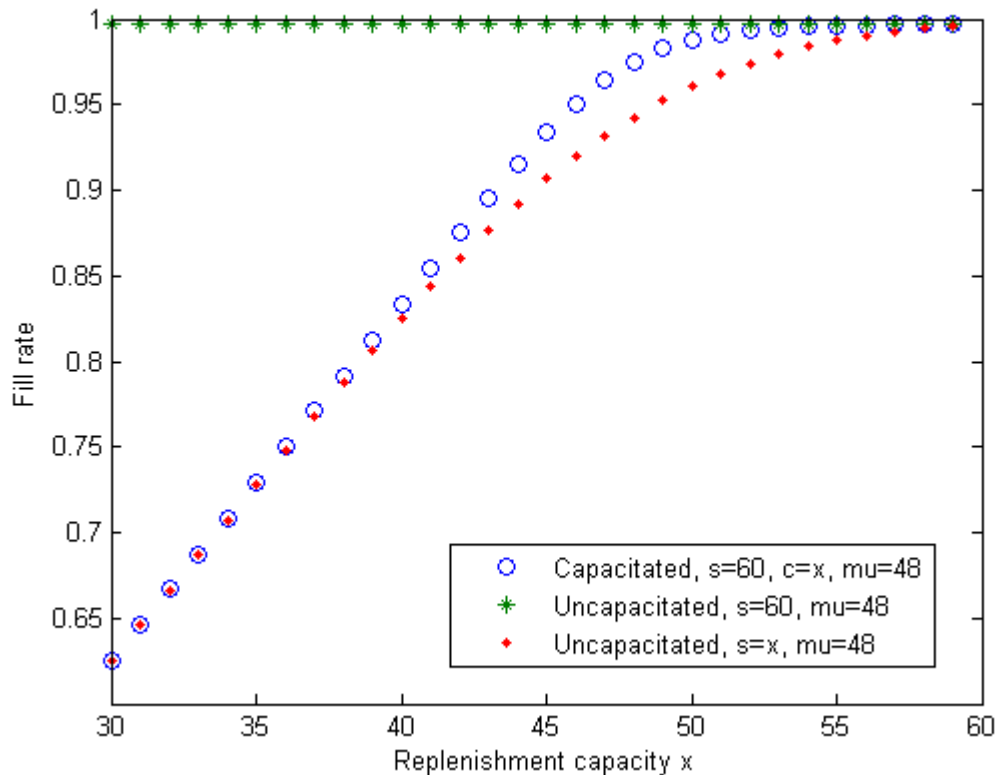
Figure 6.6. The fill rate as a function of the replenishment capacity.

Figure 6 shows the fill rate of a capacitated periodic review inventory system as a function of the replenishment capacity for a constant order-up-to level (s=60) and a constant review period demand distribution (Poisson distributed with mean μ=48). This capacitated system is compared with two uncapacitated systems: one with order-up-to level 60 and one with order-up-to level equal to the replenishment capacity of the capacitated system. For small values of the capacity of the capacitated system the fill rate of the capacitated system comes close to the fill rate of the second uncapacitated system in figure 6 and for large values of the capacity of the capacitated system the fill rate of the capacitated system comes close to the fill rate of the first uncapacitated system.
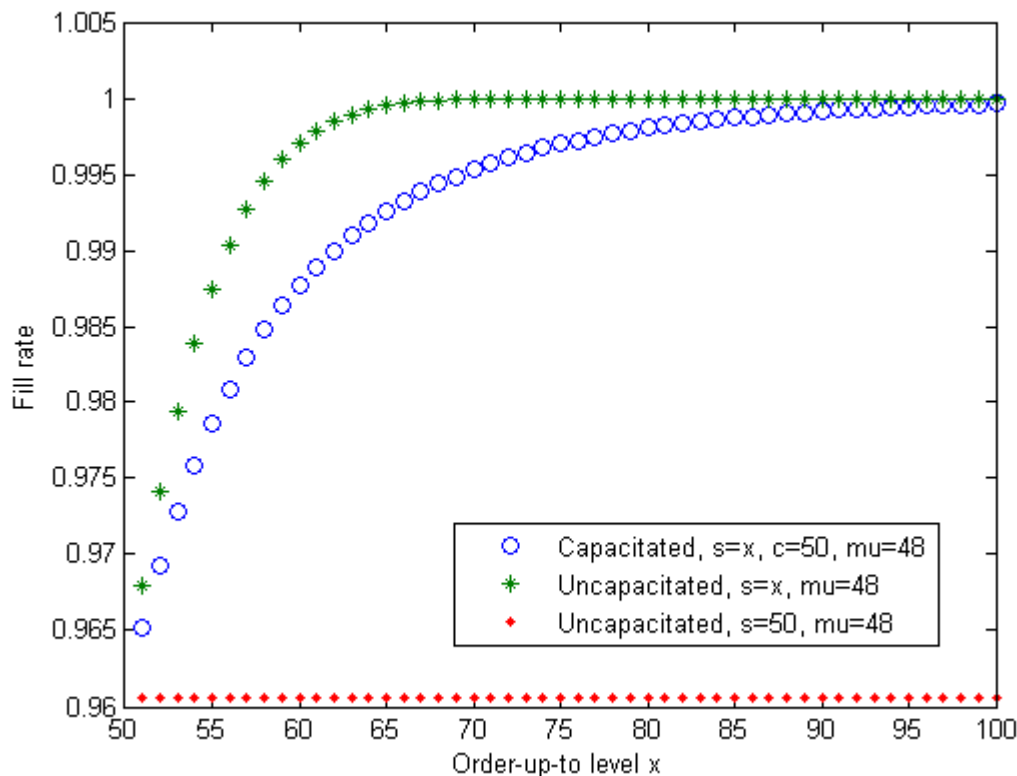
Figure 6.7. The fill rate as a function of the order-up-to level.

Figure 7 shows the fill rate of a capacitated periodic review inventory system with Poisson distributed review period demand (μ=48) as a function of the order-up-to level for a constant replenishment capacity (c=50). This capacitated system is compared with two uncapacitated systems: one with order up to level equal to the order-up-to level of the capacitated system, one with order-up-to level 50. In figure 7 the fill rate of the uncapacitated system (for order-up-to levels greater than the capacity) is smaller than the fill rate of the first uncapacitated system and larger than the fill rate of the second uncapacitated system.

6.4. Conclusion and further research

We proved four theorems that allow us to determine the fill rate of every periodic review order-up-to inventory system with capacitated replenishments, lost sales and zero lead time, for any demand probability mass function. The method is exact given the used definitions and assumptions. Extensions of this research are the study of capacitated periodic review inventory systems with positive lead times and the study of a series of periodic review inventory systems with joint capacitated replenishments. Allowing the lead time to be positive increases the complexity. For example, if the positive lead time is smaller than the review period, the stock level at the beginning of review period t will depend on the stock level at the beginning of review period t-1, the demand during review period t-1 and the demand during review period t-1 before replenishment, for the zero lead time case, see (1), the stock level at the beginning of review period t depends only on the stock level at the beginning of review period t-1 and the demand during review period t-1. If the lead time is greater than the review

period but not greater than two review periods, the stock level at the beginning of review period t will depend also on the stock level at the beginning of review period t-2, therefore the Markov chain will have order 2, for the zero lead time case, the order is 1.

References

[1] M.E. Johnson, H.L. Lee, T. Davis, R. Hall, Expressions for item fill rates in periodic inventory systems, Naval Research Logistics 42 (1995) 57-80.

[2] M.J. Sobel, Fill rates of single-stage and multistage supply systems, Manufacturing and Service Operations Management 6 (1) (2004) 41-52.

[3] J. Zhang, J. Zhang, Fill rate of single-stage general periodic review inventory systems, Operations Research Letters 35 (4) (2007) 503-509.

[4] E.A. Silver, D.P. Bischak, The exact fill rate in a periodic review base stock system under normally distributed demand, Omega International Journal of Management Science 39 (2011) 346-349.

[5] E. Guijarro, M. Cardos, E. Babiloni, On the exact calculation of the fill rate in a periodic review inventory policy under discrete demand patterns, European Journal of Operational Research 218 (2012) 442-447.

[6] J. Chen, D.K.J. Lin, D.J. Thomas, On the single item fill rate for a finite horizon, Operations Research Letters 31 (2) (2003) 119-123.

[7] A. Banerjee, A. Paul, Average fill rate and horizon length, Operations Research Letters 33 (5) (2005) 525-530.

[8] J. Mapes, The effect of capacity limitations on safety stock, International Journal of Operations & Production Management 13 (10) (1993) 26-33.

[9] R.H. Teunter, Note on the fill rate of single-stage general periodic review inventory systems, Operations Research Letters 37 (1) (2009) 67-68.

[10] R.G. Gallager, Discrete stochastic processes, Kluwer Academic Publishers, Boston, 1996.

7

Inventory management, capacitated replenishment and customer satisfaction: a search for the optimal order-up-to level

Abstract

This paper studies periodic review inventory systems in which replenishments are capacitated. This capacity restriction implies that the order-up-to level may not always be reached at each replenishment, such that additional safety stock is needed to achieve the same service level as in the uncapacitated case considered in the literature so far. To determine the required level of safety stock, and hence the order-up-to level, an iterative procedure is proposed. A computational experiment is reported that illustrates both the impact of a restricted replenishment capacity on the required safety stock level, and the effectiveness of the proposed iterative method at determining this.

Keywords: inventory, periodic review, order-up-to level, capacitated replenishment, lost sales.

7.1 Introduction and problem description

In this paper, we consider periodic review inventory systems with capacitated replenishments. In such systems, inventory is checked at regular periodic intervals (e.g. once per day or week) and a replenishment order is placed to raise the inventory level to a specified threshold, called the order-up-to level. This order-up-level is supposed to cover demand until the next replenishment and therefore consists of the average (or expected) demand during an interval, plus safety stock to buffer demand and supply uncertainties. The amount of safety stock is chosen such that a predefined fill rate (i.e. the proportion of demand satisfied directly from inventory) is obtained.

The contribution of this paper consists of developing a procedure for taking replenishment capacity into account in calculating the required safety stock. Because of the capacity limitation, it may be impossible in some periods to replenish inventory all the way up to the order-up-to level, and inventory will be slightly below this level. As a result, the probability of running out of stock in the next period is increased, and service level may decrease.

Therefore, a capacity restriction on the replenishment capacity will lead to an increase in the required safety stock level to achieve the same service level.

Mapes [1] was the first to write on the effect of capacity restrictions on the required level of safety stocks. To illustrate the effect, and also to determine the required safety stock level for a given service level, he reverts to extensive simulations. We will also make use of simulation experiments in this paper, but we will also come up with an analytical method to determine the required safety stock level for a given situation. To the best of our knowledge, we are the first to use an analytical rather than a simulation approach to this problem. Some articles such as [2], [3] and [4], deal with capacitated replenishment but in fact focus on other problems.

The remainder of this paper is organized as follows. An iterative procedure is proposed in Section 2 for determining the safety stock level in a capacitated periodic review inventory system, followed by an illustrative example. Section 3 presents a computational experiment and Section 4, finally, gives the conclusions and further research suggestions.

7.2 Solution approaches

In this section we will calculate the fill rate (*fr*) of a periodic review inventory system with restricted replenishment capacity. The fill rate is the proportion of the expected satisfied demand per replenishment cycle to the expected demand per replenishment cycle. Afterwards we propose a method for finding the order-up-to-level given a required fill rate.

7.2.1 Calculation of the probability distribution of the inventory level just after replenishment

We assume that the probability function of the demand during a replenishment cycle is given and denoted by *f*. We assume that the possible values of the inventory level are equidistant. If for example the distance between two neighboring possible values of the inventory level is ε then the probability that the demand is *d* during one cycle is $\int_{d-\varepsilon/2}^{d+\varepsilon/2} f(x)\,\mathrm{d}x$. We use *L* for the order-up-to level and *Q* for the replenishment capacity. We assume that *Q<L*.

We assume that there are *n*-2 possible values of the inventory level between *Q* and *L*: $Q+\varepsilon$, $Q+2\varepsilon$, ..., *L*-ε with $\varepsilon=(L-Q)/(n-1)$.

If we use $P_I(Q+i\varepsilon)$ for the probability that the inventory level just after replenishment is $Q+i\varepsilon$ and $P(Q+i\varepsilon|Q+j\varepsilon)$ for the probability that the inventory level just after replenishment is $Q+i\varepsilon$ given that the inventory level just after the previous replenishment was $Q+j\varepsilon$ (*i, j* $\in$ {0,1,...,n-1}), we find:

$$
\begin{cases}
P_I(Q) = P(Q|Q)P_I(Q) + P(Q|Q+\epsilon)P_I(Q+\epsilon) + P(Q|Q+2\epsilon)P_I(Q+2\epsilon) + ... + P(Q|L)P_I(L) \\
P_I(Q+\epsilon) = P(Q+\epsilon|Q)P_I(Q) + P(Q+\epsilon|Q+\epsilon)P_I(Q+\epsilon) + P(Q+\epsilon|Q+2\epsilon)P_I(Q+2\epsilon) + ... + P(Q+\epsilon|L)P_I(L) \\
... \\
P_I(L) = P(L|Q)P_I(Q) + P(L|Q+\epsilon)P_I(Q+\epsilon) + P(L|Q+2\epsilon)P_I(Q+2\epsilon) + ... + P(L|L)P_I(L) \\
1 = P_I(Q) + P_I(Q+\epsilon) + P_I(Q+2\epsilon) + ... + P_I(L)
\end{cases}
$$

$$(1)$$

Chapter 7
Inventory management, capacitated replenishment and customer satisfaction: a search for the
optimal order-up-to level

In matrix notation this becomes:

$$
\begin{bmatrix}
P_I(Q) \\
P_I(Q+\epsilon) \\
... \\
P_I(L) \\
1
\end{bmatrix}
=
\begin{bmatrix}
P(Q|Q) & P(Q|Q+\epsilon) & ... & P(Q|L) \\
P(Q+\epsilon|Q) & P(Q+\epsilon|Q+\epsilon) & ... & P(Q+\epsilon|L) \\
... & ... & ... & ... \\
P(L|Q) & P(L|Q+\epsilon) & ... & P(L|L) \\
1 & 1 & ... & 1
\end{bmatrix}
\begin{bmatrix}
P_I(Q) \\
P_I(Q+\epsilon) \\
... \\
P_I(L)
\end{bmatrix}
\tag{2}
$$

Which is equivalent to:

$$
\begin{bmatrix}
0 \\
0 \\
... \\
0 \\
1
\end{bmatrix}
=
\begin{bmatrix}
P(Q|Q)-1 & P(Q|Q+\epsilon) & ... & P(Q|L) \\
P(Q+\epsilon|Q) & P(Q+\epsilon|Q+\epsilon)-1 & ... & P(Q+\epsilon|L) \\
... & ... & ... & ... \\
P(L|Q) & P(L|Q+\epsilon) & ... & P(L|L)-1 \\
1 & 1 & ... & 1
\end{bmatrix}
\begin{bmatrix}
P_I(Q) \\
P_I(Q+\epsilon) \\
... \\
P_I(L)
\end{bmatrix}
\tag{3}
$$

The first equation is a linear combination of the others except the last one because the sum of all the equations except the last one is $0 = 0$. Therefore we can drop the first equation. With the last equation first this becomes:

$$
\begin{bmatrix}
1 \\
0 \\
... \\
0
\end{bmatrix}
=
\begin{bmatrix}
1 & 1 & ... & 1 \\
P(Q+\epsilon|Q) & P(Q+\epsilon|Q+\epsilon)-1 & ... & P(Q+\epsilon|L) \\
... & ... & ... & ... \\
P(L|Q) & P(L|Q+\epsilon) & ... & P(L|L)-1
\end{bmatrix}
\begin{bmatrix}
P_I(Q) \\
P_I(Q+\epsilon) \\
... \\
P_I(L)
\end{bmatrix}
\tag{4}
$$

This is a system of n linear equations and n variables.
Now we calculate $P(Q+i\varepsilon|Q+j\varepsilon)$ with $Q+i\varepsilon \leq L$ and $Q+j\varepsilon \leq L$:

- if $i \neq 0$ and $Q+i\varepsilon \neq L$:

$$
P(Q+i\epsilon|Q+j\epsilon) = \int_{Q+j\epsilon-i\epsilon-\epsilon/2}^{Q+j\epsilon-i\epsilon+\epsilon/2} f(x)\,dx
\tag{5}
$$

- if $Q+i\varepsilon = L$:

$$
P(L|Q+j\epsilon) = \int_{-\infty}^{2Q-L+j\epsilon+\epsilon/2} f(x)\,dx
\tag{6}
$$

We can now solve the system of linear equations and obtain the probability distribution of the inventory level just after replenishment.

Chapter 7
Inventory management, capacitated replenishment and customer satisfaction: a search for the
optimal order-up-to level

7.2.2 Formula for the fill rate

The expected satisfied demand per replenishment cycle is the expected demand per replenishment cycle $E(D)$ minus the expected shortage per replenishment cycle ESC. Therefore we find for the fill rate:

$$fr = \frac{E(D) - ESC}{E(D)} \tag{7}$$

Which is equivalent with:

$$fr = 1 - \frac{ESC}{E(D)} \tag{8}$$

If we use ESC_i for the expected shortage per replenishment cycle that starts with an inventory level of $Q+i\varepsilon$ just after replenishment, we find:

$$ESC_i = \int_{Q+i\epsilon}^{\infty} f(x)(x - Q - i\epsilon)\,\mathrm{d}x \tag{9}$$

For the expected shortage per replenishment cycle we find:

$$ESC = \sum_{i=0}^{n-1} P_I(Q + i\epsilon)ESC_i \tag{10}$$

The expected demand per replenishment cycle is:

$$E(D) = \int_{-\infty}^{\infty} f(x)x\,\mathrm{d}x \tag{11}$$

Using equation 8, 9, 10 and 11 we find for fr:

$$fr = 1 - \frac{\sum_{i=0}^{n-1} P_I(Q + i\epsilon) \int_{Q+i\epsilon}^{\infty} f(x)(x - Q - i\epsilon)\,\mathrm{d}x}{\int_{-\infty}^{\infty} f(x)x\,\mathrm{d}x} \tag{12}$$

In the case that the replenishment capacity is not restricted or $Q \geq L$, the expected shortage per replenishment cycle becomes:

$$ESC = \int_{L}^{\infty} f(x)(x - L)\,\mathrm{d}x \tag{13}$$

Chapter 7
Inventory management, capacitated replenishment and customer satisfaction: a search for the
optimal order-up-to level

and the fill rate:

$$fr = 1 - \frac{\int\limits_{L}^{\infty} f(x)(x - L)\,\mathrm{d}x}{\int\limits_{-\infty}^{\infty} f(x)x\,\mathrm{d}x} \tag{14}$$

7.2.3 Example

In this section we will calculate the fill rate for a periodic review inventory system with restricted replenishment capacity with $Q = 110$, $L = 120$, $\varepsilon = 1$ and f as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{15}$$

with $\mu = 100$ and $\sigma = 20$.

Because of the values of Q, L and ε, $n = 11$. Equation (4) becomes thus a linear system of 11 equations with 11 variables. By solving it we obtain the probability distribution of the inventory level just after replenishment. Figure 1 is the graphical representation of this solution.
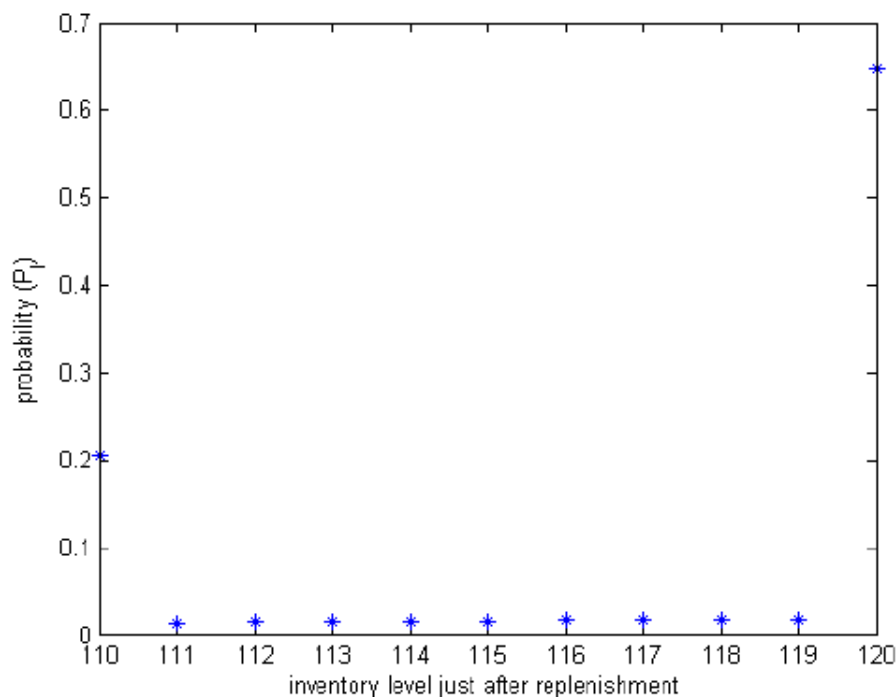
Figure 7.1. Graphical representation of the probability distribution of the inventory level just after replenishment for $n = 11$.

For the fill rate we find:

Chapter 7
Inventory management, capacitated replenishment and customer satisfaction: a search for the
optimal order-up-to level

$$fr = 1 - \frac{\sum\limits_{i=0}^{10} P_I(Q + i\epsilon) \int\limits_{Q+i\epsilon}^{\infty} f(x)(x - Q - i\epsilon)\,\mathrm{d}x}{\int\limits_{-\infty}^{\infty} f(x)x\,\mathrm{d}x} \tag{16}$$

$$= 1 - \frac{\sum\limits_{i=0}^{10} P_I(Q + i\epsilon) \int\limits_{Q+i\epsilon}^{\infty} f(x)(x - Q - i\epsilon)\,\mathrm{d}x}{\mu} \tag{17}$$

$$\approx 0.9772122 \tag{18}$$

7.2.4 Finding the order-up-to level for a given required fill rate

In this section, we want to find the minimal value of the order-up-to level, denoted L_{min} that results in a predetermined, required fill rate, denoted fr^*. We can do this iteratively by using the bisection method.

Suppose we search the minimal order up to level L_{min} that results in a fill rate of at least $fr^* = 0.99$ for the previous example. Table 1 shows the different values of L that are evaluated during the bisection method, together with the resulting fill rates. The bisection method starts with the interval [120,140] and iteratively reduces the interval, rounding to integer values where necessary. The conclusion is that 135 is the smallest value of L that gives a fill rate of at least 99%: $L_{min} = 135$.

Table 7.1. Illustration of the bisection method for finding L_{min}.

L	fr
120	0.9772
140	0.9924
130	0.9870
135	0.9901
133	0.9889
134	0.9895

7.3 Computational experiment

In order to efficiently carry out the computational experiments, the procedure described above has been implemented and executed in the Matlab software environment, version 7.5.0 (R2007b) on a computer with 2.4 GHz Intel Core i5 520M (dual-core) processor and 4 GB RAM.

To assess the performance of the method presented above, we return to the illustrative example. This example, where demand per cycle is normally distributed with $\mu = 100$ and $\sigma =$

Chapter 7
Inventory management, capacitated replenishment and customer satisfaction: a search for the
optimal order-up-to level

20, has a replenishment capacity $Q = 110$ and the parameter ε is 1. For the order-up-to level varying from $L = 111$ (= $Q+1$) to $L = 170$, the resulting fill rate was determined in two different ways: using our procedure and using Monte-Carlo simulation as in Mapes [1993]. In the simulation runs, 100,000 cycles were simulated. Figure 2 shows the fill rate as a function of the order-up-to level for uncapacitated and capacitated replenishment. Notice how close the graph of the simulation and the graph of our method are to each other. We can see that for increasing fill rates the difference between the order-op-to levels for the uncapacitated and capacitated case increases. Table 2 shows the fill rates and calculation times for some order-up-to levels. We see that our procedure is faster but the calculation time increases faster with increasing order-up-to levels.
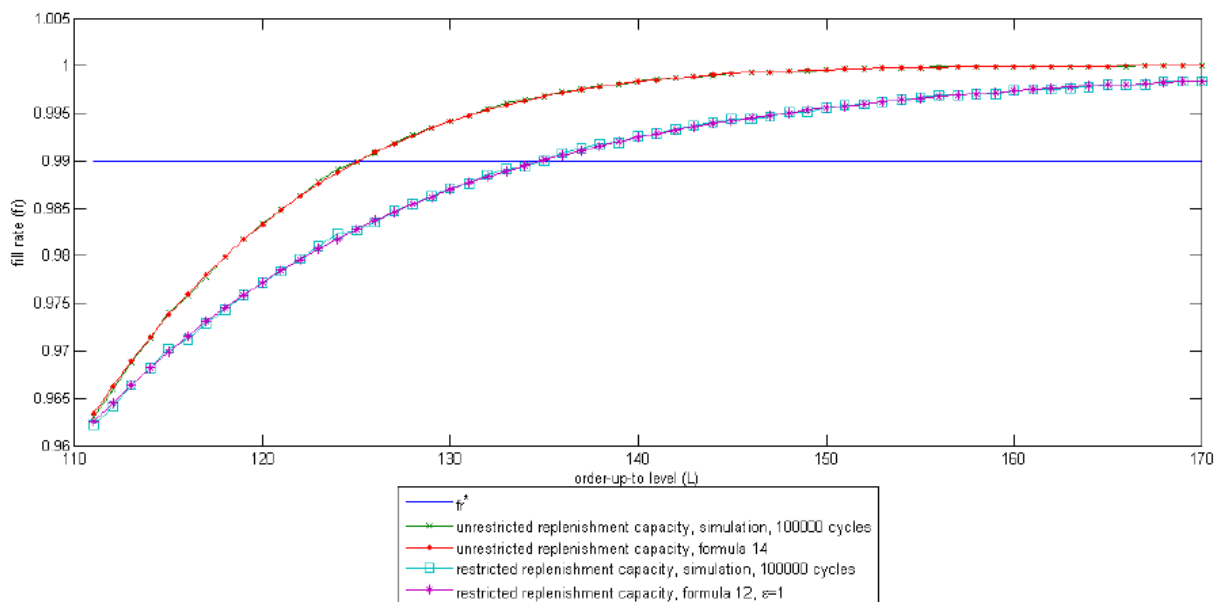
Figure 7.2. The fill rate as a function of the order-up-to level in the case of simulation and the method developed in section 2.

Table 7.2. Fill rates and calculation times for various order-up-to levels in the case of restricted replenishment capacities.

	Simulation		Analytical approach	
L	fr	time [s]	fr	time [s]
120	0.9774	1.9271	0.9772	0.0196
130	0.9870	1.9497	0.9870	0.0471
140	0.9924	1.9558	0.9924	0.0888
150	0.9955	1.9296	0.9956	0.1491
160	0.9974	1.9457	0.9974	0.2239
170	0.9984	1.9512	0.9984	0.3066

7.4 Conclusion and further research

When replenishments are capacitated, the order-up-to level in a periodic review inventory system has to be raised above the level suggested by uncapacitated inventory replenishment models. To the best of our knowledge, this paper is the first to offer a method different from simulation for determining the required order-up-to level for a given fill rate and for determining the fill rate for a given order-up-to level.

The current approach assumes that sales are lost when there is a stockout. Further research could focus among others on allowing demand to be backlogged during stockouts and replenishing multiple customers in a single distribution route (see e.g. [5]).

References

[1] J. Mapes, The effect of capacity limitations on safety stock, International Journal of Operations & Production Management 13 (10) (1993) 26-33.

[2] R. Ernst, D.F. Pyke, Optimal base stock policies and truck capacity in a two-echelon system, Naval Research Logistics 40 (7) (1993) 879-903.

[3] M. Kijima, T. Takimoto, A (T,S) inventory/production system with limited production capacity and uncertain demands, Operations Research Letters 25 (2) (1999) 67-79.

[4] C. Sitompul, E.-H. Aghezzaf, W. Dullaert, H. van Landeghem, Safety stock placement problem in capacitated supply chains, International Journal of Production Research 46 (17) (2008) 4709-4727.

[5] B. Raa, E.-H. Aghezzaf, A practical solution approach for the cyclic inventory routing problem, European Journal of Operational Research 192 (2) (2009) 429-441.

8

Inventory management, capacitated replenishment, customer satisfaction and joint transportation

Based on:
- T. Dubois, F. Witlox, G. Allaert, A periodic review inventory policy for joint capacitated replenishments, in: G. Gürsoy, M. Tanyas (eds.), 10th International Logistics and Supply Chain Congress 2012 Proceedings, 2012, pp. 641-646.
- T. Dubois, B. Raa, W. Dullaert, Managing multiple inventories with a shared restricted replenishment capacity, in: F. Ülengin (ed.), 32nd National Operations Research and Industrial Engineering Congress Proceedings, 2012, p. 226.

Abstract

In this article we consider multiple inventories with joint capacitated replenishments, for example multiple retailers replenished by one truck. The demands are considered stochastic with known probability distributions. A periodic review inventory policy is introduced in this article with the objective of guaranteeing given fill rates for the inventories. The order-up-to levels of the inventories are determined by an algorithm that simulates a large number of replenishment cycles. This solution method is illustrated with some computational experiments in which we find that sharing the capacity gives lower order-up-to levels than dividing the capacity into individual capacities.

Keywords: capacitated replenishment, fill rate, inventory management, joint replenishment.

8.1. Problem description

We consider n inventory systems (e.g. retailers) which all apply periodic review inventory policies with review at the same moment: at regular periodic intervals the inventory levels of all retailers are checked and replenishment orders are placed to raise the inventory levels to certain fixed values, the order-up-to levels ($L_1, L_2, ..., L_n$). The probability distribution f_i of the demand during a replenishment cycle of each retailer i is given. So is the required fill rate fr^*_i (the fill rate is the proportion of demand that is satisfied directly from inventory) of each retailer. The total amount of replenishment is capacitated, e.g.: the supplier uses one truck to replenish all retailers. If the sum of all order amounts is greater then the replenishment capacity Q (the truck capacity), fair share is applied: each retailer i receives a part of Q proportionate to $L_i - I_i$, with I_i the inventory level of retailer i just before replenishment. The research problem is to determine all order-up-to levels such that the fill rate fr_i of each retailer i is greater than or equal to the required fill rate fr^*_i of this retailer.

8.2. Literature

In this section we give an overview of related literature. This overview consists of three parts. The first part is the literature concerning a special case of the problem of this article, the case in which the number of inventories is one. In the second part we give literature about methods to divide scarce resources. In the third part we focus on literature about capacitated replenishments.

Mapes (1993) [1] was the first one to publish about the effect of capacity restrictions on the order-up-to level. He considered only one inventory system and used simulation as solution method. In Dubois et al. (2011) [2] also only one inventory is considered but an analytical solution method is presented. In Raa et al. (2011) [3] we find a faster approximate solution method. Zhang et al. (2007) [4] also treats the same topic (also one inventory) but with uncapacitated replenishments.

When the total amount of ordered products is more than the replenishment capacity, the capacity has to be divided. There are a different methods to do this. We can change the dividing method easily by changing a small part of the program code. Cachon et al. (1999) [5] discuss three allocation schemes for dividing scarce capacity: proportional, linear and uniform allocation. In proportional allocation, which we use in this article, the proportion of the quantity a retailer receives to the total capacity is equal to the order of this retailer to the sum of all orders, if the sum of all orders is greater than the capacity. In linear allocation every retailer receives the ordered quantity minus the shortage divided by the number of retailers (if this quantity is smaller than zero for a retailer, this retailer receives nothing and the quantities for the other retailers are recalculated). Uniform allocation means that the capacity is divided equally among the retailers. If a retailer ordered less than this quantity, this retailer receives the ordered quantity and the remaining part of the capacity is divided equally among the remaining retailers.

In Levi et al. (2008) [6] capacitated replenishments are considered and orders are managed such that the total cost is minimized. We manage the orders such that the fill rates have their required value. Other differences with our research is that in [6] there can be a correlation between the demand probabilities of different replenishment cycles, backlogging is allowed, there can be a nonzero lead time between the time an order is placed and the time it arrives and only one inventory is considered. Also in Narayanan et al. (2010) [7] the total cost is minimized. That paper proposes heuristics for the coordinated capacitated lot-size problem and considers only deterministic demand.

8.3. The algorithm

In this section we present an algorithm to find all order-up-to levels such that the fill rate fr_i of each retailer i is greater than or equal to the corresponding required fill rate fr^*_i. In this algorithm we use the uncapacitated order-up-to levels as a starting point to find the order-up-

to levels. If there is no capacity restriction, following relations hold for the order-up-to level L_{i0} of retailer i:

$$fr_i = 1 - \frac{ESC_{i0}}{\mu_i} \tag{1}$$

$$ESC_{i0} = \int_{L_{i0}}^{\infty} (x - L_{i0}) f_i(x) dx \tag{2}$$

$$\mu_i = \int_{-\infty}^{\infty} x f_i(x) dx \tag{3}$$

With ESC_{i0} the expected shortage per cycle of retailer i and μ_i the expected demand of retailer i during one replenishment cycle. In Silver et al. (1998) [8], appendix C (pages 735-736) a very fast approximation is presented for calculating the order-up-to level given the required fill rate for the uncapacitated case.

The algorithm starts with setting all order-up-to levels to $L_i = L_{i0}$. Then we do while:
$$fr_1 < fr^*_1 \text{ or } fr_2 < fr^*_2 \text{ or ... or } fr_n < fr^*_n \tag{4}$$
the following loop:
1. Do a simulation: we determine the fill rates $fr_1, fr_2, ..., fr_n$ by simulating c replenishment cycles by generating random numbers consistent with the probability distributions $f_1, f_2, ... f_n$ for the demands during a cycle.
2. If $fr_i < fr^*_i + fr_{extra}$, then set the order-up-to level to $L_i = L_i + \alpha L_{i0}$, for $i = 1, 2, ..., n$. fr_{extra} is a parameter with value e.g. 0.05/100 and α is a parameter with value e.g. 0.05/100.

We can make this algorithm faster by first taking large steps for every retailer ($\alpha = \alpha_{large}$, e.g. 0.5/100). If a large step results in a fill rate larger than the required fill rate for that retailer, a small step ($\alpha = \alpha_{small}$, e.g. 0.05/100) is taken instead of the last large step for that retailer. From that moment on only small steps are taken for that retailer or no step when last simulation resulted in a fill rate larger than or equal to $fr^*_i + fr_{extra}$. We stop if for all retailers the steps are not large anymore and
$$fr_1 \geq fr^*_1 \text{ and } fr_2 \geq fr^*_2 \text{ and ... and } fr_n \geq fr^*_n. \tag{5}$$

8.4. First illustration: order-up-to level as a function of the number of retailers

In this section we present some results obtained by using the algorithm of section 3. We consider n equivalent retailers, i.e. n retailers with identical probability distributions of the demand during a replenishment cycle ($f_1 = f_2 = ... = f_n = f$) and identical required fill rates ($fr^*_1 = fr^*_2 = ... = fr^*_n = fr^*$). For the probability distribution f of the demand during a replenishment cycle we use a normal distribution with mean $\mu = 1000$ and standard deviation $\sigma = 0.25 \times \mu$ and for the required fill rate we use $fr^* = 0.99$. For the algorithm parameters we choose the following values: $\alpha_{large} = 0.5/100$, $\alpha_{small} = 0.05/100$, $fr_{extra} = 0.05/100$ and $c = 40,000$. The replenishment capacity is dependent on the number of retailers: $Q = 1050 \times n$.

We will compare three order-up-to levels: L_{shared}, $L_{uncapacitated}$ and $L_{allocated}$. L_{shared} is the average of the order-up-to levels obtained by the algorithm of section 3 over all retailers.

$L_{uncapacitated}$ is the order-up-to level of one of the equivalent retailers in the case that there are no capacity restrictions. $L_{allocated}$ is the order-up-to level of one of the equivalent retailers in the case that no capacity is shared and every retailer has a capacity of Q/n. $L_{allocated}$ is independent of n because of the choice of Q. $L_{uncapacitated}$ is independent of n because in the uncapacitated case the retailers are independent of each other. $L_{allocated}$ can be calculated using [3] or [2].

Figure 1 shows L_{shared}, $L_{uncapacitated}$ and $L_{allocated}$ as a function of the number of retailers n. We see that L_{shared} decreases if the number of retailers increases. If there is only one retailer, the capacity plays a role when the demand is large. If there are multiple retailers with shared capacity, the capacity plays a role when the demand of most of the retailers is large. One demand being large is more probable than multiple specific demands being large at the same time. Therefore the order-up-to levels decrease if there are more retailers and thus more sharing.

If the number of retailers is one, L_{shared} and $L_{allocated}$ are solutions of the same problem resulting from different solution methods. Because of the approximate character of the algorithm L_{shared} and $L_{allocated}$ are not necessary equal in this case but nearby, like we see in the figure.
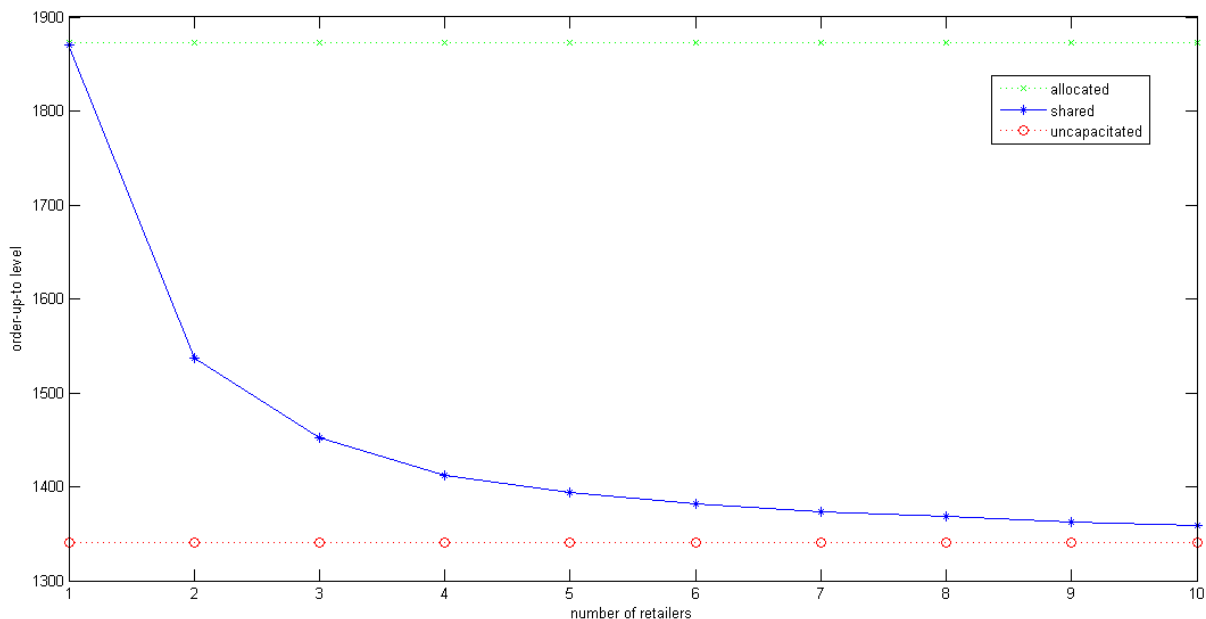
Figure 8.1. Shared, uncapacitated and allocated order-up-to level as a function of the number of retailers.

8.5. Second illustration: order-up-to level as a function of the replenishment capacity

In this section we will study the relation between the replenishment capacity Q and the order-up-to levels in the following two cases:

- The two retailers case: two retailers with required fill rates equal to fr^*=0.99 and normal distributions for the demand during a replenishment cycle with all means (μ_1 and μ_2) equal to 1000 and standard deviations σ_1=0.35×1000 and σ_2=0.15×1000 for retailers 1 and 2.
- The four retailers case: four retailers with required fill rates equal to fr^*=0.99 and normal distributions for the demand during a replenishment cycle with all means equal to 1000 and standard deviations σ_1=0.35×1000, σ_2=0.15×1000, σ_3=0.35×1000 and σ_4=0.15×1000 for retailers 1, 2, 3 and 4.

For the two retailers case we consider six different order-up-to levels: the order-up-to levels if the replenishment capacity is shared for the two retailers, the order-up-to levels if the replenishment capacity is allocated for the two retailers (see below for more details) and the uncapacitated order-up-to levels for the two retailers. For the four retailer case we get in a similar way twelve order-up-to levels which reduce to six because retailer 1 and 3 are equivalent and so are retailer 2 and 4.

The allocated order-up-to levels are the order-up-to levels if the replenishment capacity Q is not shared but every retailer i has a fixed part Q_i of the total capacity. One possible choice (method 1) is to choose each Q_i proportional to the uncapacitated order-up-to level. The disadvantage of this choice is that Q_i can be smaller than $fr^*\mu_i$, in that case the fill rate will be smaller than fr^* for every order-up-to level. Therefore we prefer to choose (method 2) each Q_i such that Q_i-$fr^*\mu_i$ is proportional to L_{i0}-$fr^*\mu_i$ (with L_{i0} the order-up-to level of retailer i in the uncapacitated case).

Figure 2 shows the allocated order-up-to levels of the two retailers of the two retailers case as a function of the replenishment capacity over the number of retailers for method 1 and 2. Because the uncapacitated order-up-to levels of retailer 1 and 2 are approximately 1529 and 1167, Q_1 and Q_2 have the following values for method 1:

$$Q_1 = \frac{1529}{1529+1167}Q \tag{6}$$

$$Q_2 = \frac{1167}{1529+1167}Q \tag{7}$$

Q_2 reaches the critical value of $fr^*\mu_2$ if Q satisfies the following equation:

$$\frac{1167}{1529+1167}Q = 0.99 \times 1000 \tag{8}$$

The solution of this equation is approximately $Q/2$=1144. Therefore smaller values of $Q/2$ give a fill rate smaller than fr^* for every order-up-to level. This is consistent with the graphs of method 1 beginning in $Q/2$=1150 in figure 2.

As an illustration we will calculate Q_1 and Q_2 for method 1 and 2 in the case of $Q/2$=1100. For method 1 we obtain for Q_1:

$$Q_1 = \frac{1529}{2696}2200 \tag{9}$$

$$Q_1 \approx 1248 \tag{10}$$

And for Q_2:

$$Q_2 = \frac{1167}{2696} 2200 \tag{11}$$

$$Q_1 \approx 952 \tag{12}$$

This is smaller than $fr^*\mu_2 = 990$. For method 2 we obtain for Q_1:

$$Q_1 = 990 + (2200 - 1980)\frac{1529 - 990}{2696 - 1980} \tag{13}$$

$$Q_1 \approx 1156 \tag{14}$$

And for Q_2:

$$Q_1 = 990 + (2200 - 1980)\frac{1167 - 990}{2696 - 1980} \tag{15}$$

$$Q_1 \approx 1044 \tag{16}$$

Q_1 and Q_2 of method 2 are both greater than 990.
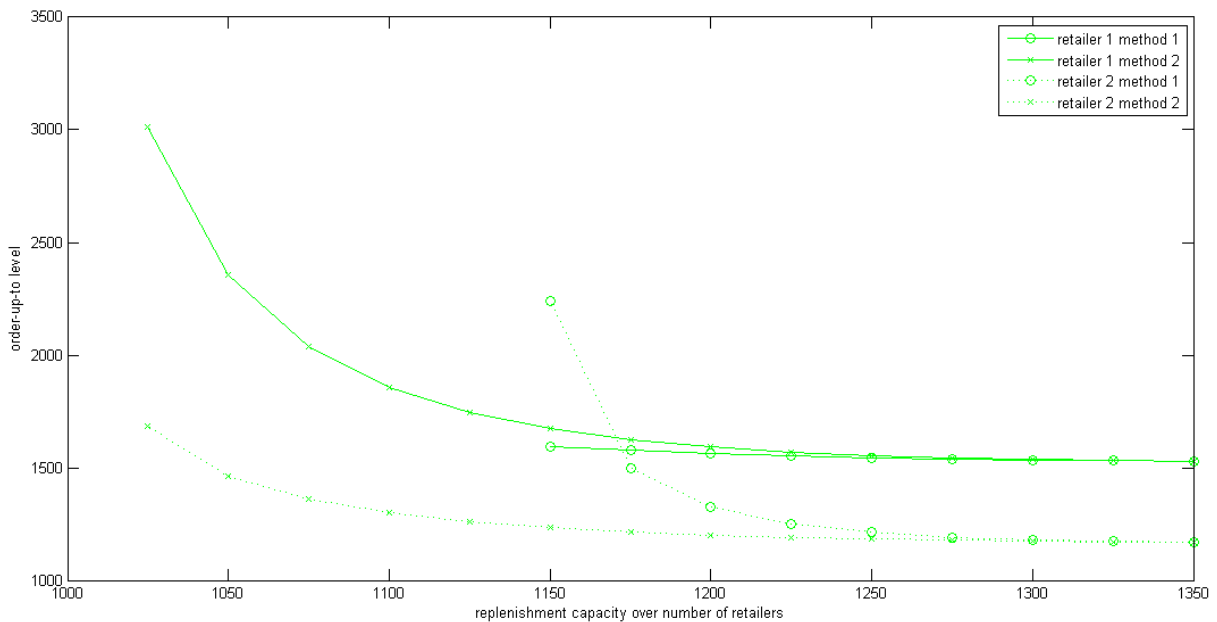
Figure 8.2. Allocated order-up-to levels of the two retailers as a function of the replenishment capacity over the number of retailers for method 1 and 2.

If we put the replenishment capacity over the number of retailers on the horizontal axis and the order-up-to level on the vertical axis the two times six order-up-to levels reduce to eight graphs because the allocated graphs of the two retailers case and the four retailers case coincide and the uncapacitated order-up-to levels of the two cases also coincide. Figure 3 shows these eight order-up-to levels. For the shared order-up-to levels we used the algorithm of section 3 with parameters: $\alpha_{large}=0.5/100$, $\alpha_{small}=0.05/100$, $fr_{extra}=0.05/100$ and $c=40,000$.
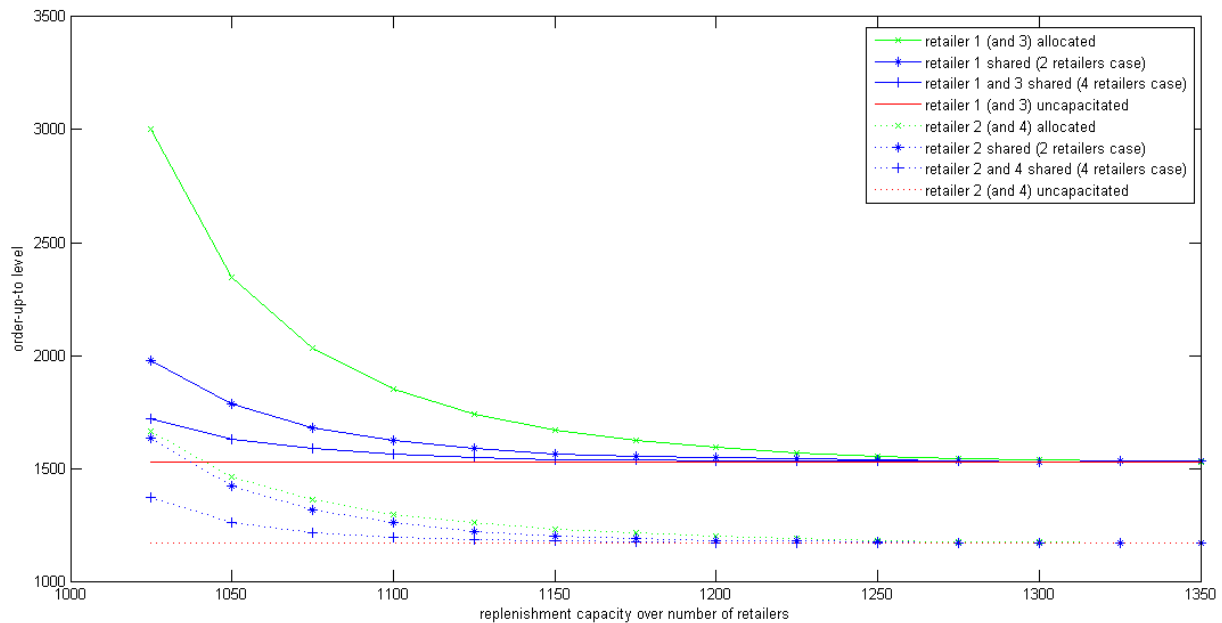
Figure 8.3. Shared and allocated (method 2) order-up-to levels as a function of the replenishment capacity over the number of retailers for the two retailers and the four retailers case.

We see that the retailers 1 and 3 have higher order-up-to levels than the retailers 2 and 4, this is because they have larger standard deviations and therefore need more safety stock. As expected the order-up-to levels decrease if the replenishment capacity increases. Using the formula of appendix C in [8] we find for the uncapacitated order-up-to level of retailer 1 and 2 approximately 1529 and 1167. Therefore larger replenishment capacities than 1348 (the average of 1529 and 1167) times the number of retailers results in all order-up-to levels being equal to the corresponding uncapacitated order-up-to levels. This is consistent with the convergence at the right side of the figure. Also notice the lower position of the graphs of the four retailers case relative to the graphs of the two retailers case. This is consistent with figure 1.

8.6. Conclusion

If there are multiple inventories with capacitated replenishments the situation is much more complex than if there is just one inventory. This reflects in simulation as solution method instead of an analytical solution. In this article an algorithm is proposed that generates approximate solutions. In the illustrations sharing the capacity resulted in clearly lower order-up-to levels than dividing the capacity into individual capacities.

References

[1] J. Mapes, The effect of capacity limitations on safety stock, International Journal of Operations & Production Management 13 (10) (1993) 26-33.

[2] T. Dubois, B. Raa, W. Dullaert, Taking Replenishment Capacity into Account in a Base Stock Inventory Policy, in: T. Notteboom (Ed.), Current Issues in Shipping, Ports and Logistics, Academic & Scientific Publishers, 2011.

[3] B. Raa, T. Dubois, W. Dullaert, Managing periodic review inventory systems with capacitated replenishments, working paper, 2011.

[4] J. Zhang, J. Zhang, Fill rate of single-stage general periodic review inventory systems, Operations Research Letters 35 (4) (2007) 503-509.

[5] G.P. Cachon, M.A. Lariviere, An equilibrium analysis of linear, proportional and uniform allocation of scarce capacity, IIE Transactions 31 (9) (1999) 835-849.

[6] R. Levi, R.O. Roundy, D.B. Shmoys, V.A. Truong, Approximation Algorithms for Capacitated Stochastic Inventory Control Models, Operations Research 56 (5) (2008) 1184-1199.

[7] A. Narayanan, P. Robinson, Efficient and effective heuristics for the coordinated capacitated lot-size problem, European Journal of Operational Research 203 (3) (2010) 583-592.

[8] E. Silver, D. Pyke, R. Peterson, Inventory Management and Production Planning and Scheduling, Wiley, 1998.

9

Urban distribution centres and joint transportation: Istanbul case study

Abstract

In this paper we study the possibility of road transshipment centres as an alternative to distribution systems with smaller distribution centres. We consider a road transshipment centre to be a large facility where goods from large supplier trucks are transferred to smaller city delivery trucks. We make a case study on the distribution system of the Asian side of Istanbul and we compare the sum of the transportation costs for six current distribution centres to the transportation cost for one road transshipment centre that supplies all customers of these six distribution centres. For estimating the transportation costs we assume the number of customers in a district to be proportional to the population of that district. We found that the fuel costs are lower for the road transshipment centre under study than for the six distribution centres under study.

Keywords: city logistics, transshipment centre, road transportation, distribution centre.

9.1. Introduction

Based on [1] we consider city logistics or urban logistics to be the study of the optimization of logistic activities in urban areas with the support of advanced information systems, considering the traffic environment, its congestion, safety and energy savings. We will study the effects of integrating several small distribution centres (DCs) to a large road transshipment centre (RTC), a large facility where goods from large supplier trucks are transferred to smaller city delivery trucks. The study of the optimization of city distribution is a part of urban logistics and is important for the decrease of urban problems such as traffic congestion, air pollution and traffic noise.

In [2] the impact of urban distribution centres is studied by comparing the situation before and after the start date of an urban distribution centre, this is done by determining the difference in vehicle miles travelled, the reduction in the average distance between drops, the reduction in the number of trips, the reduction in the number of deliveries per retailer, the reduction in total journey time, the increase in the load factor of vehicles, the increase of delivery weight

per drop, the reduction in the number of parking operations, the decrease of the total delivery parking time, the energy or fuel savings or the difference in CO_2 emissions.

This study [2] contains data for several case studies of urban distribution centres. For an urban distribution centre in Paris (La petite Reine) with start date 2003, the first 24 months the savings included 156000 km of diesel vans and 112 tons CO_2. For an urban distribution centre in Stockholm (start date 2000) there was an estimated 17% reduction in energy consumption and pollutant emissions and the vehicle miles travelled reduction was estimated to be 65% and for an urban distribution centre in Bristol (Broadmead) with start date 2004, there was a 68% reduction in number of trips.

We will estimate the difference in fuel costs with or without road transmission centre for a case study in Istanbul. First, in section 2 we will study the workflow and layout of road transshipment centres. Then, in section 3, we will calculate the total driving distance and the total fuel cost of delivery trucks and supplier trucks for two cases: for six distribution centres and for one road transshipment centre, all located in the Tuzla district of Istanbul. The distribution centre system is the current distribution system in Istanbul. We estimate the number of customers in a district by assuming that the number of costumers in a district is proportional to the population of that district.

9.2. Road transshipment centre and distribution centre workflow and layout

In this section we first give workflows of distribution centres and road transshipment centres and then we show a road transshipment centre layout. This section is based on interviews with logistic managers in Istanbul.

Distribution centre workflow
We describe the workflow of a distribution centre, based on interviews with logistic managers in Istanbul, as follows:
1. Products are brought to the distribution centre by supplier trucks. Imported products pass through customs. Most domestic products are shipped in bulk from production centres and factories.
2. Incoming products are cross-docked to delivery vehicles or stored depending on customer pre-orders.
3. While loading the delivery vehicles the condition of the products is checked visually. Sensitive and fragile products are controlled individually. Product quantities are checked and compared with customer orders. Products that are planned to be delivered first in the delivery round trip are loaded last.
4. The products that were cross-docked are distributed to the customers by the delivery vehicles in accordance with the used distribution strategy.
5. When orders come from customers, stored products are loaded to delivery vehicles and are distributed in accordance with the used distribution strategy. Product condition and product quantities are checked.
6. Distribution vehicles are parked in the parking place of the distribution centre at the end of the day.

Road transshipment centre workflow

We describe the workflow of a road transshipment centre, based on interviews with logistic managers in Istanbul, as follows:

1. Supplier trucks bring products to the road transshipment centre, located close to the city border. Imported products pass through customs. Most domestic products are shipped in bulk from production centres and factories.

2. Documents of incoming supplier trucks are checked and properties of the vehicles, incoming products and distribution conditions (e.g. refrigerated distribution) are entered in the RTC computer program. It is determined if the the products need to be cross-docked or stored. The truck driver receives a document.

3. Supplier trucks are directed to the cross-dock area or to the warehouses and the products are cross-docked to delivery vehicles or stored. Some delivery vehicles are also loaded from warehouses. The labour is supplied from the available workforce. The organization of the unloading and loading is based on the output of the RTC computer program.

4. While loading the delivery vehicles the condition of the products is checked visually. Sensitive and fragile products are controlled individually. Product quantities are checked and compared with customer orders. Products that are planned to be delivered first in the delivery round trip are loaded last.

5. The products are distributed to the customers by the delivery vehicles in accordance with the output of the RTC computer program regarding the distribution routes.

6. Distribution vehicles are parked in the parking place of the RTC when they are not used.

Road transshipment centres handle more products per day and a larger variety of products than distribution centres. RTCs also have more incoming and outgoing vehicles per day. Therefore the organization of RTCs is more complex and information systems are more important. Most RTCs are also per day longer operational than DCs. Some road transshipment centres use driver shift systems and make deliveries 24 hours per day.

Road transshipment centre layout

Figure 1 is a layout plan of a road transshipment centre that we made, it includes entry gates, parking places, warehouses and administrative and commercial buildings. The plan is based on interviews with logistics managers in Istanbul.

Figure 9.1. Road transshipment centre layout.

The transshipment ramps are the places where the cross-docking happens. The warehouses are for goods that are not immediately going from supplier truck to delivery truck. There are parking areas for the large supplier trucks and the smaller delivery trucks.

9.3. Istanbul case study

In this section we will compare the transportation costs for distributing goods from suppliers outside of Istanbul to customers in the Asian side of Istanbul for the cases: (i) distribution via several distribution centres (DCs), this is the current system and (ii) distribution via one road transshipment centre (RTC). More specifically we will study the fuel costs in the case of six distribution centres in the Tuzla district and one road transshipment centre in Tuzla. The Tuzla district is suitable for transshipment because of its location at the boundary of Istanbul and the inclusion of parts of the E-80 (TEM) road and the D-100 (E-5) road. We assume that the vehicles of the suppliers of the DCs and RTCs enter Istanbul from the Asian side via the E-80 road. The locations of the six DCs and the RTC are shown in figure 2. Transport from the DCs/RTC to the customers is done by small delivery trucks and up to ten customers are supplied in one round trip or milk run. Most transport from the suppliers to the DCs/RTC is done by semi-trailer trucks. We assume that the number of customers per workday is 100 for a DC and 600 for the RTC. We also assume that all goods are delivered to the DC/RTC customers the same day as they are delivered to the DC/RTC.

Figure 9.2. Map of the Asian side of Istanbul with different districts in different colours and with the locations of the six distribution centres and the road transshipment centre. Source: Istanbul Metropolitan Municipality website [3].

9.3.1. Delivery trucks fuel cost

Goods are delivered from the DCs/RTC to customers in the city with small delivery trucks. Because we have no data about the exact location of every customer, we use a method similar
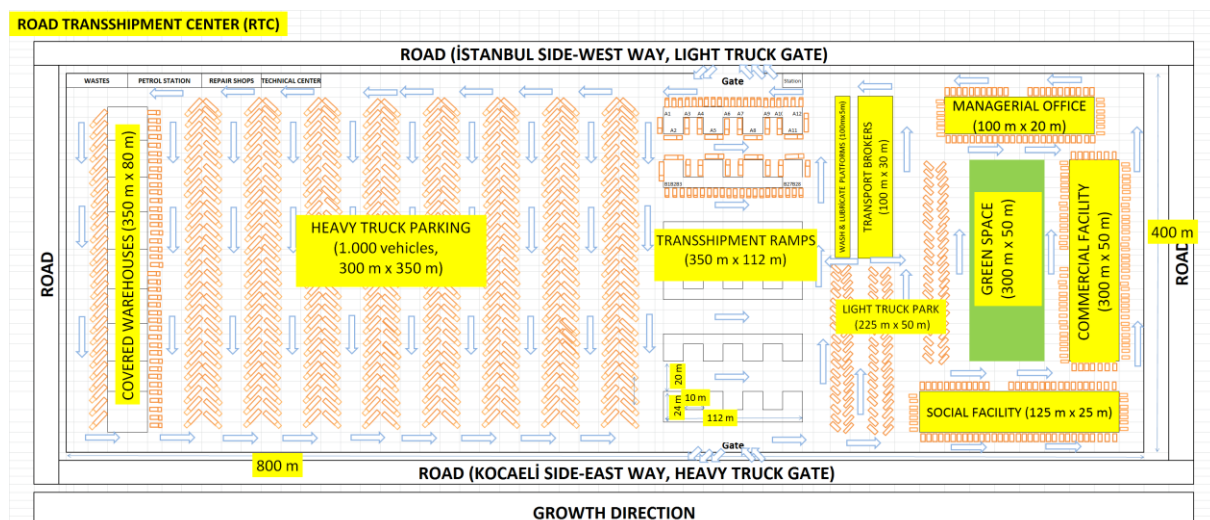
to [6], [7] and [8] for calculating the delivery trucks fuel cost: we divide the part of the city where the customers are located in districts, we determine one point for every district (district centre), we determine the number of customers per district by assuming that the district population is proportional to the number of customers in that district and we locate all customers in a district at the centre of that district.

We estimate the delivery truck fuel cost for the RTC and for the DCs in the following way:
1) We divide the Asian side of Istanbul into districts (for the district boundaries, we refer to [4] or figure 2). For every district we determine a location, which we call the district centre, by estimating the geometric centre of the populated area of that district. By assuming the number of customers per district to be proportional to the population of that district, we calculate the number of customers in every district. For the Adalar district (the islands), we locate the centre in the Bostanci neighbourhood of the Kadiköy district, which has a ferryboat connection with the islands. Table 1 (third and fourth column) shows the coordinates of all district centres of the Asian side of Istanbul. We use the same coordinate system as in the map of Istanbul on the site of the Istanbul Metropolitan Municipality [4] and we refer to coordinates according to this coordinate system as ibb-coordinates. The 2013 population of each district according to [5] is shown in the fifth column of table 1 and the number of RTC and DC customers per workday is shown for each district in the eighth and twelfth column of table 1. The ibb-coordinates of the DCs and the RTC are shown in the third and fourth column of table 2.
2) For the RTC:
We calculate the Euclidean distance between two points with coordinates (x,y) and (x',y') with the formula:
$$\sqrt{(x-x')^2+(y-y')^2} \; . \qquad\qquad (1)$$
The Euclidean distances between the districts and the RTC are shown in the seventh column of table 1. We estimate the number of milk runs per workday from the RTC to a district and back as the number of customers per workday of that district divided by ten, because we assume that one milk run includes ten deliveries. This number of milk runs is shown in the ninth column of table 1 per district (with the Adalar district together with the Kadiköy district and part of the Tuzla district in one milk run with the Sile district). We estimate the total length of a milk run as two times the Euclidean distance between the RTC and the district centre plus a correction term because not all customers are located in the district centre. As correction term we use for a district the value of the tenth column divided by the value of the ninth column. Then we multiply the result by 1.4 for estimating not the Euclidean distance but the road network travel distance, which we also call the driving distance. The factor 1.4 is based on the appendix of [7]. The eleventh column of table 1 shows the total length (using Euclidean distance) per workday of all RTC milk runs per district. The total RTC delivery trucks driving distance per workday is shown in the eight column of table 2: 3.8×10^3 km. For the fuel cost per workday, we multiply the driving distance per workday by 0.7 TL/km, based on interviews with logistics managers in Istanbul. This factor is the estimated fuel cost per km for a small delivery truck (average of with and without cargo) and this factor is similar to the factor used in [6], [7] and [8]: 0.6 TL/km. The estimates of table 2 concerning vehicle properties (the last three rows of table 2) are based on interviews with logistics managers in Istanbul. We estimate the total RTC delivery trucks fuel cost per workday to be 2.7×10^3 TL (table 2, column 10).

Table 9.1. Illustration of the fuel cost calculations for the road transshipment centre and the six distribution centres, part 1. Coordinates according to [4] and the source of the population data is [5].

Costumer locations

	District	District centre ibb-x-coordinate* [km]	District centre ibb-y-coordinate [km]	District population in 2013**	Percent of population Asian side Istanbul 2013	D_{oi}: Euclidean distance between district centre and road transshipment centre [km]	Number of road transshipment centre customers per workday	N_{ci}: number of road transshipment centre milk runs per workday (Kadiköy and Adalar together, part of Tuzla in milk run with Sile), 10 deliveries per milk run	D_i: Estimated length of the part of the milk run inside the district if only one road transshipment centre milk run delivers to the district [km]	Total length of milk run from road transshipment centre to district and back $(=2D_{io}+D_i/N_{ci})$ times the number of road transshipment centre milk runs to that district [km]	Number of distribution centre customers per workday
1	Ümraniye	426,3	4543,3	660125	13,21	27,0	79	8	4,0	433	13
2	Pendik	439,6	4530,7	646375	12,93	9,1	78	8	8,0	149	13
3	Üsküdar	420,7	4544,8	534636	10,70	32,0	64	6	4,5	415	11
4	Kadiköy	421,1	4539,2	506293	10,13	28,0	61	6	4,0	356	10
5	Maltepe	427,5	4534,1	471059	9,43	19,9	57	6	3,5	228	9
6	Kartal	432,7	4530,5	447110	8,95	13,5	54	5	3,0	148	9
7	Atasehir	426,2	4539,7	405974	8,12	24,5	49	5	4,0	243	8
8	Sultanbeyli	439,0	4537,5	309347	6,19	15,6	37	4	2,5	119	6
9	Sancaktepe	436,0	4540,8	304406	6,09	19,8	37	4	3,0	147	6
10	Beykoz	429,4	4554,9	248056	4,96	35,3	30	3	7,5	218	5
11	Tuzla	444,7	4527,2	208807	4,18	4,7	25	3	9,5	33	4
12	Çekmeköy	434,9	4547,2	207476	4,15	26,1	25	2	5,5	103	4
13	Sile	469,5	4555,2	31718	0,63	41,6	4	1	15,5	99	1
14	Adalar	423,8	4535,8	16166	0,32	23,9	2				0
				Total:	Total:		Total:	Total:		Total:	Total:
				4997548	100,00		600	60		2690	100

Suppliers location

Location	ibb-x-coordinate [km]	ibb-y-coordinate [km]	Euclidean distance between suppliers location and road transshipment centre [km]
Where E-80 road and D-100 road cross in Gebze district of Kocaeli province	457,2	4519,1	13,9

*: coordinates from Istanbul map on site of Istanbul Metropolitan Municipality: http://sehirrehberi.ibb.gov.tr/map.aspx, August 2014

**: data from Turkish Statistical Institute (TURKSTAT) website (http://www.turkstat.gov.tr): http://rapor.tuik.gov.tr/reports/rwservlet?adnksdb2 &ENVID=adnksdb2Env&report=wa_turkiye_ilce_koy_sehir.RDF&p_il1=34&p_kod=1&p_yil=2013&p_dil=2&desformat=html, August 2014

Table 9.2. Illustration of the fuel cost calculations for the road transshipment centre and the six distribution centres, part 2.

	District	Ibb-x-coordinate [km]	Ibb-y-coordinate [km]	Number of custumers per workday	Number of delivery milk runs per workday	Number of supplier vehicles (semi-trailer trucks) per workday	Total delivery trucks driving distance per workday (factor 1,4 for conversion from Euclidean distance to driving distance) [km]	Total supplier trucks driving distance per workday (factor 1,4 for conversion from Euclidean distance to driving distance) [km]	Total delivery trucks fuel cost per workday [TL]	Total supplier trucks fuel cost per workday [TL]	Total fuel cost per workday [TL]
Road transshipment centre location											
	Tuzla	443,7	4522,6	600	60	10	**3766**	**373**	**2734**	**549**	**3284**
Distribution centres locations											
	District	Ibb-x-coordinate [km]	Ibb-y-coordinate [km]	Number of custumers per workday	Number of delivery milk runs per workday	Number of supplier vehicles (semi-trailer trucks) per workday	Total delivery trucks driving distance per workday (factor 1,4 for conversion from Euclidean distance to driving distance) [km]	Total supplier trucks driving distance per workday (factor 1,4 for conversion from Euclidean distance to driving distance) [km]	Total delivery trucks fuel cost per workday [TL]	Total supplier trucks fuel cost per workday [TL]	Total fuel cost per workday [TL]
1	Tuzla	448,4	4529,9	100	10	2	830	62	603	91	694
2	Tuzla	447,6	4527,4	100	10	2	842	57	612	83	695
3	Tuzla	448,7	4528,5	100	10	2	852	56	619	83	702
4	Tuzla	447,8	4528,1	100	10	2	837	58	608	85	693
5	Cayirova (Kocaeli)	446,5	4524,1	100	10	2	875	53	635	77	713
6	Tuzla	449,1	4529,7	100	10	2	848	59	615	88	703
				Total: 600	Total: 60	Total: 10	Total: **5085**	Total: **345**	Total: **3692**	Total: **508**	Total: **4200**

Vehicles						
Vehicle type			Maximum cargo mass [kg]	Fuel economy (average of empty truck and full truck) [litre / 100km]	Fuel cost (average of empty truck and full truck) (4,4 TL/litre) [TL/km]	
Small delivery truck			3500	17	0,73	
Large rigid truck			15000	27	1,17	
Semi-trailer truck			22000	34	1,47	

3) For the DCs:

The number of DC customers per workday is shown for each district in the twelfth column of table 1. Because the customers per workday are smaller than ten for most districts, some of the milk runs will deliver to more than one district. We construct efficient milk runs with ten deliveries or less per milk run (we also allow eleven deliveries per milk run). Table 3 shows the DC milk runs for one of the DCs. Four of the ten milk runs deliver to more than one district. For the construction of the milk runs we put the Adalar district and the Kadiköy district together. We estimate the total length of all milk runs (using Euclidean distance) per workday to be 0.59×10^3 km for DC 1 (table 3, column 8). The total delivery trucks driving distance then becomes 0.83×10^3 km for DC 1 (table 2, column 8). For all DCs together, we estimate the total delivery trucks driving distance to be 5.1×10^3 km (table 2, column 8). The total DCs delivery trucks fuel cost per workday then becomes 3.7×10^3 TL (table 2, column 10). The total RTC delivery trucks fuel cost per workday is 74% of this value.

Table 9.3. Illustration of the calculation of the total milk run length for one distribution centre.

Calculation of the total route length in km of the distribution centre 1 delivery truck routes						Total milk run length	Number of deliveries
Milk runs for DC1:							
1 Üsküdar	DC1-Üsküdar	Üsküdar	Üsküdar-DC1				
length [km]:	31,5	4,5	31,5			67,4	11
2 Maltepe	DC1-Maltepe	Maltepe	Maltepe-DC1				
length [km]:	21,3	3,5	21,3			46,1	9
3 Kartal	DC1-Kartal	Kartal	Kartal-DC1				
length [km]:	15,7	3,0	15,7			34,4	9
4 Sultanbeyli-Beykoz	DC1-Sultanbeyli	Sultanbeyli	Sultanbeyli-	Beykoz	Beykoz-		
length [km]:	12,1	2,5	19,9	7,5	31,4	73,4	11
5 Tuzla-Sancaktepe	DC1-Tuzla	Tuzla	Tuzla-Sancaktepe	Sancaktepe	Sancaktepe-DC1		
length [km]:	4,6	9,5	16,1	3,0	16,5	49,7	10
6 Adalar / Kadiköy	DC1-Adalar / Kadiköy	Adalar / Kadiköy	Adalar / Kadiköy-DC1				
length [km]:	28,8	4,0	28,8			61,7	10
7 Pendik (10 deliveries)-Sile	DC1-Pendik	Pendik	Pendik-Sile	Sile	Sile-DC1		
length [km]:	8,8	8,0	38,7	0,0	32,9	88,4	11
8 Ümraniye (3 customers)-	DC1-Ümraniye	Ümraniye	Ümraniye-Çekmeköy	Çekmeköy	Çekmeköy-DC1		
length [km]:	25,8	0,0	9,4	5,5	21,9	62,7	7
9 Ümraniye (10 customers)	DC1-Ümraniye	Ümraniye	Ümraniye-DC1				
length [km]:	25,8	4,0	25,8			55,7	10
10 Pendik (3 customers)-Atasehir	DC1-Pendik	Pendik	Pendik-Atasehir	Atasehir	Atasehir-DC1		
length [km]:	8,8	0,0	16,1	4,0	24,3	53,2	11
						Total:	Total:
						593	100

Figure 3 and 4 show the milk runs for DC 1 (figure 3) and for the RTC (figure 4). Figure 3 shows that some of the DC milk runs deliver to more than one district. Because the RTC has six times more customers than one DC, it is possible to organize RTC milk runs more efficiently than DC milk runs, with less milk runs delivering to more than one district. Therefore we expect the total delivery trucks driving distance and the total delivery trucks fuel cost to be less for the RTC than for the DCs. This corresponds with our calculations: the total RTC delivery trucks fuel cost per workday is estimated to be 74% of the total DCs delivery trucks fuel cost per workday.
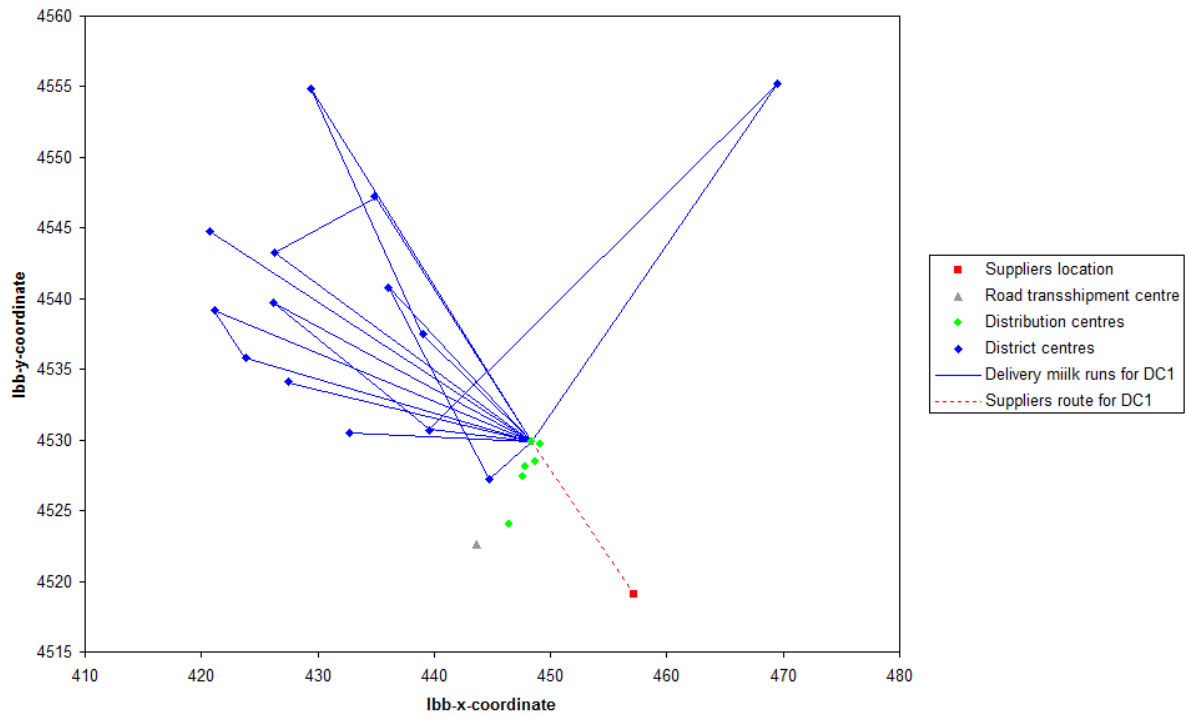
Figure 9.3. Distribution centre delivery truck routes and supplier trucks route.
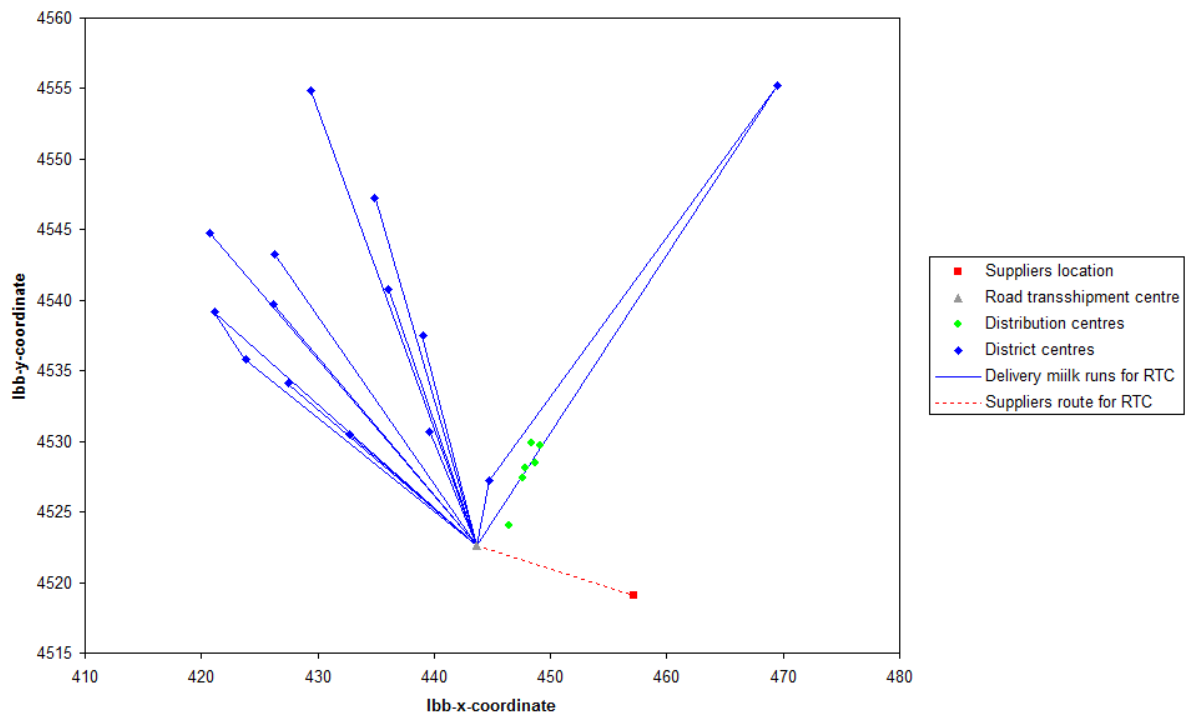

Figure 9.4. Road transshipment centre delivery truck routes and supplier trucks route.

9.3.2. The supplier trucks fuel cost

We assume that the trucks of the suppliers of the DCs and the RTC enter Istanbul from the Asian side via the E-80 road. The supplier trucks are mostly semi-trailer trucks and we estimate the fuel cost per km to be 1.5 TL/km (average of with and without cargo), based on interviews with logistics managers in Istanbul. Because we have no data about the exact location of every supplier, we locate all suppliers where the E-80 road and the D-100 road cross in the Gebze district of the Kocaeli province. This allows us to calculate the difference between the DCs supplier trucks fuel cost and the RTC supplier trucks fuel cost. The ibb-coordinates of the suppliers location are shown in the third and fourth column of table 1. Table 2 shows estimates of the maximum cargo mass for the delivery trucks and the supplier trucks, based on interviews with logistics managers in Istanbul. We use these numbers to estimate the ratio of the number of delivery milk runs to the number of supplier trucks: 22000/3500, which is more or less six. The seventh column of table 2 shows the number of supplier trucks per workday for all DCs and the RTC. The ninth column of table 2 shows the total driving distance of the supplier trucks per workday for all DCs and the RTC. This is estimated by multiplying two times the Euclidean distance between the suppliers location and the DC/RTC location by 1.4 (factor for estimating not the Euclidean distance but the road network travel distance) and by the number of supplier trucks per workday. Multiplying the supplier trucks driving distances by 1.5 TL/km gives us the supplier trucks fuel costs per workday (table 2, column 11). We estimate the total RTC suppliers trucks fuel cost per workday to be 41 TL more than the total DCs suppliers trucks fuel cost per workday.

9.3.3. The total fuel cost

For calculating the total (delivery trucks and supplier trucks) fuel cost we add the total delivery trucks fuel cost to the supplier trucks fuel cost. We estimate the total DCs fuel cost per workday to be 0.92×10^3 TL more than the total RTC fuel cost. We conclude this section by listing some of our results:
- We estimate the total delivery trucks driving distance per workday to be 5.1×10^3 km for the DCs and 3.8×10^3 km for the RTC. The total RTC delivery trucks driving distance per workday is 74% of the total DC delivery trucks driving distance per workday and 1.3×10^3 km shorter.
- We estimate the total delivery trucks fuel cost per workday to be 3.7×10^3 TL for the DCs and 2.7×10^3 TL for the RTC. The total RTC delivery trucks fuel cost per workday is 74% of total DC delivery trucks fuel cost per workday and 0.96×10^3 TL lower.
- We estimate the total supplier trucks driving distance per workday to be 28 km longer for the RTC than for the DCs.
- We estimate the total supplier trucks fuel cost per workday to be 41 TL higher for the RTC than for the DCs.
- We estimate the total fuel cost per workday to be 0.92×10^3 TL lower for the RTC than for the DCs.

9.4. Conclusion

In this article we made a case study on city distribution in Istanbul. We compared the sum of the transportation costs of six distribution centres to the transportation cost of one large

transshipment centre that supplies the same customers. We found that the transportation cost is lower for the road transshipment centre. The main reason for that is that integrating different distribution centres allows us to organize the city distribution more efficiently. More specifically, in the case of the road transshipment centre it was possible to organize the distribution such that most milk runs only supplied to one district. In the case of the six distribution centres more milk runs supplied to more than one district and therefore the total delivery trucks driving distance was longer and the total transportation cost higher.

References

[1] E. Taniguchi, R.G. Thompson, T. Yamada, R. Van Duin, City Logistics: Network Modelling and Intelligent Transport Systems, Pergamon, Oxford, 2001.

[2] M. Panero, H.-S. Shin, D.P. Lopez, Urban distribution centers: a means to reducing freight vehicle miles traveled, https://www.dot.ny.gov/divisions/engineering/technical-services/trans-r-and-d-repository/C-08-23_0.pdf (21.08.2014).

[3] http://www.ibb.gov.tr/tr-TR/SiteImages/Haber/nisan2009/04052009idarisinirharitasi.jpg (22.08.2014).

[4] http://sehirrehberi.ibb.gov.tr/map.aspx (22.08.2014) (Istanbul map on site of Istanbul Metropolitan Municipality).

[5] http://rapor.tuik.gov.tr/reports/rwservlet?adnksdb2&ENVID=adnksdb2Env&report=wa_turkiye_ilce_koy_sehir.RDF&p_il1=34&p_kod=1&p_yil=2013&p_dil=2&desformat=html (21.08.2014) (2013 Istanbul district population data on site of Turkish Statistical Institute).

[6] T. Dubois, M. Bamyaci, M. Tanyas, L. Aksoy, H. Vanli, F. Witlox, G. Allaert, A location comparison based on fuel costs for one of the fruit and vegetable markets in Istanbul, in: G. Gürsoy, M. Tanyas (eds.), 10th International Logistics and Supply Chain Congress 2012 Proceedings, Istanbul Kemerburgaz University & Logistics Association (LODER) & Ideal Kultur Publishing, Istanbul, 2012, pp. 179-184.

[7] T. Dubois, M. Tanyas, M. Bamyaci, H. Vanli, L. Aksoy, F. Witlox, G. Allaert, A search for the optimal location regarding the supply chain fuel costs for one of the fruit and vegetable markets in Istanbul, in: G. Gürsoy, M. Tanyas (eds.), 10th International Logistics and Supply Chain Congress 2012 Proceedings, Istanbul Kemerburgaz University & Logistics Association (LODER) & Ideal Kultur Publishing, Istanbul, 2012, pp. 544-552.

[8] T. Dubois, M. Bamyaci, H. Vanli, L. Aksoy, M. Tanyas, G. Allaert, F. Witlox, Case study on different distribution methods for an urban wholesale market, in: C. Cebeci, G. Büyüközkan (eds.), 11th International Logistics and Supply Chain Congress 2013 Proceedings - Managing Opportunities and Threats in Supply Chain, 2013, pp. 313-319.

10
Conclusion and discussion

In this chapter we conclude the dissertation by putting together the results of previous chapters and discussing them. Section 1 and 2 concludes the Istanbul fruit and vegetable wholesale market case study. Section 3 concludes the research on inventory management with capacitated replenishment. In section 4 the research results and possibilities for future research are discussed. The last section of this chapter is a summary of the research presented in this dissertation.

10.1. Istanbul fruit and vegetable wholesale market case study: conclusion

In this section we put together the results of the chapters 2, 3, 4 and 5. In [1] and [2] location problem models are categorized in four main categories: analytic models, continuous models, network models and discrete models. Table 1 shows the differences between these models.

Table 10.1. Location problem models.

	Customer locations and supplier locations are given by	Facility location is restricted to
Analytic models	a density function over an area or	
	a density function over an area and a finite number of points	an area
Continuous models	a finite number of points	an area
Network models	a density function over a network or	
	a finite number of points on a network or	
	a density function over a network and a finite number of points on this network	a network
Discrete models	a finite number of points	a finite number of points

In chapter 2 an analytic location problem model is used to determine the location of an urban facility that minimizes the supply chain fuel cost. This model is not immediate applicable to the Istanbul fruit and vegetable wholesale market because the shape of the Asian side of Istanbul and the urban population density are not accurately described by a disc and the used population density function in chapter 2. Chapter 2 is relevant for the Istanbul fruit and vegetable wholesale market study because of the introduced parameter α and the dependence of the optimal facility location on α. α is defined in chapter 2 as the ratio of the number of supplier vehicles (N_s) times the supplier vehicles cost per km (c_s) to the number of costumer vehicles (N_c) times the costumer vehicle cost per km (c_c): $\alpha = \dfrac{N_s c_s}{N_c c_c}$. α is greater if the customers transportation is more efficient (with less vehicles, with vehicles with less fuel cost per km) given the suppliers transportation is constant. In the model of chapter 2 the optimal facility location regarding the supply chain fuel cost is the optimal facility location for the customers (city centre) if α is zero and the optimal facility location is the suppliers location if α is 1. In this chapter we will study the dependence of the optimal Istanbul fruit and vegetable wholesale market location regarding the supply chain fuel cost on α.
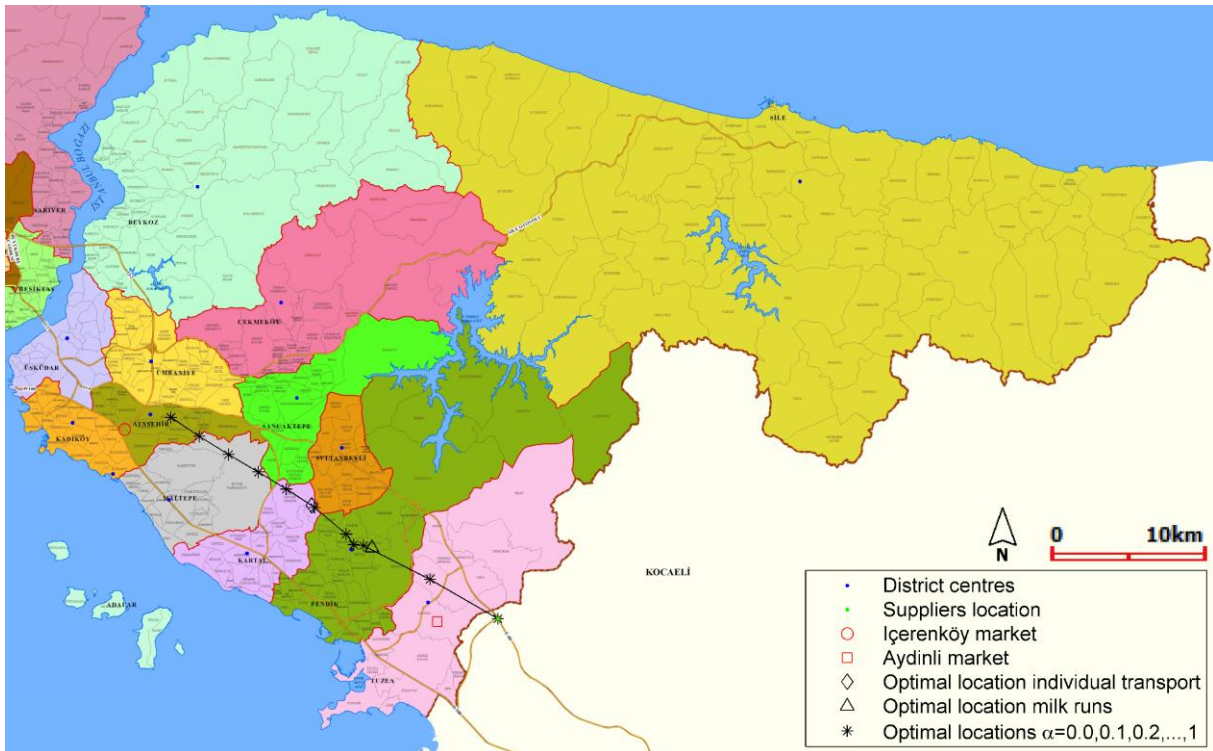
In chapter 4 a continuous location problem model is used to find the location of the Istanbul fruit and vegetable wholesale market that minimizes the supply chain fuel cost, assuming the customer routes and supplier routes go via straight lines and assuming each customer vehicle goes to the market without cargo and back to the costumer location with goods for one retailer (individual transport). Chapter 5 studies the fuel cost for delivering the goods to the customers with trucks making milk runs (round trips). These two delivery methods (individual transport and milk run distribution) correspond to two different values of α: $\alpha=0.49$ for individual transport and $\alpha=0.83$ for milk run distribution (for milk run distribution we define α as $\alpha = \dfrac{N_s c_s}{\beta N_c c_c}$ with β the correction factor to take into account that milk run routes are longer than individual transport routes because there are more drops than one. The used value for β is 1.1 in chapter 5. Figure 1 shows the optimal market location for different values of α including the values that correspond to individual transport distribution and milk run distribution. The source of the map of Istanbul used in this figure is the Istanbul Metropolitan Municipality website [3]. The optimal locations are found by using the Weiszfeld algorithm (see [4] pp. 14-15) with 10000 iterations. The figure shows that if α is greater, the optimal market location is closer to the suppliers location and the border of the city. For smaller values of α the optimal market location is closer to densely populated areas. Therefore making the customers transportation more efficient has as the extra advantage that the optimal market location regarding fuel costs moves to a location closer to the city border which is considered to be an environmental advantage (less traffic close to the city centre, less air pollution and less traffic noise (at night) close to the city centre and more space available close to the city centre for other purposes) and a financial advantage (lower land cost).

Figure 10.1. The location that optimizes the supply chain fuel cost calculated by using Euclidean distances for customers and suppliers, for $\alpha=0.0, 0.1, 0.2, ..., 1$.

In chapter 3 a discrete location problem model is used to compare two locations for the Istanbul fruit and vegetable wholesale market: Içerenköy, the actual location and Aydinli, the planned new market location. The used distances for the customer routes and supplier routes are the road network travel distances (driving distances). Chapter 3 only considers individual transport distribution. We extend this study by also considering milk run distribution for these two market locations. The result is table 2. The table shows that the Içerenköy market location is optimal regarding supply chain fuel costs for individual transport, for milk run distribution the Aydinli location is optimal. The supply chain fuel cost for the Aydinli location with milk run distribution is lower than the supply chain fuel cost for the Içerenköy location (with or without milk runs). But the supply chain fuel cost for the Aydinli location with individual transport distribution is higher than the supply chain fuel cost for the Içerenköy location (with or without milk runs). If the market moves to Aydinli, we therefore advise to consider milk run distribution. Delivery to the customers also solves the problem that the average driving distance from a customer to the Aydinli market location is more or less the double of the average driving distance from a customer to the Içerenköy market location.

Table 10.2. Calculation of the supply chain fuel cost during one workday, using driving distances for customers and suppliers, for the market located in Içerenköy and Aydinli and for individual transport distribution and milk run distribution.

District	Number of	Number of	Driving distance	Driving distance	Transportation cost during one workday			
					Individual transport		Milk run distribution	
(customers)	customer vehicles	milk runs	to Içerenköy market	to Aydinli market	Içerenköy	Aydinli	Içerenköy	Aydinli
	per day (N_{c1})	per day (N_{c2})	[km]	[km]	[TL]	[TL]	[TL]	[TL]
Ümraniye	283	77	9,7	37,4	3315	12719	1969	7555
Pendik	275	74	21,2	15,3	6993	5050	4154	3000
Kadiköy	250	68	4,4	35,0	1313	10514	780	6245
Üsküdar	247	67	9,6	36,7	2852	10913	1694	6482
Maltepe	206	56	6,0	24,9	1475	6156	876	3657
Kartal	203	55	15,3	16,7	3723	4058	2212	2410
Atasehir	176	48	2,4	35,9	512	7590	304	4509
Sultanbeyli	137	37	16,6	14,0	2725	2297	1619	1365
Sancaktepe	120	33	17,4	28,2	2512	4071	1492	2418
Beykoz	116	31	23,8	56,2	3298	7796	1959	4631
Tuzla	87	24	29,0	4,6	3038	484	1804	288
Çekmeköy	79	21	20,1	36,0	1910	3421	1135	2032
Sile	13	4	67,7	76,4	1073	1211	638	719
Adalar	7	2	4,1	28,7	33	230	19	137
	Total:	Total:	Weighted average:	Weighted average:	Customers total:	Customers total:	Customers total:	Customers total:
	2200	594	13,2	29,0	3,48E+04	7,65E+04	2,07E+04	4,54E+04
District	Number of		Driving distance	Driving distance	Suppliers total:	Suppliers total:	Suppliers total:	Suppliers total:
(suppliers)	supplier vehicles		to Içerenköy market	to Aydinli market	4,27E+04	9,15E+03	4,27E+04	9,15E+03
	per day (N_s)		[km]	[km]	Total:	Total:	Total:	Total:
Cayirova (Kocaelli)	540		33,0	7,1	7,75E+04	8,57E+04	6,34E+04	5,46E+04
(on TEM road)					Relative:	Relative:	Relative:	Relative:
					0	8,18E+03	-1,41E+04	-2,29E+04
Fuel cost per km:	0,6	TL/km	for individual transport vehicles (c_{c1})					
	1,2	TL/km	for supplier vehicles (c_s) and milk run distribution vehicles (c_{c2})					
alpha=	$N_s c_s/(N_{c1} c_{c1})$=		0,49	for individual transport				
	$N_s c_s/(1,1 * N_{c2} c_{c2})$=		0,83	for milk run distribution				

We extend this study further by calculating the supply chain fuel cost in function of α for the Içerenköy and Aydinli location. We consider the same suppliers transportation for every value of α. The results are shown in table 3 and figure 2. Table 3 and figure 2 show that for α greater than 0.7 the Aydinli market location has lower supply chain fuel cost.

Table 10.3. The supply chain fuel cost during one workday in function of α with $N_s c_s$ constant, for the market location in Içerenköy and Aydinli.

alpha ($N_s c_s$=540*1,2 TL/km)	Transportation cost during one workday	
	Içerenkoy [TL]	Aydinli [TL]
0,1	2,13E+05	3,85E+05
0,2	1,28E+05	1,97E+05
0,3	9,96E+04	1,34E+05
0,4	8,54E+04	1,03E+05
0,49	7,75E+04	8,57E+04
0,5	7,69E+04	8,43E+04
0,6	7,12E+04	7,18E+04
0,7	6,71E+04	6,28E+04
0,83	6,34E+04	5,46E+04
0,8	6,41E+04	5,61E+04
0,9	6,17E+04	5,09E+04
1	5,98E+04	4,67E+04
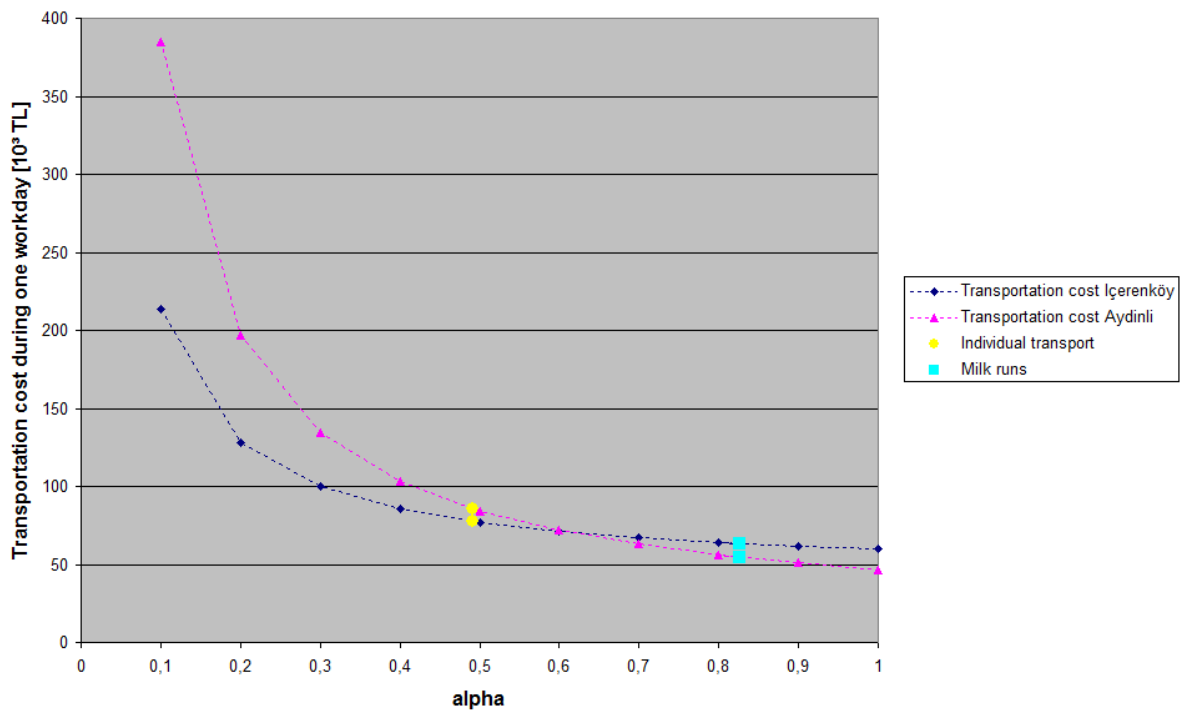
Figure 10.2. The supply chain fuel cost during one workday in function of α with $N_s c_s$ constant, for the market location in Içerenköy and Aydinli.

We conclude by summarizing that if α is large enough the Aydinli market location is more favourable than the Içerenköy market location regarding the supply chain fuel cost. α increases if the customers transportation efficiency increases, e.g. distribution by milk runs instead of individual transport or distribution by electric vehicles instead of internal combustion vehicles. The Aydinli location is more favourable than the Içerenköy location regarding the supply chain fuel cost if the goods are distributed to the customers by milk runs. Delivery to the customers also solves the problem that the average driving distance from a

customer to the Aydinli market location is more or less the double of the average driving distance from a customer to the Içerenköy market location. For individual transport distribution the Aydinli location is less favourable than the Içerenköy location regarding the supply chain fuel cost. If Aydinli is favourable regarding fuel costs, this location is likely also favourable regarding air pollution. Because Aydinli is close to the border of the city, environmental advantages of this market location are: less traffic in the densely populated areas of the city, less air pollution and less traffic noise in the densely populated areas of the city and more space available in the densely populated areas of the city for other purposes. Other advantages of the Aydinli location are the lower land cost and the availability of a larger area.

10.2. Istanbul fruit and vegetable wholesale market case study: sensitivity analysis

In this section we make a sensitivity analysis regarding the Istanbul fruit and vegetable wholesale market case study. We study how the costs saved by moving the market to Aydinli and applying milk run distribution (y) change, if one of the inputs (x) change (one input at a time). We consider y, the average customers and suppliers transportation fuel costs during one workday of the situation with the market in Içerenköy and individual transport minus the average customers and suppliers transportation fuel costs during one workday of the situation with the market in Aydinli and milk run distribution, to be the output. Table 4 shows in green the different inputs that we consider and illustrates how the output (in red) is calculated. We notice that for every input x, $y=ax+b$, with a and b dependent on the other inputs but not on x. Therefore if input x changes from x_1 to x_2, the output y changes from y_1 to y_2 with $\Delta y/\Delta x=a$ ($\Delta y=y_2-y_1$ and $\Delta x=x_2-x_1$).

Table 10.4. The inputs (green) and output (red) considered in the sensitivity analysis.

District (customers)	Number of customer vehicles per day (N_{c1})	Number of milk runs per day (N_{c2})	Driving distance to Içerenköy market [km]	Driving distance to Aydinli market [km]	Transportation cost during one workday			
					Individual transport		Milk run distribution	
					Içerenköy [TL]	Aydinli [TL]	Içerenköy [TL]	Aydinli [TL]
Ümraniye	283	77	9,7	37,4	3315	12719	1969	7555
Pendik	275	74	21,2	15,3	6993	5050	4154	3000
Kadiköy	250	68	4,4	35,0	1313	10514	780	6245
Üsküdar	247	67	9,6	36,7	2852	10913	1694	6482
Maltepe	206	56	6,0	24,9	1475	6156	876	3657
Kartal	203	55	15,3	16,7	3723	4058	2212	2410
Atasehir	176	48	2,4	35,9	512	7590	304	4509
Sultanbeyli	137	37	16,6	14,0	2725	2297	1619	1365
Sancaktepe	120	33	17,4	28,2	2512	4071	1492	2418
Beykoz	116	31	23,8	56,2	3298	7796	1959	4631
Tuzla	87	24	29,0	4,6	3038	484	1804	288
Çekmeköy	79	21	20,1	36,0	1910	3421	1135	2032
Sile	13	4	67,7	76,4	1073	1211	638	719
Adalar	7	2	4,1	28,7	33	230	19	137
	Total: 2200	Total: 594	Weighted average: 13,2	Weighted average: 29,0	Customers total: 3,48E+04	Customers total: 7,65E+04	Customers total: 2,07E+04	Customers total: 4,54E+04
District (suppliers)	Number of supplier vehicles per day (N_s)		Driving distance to Içerenköy market [km]	Driving distance to Aydinli market [km]	Suppliers total: 4,27E+04	Suppliers total: 9,15E+03	Suppliers total: 4,27E+04	Suppliers total: 9,15E+03
Cayirova (Kocaelli) (on TEM road)	540		33,0	7,1	Total: 7,75E+04	Total: 8,57E+04	Total: 6,34E+04	Total: 5,46E+04
Fuel cost per km:	0,6 TL/km		for individual transport vehicles (c_{c1})		Relative: 0	Relative: 8,18E+03	Relative: -1,41E+04	Relative: -2,29E+04
	1,2 TL/km		for supplier vehicles (c_s)		Transportation fuel costs for market in Içerenköy with individual transport minus transportation fuel costs for market in Aydinli with milk run distribution (during one workday):			
	1,2 TL/km		milk run distribution vehicles (c_{c2})					
Number of milk runs over number of deliveries to customers:				0,27				
Average number of deliveries per milk run:				3,7			2,29E+04 TL	

Tables 5 and 6 show $\Delta y/\Delta x$ and $(\Delta y/y)/(\Delta x/x)$ for the different inputs. In column 1 the inputs are numbered from 1 to 49. Table 5 contains the sensitivity analysis of the number of customer vehicles of the different districts (inputs 1 to 14). For example (input 3) if there is one more customer vehicle added for the Kadiköy district, the output decreases with 20 TL and if the average number of customers vehicles for the Kadiköy district increases with 1%, the output decreases with 0.22%. We notice that the districts close to the Kocaeli province have positive $\Delta y/\Delta x$ with x the number of customer vehicles and the districts close to the European side of Istanbul have negative $\Delta y/\Delta x$ with x the number of customer vehicles.

The $(\Delta y/y)/(\Delta x/x)$ columns of tables 5 and 6 contain an overview of the sensitivity of the output to changes in the different inputs. Unlike $\Delta y/\Delta x$, $(\Delta y/y)/(\Delta x/x)$ is a dimensionless quantity. The largest $|(\Delta y/y)/(\Delta x/x)|$ values in the tables are for the inputs 43 to 49. These inputs are not regarding a particular district but regarding all customers or all suppliers. Therefore for improving the estimation of the predicted cost difference (y), most important is to improve the estimations of inputs 43 to 49. Especially, accurate estimations of the average fuel cost per km for the milk run vehicles (input 48) and the number of milk runs over the number of deliveries to customers (input 49, this is one over the average number of deliveries per milk run) are important for getting an accurate value for the output y.

The sensitivity of the output to the number of customer vehicles in the case of individual transport is shown in the last row of table 6. For calculating $\Delta y/\Delta x$ and $(\Delta y/y)/(\Delta x/x)$ we assumed the number of costumer vehicles for a district over the total number of customer vehicles to be constant.

Table 10.5. Change in output over change in input (Δ*y*/Δ*x*) and relative change in output over relative change in input ((Δ*y*/*y*)/(Δ*x*/*x*)) for different inputs, part 1.

				y	unit
Transportation fuel costs for market in Içerenköy with individual transport minus transportation fuel costs for market in Aydinli with milk run distribution (during one workday):				2,29E+04	TL
		x	unit	Δy/Δx	unit	(Δy/y)/(Δx/x)
	Number of customer vehicles (individual transport)					
1	Ümraniye	283		-15	TL	-0,19
2	Pendik	275		15	TL	0,17
3	Kadiköy	250		-20	TL	-0,22
4	Üsküdar	247		-15	TL	-0,16
5	Maltepe	206		-11	TL	-0,10
6	Kartal	203		6	TL	0,06
7	Atasehir	176		-23	TL	-0,17
8	Sultanbeyli	137		10	TL	0,06
9	Sancaktepe	120		1	TL	0,004
10	Beykoz	116		-12	TL	-0,06
11	Tuzla	87		32	TL	0,12
12	Çekmeköy	79		-2	TL	-0,01
13	Sile	13		27	TL	0,02
14	Adalar	7		-16	TL	-0,005
	Driving distance from district centre to Içerenköy market					
15	Ümraniye	9,7	km	340	TL/km	0,14
16	Pendik	21,2	km	330	TL/km	0,31
17	Kadiköy	4,4	km	300	TL/km	0,06
18	Üsküdar	9,6	km	297	TL/km	0,12
19	Maltepe	6,0	km	247	TL/km	0,06
20	Kartal	15,3	km	244	TL/km	0,16
21	Atasehir	2,4	km	211	TL/km	0,02
22	Sultanbeyli	16,6	km	164	TL/km	0,12
23	Sancaktepe	17,4	km	145	TL/km	0,11
24	Beykoz	23,8	km	139	TL/km	0,14
25	Tuzla	29,0	km	105	TL/km	0,13
26	Çekmeköy	20,1	km	95	TL/km	0,08
27	Sile	67,7	km	16	TL/km	0,05
28	Adalar	4,1	km	8	TL/km	0,001

Table 10.6. Change in output over change in input ($\Delta y/\Delta x$) and relative change in output over relative change in input (($\Delta y/y)/(\Delta x/x)$) for different inputs, part 2.

		x	unit	$\Delta y/\Delta x$	unit	($\Delta y/y)/(\Delta x/x)$
	Driving distance from district centre to Aydinli market					
29	Ümraniye	37,4	km	-202	TL/km	-0,33
30	Pendik	15,3	km	-196	TL/km	-0,13
31	Kadiköy	35,0	km	-178	TL/km	-0,27
32	Üsküdar	36,7	km	-176	TL/km	-0,28
33	Maltepe	24,9	km	-147	TL/km	-0,16
34	Kartal	16,7	km	-145	TL/km	-0,11
35	Atasehir	35,9	km	-126	TL/km	-0,20
36	Sultanbeyli	14,0	km	-97	TL/km	-0,06
37	Sancaktepe	28,2	km	-86	TL/km	-0,11
38	Beykoz	56,2	km	-82	TL/km	-0,20
39	Tuzla	4,6	km	-62	TL/km	-0,01
40	Çekmeköy	36,0	km	-56	TL/km	-0,09
41	Sile	76,4	km	-9	TL/km	-0,03
42	Adalar	28,7	km	-5	TL/km	-0,01
43	Number of supplier vehicles	540		62	TL	1,47
44	Driving distance from suppliers entry point to İçerenköy market	33,0	km	1,30E+03	TL/km	1,87
45	Driving distance from suppliers entry point to Aydinli market	7,1	km	-1,30E+03	TL/km	-0,40
46	Fuel cost per km for customer vehicles (individual transport)	0,6	TL/km	5,80E+04	TL/(TL/km)	1,52
47	Fuel cost per km for supplier vehicles	1,2	TL/km	2,80E+04	TL/(TL/km)	1,47
48	Fuel cost per km for milk run vehicles	1,2	TL/km	-3,79E+04	TL/(TL/km)	-1,99
49	Number of milk runs over number of deliveries to customers	0,27		-1,68E+05	TL	-1,99
	Number of customer vehicles (individual transport)	2200		-5	TL	-0,47

10.3. Inventory management with joint capacitated replenishment: conclusion

In this section we conclude the research done in chapters 6, 7 and 8 in the field of inventory management by comparing individual capacitated replenishment and joint capacitated replenishment. We consider different retail shops and study how the optimal inventory policy parameters and the inventory holding costs change if the transportation of goods from a supplier to different retail shops is reorganized from individual transportation (one retailer per trip) to joint transportation (milk run distribution, multiple retailers per round trip). We put together the results of the chapters 6, 7 and 8 and we illustrate by making extra simulations, which enable us to calculate the inventory costs.

Let us consider five retail shops that apply period review order-up-to level inventory policies. The required fill rate for each retail shop is 99% and the review period is one week for each retail shop. We assume zero lead time and we simulate the demand as a Poisson process with on average 16 demand occurrences per week. The discrete version of (14) of chapter 7,

$$\beta_{uncap} = 1 - \frac{\sum\limits_{i=s_{uncap}}^{\infty}(i - s_{uncap})f(i)}{\sum\limits_{i=0}^{\infty}if(i)}, \tag{1}$$

enables us to calculate the fill rate β_{uncap} if replenishment is not capacitated. s_{uncap} is the order-up-to level and f is the probability mass function of the demand during one review period. In our example the demand during one review period is Poisson distributed with a mean of 16 products. We notice that (1) is equal to (24) from chapter 6 with the capacity c equal to the order-up-to level s. By using the method illustrated in chapter 7 (the bisection method) we use formula (1) to find the smallest order-up-to level $s_{uncap}*$ such that the fill rate is greater than or equal to 99%, if replenishment is not capacitated. The result is:

$$s_{uncap}* = 22 .$$ (2)

If replenishment is individual and capacitated, chapter 6 enables us to find the fill rate, given the capacity, the order-up-to level and the probability mass function of the demand during one review period. Let us consider a capacity of 17 products for each retailer (c_{ind}=17) and as before Poisson distributed demand during a review period with a mean of 16 products. An order up-to-level of 22 gives now a fill rate that is less than 99%. We search for the smallest order-up-to level $s_{cap,ind}*$ such that the fill rate is greater than or equal to 99% by using chapter 6 and the bisection method. The result is:

$$s_{cap,ind}* = 29 .$$ (3)

If replenishment is joint and capacitated with a capacity of 5×17 products (c_{joint}=5×17), a discrete version of the algorithm of chapter 8 enables us to find the smallest order-up-to level $s_{cap,joint}*$ for the retailers such that the fill rate is greater than or equal to 99%. We find:

$$s_{cap,joint}* = 24 .$$ (4)

We notice that in the example the order-up-to level decreases from 29 to 24 by reorganizing the replenishment from individual replenishment to joint replenishment. Therefore, by reorganizing the transportation from the supplier to the retailers from individual transportation with vehicles with a capacity of 17 products to milk run (round trip) distribution with one large vehicle with a capacity of 5×17 products, the retailers order-up-to level is reduced by more or less 17%.

Now let us study how the inventory holding cost changes if the replenishment is reorganized from individual replenishment to joint replenishment. We are still considering the same example with five retailers and we assume that the inventory holding cost of holding n products during m weeks in a retail shop is $n×m×h$. For this study we simulate 10000 weeks for the three cases (uncapacitated replenishment, individual capacitated replenishment and joint capacitated replenishment) and calculate the average inventory holding cost per week for one retailer. The simulation is done by generating random numbers consistent with the exponential distribution with a mean of 1/16. These random numbers are the times in weeks between consecutive demand occurrences in the simulation. The stock on hand is updated in accordance with the used inventory policy after every demand occurrence and after every replenishment. The average inventory holding cost per week for retailer i is calculated with the following formula:

average inventory cost per week for retailer $i = \dfrac{\int_{0}^{10000} I_i(t)dt \times h}{10000} ,$ (5)
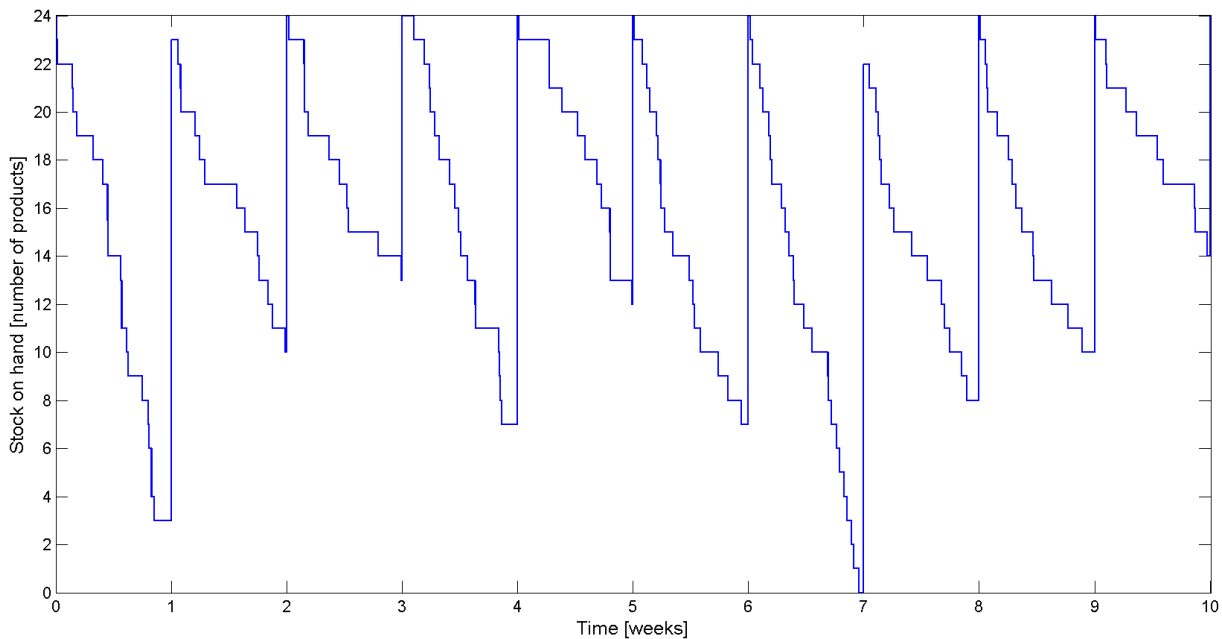
with $I_i(t)$ the stock on hand of retailer i at time t (time in weeks). The results of the simulations are:

- average inventory holding cost per week per retailer in the uncapacitated case: $14.0 \times h$
- average inventory holding cost per week per retailer in the individual capacitated replenishment case ($c_{ind}=17$ for every retailer): $17.5 \times h$
- average inventory holding cost per week per retailer in the joint capacitated replenishment case ($c_{joint}=5 \times 17$): $14.9 \times h$.
(We rounded the values of the inventory holding costs to integer multiples of $0.1 \times h$.)
Figure 3 shows the stock on hand as a function of time for the first ten weeks of the simulation. The inventory holding cost is proportional to the area under the graph and above the time axis.

Figure 10.3. The stock on hand of retailer 1 as a function of time in the case of joint capacitated replenishment (with a capacity of 85 products) of five retailers. Demand is simulated as a Poisson process with on average 16 demand occurrences per week. Each retailer applies the periodic review order-up-to level inventory policy with an order-up-to level of 24 products and a review period of one week.

We conclude that in this example by reorganizing the distribution from individual capacitated replenishment with a capacity of 17 products per retailer to joint capacitated replenishment with a capacity of 5×17 products, the expected inventory holding cost per week is reduced by more or less 15%. Therefore we found that if milk runs (round trips) and individual transportation are compared (e.g. for urban distribution), besides a difference in transportation costs, it is possible that there is also a difference in inventory costs. Chapters 6, 7 and 8 and this section contain methods to determine this difference in inventory holding costs and also the difference in optimal order-up-to levels for periodic review order-up-to level inventory policies under fill rate constraints (minimal required fill rates).

10.4. Discussion and future research

In this dissertation we studied facility location optimization, transportation optimization, inventory policy optimization and how the optimal facility locations and the optimal inventory policy parameters change if the urban freight distribution is reorganized. Research on facility location optimization is presented in chapters 3 and 4. In chapter 3 a discrete facility location problem is studied: two candidate locations are compared for the Istanbul fruit and vegetable wholesale market and the transportation fuel costs for both locations are estimated. In chapter 4 a continuous facility location problem is studied: the location of the Istanbul fruit and vegetable wholesale market that minimizes the transportation fuel costs is searched for over the Asian side of Istanbul. Research on transportation optimization is presented in chapters 5 and 9. In chapter 5 individual urban freight transportation (independently organized by every customer) and urban freight distribution by milk runs (round trips in which different customers are supplied) are compared for the Istanbul fruit and vegetable wholesale market and the transportation costs are estimated. In chapter 9 we study how the transportation costs change if nearby distribution centres merge. For the transportation costs the merger of nearby distribution centres is similar to the possibility of including different stops during one round trip at different distribution centres where products are moved from the distribution centre to the vehicle. In chapter 9 urban freight distribution independently organized by different distributers is compared to urban freight distribution jointly organized by different distributers in a case study of urban freight distribution in Istanbul. The transportation fuel costs are estimated for both distribution methods. Inventory policy optimization is studied in chapters 6 and 7 in which the order-up-to level that minimizes the expected inventory holding cost per unit of time under a fill rate constraint is determined for a periodic review inventory policy applied to a retail shop with capacitated replenishment. In chapter 2 and section 1 of chapter 10 we study how the optimal facility location changes if the urban freight distribution is reorganized. In section 1 of chapter 10 this is studied for the Istanbul fruit and vegetable wholesale market and in chapter 2 this is studied more generally for simplified conditions. In these two studies we found that if the organization of the urban freight distribution is more efficient (e.g. milk run distribution instead of individual transportation), the optimal facility location is closer to the border of the city, closer to where the supplier vehicles enter the city. In chapter 8 and section 3 of chapter 10 we study how the optimal inventory policy parameters (the order-op-to levels that minimize the expected inventory holding costs per unit of time under fill rate constraints) change if the urban freight distribution is reorganized. We found that the reorganization of the urban freight distribution from individual transportation with n ($n>1$) vehicles with capacity c to milk run distribution with a shared capacity $n \times c$ lowers the optimal order-up-to levels and lowers the expected inventory holding cost per unit of time, in the studied cases.

The advantages met in this dissertation of reorganizing the urban fright distribution more jointly are: (i) in case the transportation is organized more jointly by the customers or by the suppliers: less kilometres travelled, less transportation costs, less urban traffic, less air pollution and less traffic noise; (ii) in case of locating an urban facility with customers in the city and with all suppliers vehicles entering the city through the same road (the optimal facility location moves towards the city border): less traffic in the city centre, less air pollution in the city centre, less traffic noise in the city centre, more space available in the city centre for other purposes, lower land cost of the optimal facility location and probably more space available to extend the facility around the optimal facility location; (iii) in case n ($n>1$) facilities are replenished with one vehicle that has n times more capacity than the vehicles

used for individual transportation: less expected inventory holding cost per unit of time and less space necessary in the facilities for inventory because of smaller order-up-to levels.

The research in this dissertation concerning facility location analysis considered only one facility. A possible extension of this research is to consider p facilities with p greater than one and search for the optimal locations of these facilities such as in the p-median problem. Another extension is to search the optimal number of facilities and the optimal location of these facilities, such as in the simple plant location problem. We discussed the p-median problem and the simple plant location problem in the introduction (section 3 of chapter 1).

Other ideas for further research are to study urban freight distribution with electric vehicles and to estimate the costs of distribution with electric vehicles. In [5] and [6] research is done on the costs of electric vehicles. We found in chapter 2 that organizing the urban freight distribution more efficiently changes the optimal location of an urban facility. An extension to this research is to study the change in the optimal facility location if urban freight distribution is done with electric vehicles instead of internal combustion vehicles. Another extension is to study more quantitatively the environmental advantages or disadvantages of candidate facility locations and possible urban freight distribution methods.

An extension of the research presented in this dissertation in the field of inventory management is replenishment with a positive lead time. In chapters 6, 7 and 8 the lead time, the time between the placement of an order and the arrival of the ordered products, is assumed to be zero. Allowing the lead time to be positive increases the complexity. E.g., for the inventory policy studied in chapter 6, if the positive lead time is smaller than the review period, the stock on hand at the beginning of a replenishment cycle (just after ordering) depends on the stock on hand at the beginning of the previous replenishment cycle, the demand during the previous replenishment cycle and the demand during the previous replenishment cycle before replenishment, for the zero lead time case, the stock on hand at the beginning of a replenishment cycle depends only on the stock on hand at the beginning of the previous replenishment cycle and the demand during the previous replenishment cycle. Other ideas for further research are to study also inventory policies with backorders instead of lost sales and to study inventory policies with a reorder level. In inventory policies with a reorder level no orders are placed if the stock on hand is greater than the reorder level. In this dissertation we studied period review order-up-to level inventory policies. An idea for further research is to study the changes in costs if a reorder level is added to this inventory policy for uncapacitated replenishment, individual capacitated replenishment and joint capacitated replenishment.

10.5. Summary

The chapters 2, 3, 4 and 5 and section 1 of chapter 10 contain research on urban facility location analysis and urban freight distribution. Chapter 2 is a more general location analysis of urban facilities while chapters 3, 4 and 5 and section 1 of chapter 10 are case studies on a specific urban facility: the Istanbul fruit and vegetable wholesale market. Chapter 5 showed that reorganizing the urban freight distribution of the Istanbul fruit and vegetable wholesale market from individual transportation to joint transportation (milk run distribution) reduces the transportation fuel cost by more or less 40%. Chapter 2 showed that for an urban facility with customers in the city, suppliers outside the city and all suppliers vehicles entering the

city through the same entry point, reorganizing the transportation changes also the optimal facility location (optimal regarding the transportation cost): if the customer transportation is organized more efficiently, the optimal urban facility location moves towards the city border, towards the supplier vehicles entry point. This is also shown in the Istanbul fruit and vegetable wholesale market case study in section 1 of chapter 10: by reorganizing the customers transportation from individual transportation to milk run distribution, the Aydinli market location changed from being less favourable than the İçerenköy location to more favourable. Reorganizing the urban freight distribution for the Istanbul fruit and vegetable wholesale market from individual transportation to joint transportation decreases the transportation cost and moves the optimal location towards the city border. A market location closer to the city border is considered an environmental advantage (less traffic and pollution in the city centre), a financial advantage (lower land cost) and an operational advantage (possibility to enlarge the facility which is desired by the market customers).

The chapters 6, 7 and 8 and section 3 of chapter 10 contain research on inventory management of periodic review inventory systems with capacitated replenishment. We presented methods to determine the optimal order-up-to levels (optimal regarding the expected inventory holding costs per unit of time) if required degrees of customer satisfaction (required fill rates) are given, for individual capacitated replenishment and joint capacitated replenishment. For the studied cases we found that reorganizing the urban freight distribution from individual transportation (supplying n ($n>1$) retailers individually with vehicles with capacity c) to joint transportation (supplying n retailers in one milk run with a vehicle with capacity $n \times c$) reduces the optimal order-up-to levels (chapters 8 and 10) and the expected inventory holding cost per unit of time (chapter 10) for a fixed required degree of customer satisfaction (fixed required fill rate). In the example in section 4 of chapter 8 the reorganization of the transportation reduces the optimal order-up-to levels with up to more or less 25%. The reduction is dependent on the number of retailers in one milk run: if there are more retailers in one milk run, the order-up-to level is reduced more. In the example of section 3 of chapter 10 the reorganization of the transport reduces the order-up-to levels with more or less 17% and the expected inventory holding cost per unit of time with more or less 15%.

Chapter 9 contains research on the merger of distribution centres located in the same part of the city. This chapter is a case study on six distribution centres and a possible road transshipment centre in the Tuzla district of Istanbul. We compare the urban freight distribution of the six distribution centres before the merger with the integrated urban freight distribution after the merger. We found also in chapter 9 that organizing the transportation more jointly reduces the costs. More specifically we found in the case study that merging the six distribution centres, together with reorganizing the distribution, reduces the urban distribution fuel cost by up to more or less 25%.

We conclude by noticing that in our case studies and examples we found that organizing the urban freight distribution more jointly (by integration of urban delivery trips to different customers or integration of urban delivery trips from different facilities), reduces the transportation costs, reduces the inventory holding costs, reduces the optimal order-up-to levels, changes the optimal urban facility locations (in our examples to locations closer to the city border) and has environmental advantages.

References

[1] M.S. Daskin, What you should know about location modeling, Naval Research Logistics 55 (4) (2008) 283-294.
[2] C.S. ReVelle, H.A. Eiselt, M.S. Daskin, A bibliography for some fundamental problem categories in discrete location science, European Journal of Operational Research 184 (3) (2008) 817-848.
[3] http://www.ibb.gov.tr/tr-TR/SiteImages/Haber/nisan2009/04052009idarisinirharitasi.jpg (22.08.2014).
[4] R.F. Love, J.G. Morris, G.O. Wesolowsky, Facilities location: models and methods, Elsevier Science, New York, 1988.
[5] B.A. Davis, M.A. Figliozzi, A methodology to evaluate the competitiveness of electric delivery trucks, Transportation Research Part E 49 (2013) 8-23.
[6] D.-Y. Lee, V.M. Thomas, M.A. Brown, Electric urban delivery trucks: energy use, greenhouse gas emissions, and cost-effectiveness, Environmental Science & Technology 47 (2013) 8022-8030.

Samenvatting

In dit proefschrift wordt onderzoek gedaan naar het optimaliseren van locaties van faciliteiten (bijvoorbeeld distributiecentra, markten of winkels) in de stad, het optimaliseren van het beheer van voorraden en het optimaliseren van de distributie van goederen naar klanten in de stad. Ook onderzoeken we hoe de optimale locatie van faciliteiten en het optimale beheer van voorraden wijzigt, als de distributie van goederen naar klanten in de stad wordt gewijzigd.

Het proefschrift bevat een uitgebreide casestudy van de fruit- en groentegroothandelsmarkt van Istanbul. In het eerste deel van deze casestudy worden twee locaties vergeleken: de huidige locatie van de fruit- en groentegroothandelsmarkt en de geplande nieuwe locatie van de fruit- en groentegroothandelsmarkt. Voor beide locaties worden de totale brandstofkosten van de voertuigen van de klanten en de leveranciers geschat. In het tweede deel van de casestudy wordt naar de locatie gezocht waarbij de totale brandstofkosten van de voertuigen van de klanten en de leveranciers minimaal zijn, er wordt nu gezocht over heel het Aziatische deel van Istanbul. In het derde deel van de casestudy vergelijken we twee methodes van stedelijke distributie voor de fruit- en groentegroothandelsmarkt van Istanbul: afzonderlijk transport waarbij de klanten zelf de gewenste goederen komen halen en rondes waarbij de goederen geleverd worden aan de klanten (met verschillende leveringen in één ronde). We berekenen het verschil in transportkosten voor beide distributiemethodes. In het vierde deel van de casestudy leggen we alle resultaten samen. Het resultaat van de casestudy is dat de huidige locatie voordeliger is als de belevering van de klanten gebeurt met afzonderlijk transport en de geplande nieuwe locatie, verder van het centrum van de stad, voordeliger is als de belevering van de klanten gebeurt met rondes. Ook onderzoeken we meer algemeen hoe de optimale locatie van een stedelijke faciliteit met klanten in de stad en leveranciers buiten de stad waarvan de voertuigen via een enkel punt de stad binnenkomen, wijzigt, als de distributiemethode wordt gewijzigd. De conclusie is dat als de stedelijke goederendistributie efficiënter wordt, de optimale locatie van de faciliteit verschuift in de richting van het punt waar de voertuigen van de leveranciers de stad binnenkomen (de optimale locatie van de faciliteit verschuift naar de rand van de stad).

In het proefschrift bestuderen we ook de optimalisatie van het beheer van voorraden waarbij de belevering beperkt is. We beschouwen een detailhandelaar die voorraad houdt in een winkel. Het aantal producten in voorraad daalt door verkoop aan klanten en stijgt door levering. Een beleid betreffende wanneer te bestellen en hoeveel te bestellen noemen we een voorraadbeleid. We bestuderen een voorraadbeleid waarbij op regelmatige tijdstippen producten worden besteld en waarvoor geldt dat bij elke bestelling het aantal producten besteld gelijk is aan een vast aantal, de order-up-to level, vermindert met het aantal producten in voorraad. In het eerste deel van ons onderzoek op het gebied van voorraadbeheer bepalen we de fill rate (dit is de verwachting van het percentage van de klantenvraag dat direct wordt ingewilligd door producten uit de voorraad, beschouwd over een lange tijd) voor een voorraadsyteem waarbij de belevering beperkt is, d.w.z. het aantal producten geleverd is kleiner dan of gelijk aan een vast aantal, de capaciteit. In het tweede deel van ons onderzoek op het gebied van voorraadbeheer bepalen we de order-up-to level waarvoor de verwachte voorraadkosten per tijdseenheid minimaal zijn en de fill rate groter dan of gelijk aan een gegeven waarde is, voor een voorraadsysteem met beperkte levering. In het derde deel van ons onderzoek op het gebied van voorraadbeheer beschouwen we verschillende winkels die gezamenlijk beleverd worden door een voertuig dat niet meer producten dan een vast aantal

levert. We presenteren een algoritme om de optimale order-up-to levels (de minimale order-up-to levels zodat de fill rates groter dan of gelijk aan de vereiste waarden zijn) voor de verschillende winkels te bepalen. Een resultaat van dit onderzoek is dat de herorganisatie van de distributie van individuele belevering van n ($n>1$) winkels met n voertuigen met capaciteit c naar gemeenschappelijke belevering met één voertuig met capaciteit $n\times c$ (dat een ronde maakt) de optimale order-up-to levels en de verwachte voorraadkosten per tijdseenheid reduceert in de bestudeerde gevallen.

Het proefschrift bevat ook een casestudy waarin we verschillende nabijgelegen distributiecentra beschouwen en we vergelijken de distributiekosten van deze distributiecentra met de distributiekosten in het geval dat deze distributiecentra worden geïntegreerd tot één groot distributiecentrum. We doen dit voor zes distributiecentra in het district Tuzla van Istanbul. Het resultaat van dit onderzoek is dat ook in deze casestudy gemeenschappelijke organisatie van de distributie van goederen naar klanten in de stad goedkoper is dan individuele distributie (hier door de zes distributiecentra afzonderlijk).

Curriculum vitae

Thomas Dubois (1983) received his Master's degree in Engineering Physics in 2007 at Ghent University. In 2010 he started his PhD at Ghent University. During his PhD studies he was also a visiting researcher at Maltepe University, Istanbul.

This dissertation contributes to the research in the fields of urban logistics, supply chain management, operations research and applied mathematics. In the dissertation the distribution of goods to customers in the city is studied and optimized regarding the transportation costs. The relations between urban freight distribution reorganizations and different aspects of the supply chain, such as the optimal locations of facilities regarding transportation costs and the optimal inventory management regarding inventory holding costs under fill rate constraints, are studied. In the dissertation mathematical proofs and methods are presented related with facility location problems and inventory management. The research also includes case studies on the location and the urban freight distribution of the Istanbul fruit and vegetable wholesale market and on the urban freight distribution of distribution centres and a possible road transshipment centre in Istanbul. Environmental aspects are also discussed.