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# Essays on non-welfaristic redistribution

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by

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Roland Iwan Luttens

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# Contents

# Non-technical summary and conclusions

This dissertation deals with non-welfaristic redistribution and exists out of three papers:

**Paper 1: Lorenz dominance and non-welfaristic redistribution.**

**Paper 2: Is it fair to 'make work pay'?**

**Paper 3: Minimal rights based solidarity.**

The first section presents non-welfaristic redistribution from a broad perspective. The main settings of the different papers and the most important similarities and differences between them are discussed in section two. Section three explicitly turns to the research questions posed in this dissertation and summarizes our main answers to them. Section four explores possible directions for future research.

## 1 Broad orientation

Opinions tend to differ strikingly concerning the redistributive role of the government. Individuals who think that economic success exemplified by a high income primarily depends on hard work, being flexible, being mobile, willing to take high risks, etc, usually oppose redistribution. Conversely, individuals who believe that sex, race, family background, networking, etc, primarily determine income differences usually favor redistribution (Fong (2001), Bowles and Gintis (2002)). Politicians and central planners alike have to base the design of redistributive mechanisms on important choices about which sources of income inequalities entail a reason for redistribution because they are considered offensive and should be eliminated and about which sources of income inequalities are socially acceptable. Therefore, a responsibility cut has to be drawn, i.e. a fundamental distinction has to be made between factors for which individuals should or should not be held responsible for (Dworkin (1981)). Here again, political disagreement arises on the location of the responsibility cut. Right wing parties usually hold people responsible for a larger set of factors than left wing parties do. Given these views, there is a clear tendency for the right wing to be less supportive of redistribution, although on a theoretical level there is not necessarily a monotonic relationship between the degree of responsibility assigned to individuals and the ideal level of redistribution (Cappelen and Tungodden (2005)).

Exactly the idea of incorporating notions of responsibility into the design of redistributive mechanisms constitutes the foundation of non-welfaristic theories of distributive justice (see, among others, Dworkin (1981), Arneson (1989), Cohen (1989), Roemer (1993, 1996, 1998), Fleurbaey (1994, 1995a,b,c,d), Bossert (1995), Bossert and Fleurbaey (1996), Fleurbaey and Maniquet (1996, 1999), Maniquet (1998)). These relatively recent theories have to be contrasted with the traditional welfaristic redistributive theories which are based on the measurement of subjective satisfaction in an interpersonally comparable way to judge the desirability of different social states. Departing from the prevailing ideal of equalizing subjective welfare, a philosophical discussion originated on what exactly constituted the proper equalisandum of non-welfaristic theories. Rawls (1971) first proposed to equalize resources. His resourcist theory was supported by Dworkin (1981) but also different alternatives in between resources and subjective welfare such as functionings (Sen (1985)) or midfare (Cohen (1989)) were proposed. Throughout this dissertation our equalisandum is income. Given that the responsibility cut has been decided upon, the goal of non-welfaristic redistribution of income is perhaps best summarized as an attempt to design a redistribution mechanism that simultaneously satisfies two requirements (Fleurbaey (1995a) and Bossert and Fleurbaey (1996)). On the one hand, there is the *principle of compensation* (Fleurbaey (1995a)): two individuals who are identical on all characteristics for which they are held responsible, and hence only differ with respect to characteristics for which they must be compensated, should receive an equal income after redistribution. This principle implies that when all unequal characteristics are to be compensated, a completely equal distribution of income is the goal. On the other hand, there is the *principle of responsibility* (Barry (1991)), also called the *principle of natural reward* (Fleurbaey (1995a)): two individuals with identical compensation characteristics who only differ with respect to characteristics for which they are held responsible, should not be affected by the redistribution process. This principle implies that no individual should be extra rewarded by redistribution for exercising her responsibility in a particular way. It also implies that if all individuals have identical compensation characteristics there is no reason to perform any redistribution at all.

At the heart of the literature on non-welfaristic income redistribution is the observation that a tension exists between the principle of compensation and the principle of responsibility (see Fleurbaey and Maniquet (2004) and the references cited therein). In fact, in various contexts, axioms representing the principle of compensation turn out to be incompatible with axioms representing the principle of responsibility. Especially when the influence of compensation characteristics is not separable from the influence of responsibility characteristics in the determination of an individual's income, transfers designed to fully compensate for offensive inequalities may alter the distribution of income in a way that is no longer neutral with respect to rewarding different degrees of responsibility. As a result an ethical dilemma arises on which principle should be given priority and which principle should be, possibly considerably, weakened in order to allow for the existence of a redistribution mechanism. The literature has produced a variety of (families of) redistribution mechanisms that either satisfy the principle of compensation or the principle of responsibility.

Although theories of non-welfaristic redistribution of income are clearly based on solid ethical principles, they take up a relatively small share of normative public economics. Their modest popularity could be attributed to the fact that most results are predominantly theoretical and that the implications and workability of implementing non-welfaristic theory are not clearly understood by more traditional analysts. As we clarify below, this dissertation studies in depth some redistributive properties of particular non-welfaristic redistribution mechanisms using traditional concepts from the analysis of income inequalities and poverty. Furthermore, we show how feasible tax-benefit schemes could be designed on the basis of social preferences that take ethical principles on the compensation-responsibility tradeoff into account. We also present some new non-welfaristic redistribution mechanisms and study the corresponding income distributions. By doing so we hope to have contributed in enlarging the set of options available to policy makers confronted with compensation issues.

# 2 A brief exploration into non-welfaristic redistribution

We do not address in this dissertation the question of how to sort individual characteristics into responsibility and compensation factors and hence all papers assume that a democratic process has fixed the responsibility cut. For theories determining the location of the responsibility cut, we refer to Rawls (1971, 1975), Sen (1980), Dworkin (1981), Arneson (1989), Cohen (1989), Temkin (1993), Fleurbaey (1998) and Devooght (2003) for an overview.

The environment in which non-welfaristic redistribution is studied in **paper 1** and **paper 3** is different from the environment used in **paper 2**. The particular choice of framework crucially hinges upon the question whether behavioral responses to redistribution are modelled or not. **Paper 1** and **paper 3** ignore these incentive issues and the model basically describes the case where there is no production and income can be freely transferred in order to compensate for differences in compensation characteristics. In **paper 2** individuals do respond to redistribution and the basic model, which dates back to seminal contributions of Mirrlees (1971) and Pazner and Schmeidler (1974), describes a production environment. Individuals transform their input, in this case their labor, into one output, in this case their income, sharing the same technology, i.e. having the same pre-tax income function. Individuals differ with respect to their productive skills and with respect to their preferences towards labor time and consumption. We briefly set up this basic production framework in a simplified version, as it allows us to sketch some key concepts used in the different papers.

Consider a four type economy in which individuals $\{1, ..., n\}$ have either a low or a high innate skill $s \in S = \{s_L, s_H\}, 0 < s_L < s_H$ and a low or a high taste for working $t \in T = \{t_L, t_H\}, 0 < t_L < t_H$. Types are abbreviated $st \in ST$. Given an amount of work $\ell \in [0, 1]$, income before redistribution $y$ equals $s\ell$ in the usual multiplicative way. Income after redistribution (which is used for consumption) $c$ equals $y - \tau(y)$ with $\tau(y)$ the redistributive mechanism of the government which is an income tax as the government typically only observes $y$ (and not $s$ and $t$). Taste for working defines the marginal rate of substitution between $c$ and $\ell$ for preferences represented by a utility function $u_t(c, \ell)$. Graphically, $t$ determines the curvature of the individual's indifference curves. The indifference curves of a 'lazy' individual are steeper than the indifference curves of a 'hard-working' individual as the former requires a larger increase in consumption in order to be willing to work a little more. The following figure illustrates the initial income distribution in the laisser-faire when there is no government and hence no redistribution.

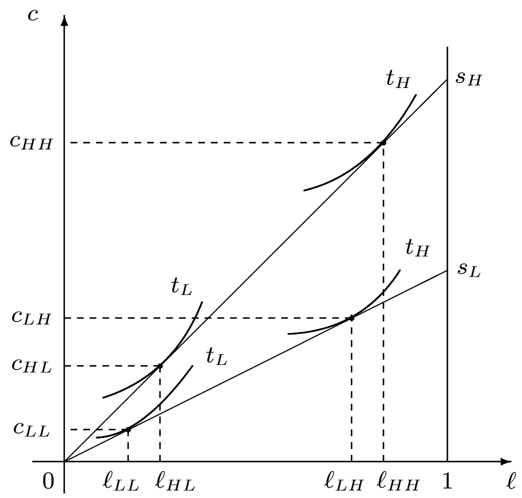


Figure 1: Laisser-faire in a production economy

From a non-welfaristic point of view, this initial income distribution can only be optimal when both $s$ and $t$ belong to the set of responsibility characteristics of the individual and no compensation is needed. The ethical assumption that is usually made is that individuals are held responsible for $t$ but not for differences in $s$. Hence, the principle of compensation applies to skills and demands to redistribute income such that the incomes after redistribution of types $LL$ and $HL$ and the incomes after redistribution of types $LH$ and $HH$ are equalized respectively. The principle of responsibility demands that different preferences do not obtain any differential treatment. In the construction of social preferences where, besides fairness conditions, also efficiency considerations (like the Pareto principle) are taken into account and which are applied to design $\tau(y)$, incompatibilities between both principles arise. The contributions of, among others, Fleurbaey and Maniquet (1996, 1999, 2005a,b,c), Kolm (1996), Gaspart (1998) and Maniquet (1998) are, like **paper 2**, devoted to the proposal of different solutions out of these negative results.

In the setting of **paper 1** and **paper 3** there is no production and redistribution is made via lump-sum transfers, here denoted $\lambda$. Hence, for an individual $i$, $c_i$ equals $\lambda_i + y_i$ and the government's budget constraint is simply $\sum_{i=1}^{n} \lambda_i = 0$. Note that we can move from the setting of **paper 2** to the setting of **paper 1** and **paper 3** by assuming that in our production environment described above utility functions $u_t(c, \ell)$ are quasi-linear in $c$, i.e. $u_t(c, \ell) = c - v_t(\ell)$. Whatever the amount of consumption the individual receives after redistribution, the individual holds her labor supply fixed at her optimal labor time. Hence the total sum of income before redistribution remains constant, irrespective of the way income is redistributed over the population. This explains why the basic model of **paper 1** and **paper 3** is often called the quasi-linear model. Important contributions using quasi-linear models are, among others, Moulin (1994), Bossert (1995), Bossert and Fleurbaey (1996), Iturbe (1997), Sprumont (1997), Bossert, Fleurbaey and Van de gaer (1999) and Tungodden (2005). Let us continue using our four type economy. The compensation characteristic is $s$ and the responsibility characteristic is $t$ and both determine income before redistribution. Assume that a higher skill or a higher taste for working leads to a higher income, hence $y_{LL}$ is strictly smaller than $y_{HL}$ or $y_{LH}$ and $y_{HH}$ is strictly larger than $y_{HL}$ or $y_{LH}$. The principle of compensation requires that individuals with the same taste for working but a different skill end up having the same income after redistribution, whereas the principle of responsibility requires that individuals with the same skill but a different taste for working receive the same transfer. The tension between both principles is easily demonstrated for our four type economy. Consider the following linear system, where the first and second equation represent the principle of compensation, the third and fourth equation the principle of responsibility and the fifth equation the government's budget constraint:

$$\begin{cases} \lambda_{LL} + y_{LL} = \lambda_{HL} + y_{HL} \\ \lambda_{LH} + y_{LH} = \lambda_{HH} + y_{HH} \\ \lambda_{LL} = \lambda_{LH} \\ \lambda_{HL} = \lambda_{HH} \\ \sum_{st} \lambda_{st} = 0 \end{cases}$$

This system has no solution for almost all combinations of parameter values for the different $y$'s.

Two important families of non-welfaristic redistribution mechanisms that play a central role in **paper 1**, the family of *Egalitarian Equivalent mechanisms* (Bossert and Fleurbaey (1996)), named after the concept of egalitarian equivalence introduced by Pazner and Schmeidler (1978) and Fleurbaey (1995b), and the family of *Conditionally Egalitarian mechanisms* (Bossert and Fleurbaey

(1996)), named after the concept of conditional equality introduced by Roemer (1993), respectively keep one principle intact and relax the other principle to only hold for one specific value of the compensation or responsibility characteristic, which is called the *reference* characteristic. In our four type economy and assuming, for the sake of the argument, that the reference characteristic is observed in the population, this is exemplified by dropping one of the first four equations from the system. The $s_L(s_H)$-Egalitarian Equivalent mechanism keeps the principle of compensation intact but the principle of responsibility is only satisfied for individuals having a skill equal to the reference skill, in this case $s_L(s_H)$ (i.e. the fourth (third) equation is dropped from the system). Similarly, the $t_L(t_H)$-Conditionally Egalitarian mechanism keeps the principle of responsibility intact but the principle of compensation is only satisfied for individuals having a taste for working equal to the reference taste, in this case $t_L(t_H)$ (i.e. the first (second) equation is dropped from the system).

Many propositions of redistribution mechanisms in the literature are accompanied with axiomatic characterizations, showing the logical links between different ethical principles expressed in different axioms. An axiomatic approach is adopted in **paper 3**. It can be shown that the principle of compensation underlies the idea that changes in the population profile of compensation characteristics should harm or benefit all individuals in the same direction. Different characterizations of different redistribution mechanisms depend on the precise way in which this *solidarity principle* is performed. A central observation to the analysis in **paper 3** is the relationship between the literature on fair income redistribution and the literature on competing claims and bankruptcy problems (see Thomson (2003) and the references cited therein). In a competing claims problem a fixed amount of money must be allocated on the basis of monetary claims that sum up to more than can be divided. The objective is to design allocation mechanisms that associate with each claims problem a division of the amount available over the claimants. In the context of fair income redistribution, a claim is interpreted as a *reference income level*, which equals the income before redistribution an individual would receive when having the reference compensation characteristic instead of her own compensation characteristic. For example, in our four type economy, low skill type individuals $LL$ and $LH$ could respectively claim the high skill type's income levels $y_{HL}$ and $y_{HH}$, when $s_H$ is the reference skill. In this example the total sum of claims exceeds the total sum of income before redistribution and a competing claims problem arises. We show how the interplay between both strands of the literature in giving a precise interpretation to the solidarity principle leads to the characterization of new families of redistribution mechanisms.

# 3 Research questions ... and answers

In order to implement a non-welfaristic redistribution mechanism, the normative choice for a non-welfaristic social planner is *not* limited to deciding which fairness principle, the principle of compensation or the principle of responsibility, is kept intact and which principle is weakened. Also a reference characteristic needs to be chosen. **Paper 1** studies the effect of the choice of reference characteristic on the income distributions resulting from Egalitarian Equivalent and Conditionally Egalitarian mechanisms. The reason is that different choices of the reference characteristic might lead to considerable differences in income inequalities, even for mechanisms belonging to the *same* family. Whether these income inequalities matter to a non-welfaristic social planner is a normative choice, but when there exist lower or upper bounds to the amount of redistribution we want to perform –think for example of an undesirable redistribution mechanism that forces a part of the population into deprivation– our analysis may help to restrict the range of acceptable reference characteristics. Using the Lorenz dominance partial ordering, the following research question is addressed:

**Is it possible to Lorenz order different Egalitarian Equivalent mechanisms and/or different Conditionally Egalitarian mechanisms?,*

and if the answer to this question is affirmative,

**Which (empirical) information is needed to do so?*

Only requiring that responsibility and compensation characteristics are complements or substitutes in the income generating process, we show that all Egalitarian Equivalent mechanisms can be Lorenz ordered. Conversely, different Conditionally Egalitarian mechanisms cannot be Lorenz ordered, nor can they Lorenz dominate Egalitarian Equivalent mechanisms. Some Egalitarian Equivalent mechanisms can Lorenz dominate all Conditionally Egalitarian mechanisms but in the specific comparison of these mechanisms, different income distributions can only be thoroughly compared when accurate empirical estimates of the pre-tax income function and of the distributions of responsibility and compensation characteristics are available.

In **paper 2** we suggest that a fair allocation would arise if all individuals divided and shared equally all resources, including productive skills. Of course, in the context of income redistribution, labor market productivities such as intelligence or handicaps are typically inalienable. Hence, we ask the question whether an efficient allocation exists where every individual is indifferent between her bundle and sharing all resources equally, or:

**Does a Pareto Efficient and Shared Resources Equivalent allocation exist?*

We show that such an allocation indeed exists and we use this allocation to construct social preferences. Subsequently, these social preferences are applied to design an optimal *fair* non-linear income tax scheme. We then ask:

**Does a fair income tax scheme exhibit negative marginal tax rates for the lower incomes?*

This is certainly a relevant policy question given the increased importance many governments attach to 'making work pay' programs that aim at subsidizing low income earners. We show that, for a given budget constraint, negative marginal tax rates cannot be optimal in the reasonable case in which at least some unemployed are willing to work but cannot due to exogenous labor market constraints.

In **paper 3** we propose to use individuals' *minimal rights* to divide an extra amount of income generated by a change in the population profile of compensation characteristics. The minimal right of an individual equals the amount that remains from the sum of income before redistribution when all other individuals receive their claim. Priority is given to individuals with a higher claim once the sum of income before redistribution exceeds threshold levels where individuals' minimal rights become positive. We call this solidarity principle *minimal rights based solidarity* and ask:

* *Which families of non-welfaristic redistribution mechanisms satisfy minimal rights based solidarity?*

We characterize two different families of Minimal Rights based Egalitarian mechanisms. One family guarantees each individual her claim when claims are feasible. The other family guarantees a non-negative income after redistribution for all individuals.

# 4 Back to the future

As every new answer usually raises a multiple of new questions, it is not surprising that the topics dealt with in this dissertation still leave open some avenues for future research.

As shown in **paper 1** and **paper 3**, many non-welfaristic redistribution mechanisms take the form of families. These families contain in themselves an infinity of different solutions with possibly very different properties. The precise choice of a member from such a family usually depends on the choice of a specific reference characteristic, which invokes additional ethical principles. In **paper 1** there is a concern for the way responsibility is rewarded, as some non-welfaristic redistribution mechanisms might bring about income inequalities that are considered too severe. However, very different principles might be considered. Therefore, our analysis in **paper 1** might add some new insights, but a full-blown theory about the choice of reference characteristic still needs to be developed.

The Pareto Efficient and Shared Resources Equivalent allocation presented in **paper 2** is in itself an interesting research topic. A characterization of this solution might reveal its ethical qualities and lead to a confrontation with related solutions proposed in the literature. Furthermore, the Walrasian equilibrium properties of sharing productive skills equally need to be studied. Our theoretical results in **paper 2** on the optimal second best allocation in a four type economy could be extended to the case of a general population. The calibration of the responsibility and compensation characteristic used in the simulation is probably only one of many possible ways to turn specific concepts from non-welfaristic theory into a computable framework. A general procedure to generate statistically independent (indexes of) responsibility and compensation characteristics has not yet been proposed.

We use specific fairness principles in **paper 2** to derive an *optimal* income tax scheme. However, one could start from an actual tax scheme and the distributions of personal characteristics and introduce fairness principles in *marginal tax reform.*

The established similarities in **paper 3** between the literature on non-welfaristic redistribution and the literature on competing claims problems obviously open up an array of new questions into finding further links. It is for example not hard to show that, when there are only two claimants, the two families of Minimal Rights based Egalitarian mechanisms presented in **paper 3** are equivalent with a mechanism called Concede-and-Divide. This mechanism is a widely accepted procedure to adjudicate two conflicting claims and many well known $n$-person rules coincide with Concede-and-Divide in the two claimants case (see Thomson (2003) for an overview). We believe that solidarity based on minimal rights could also be relevantly applied in a competing claims context: think for example of correcting an allocation that was originally based on a wrong estimation of the liquidation value of a bankrupt firm. The fact that our two families of Minimal Rights based Egalitarian mechanisms have not yet been proposed in the context of a competing claims problem illustrates that, although both strands of the literature abundantly use axiomatic characterizations in proposing new solutions, most axioms express very different principles. A further examination into which ethical principles could be transferred from one context to the other might lead to proposals of new income redistribution mechanisms and/or new bankruptcy rules.

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# Lorenz dominance and non-welfaristic redistribution*

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**Abstract**

Our concern is for income inequalities that may result from non-welfaristic redistribution schemes. We show that for large classes of income functions Lorenz dominance results can be found in the comparison of two egalitarian equivalent mechanisms. Comparisons of different conditionally egalitarian mechanisms only yield poverty dominance results. In general, no egalitarian equivalent mechanism can be Lorenz dominated by a conditionally egalitarian mechanism. Our analysis stresses the need for accurate empirical estimates of the pre-tax income function and of the distributions of responsibility and compensation characteristics.

JEL classification: D31, D63, H21, I32.
Keywords: non-welfaristic redistribution, Lorenz dominance.

## 1 Motivation

Recently, non-welfaristic income redistribution schemes are presented (Fleurbaey (1994, 1995a), Bossert (1995), Bossert and Fleurbaey (1996)). These axiomatically founded mechanisms are designed to fulfil to different degrees the fairness goal of compensating individuals for factors beyond their responsibility. In that, they follow ideas of political philosophers like Dworkin (1981a,b), Arneson (1989) and Cohen (1989), who motivate egalitarians to criticize traditional welfaristic theory, where the objective of the government is to maximize a social welfare function based on the aggregation of individual utilities only, for ignoring the issue of personal responsibility. Following Cohen, different outcomes should only be equalized when they are due to factors beyond control of the

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individual. Conversely, differences due to differential effort are morally acceptable. These ideas have smoothed the path for non-welfaristic theory, where one has to collect non-utility information such as expended effort to express a social judgement. However, these considerations have given rise to the compensation problem (extensively surveyed in Fleurbaey and Maniquet (2004)). Within the context of first best income redistribution and without separability assumptions on the pre-tax income function, no solution can at the same time assure equal incomes to individuals with equal responsibility characteristics, while guaranteeing equal transfers to individuals with equal compensation characteristics (Fleurbaey (1995b), Bossert (1995)). The redistribution mechanisms of Bossert and Fleurbaey (1996), namely the *egalitarian equivalent* mechanism and the *conditionally egalitarian* mechanism, following Pazner and Schmeidler (1978) and Fleurbaey (1995a), are derived from relaxing either one of these axioms.

There exists a related literature on equality of opportunity, that proposes criteria to evaluate redistribution schemes. Roemer's (1993, 1998) criterion advocates redistribution schemes that maximize a weighted average of the lowest income achieved for each value of the responsibility characteristic. Van de gaer's (1993) criterion computes for every group of individuals with an equal compensation characteristic the weighted average of their incomes over all values of the responsibility characteristic and advocates redistribution schemes that maximize this lowest weighted average. With these criteria a complete ordering of different redistribution schemes is obtained and axiomatic characterizations of these criteria have been provided (Bossert et al. (1999), Ooghe et al. (2005)). Based on opportunity equalizing transfers, Peragine (2004) develops dominance criteria that have a strong analogy with the traditional Lorenz dominance ordering. His methodology provides an incomplete ranking of redistribution mechanisms from a perspective of equality of opportunity.

The main goal of our analysis is different. We examine the consequences of egalitarian equivalent and conditional egalitarian redistribution on income inequalities. We believe that a concern for income inequality is – even for opportunity egalitarians – legitimate for a number of reasons. Performing a non-welfaristic redistribution exercise may result in inequalities that are too harsh. Our ethical intuition may lead us to consider a limit to the kind and amount of penalty individuals should endure for exercising low responsibility. At least the satisfaction of their basic needs should be guaranteed (Fleurbaey (1995b)). Moreover, some caution on the amount of redistribution is advisable whenever informational difficulties hamper the determination of personal responsibility. Statistical difficulties may render the latter even more difficult in empirical applications. Furthermore, the implementation of an egalitarian equivalent mechanism or a conditionally egalitarian mechanism requires the choice of a reference characteristic. Given the above-mentioned reasons, a non-welfaristic social planner may take income inequalities or poverty into consideration when choosing a precise reference characteristic. Our analysis may therefore help to restrict the range of admissible reference characteristics and add some new insights to the theory about the choice of particular reference characteristics within different families of non-welfaristic redistribution schemes.

In this paper we attempt to order egalitarian equivalent mechanisms and conditionally egalitarian mechanisms on the basis of standard distributional criteria such as Lorenz dominance (following the fundamental contribution of Atkinson (1970)) and poverty dominance (following the pioneering work by Sen (1976)). As explained before, our particular concern for the poorest is prompted by the harsh penalties non-welfaristic redistribution may provoke.

This paper proceeds as follows. In section 2 we introduce the pre-tax income function, discuss the goal of non-welfaristic redistribution, present the egalitarian equivalent mechanism and the conditionally egalitarian mechanism and explain the criteria of Lorenz dominance and poverty dominance to compare different income distributions. In section 3 we impose some minimal assumptions on the pre-tax income function in order to keep the analysis as general as possible. Without making distributional assumptions on individual's responsibility and compensation characteristics, we compare the income distributions of two different egalitarian equivalent mechanisms, two different conditionally egalitarian mechanisms and an egalitarian equivalent mechanism versus a conditionally egalitarian mechanism. We check whether under one of the two mechanisms both the poorest do not get poorer while the richest do not get richer. After taking this necessary condition for Lorenz dominance into consideration, only the comparison of an egalitarian equivalent mechanism with a conditionally egalitarian mechanism remains undecided. Additional information is needed, so in section 4 we present a specific example, which illustrates the results derived in the previous section but at the same time allows us to draw more precise conclusions: possible poverty dominance between two conditionally egalitarian mechanisms is examined and also the comparison of an egalitarian equivalent mechanism with a conditionally egalitarian mechanism is revisited. Section 5 summarizes our main conclusions.

## 2 Non-welfaristic redistribution mechanisms

The model presented here is taken from Bossert (1995). Suppose that in a population $N = \{1, ..., n\}$, $n \geq 2$, person $i$'s ($i \in N$) individual characteristics vector $\mathbf{a}_i$ can be partitioned into two vectors: $\mathbf{a}_i^R \in \Omega_R$, representing the individual's responsibility characteristics, and $\mathbf{a}_i^S \in \Omega_S$, representing her compensation characteristics. The set of all possible characteristics vectors is $\Omega = \Omega_R \times \Omega_S$, where $\Omega_R \subseteq \mathbb{R}^r$, $\Omega_S \subseteq \mathbb{R}^s$ and $\Omega_R$, $\Omega_S \neq \emptyset$. The characteristics profile is given by $\mathcal{A} = (\mathbf{a}_1, ..., \mathbf{a}_n) \in \Omega^n$. Denote $f : \Omega \to \mathbb{R}_{++} : \mathbf{a} = (\mathbf{a}^R, \mathbf{a}^S) \to f(\mathbf{a})$ an income function, assigning pre-tax income to each possible characteristics vector and let $\mathcal{F}$ be the set of all possible pre-tax income functions. An economy is described by $\mathcal{E} = (\mathcal{A}, f)$. Denote $D = \Omega^n \times \mathcal{F}$ the set of all economies. A redistribution mechanism $F : D \to \mathbb{R}^n : \mathcal{E} \to [F_1(\mathcal{E}), ..., F_n(\mathcal{E})]$ maps an economy into an income distribution, such that $\sum_{i=1}^n F_i(\mathcal{E}) = \sum_{i=1}^n f(\mathbf{a}_i), \forall \mathcal{E} \in D$. The set of all possible redistribution mechanisms is denoted $\digamma$.

The goal of non-welfaristic income redistribution is to preserve the effect of responsibility characteristics and to eliminate the influence of compensation characteristics. Revelatory work of Fleurbaey (1994, 1995b) and Bossert (1995) elucidates the compensation problem: unless the income function is additively separable in $\mathbf{a}^R$ and $\mathbf{a}^S$, no redistribution mechanism can only but fully compensate for differentials in $\mathbf{a}^S$. Bossert and Fleurbaey (1996) relax some of the axioms underlying this impossibility result to derive the two families of solutions at the heart of our research, the egalitarian equivalent mechanism and the conditionally egalitarian mechanism. They are designed to fully accomplish one of the two goals (respectively compensation and responsibility), while the other goal is only fulfilled for a so called 'reference' vector. Denote $F^{EE,\tilde{\mathbf{a}}^S}(\mathcal{E})$ the egalitarian equivalent mechanism where $\tilde{\mathbf{a}}^S \in \Omega_S$ is the reference compensation characteristics vector. Similarly, $F^{CE,\tilde{\mathbf{a}}^R}(\mathcal{E})$ is the conditionally egalitarian mechanism where $\tilde{\mathbf{a}}^R \in \Omega_R$ is the reference responsibility characteristics vector. Our purpose is to evaluate these redistribution mechanisms, using the traditional concepts of Lorenz dominance and poverty dominance. The following subsections explain $F^{EE,\tilde{\mathbf{a}}^S}(\mathcal{E})$, $F^{CE,\tilde{\mathbf{a}}^R}(\mathcal{E})$ and the criteria of Lorenz dominance and poverty dominance in more detail.

## 2.1 The egalitarian equivalent mechanism

The egalitarian equivalent mechanism $F^{EE,\tilde{\mathbf{a}}^S}(\mathcal{E})$ gives, for all $\mathcal{E} \in D$, each individual $k \in N$ the following income:

$$F_k^{EE,\tilde{\mathbf{a}}^S}(\mathcal{E}) = f(\mathbf{a}_k^R, \tilde{\mathbf{a}}^S) - \frac{1}{n}\sum_{i=1}^{n}\left(f(\mathbf{a}_i^R, \tilde{\mathbf{a}}^S) - f(\mathbf{a}_i^R, \mathbf{a}_i^S)\right). \tag{1}$$

With this mechanism, every individual has a post-tax income equal to the pre-tax income she would earn if her compensation characteristics were $\tilde{\mathbf{a}}^S$, plus a uniform transfer. Two egalitarian equivalent mechanisms only differ in the choice of $\tilde{\mathbf{a}}^S$. This choice determines the reward scheme for responsibility, i.e. it determines the magnitude of income differences due to differences in $\mathbf{a}^R$.

This mechanism satisfies the strong compensation axiom of 'group solidarity in $\mathbf{a}^S$': any variation in some individual's compensation characteristics is equally borne by all individuals. At the same time, only the weaker responsibility axiom of 'equal transfer for $\tilde{\mathbf{a}}^S$' is satisfied. This implies that there is no reason to perform any redistribution only when everybody's compensation characteristics are equal to $\tilde{\mathbf{a}}^S$.

## 2.2 The conditionally egalitarian mechanism

The conditionally egalitarian mechanism $F^{CE,\tilde{\mathbf{a}}^R}(\mathcal{E})$ gives, for all $\mathcal{E} \in D$, each individual $k \in N$ the following income:

$$F_k^{CE,\tilde{\mathbf{a}}^R}(\mathcal{E}) = f(\mathbf{a}_k^R, \mathbf{a}_k^S) - f(\tilde{\mathbf{a}}^R, \mathbf{a}_k^S) + \frac{1}{n}\sum_{i=1}^{n} f(\tilde{\mathbf{a}}^R, \mathbf{a}_i^S). \tag{2}$$

With this mechanism, every individual has a post-tax income equal to the average pre-tax income that would prevail if everyone in society has $\mathbf{a}^R = \tilde{\mathbf{a}}^R$. If the individual deviates from this reference level, she alone bears the resulting difference. Two conditionally egalitarian mechanisms only differ in the choice of $\tilde{\mathbf{a}}^R$. This choice determines the magnitude of differences in transfers due to differences in $\mathbf{a}^S$.

This mechanism only satisfies the weaker compensation axiom of 'equal income for $\tilde{\mathbf{a}}^R$': income equality is only required if all individuals have responsibility characteristics equal to $\tilde{\mathbf{a}}^R$. This mechanism also satisfies the strong responsibility axiom of 'individual monotonicity in $\mathbf{a}^R$'. This means that a change in one individual's responsibility characteristics only affects this person's post-tax income.

## 2.3 Lorenz dominance and poverty dominance

We briefly discuss the concepts of Lorenz dominance and poverty dominance (see respectively Lambert (2001) and Zheng (2000) for details).[1] The set of income distributions with support $[\underline{y}, \overline{y}]$ is $\Gamma = \{G : [\underline{y}, \overline{y}] \to [0,1] | G$ is nondecreasing, $G(\underline{y}) = 0$ and $G(\overline{y}) = 1\}$. Let $G_{\mathcal{E}}^X \in \Gamma$ be the income distribution that results from redistribution mechanism $F^X(\mathcal{E}) \in \digamma$. Mean income $\mu(\mathcal{E}) = \int_{\underline{y}}^{\overline{y}} y dG_{\mathcal{E}}^X(y)$ is independent of the redistribution mechanism. With every income level $y$ there corresponds a rank $p \in [0,1]$ identified by $p = G_{\mathcal{E}}^X(y)$. For every $p$-value, define $y_{\mathcal{E}}^X(p)$ as the inverse function of $G_{\mathcal{E}}^X$ evaluated at $p$. The Lorenz curve $L_{\mathcal{E}}^X(p) = \frac{1}{\mu(\mathcal{E})} \int_{\underline{y}}^{y_{\mathcal{E}}^X(p)} x dG_{\mathcal{E}}^X(x)$. An income distribution $G_{\mathcal{E}}^A \in \Gamma$ Lorenz dominates an income distribution $G_{\mathcal{E}}^B \in \Gamma$ if and only if $L_{\mathcal{E}}^A(p) \geq L_{\mathcal{E}}^B(p)$ for all $p$ and $L_{\mathcal{E}}^A \neq L_{\mathcal{E}}^B$. Let $F^A(\mathcal{E}), F^B(\mathcal{E}) \in \digamma$. Define the function $S_{\mathcal{E}}^{A;B} : [\underline{y}, \overline{y}] \to \mathbb{R} : y \to S_{\mathcal{E}}^{A;B}(y) = \int_{\underline{y}}^{y} (G_{\mathcal{E}}^B(x) - G_{\mathcal{E}}^A(x))dx$. The Lorenz dominance partial ordering $\succ_L$ is a binary relation on $\digamma$ applied to a given economy $\mathcal{E}$ such that $F^A(\mathcal{E}) \succ_L F^B(\mathcal{E})$ if and only if $S_{\mathcal{E}}^{A;B}(y) \geq 0$ for all $y \in [\underline{y}, \overline{y}]$ and there exists $y \in [\underline{y}, \overline{y}]$ such that $S_{\mathcal{E}}^{A;B}(y) > 0$. A necessary condition for Lorenz dominance is that, at the same time, the poorest are not poorer and the richest are not richer under $F^A(\mathcal{E})$ compared to $F^B(\mathcal{E})$. If we fail to establish Lorenz dominance results, one next step could be to look for more complete orderings.[2] Instead, we look at poverty dominance.

[1] The normative implications of Lorenz dominance results in terms of social welfare are well known since the influential work of Kolm (1969) and Atkinson (1970). However, for a non-welfarist concerned with income inequality this welfarist underpinning is not essential.

[2] This requires the imposition of axioms, such as the transfer sensitivity axiom (Shorrocks and Foster (1987)), that are less generally accepted (Lambert (2001)).

Denote the poverty line separating the poor from the nonpoor by $z \in [\underline{y}, \overline{y}]$. Let $\overline{z}$ be the highest poverty line considered. The first order poverty dominance partial ordering $\succ_{P_1,\overline{z}}$ is a binary relation on $\digamma$ applied to a given economy $\mathcal{E}$ such that $F^A(\mathcal{E}) \succ_{P_1,\overline{z}} F^B(\mathcal{E})$ if and only if $G^A_{\mathcal{E}}(y) \leq G^B_{\mathcal{E}}(y)$ for all $y \in [\underline{y}, \overline{z}]$ and there exists $y \in [\underline{y}, \overline{z}]$ such that $G^A_{\mathcal{E}}(y) \neq G^B_{\mathcal{E}}(y)$. The second order poverty dominance partial ordering $\succ_{P_2,\overline{z}}$ is a binary relation on $\digamma$ applied to a given economy $\mathcal{E}$ such that $F^A(\mathcal{E}) \succ_{P_2,\overline{z}} F^B(\mathcal{E})$ if and only if $S^{A;B}_{\mathcal{E}}(y) \geq 0$ for all $y \in [\underline{y}, \overline{z}]$ and there exists $y \in [\underline{y}, \overline{z}]$ such that $S^{A;B}_{\mathcal{E}}(y) > 0$. Clearly, first order poverty dominance implies second order poverty dominance. Furthermore, Lorenz dominance implies second order poverty dominance but not the other way around. Therefore, and since non-welfarists might be concerned about poverty (see section 1), if we fail to establish Lorenz dominance results between two mechanisms, we see whether we can prove poverty dominance or not.

# 3 Distributional analysis: general framework

In this section, we state Lorenz dominance results with respect to the different non-welfaristic redistribution mechanisms in general terms. First, we define two large classes of pre-tax income functions which we use to define domains for redistribution mechanisms. Successively, we compare the income distributions of two egalitarian equivalent mechanisms with different reference compensation characteristics, the income distributions of two conditionally egalitarian mechanisms with different reference responsibility characteristics and the income distributions of an egalitarian equivalent mechanism and a conditionally egalitarian mechanism. We search for dominance results that hold for any economy in the domains defined. This implies that the results we obtain are valid over all distributions of responsibility and compensation characteristics, since no explicit assumptions on these distributions are stated. We conclude this section with some remarks.

## 3.1 Assumptions on the distributions of $a^R$ and $a^S$ and on the pre-tax income function

We consider the case where $a^R$ and $a^S$ are scalars instead of vectors. The lowest and highest values of $a^R$ are respectively denoted $\underline{a}^R$ and $\overline{a}^R$. Similarly, the lowest and highest values of $a^S$ are respectively denoted $\underline{a}^S$ and $\overline{a}^S$. Furthermore, we require that $\underline{a}^R$ and $\underline{a}^S$ are not equal to $\overline{a}^R$ and $\overline{a}^S$ respectively. The set of all possible characteristics vectors $(a^R, a^S)$ that have these properties is $\Omega'$. The characteristics profile is given by $\mathcal{A}' \in \Omega'^n$.

The pre-tax income function $f$ is continuously differentiable to the required degree. In addition, for all $(a^R, a^S) \in [\underline{a}^R, \overline{a}^R] \times [\underline{a}^S, \overline{a}^S]$,

$$\frac{\partial f(a^R, a^S)}{\partial a^R} > 0 \tag{3}$$

and[3]

$$\frac{\partial f(a^R, a^S)}{\partial a^S} > 0. \tag{4}$$

For all pre-tax income functions having properties (3) and (4), the poorest have characteristics $(\underline{a}^R, \underline{a}^S)$, while the richest have characteristics $(\overline{a}^R, \overline{a}^S)$.

If $\frac{\partial^2 f(a^R,a^S)}{\partial a^R \partial a^S}$ equals 0 for all $(a^R, a^S) \in [\underline{a}^R, \overline{a}^R] \times [\underline{a}^S, \overline{a}^S]$, both the egalitarian equivalent mechanism and the conditionally egalitarian mechanism reduce to 'the natural redistribution mechanism', which assigns the income due to an individual's $a^R$ entirely to that individual and divides the total income due to $a^S$ equally among all individuals (Bossert (1995)). In this case all egalitarian equivalent mechanisms and all conditionally egalitarian mechanisms lead to the same income distribution.

Denote by $\mathcal{F}^+$ the class of income functions that satisfy (3), (4) and

$$\frac{\partial^2 f(a^R, a^S)}{\partial a^R \partial a^S} \geq 0 \tag{5}$$

and for at least one value of $(a^R, a^S) \in [\underline{a}^R, \overline{a}^R] \times [\underline{a}^S, \overline{a}^S]$ the inequality holds strict.

Denote by $\mathcal{F}^-$ the class of income functions that satisfy (3), (4) and

$$\frac{\partial^2 f(a^R, a^S)}{\partial a^R \partial a^S} \leq 0 \tag{6}$$

and for at least one value of $(a^R, a^S) \in [\underline{a}^R, \overline{a}^R] \times [\underline{a}^S, \overline{a}^S]$ the inequality holds strict.

$\mathcal{F}^+$ ($\mathcal{F}^-$) is the class of income functions for which $a^R$ and $a^S$ are complements (substitutes) in the income generating process.[4] As such, these classes are easy to interpret. Moreover, whether $f$ belongs to $\mathcal{F}^+$, $\mathcal{F}^-$ or neither is essentially an empirical issue. For these classes of income functions, it is straightforward to test statistically to which class the income function belongs.

Our goal is to look for Lorenz dominance results for different non-welfaristic redistribution mechanisms over all economies $\mathcal{E}' = (\mathcal{A}', f)$ in the domains $D^+ = {\Omega'}^n \times \mathcal{F}^+$ or $D^- = {\Omega'}^n \times \mathcal{F}^-$.

[3] Conditional upon monotonicity, assumptions (3) and (4) are a matter of measurement. If, for example, we only have information on handicaps, assumption (4) can be satisfied by redefining the information in terms of '*lack of* ... (handicap)'.

[4] Restrictions on cross derivatives are not uncommon in the dominance literature, e.g., Hadar and Russell (1974) or Atkinson and Bourguignon (1982).

## 3.2 Two egalitarian equivalent mechanisms

First, the following lemma identifies the poorest and richest individuals under an egalitarian equivalent mechanism $F^{EE,\tilde{a}^S}(\mathcal{E}')$.

**Lemma 1** *: For an economy $\mathcal{E}'$ with $\mathcal{A}' \in \Omega'^n$ and $f$ satisfying (3), under $F^{EE,\tilde{a}^S}(\mathcal{E}')$ the poorest have responsibility characteristic $\underline{a}^R$ and the richest have responsibility characteristic $\overline{a}^R$, irrespective of their compensation characteristic $a^S$.*

**Proof.** $F^{EE,\tilde{a}^S}(\mathcal{E}')$ consists of an individual specific part $f(a_k^R, \tilde{a}^S)$ plus a uniform transfer. By (3), $f(a^R, \tilde{a}^S)$ is increasing in $a^R$. ■

The following proposition states that, depending on the economy concerned, the egalitarian equivalent mechanism with the lower (higher) reference compensation characteristic Lorenz dominates the egalitarian equivalent mechanism with the higher (lower) reference compensation characteristic.

**Proposition 1** *: $\forall \tilde{a}_1^S, \tilde{a}_2^S \in [\underline{a}^S, \overline{a}^S]$:*

$$\forall \mathcal{E}' \in D^+: \tilde{a}_1^S < \tilde{a}_2^S \Leftrightarrow F^{EE,\tilde{a}_1^S}(\mathcal{E}') \succ_L F^{EE,\tilde{a}_2^S}(\mathcal{E}'),$$
$$\forall \mathcal{E}' \in D^-: \tilde{a}_1^S < \tilde{a}_2^S \Leftrightarrow F^{EE,\tilde{a}_2^S}(\mathcal{E}') \succ_L F^{EE,\tilde{a}_1^S}(\mathcal{E}').$$

**Proof.** ($\Rightarrow$) Compare $F^{EE,\tilde{a}_1^S}(\mathcal{E}')$ and $F^{EE,\tilde{a}_2^S}(\mathcal{E}')$ with $\tilde{a}_1^S, \tilde{a}_2^S \in [\underline{a}^S, \overline{a}^S]$ and $\tilde{a}_1^S < \tilde{a}_2^S$. Using (1), the income difference for an individual $k$ between the two mechanisms $F_k^{EE,\tilde{a}_1^S}(\mathcal{E}') - F_k^{EE,\tilde{a}_2^S}(\mathcal{E}')$ equals
$f(a_k^R, \tilde{a}_1^S) - f(a_k^R, \tilde{a}_2^S) + A = \phi(a_k^R)$
where $A = \left(\frac{1}{n}\sum_{i=1}^n f(a_i^R, \tilde{a}_2^S) - \frac{1}{n}\sum_{i=1}^n f(a_i^R, \tilde{a}_1^S)\right)$.
Note that:
$(i)$ redistribution in the first best means that: $\sum_{i=1}^n \phi(a_i^R) = 0$,
$(ii)$ by (4) and ($\tilde{a}_1^S < \tilde{a}_2^S$): $A > 0$ and $f(a_k^R, \tilde{a}_1^S) - f(a_k^R, \tilde{a}_2^S) < 0$,
$(iii)$ for $D^+$: by (5), $\phi(a_k^R)$ is non increasing in $a_k^R$, and
$(iv)$ for $D^-$: by (6), $\phi(a_k^R)$ is non decreasing in $a_k^R$.
Combining $(i)$, $(ii)$ and $(iii)$ ensures that for $D^+$ there exists $a_+^R \in (\underline{a}^R, \overline{a}^R)$, such that, if $a_k^R < (>)\ a_+^R$, then $\phi(a_k^R) > (<)\ 0$. Hence, individuals with $a_k^R < (>)\ a_+^R$ receive more (less) income under $F^{EE,\tilde{a}_1^S}(\mathcal{E}')$ than under $F^{EE,\tilde{a}_2^S}(\mathcal{E}')$. So there are transfers of income from individuals with a high $a^R$ to individuals with a low $a^R$. Since $F^{EE,\tilde{a}^S}(\mathcal{E}')$ exists of an individual specific part $f(a_k^R, \tilde{a}^S)$ plus a uniform transfer, income is increasing in $a^R$. This implies that all transfers go from rich to poor. As a result $F^{EE,\tilde{a}_1^S}(\mathcal{E}')$ Lorenz dominates $F^{EE,\tilde{a}_2^S}(\mathcal{E}')$.
Combining $(i)$, $(ii)$ and $(iv)$ ensures that for $D^-$ there exists $a_-^R \in (\underline{a}^R, \overline{a}^R)$, such that, if $a_k^R < (>)\ a_-^R$, then $\phi(a_k^R) < (>)\ 0$. Hence, individuals with $a_k^R < (>)\ a_-^R$ receive less (more) income under $F^{EE,\tilde{a}_1^S}(\mathcal{E}')$ than under $F^{EE,\tilde{a}_2^S}(\mathcal{E}')$. So there are transfers of income from individuals with a low $a^R$ to individuals with a high

$a^R$, that is all transfers go from poor to rich. As a result $F^{EE,\tilde{a}_2^S}(\mathcal{E}')$ Lorenz dominates $F^{EE,\tilde{a}_1^S}(\mathcal{E}')$.

($\Leftarrow$) For $D^+$. When $F^{EE,\tilde{a}_1^S}(\mathcal{E}') \succ_L F^{EE,\tilde{a}_2^S}(\mathcal{E}')$ and $\tilde{a}_1^S \neq \tilde{a}_2^S$, the poorest have a higher income under $F^{EE,\tilde{a}_1^S}(\mathcal{E}')$ than under $F^{EE,\tilde{a}_2^S}(\mathcal{E}')$ and the richest have a higher income under $F^{EE,\tilde{a}_2^S}(\mathcal{E}')$ than under $F^{EE,\tilde{a}_1^S}(\mathcal{E}')$. Both conditions only hold together when $f(\bar{a}^R, \tilde{a}_1^S) - f(\underline{a}^R, \tilde{a}_1^S) < f(\bar{a}^R, \tilde{a}_2^S) - f(\underline{a}^R, \tilde{a}_2^S)$. Given (5), this can only be true when $\tilde{a}_1^S < \tilde{a}_2^S$. The proof for $D^-$ is analogous. ■

Note that proposition 1 is instructive for the choice of $\tilde{a}^S$, since this choice determines the inequality in the resulting income distribution.

### 3.3 Two conditionally egalitarian mechanisms

First, the following lemma identifies the poorest and richest individuals under a conditionally egalitarian mechanism $F^{CE,\tilde{a}^R}(\mathcal{E}')$.

**Lemma 2** *: For an economy $\mathcal{E}'$ in $D^+$, under $F^{CE,\tilde{a}^R}(\mathcal{E}')$ the poorest have characteristics $(\underline{a}^R, \overline{a}^S)$ and the richest have characteristics $(\overline{a}^R, \overline{a}^S)$. For an economy $\mathcal{E}'$ in $D^-$, under $F^{CE,\tilde{a}^R}(\mathcal{E}')$ the poorest have characteristics $(\underline{a}^R, \underline{a}^S)$ and the richest have characteristics $(\overline{a}^R, \underline{a}^S)$.*

**Proof.** For $D^+$ or $D^-$, employing (3), $\frac{\partial F^{CE,\tilde{a}^R}(\mathcal{E}')}{\partial a^R} > 0$, so the poorest have $\underline{a}^R$ and the richest have $\overline{a}^R$. Under $F^{CE,\tilde{a}^R}(\mathcal{E}')$ the poorest have the smallest value of $f(\underline{a}^R, a^S) - f(\tilde{a}^R, a^S)$, while the richest have the highest value of $f(\overline{a}^R, a^S) - f(\tilde{a}^R, a^S)$. For $D^+$, due to (5), the poorest and the richest have $\overline{a}^S$. For $D^-$, due to (6), the poorest and the richest have $\underline{a}^S$. ■

From lemma 2 and lemma 1, for any economy in $D^+$ or $D^-$, the poorest and the richest under a conditionally egalitarian mechanism are also among the poorest and richest under an egalitarian equivalent mechanism.

The following proposition states that there is no Lorenz dominance between two different conditionally egalitarian mechanisms.

**Proposition 2** *: There does not exist $\mathcal{E}' \in (D^+ \cup D^-)$ and $\tilde{a}_1^R, \tilde{a}_2^R \in [\underline{a}^R, \overline{a}^R]$ : $F^{CE,\tilde{a}_1^R}(\mathcal{E}') \succ_L F^{CE,\tilde{a}_2^R}(\mathcal{E}')$.*

**Proof.** Compare $F^{CE,\tilde{a}_1^R}(\mathcal{E}')$ and $F^{CE,\tilde{a}_2^R}(\mathcal{E}')$ with $\tilde{a}_1^R < \tilde{a}_2^R$ and $\tilde{a}_1^R, \tilde{a}_2^R \in [\underline{a}^R, \overline{a}^R]$. Using (2), the income difference for an individual $k$ between the two mechanisms $F_k^{CE,\tilde{a}_1^R}(\mathcal{E}') - F_k^{CE,\tilde{a}_2^R}(\mathcal{E}')$ equals
$f(\tilde{a}_2^R, a_k^S) - f(\tilde{a}_1^R, a_k^S) + B = \varphi(a_k^S)$
where $B = \left(\frac{1}{n}\sum_{i=1}^n f(\tilde{a}_1^R, a_i^S) - \frac{1}{n}\sum_{i=1}^n f(\tilde{a}_2^R, a_i^S)\right)$.
Note that:

$(i)$ redistribution in the first best means that: $\sum_{i=1}^n \varphi(a_i^S) = 0$,

$(ii)$ by (3) and $(\tilde{a}_1^R < \tilde{a}_2^R)$: $B < 0$ and $f(\tilde{a}_2^R, a_k^S) - f(\tilde{a}_1^R, a_k^S) > 0$,

$(iii)$ for $D^+$: by (5), $\varphi(a_k^S)$ is non decreasing in $a_k^S$,
$(iv)$ for $D^-$: by (6), $\varphi(a_k^S)$ is non increasing in $a_k^S$.
Combining $(i)$, $(ii)$ and $(iii)$ ensures that for $D^+$ there exists $a_+^S \in (\underline{a}^S, \overline{a}^S)$, such that, if $a_k^S < (>)\ a_+^S$, then $\varphi(a_k^S) < (>)\ 0$. Hence, individuals with $a_k^S < (>)\ a_+^S$ receive less (more) income under $F^{CE,\tilde{a}_1^R}(\mathcal{E}')$ than under $F^{CE,\tilde{a}_2^R}(\mathcal{E}')$. So there are transfers of income from individuals with a low $a^S$ to individuals with a high $a^S$. From lemma 2, the poorest and the richest individuals gain from the transfers, making Lorenz dominance impossible.
Combining $(i)$, $(ii)$ and $(iv)$ ensures that for $D^-$ there exists $a_-^S \in (\underline{a}^S, \overline{a}^S)$, such that, if $a_k^S < (>)\ a_-^S$, then $\varphi(a_k^S) > (<)\ 0$. Hence, individuals with $a_k^S < (>)\ a_-^S$ receive more (less) income under $F^{CE,\tilde{a}_1^R}(\mathcal{E}')$ than under $F^{CE,\tilde{a}_2^R}(\mathcal{E}')$. So there are transfers of income from individuals with a high $a^S$ to individuals with a low $a^S$. From lemma 2, the poorest and the richest individuals gain from the transfers, making Lorenz dominance impossible. ■

Remark that for any economy in $D^+$ or $D^-$ the poorest gain income from the change of a conditionally egalitarian mechanism with a higher $\tilde{a}^R$ to a conditionally egalitarian mechanism with a lower $\tilde{a}^R$. This suggests that results in terms of poverty dominance might be drawn, but more assumptions on the pretax income function are needed. We come back to this issue within our specific framework in subsection 4.2.

### 3.4 EE versus CE

First, the following remark excludes one particular comparison from $D^+$ and $D^-$, because the incomes of the poorest and the richest remain unchanged.[5]

**Remark** : For $D^+$ $(D^-)$, the incomes of the poorest and the richest remain unchanged in the comparison of $F^{EE,\tilde{a}^S}(\mathcal{E}')$ with $\tilde{a}^S = \overline{a}^S$ $(\underline{a}^S)$ and $F^{CE,\tilde{a}^R}(\mathcal{E}')$ with $\tilde{a}^R$ chosen such that $f(\tilde{a}^R, \tilde{a}^S) = \frac{1}{n}\sum_{i=1}^n f(a_i^R, \tilde{a}^S) + \frac{1}{n}\sum_{i=1}^n f(\tilde{a}^R, a_i^S) - \frac{1}{n}\sum_{i=1}^n f(a_i^R, a_i^S)$.

**Proof.** Compare $F^{EE,\tilde{a}^S}(\mathcal{E}')$ with $\tilde{a}^S \in [\underline{a}^S, \overline{a}^S]$ and $F^{CE,\tilde{a}^R}(\mathcal{E}')$ with $\tilde{a}^R \in [\underline{a}^R, \overline{a}^R]$. Using (1) and (2), the income difference for an individual $k$ between the two mechanisms $F_k^{EE,\tilde{a}^S}(\mathcal{E}') - F_k^{CE,\tilde{a}^R}(\mathcal{E}')$ equals
$C + E = \psi(a_k^R, a_k^S)$
where $C = f(a_k^R, \tilde{a}^S) + f(\tilde{a}^R, a_k^S) - f(a_k^R, a_k^S)$ and $E = \frac{1}{n}\sum_{i=1}^n f(a_i^R, a_i^S) - \frac{1}{n}\sum_{i=1}^n f(a_i^R, \tilde{a}^S) - \frac{1}{n}\sum_{i=1}^n f(\tilde{a}^R, a_i^S)$.
Using lemma 1 and 2, the proof boils down to showing that:
for $D^+$: $\psi(\underline{a}^R, \overline{a}^S) = \psi(\overline{a}^R, \overline{a}^S) = 0$ when $\tilde{a}^S = \overline{a}^S$ and $\tilde{a}^R$ is chosen such that $f(\tilde{a}^R, \overline{a}^S) = -E$, and

[5] In order to derive Lorenz dominance results, further assumptions on the distributions of responsibility and compensation characteristics have to be made. We illustrate this specific comparison in the numerical example in subsection 4.3.

for $D^-$: $\psi(\underline{a}^R, \underline{a}^S) = \psi(\overline{a}^R, \underline{a}^S) = 0$ when $\tilde{a}^S = \underline{a}^S$ and $\tilde{a}^R$ is chosen such that $f(\tilde{a}^R, \underline{a}^S) = -E$. ■

Excluding the comparison described in the remark, the following proposition states that a conditionally egalitarian mechanism does not Lorenz dominate an egalitarian equivalent mechanism.

**Proposition 3** *: Excluding the comparison described in the remark, there does not exist $\mathcal{E}' \in (D^+ \cup D^-), \tilde{a}^R \in [\underline{a}^R, \overline{a}^R]$ and $\tilde{a}^S \in [\underline{a}^S, \overline{a}^S]$ :*
*$F^{CE,\tilde{a}^R}(\mathcal{E}') \succ_L F^{EE,\tilde{a}^S}(\mathcal{E}')$.*

**Proof.** Reconsider $C$, $E$ and $\psi(a_k^R, a_k^S)$ as defined in the previous proof.
Redistribution in the first best means that: $\sum_{i=1}^n \psi(a_i^R, a_i^S) = 0$.
$E$ is the same for all individuals. Without further assumptions on the pre-tax income function, $E$ can be either positive or negative. Anyhow, the individuals that gain from the change of $F^{CE,\tilde{a}^R}(\mathcal{E}')$ to $F^{EE,\tilde{a}^S}(\mathcal{E}')$ have a larger $C$ than those who lose from it.

Divide the population in two groups: group 1 comprises all individuals with $a_k^S \leq \tilde{a}^S$, group 2 all individuals with $a_k^S \geq \tilde{a}^S$. From lemma 2, the poorest and the richest belong to group 2 for any economy in $D^+$ and to group 1 for any economy in $D^-$.

For $D^+$, by (5), $\frac{\partial C}{\partial a_k^R} = \frac{\partial f(a_k^R, \tilde{a}^S)}{\partial a_k^R} - \frac{\partial f(a_k^R, a_k^S)}{\partial a_k^R} \leq 0$ for group 2. There exists $\grave{a}_+^R(\tilde{a}^R, \tilde{a}^S, \overline{a}^S) \in \mathbb{R}$ such that, if $a_k^R < (>)\ \grave{a}_+^R$, $\psi(a_k^R, \overline{a}^S) > (<)\ 0$. Hence, individuals with $a_k^R < (>)\ \grave{a}_+^R$ receive more (less) income under $F^{EE,\tilde{a}^S}(\mathcal{E}')$ than under $F^{CE,\tilde{a}^R}(\mathcal{E}')$.

For $D^-$, by (6), $\frac{\partial C}{\partial a_k^R} = \frac{\partial f(a_k^R, \tilde{a}^S)}{\partial a_k^R} - \frac{\partial f(a_k^R, a_k^S)}{\partial a_k^R} \leq 0$ for group 1. There exists $\grave{a}_-^R(\tilde{a}^R, \tilde{a}^S, \underline{a}^S) \in \mathbb{R}$ such that, if $a_k^R < (>)\ \grave{a}_-^R$, $\psi(a_k^R, \underline{a}^S) > (<)\ 0$. Hence, individuals with $a_k^R < (>)\ \grave{a}_-^R$ receive more (less) income under $F^{EE,\tilde{a}^S}(\mathcal{E}')$ than under $F^{CE,\tilde{a}^R}(\mathcal{E}')$.

If respectively $\grave{a}_+^R, \grave{a}_-^R \in [\underline{a}^R, \overline{a}^R]$, the poorest do not become poorer and the richest do not become richer under $F^{EE,\tilde{a}^S}(\mathcal{E}')$ than under $F^{CE,\tilde{a}^R}(\mathcal{E}')$. Therefore, the necessary condition for Lorenz dominance of $F^{EE,\tilde{a}^S}(\mathcal{E}')$ over $F^{CE,\tilde{a}^R}(\mathcal{E}')$ is fulfilled, implying that the necessary condition for Lorenz dominance of $F^{CE,\tilde{a}^R}(\mathcal{E}')$ over $F^{EE,\tilde{a}^S}(\mathcal{E}')$ is violated, proving the proposition. ■

The proof shows the possibility that the poorest do not become poorer and the richest do not become richer under an egalitarian equivalent mechanism than under a conditionally egalitarian mechanism. However, this condition implicitly requires further assumptions on the pre-tax income function. If respectively $\grave{a}_+^R, \grave{a}_-^R \notin [\underline{a}^R, \overline{a}^R]$, the incomes of the poorest and the richest change in the same direction making Lorenz dominance results between egalitarian equivalent mechanisms and conditionally egalitarian mechanisms impossible. We come back to this issue in subsection 4.3.

### 3.5 Remarks

Remark that the validity of propositions 1-3 only holds for any economy in $D^+$ or $D^-$, in the case where the responsibility and compensation characteristic are one-dimensional. Extending these limiting assumptions greatly enlarges the difficulty to obtain clear results.

Within the case where $a^R$ or $a^S$ are scalars, Lorenz dominance results are hard to establish. Extensions to the case where $\mathbf{a}^R$ and $\mathbf{a}^S$ are vectors lead to even fewer dominance results. Under multidimensional versions of (3), (4) and (5) or (6), requiring that the derivatives of the pre-tax income function with respect to every element in the $\mathbf{a}^R$- and $\mathbf{a}^S$-vectors are positive and the cross derivatives have equal sign, an unambiguous comparison of two different egalitarian equivalent mechanisms along the lines of proposition 1 can only be made when every element in $\tilde{\mathbf{a}}^S$ changes in the same direction.

Restricting the analysis to income functions in $\mathcal{F}^+$ and $\mathcal{F}^-$ drives the positive result of proposition 1. A natural question is whether we can derive Lorenz dominance results for the comparison of two conditionally egalitarian mechanisms by restricting the analysis to other classes of income functions. Suppose that the income function still satisfies (3) and (4) but $\frac{\partial^2 f(a^R,a^S)}{\partial a^R \partial a^S}$ changes sign: $\frac{\partial^2 f(a^R,a^S)}{\partial a^R \partial a^S} \leq 0$ for small values of $a^R$, $\frac{\partial^2 f(a^R,a^S)}{\partial a^R \partial a^S} \geq 0$ for large values of $a^R$ and $\frac{\partial^3 f(a^R,a^S)}{\partial^2 a^R \partial a^S} \geq 0$. Denote this class of income functions $\mathcal{F}^{\pm}$. The poorest gain income and the richest lose income if $\tilde{a}^R$ of the new conditionally egalitarian mechanism is lower than $\tilde{a}^R$ of the old conditionally egalitarian mechanism. Indeed, the transfers going to the poorest increase as $\tilde{a}^R$ decreases, while the transfers going to the richest decrease as $\tilde{a}^R$ decreases. However, this result only holds if, under both mechanisms, the poorest have characteristics $(\underline{a}^R, \underline{a}^S)$ and the richest have characteristics $(\overline{a}^R, \overline{a}^S)$. This does not apply for all income functions in $\mathcal{F}^{\pm}$, making Lorenz dominance results only possible over suitably defined domains.

# 4 Distributional analysis: example

In this section, we illustrate our previous results with an example, based on a framework adopted in Schokkaert et al. (2004), following Atkinson (1995). Closed form solutions for the pre-tax income function and the different non-welfaristic redistribution mechanisms are derived. We show that the pre-tax income function belongs to $\mathcal{F}^+$, which automatically enables us to draw corollaries from propositions 1-3, since we assume that the distributional assumptions of domain $D^+$ also hold true. However, this example enables us to draw more precise results with respect to poverty dominance between two conditionally egalitarian mechanisms. Irrespective of the distributions of individual characteristics, an upper bound poverty line is determined below which poverty is reduced under one of the two mechanisms. Finally, the comparison of an egalitarian equivalent mechanism with a conditionally egalitarian mechanism is revisited. We show which egalitarian equivalent mechanisms are eligible to Lorenz dominate which conditionally egalitarian mechanisms in our example. Introducing specific assumptions on the distributions of personal characteristics, a numerical example shows the necessity of making these assumptions in order to state exact dominance results.

## 4.1 Responsibility versus compensation

Suppose individuals differ in only two dimensions: their pure preference for leisure $e$ and their innate skill level $w$. We assume that individuals can be held responsible for differences in $e$, while at the same time compensation is desirable for differences in $w$. We suppose that both variables have finite support $0 < \underline{e} \leq e \leq 1$ and $0 < \underline{w} \leq w \leq 1$ respectively, but we exclude the possibility that $\underline{e}$ equals 1 and $\underline{w}$ equals 1. For convenience[6], we assume that $e$ and $w$ are distributed independently with density functions $g_E : [\underline{e}, 1] \to \mathbb{R}_+ : e \to g_E(e)$ and $g_W : [\underline{w}, 1] \to \mathbb{R}_+ : w \to g_W(w)$. All distributions of characteristics $(g_E, g_W)$ that satisfy these properties belong to $\Omega^\circ$, which is equivalent to $\Omega'$ with a continuum of individuals and the additional assumption of independence.

### 4.1.1 The pre-tax income function belongs to $\mathcal{F}^+$

In our first best setting, we rule out behavioural responses: every individual chooses her labor supply as if there were no redistribution, or alternatively the government has complete information on individual behaviour and is able to enforce this behaviour.

Suppose labor supply is iso-elastic:

$$L = e^\varepsilon w^\varepsilon \tag{7}$$

where $\varepsilon$ is the constant elasticity of labor supply, a measure for the efficiency cost of the tax and assumed to be identical for all individuals.

Using (7), pre-tax income $f$ as a function of $e$ and $w$ equals:

$$f(e, w) = wL = e^\varepsilon w^{\varepsilon+1}. \tag{8}$$

Pre-tax income is increasing in both $e$ and $w$, implying that the laziest, lowest skilled people $(\underline{e}, \underline{w})$ are poorest. Since $\frac{\partial f(e,w)}{\partial e}$, $\frac{\partial f(e,w)}{\partial w}$ and $\frac{\partial^2 f(e,w)}{\partial e \partial w}$ are positive, the pre-tax income function belongs to $\mathcal{F}^+$.

In this section, we search for Lorenz dominance results of different non-welfaristic redistribution mechanisms over the following domain:

$D^\circ$ : the set of all economies $\mathcal{E}^\circ$ with $(g_E, g_W) \in \Omega^\circ$ and $f(e, w)$ given by (8).

[6] Furthermore, it is doubtful that agents should be held responsible for characteristics which are determined by characteristics that require compensation.

### 4.1.2 The egalitarian equivalent mechanism

Define the $\alpha$-th moment of a variable $h$ with support $\left[\underline{h}, \overline{h}\right]$ and density function $g_H$ as $\mu_H(\alpha) = \int_{\underline{h}}^{\overline{h}} x^{\alpha} g_H(x) dx$.

For all $\mathcal{E}^{\circ} \in D^{\circ}$, each individual $k$ receives under an egalitarian equivalent mechanism $F^{EE,\tilde{w}}(\mathcal{E}^{\circ})$ an income:

$$\begin{aligned} F_k^{EE,\tilde{w}}(\mathcal{E}^{\circ}) &= f(e_k, \tilde{w}) - \int_{\underline{e}}^{1} \int_{\underline{w}}^{1} \left(f(e, \tilde{w}) - f(e, w)\right) g_E(e) g_W(w) de dw \\ &= e_k^{\varepsilon} \tilde{w}^{\varepsilon+1} - \mu_E(\varepsilon) \tilde{w}^{\varepsilon+1} + \mu_E(\varepsilon) \mu_W(\varepsilon+1) \end{aligned} \quad (9)$$

where $\tilde{w}$ denotes the reference compensation characteristic.

When $\tilde{w}$ equals 0, income is equally distributed. However, a non-welfaristic social planner will not choose $\tilde{w}$ equal to 0, since this completely eliminates the impact of responsibility on received income, which clearly conflicts with the goal of non-welfaristic redistribution. Throughout, we assume that $\tilde{w}$ is chosen between $[\underline{w}, 1]$. It deserves mentioning that in order to avoid the delicate choice of $\tilde{w}$, Fleurbaey (1995a) and Bossert and Fleurbaey (1996) propose an average version of the egalitarian equivalent mechanism ($F^{AEE}(\mathcal{E}^{\circ})$). The idea is to use every value of $w \in [\underline{w}, 1]$ successively as $\tilde{w}$ and to give each individual the average of the resulting incomes she would obtain under these different $F^{EE,\tilde{w}}(\mathcal{E}^{\circ})$. However, it can be easily shown that in this example $F^{AEE}(\mathcal{E}^{\circ})$ is equivalent to $F^{EE,\tilde{w}}(\mathcal{E}^{\circ})$ with $\tilde{w} = (\mu_W(\varepsilon+1))^{\frac{1}{\varepsilon+1}}$.

Income is increasing in $e$ but no longer a function of $w$. This implies that the laziest ($e = \underline{e}$) are poorest and the hard working ($e = 1$) are richest, regardless their skill level (cfr. lemma 1).

The following corollary from proposition 1 states that the egalitarian equivalent mechanism with the lower reference compensation characteristic Lorenz dominates the egalitarian equivalent mechanism with the higher reference compensation characteristic.

**Corollary 1** *:* $\forall \mathcal{E}^{\circ} \in D^{\circ} : \forall \tilde{w}_1, \tilde{w}_2 \in [\underline{w}, 1] : \tilde{w}_1 < \tilde{w}_2 \Leftrightarrow F^{EE,\tilde{w}_1}(\mathcal{E}^{\circ}) \succ_L F^{EE,\tilde{w}_2}(\mathcal{E}^{\circ})$.

### 4.1.3 The conditionally egalitarian mechanism

For all $\mathcal{E}^{\circ} \in D^{\circ}$, each individual $k$ receives under a conditionally egalitarian mechanism $F^{CE,\tilde{e}}(\mathcal{E}^{\circ})$ an income:

$$\begin{aligned} F_k^{CE,\tilde{e}}(\mathcal{E}^{\circ}) &= f(e_k, w_k) - f(\tilde{e}, w_k) + \int_{\underline{w}}^{1} f(\tilde{e}, w) g_W(w) dw \\ &= e_k^{\varepsilon} w_k^{\varepsilon+1} - \tilde{e}^{\varepsilon} w_k^{\varepsilon+1} + \tilde{e}^{\varepsilon} \mu_W(\varepsilon+1) \end{aligned} \quad (10)$$

where $\tilde{e}$ denotes the reference preference characteristic.

We assume that $\tilde{e}$ is chosen between $[\underline{e}, 1]$. In order to avoid the delicate choice of $\tilde{e}$, Fleurbaey (1995a) and Bossert and Fleurbaey (1996) propose an average version of the conditionally egalitarian mechanism ($F^{ACE}(\mathcal{E}^\circ)$). The idea is to use every value of $e \in [\underline{e}, 1]$ successively as $\tilde{e}$ and to give each individual the average of the resulting incomes she would obtain under these different $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$. However, it can be easily shown that in this example $F^{ACE}(\mathcal{E}^\circ)$ is equivalent to $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$ with $\tilde{e} = (\mu_E(\varepsilon))^{\frac{1}{\varepsilon}}$.

Income is increasing in $e$. Since $\tilde{e} \geq \underline{e}$, income is no longer increasing in $w$ for all individuals. More precisely, the poorest are laziest ($e = \underline{e}$) and highest skilled ($w = 1$), while the richest are hard working ($e = 1$) and highest skilled ($w = 1$)(cfr. lemma 2). The slope of the iso-income curves under a conditionally egalitarian mechanism ($-\frac{\varepsilon}{(\varepsilon+1)} w \frac{e^{\varepsilon-1}}{(e^\varepsilon - \tilde{e}^\varepsilon)}$) is larger than the slope of the iso-income curves of the pre-tax income function ($-\frac{\varepsilon}{(\varepsilon+1)} \frac{w}{e}$) and is positive for all individuals $i$ with $e_i < \tilde{e}$. Figure 1 depicts iso-income curves for $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$.

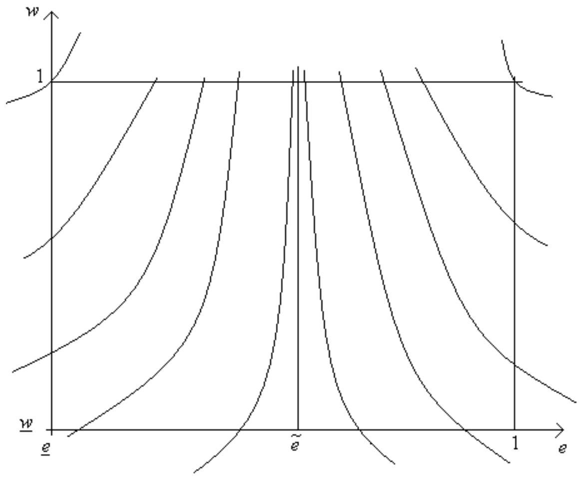


Figure 1: Iso-income curves for $F^{CE,\tilde{e}}(\mathcal{E})$

Note that income will never be equally distributed. Graphically, all individuals would have to lie on the same iso-income curve. This requires correlation between $e$ and $w$, which violates our independence assumption.[7]

The following corollary from proposition 2 states that there is no Lorenz dominance between two conditionally egalitarian mechanisms.

**Corollary 2** *: There does not exist* $\mathcal{E}^\circ \in D^\circ$ *and* $\tilde{e}_1, \tilde{e}_2 \in [\underline{e}, 1]$ :
$F^{CE,\tilde{e}_1}(\mathcal{E}^\circ) \succ_L F^{CE,\tilde{e}_2}(\mathcal{E}^\circ)$.

[7] Remark that income is equally distributed when $e_i = \tilde{e}$ for all $i$. However, this distribution does not belong to $\Sigma^\circ$.

## 4.2 Poverty dominance between two conditionally egalitarian mechanisms

Denote $F^{CE,\tilde{e}}_{(e,w)}(\mathcal{E}^\circ)$ the income that each individual with characteristics $(e,w)$ receives under $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$. As suggested at the end of subsection 3.3, the fact that $F^{CE,\tilde{e}_1}_{(\underline{e},1)}(\mathcal{E}^\circ) > F^{CE,\tilde{e}_2}_{(\underline{e},1)}(\mathcal{E}^\circ)$ suggests that we might arrive at a conclusion in terms of poverty dominance between two different conditionally egalitarian mechanisms. The following proposition states that for all poverty lines below $\underline{e}^\varepsilon \mu_W(\varepsilon+1)$, the conditionally egalitarian mechanism with the lower reference preference characteristic first order poverty dominates the conditionally egalitarian mechanism with the higher reference preference characteristic.

**Proposition 4** *:* $\forall \mathcal{E}^\circ \in D^\circ : \forall \tilde{e}_1, \tilde{e}_2 \in [\underline{e}, 1]$ *and* $\tilde{e}_1 < \tilde{e}_2$ *:*
$F^{CE,\tilde{e}_1}(\mathcal{E}^\circ) \succ_{P_1, \underline{e}^\varepsilon \mu_W(\varepsilon+1)} F^{CE,\tilde{e}_2}(\mathcal{E}^\circ)$.

**Proof.** First, note that for all $e \in [\underline{e}, \tilde{e}_1]$, the technical rate of substitution of $F^{CE,\tilde{e}_1}(\mathcal{E}^\circ)$ is higher than the technical rate of substitution of $F^{CE,\tilde{e}_2}(\mathcal{E}^\circ)$. Therefore, iso-income curves cross at most once over the subspace $[\underline{e}, \tilde{e}_1] \times [\underline{w}, 1]$. Second, remark that all individuals with characteristics $(\underline{e},\ (\mu_W(\varepsilon+1))^{\frac{1}{\varepsilon+1}})$ receive an income $y_* = \underline{e}^\varepsilon \mu_W(\varepsilon+1)$, irrespective of which conditionally egalitarian mechanism is implemented. The iso-$y_*$ curve is depicted in figure 2 for $F^{CE,\tilde{e}_1}(\mathcal{E}^\circ)$ and $F^{CE,\tilde{e}_2}(\mathcal{E}^\circ)$ with $\underline{e} < \tilde{e}_1 < \tilde{e}_2 < 1$:

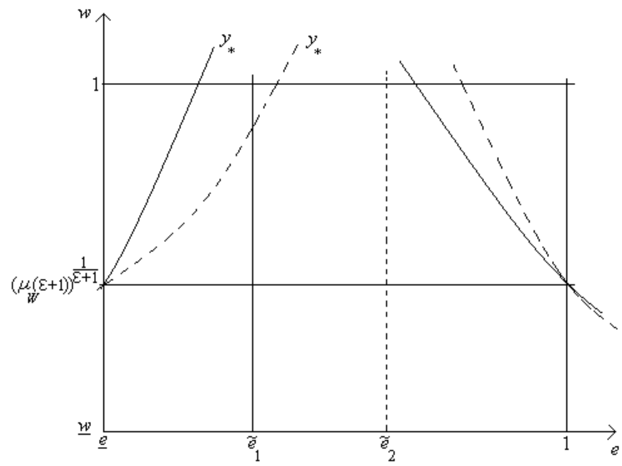


Figure 2: Iso-income curves for $F^{CE,\tilde{e}_1}(\mathcal{E})$ and $F^{CE,\tilde{e}_2}(\mathcal{E})$

Third, it remains to show that two equal iso-income curves of two different conditionally egalitarian mechanisms only cross for income levels higher than $y^*$. These crossings imply that assumptions about the marginal distributions of $w$ and $e$ have to be made to establish first order poverty dominance for income levels larger than $y^*$: we need to know exactly how many individuals

are situated at each point in the $e \times w$ space. Denote $w_y^{CE,\tilde{e}_1}(\underline{e})$ as the skill level that an individual with preference characteristic $\underline{e}$ should have to obtain an income $y$ under $F^{CE,\tilde{e}_1}(\mathcal{E}^{\circ})$. Two equal iso-income curves of two different conditionally egalitarian mechanisms cross over the subspace $[\underline{e}, \tilde{e}_1] \times [\underline{w}, 1]$ when $w_y^{CE,\tilde{e}_2}(\underline{e}) > w_y^{CE,\tilde{e}_1}(\underline{e})$.
By (10), this is equivalent to

$$(y - \tilde{e}_2^{\varepsilon}\mu_W(\varepsilon+1))^{\frac{1}{1+\varepsilon}} \left(\frac{1}{\underline{e}^{\varepsilon} - \tilde{e}_2^{\varepsilon}}\right)^{\frac{1}{1+\varepsilon}} > (y - \tilde{e}_1^{\varepsilon}\mu_W(\varepsilon+1))^{\frac{1}{1+\varepsilon}} \left(\frac{1}{\underline{e}^{\varepsilon} - \tilde{e}_1^{\varepsilon}}\right)^{\frac{1}{1+\varepsilon}}.$$

Since $(\underline{e}^{\varepsilon} - \tilde{e}_1^{\varepsilon}) < 0$ and $(\underline{e}^{\varepsilon} - \tilde{e}_2^{\varepsilon}) < 0$, this inequality can be written as

$$(y - \tilde{e}_2^{\varepsilon}\mu_W(\varepsilon+1))(\underline{e}^{\varepsilon} - \tilde{e}_1^{\varepsilon}) > (y - \tilde{e}_1^{\varepsilon}\mu_W(\varepsilon+1))(\underline{e}^{\varepsilon} - \tilde{e}_2^{\varepsilon})$$

$$\Leftrightarrow (\tilde{e}_2^{\varepsilon} - \tilde{e}_1^{\varepsilon})y > (\tilde{e}_2^{\varepsilon} - \tilde{e}_1^{\varepsilon})\underline{e}^{\varepsilon}\mu_W(\varepsilon+1).$$

Since $(\tilde{e}_2^{\varepsilon} - \tilde{e}_1^{\varepsilon}) > 0$, this requires

$$y > \underline{e}^{\varepsilon}\mu_W(\varepsilon+1) = y_*$$

which completes the proof. ■

Analogously, it can be shown that equal iso-income curves of two different conditionally egalitarian mechanisms no longer cross for all income levels higher than the income of an individual with characteristics $\left(1, (\mu_W(\varepsilon+1))^{\frac{1}{\varepsilon+1}}\right)$(see also figure 2).

Proposition 4 establishes first order poverty dominance results between two conditionally egalitarian mechanisms. The maximum value of the poverty line for which second order poverty dominance holds, is larger than $\underline{e}^{\varepsilon}\mu_W(\varepsilon+1)$. To derive its exact value, additional assumptions about $g_E$ and $g_W$ are needed. Since we do not want to make these assumptions here, we limited ourselves to first order poverty dominance.

## 4.3 EE versus CE revisited

In this model, the remark of subsection 3.4 concerns the comparison of $F^{EE,1}(\mathcal{E}^{\circ})$ and $F^{CE,(\mu_E(\varepsilon))^{\frac{1}{\varepsilon}}}(\mathcal{E}^{\circ})$. Excluding this comparison, the following corollary from proposition 3 states that a conditionally egalitarian mechanism cannot Lorenz dominate an egalitarian equivalent mechanism.

**Corollary 3** *: There does not exist* $\mathcal{E}^{\circ} \in D^{\circ}, \tilde{e} \in [\underline{e}, 1]$ *and* $\tilde{w} \in [\underline{w}, 1]$ : $F^{CE,\tilde{e}}(\mathcal{E}^{\circ}) \succ_L F^{EE,\tilde{w}}(\mathcal{E}^{\circ})$.

Finally, the remaining question is whether an egalitarian equivalent mechanism can Lorenz dominate a conditionally egalitarian mechanism. A priori, one could think that the former Lorenz dominates the latter, since the egalitarian equivalent mechanism satisfies stronger compensation axioms and weaker responsibility axioms than the conditionally egalitarian mechanism. Therefore, we examine first which egalitarian equivalent mechanisms can Lorenz dominate all conditionally egalitarian mechanisms.

Part a) of the following proposition states that if an egalitarian equivalent mechanism Lorenz dominates all conditionally egalitarian mechanisms, $\tilde{w}$ is smaller than $(\mu_W(\varepsilon+1))^{\frac{1}{\varepsilon+1}}$. Part b) states that if all egalitarian equivalent mechanisms Lorenz dominate a conditionally egalitarian mechanism, the latter has a reference preference characteristic equal to $(\mu_E(\varepsilon))^{\frac{1}{\varepsilon}}$.

**Proposition 5** *:*
*a)* $\forall \mathcal{E}^\circ \in D^\circ : \forall \tilde{e} \in [\underline{e}, 1] : F^{EE,\tilde{w}}(\mathcal{E}^\circ) \succ_L F^{CE,\tilde{e}}(\mathcal{E}^\circ) \Rightarrow \underline{w} \leq \tilde{w} \leq (\mu_W(\varepsilon+1))^{\frac{1}{\varepsilon+1}}$,
*b)* $\forall \mathcal{E}^\circ \in D^\circ : \forall \tilde{w} \in [\underline{w}, 1] : F^{EE,\tilde{w}}(\mathcal{E}^\circ) \succ_L F^{CE,\tilde{e}}(\mathcal{E}^\circ) \Rightarrow \tilde{e} = (\mu_E(\varepsilon))^{\frac{1}{\varepsilon}}$.

**Proof.** Denote $F^{EE,\tilde{w}}_{(e,w)}(\mathcal{E}^\circ)$ the income that each individual with characteristics $(e,w)$ receives under $F^{EE,\tilde{w}}(\mathcal{E}^\circ)$. The necessary condition for Lorenz dominance that the poorest have at least the same income under $F^{EE,\tilde{w}}(\mathcal{E}^\circ)$ as under $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$, while the income of the richest should not be higher under $F^{EE,\tilde{w}}(\mathcal{E}^\circ)$ than under $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$ requires that: $\forall \tilde{e}$ :
$F^{CE,\tilde{e}}_{(\underline{e},1)}(\mathcal{E}^\circ) \leq F^{EE,\tilde{w}}_{(\underline{e},\cdot)}(\mathcal{E}^\circ)$
$\Leftrightarrow \underline{e}^\varepsilon - \tilde{e}^\varepsilon + \tilde{e}^\varepsilon \mu_W(\varepsilon+1) \leq \underline{e}^\varepsilon \tilde{w}^{\varepsilon+1} - \mu_E(\varepsilon)\tilde{w}^{\varepsilon+1} + \mu_E(\varepsilon)\mu_W(\varepsilon+1)$
$\Leftrightarrow \tilde{w}^{\varepsilon+1} \leq \frac{\tilde{e}^\varepsilon(\mu_W(\varepsilon+1)-1)+\underline{e}^\varepsilon-\mu_E(\varepsilon)\mu_W(\varepsilon+1)}{(\underline{e}^\varepsilon-\mu_E(\varepsilon))} = RHS(1)$
and
$F^{CE,\tilde{e}}_{(1,1)}(\mathcal{E}^\circ) \geq F^{EE,\tilde{w}}_{(1,\cdot)}(\mathcal{E}^\circ)$
$\Leftrightarrow 1 - \tilde{e}^\varepsilon + \tilde{e}^\varepsilon \mu_W(\varepsilon+1) \geq \tilde{w}^{\varepsilon+1} - \mu_E(\varepsilon)\tilde{w}^{\varepsilon+1} + \mu_E(\varepsilon)\mu_W(\varepsilon+1)$
$\Leftrightarrow \tilde{w}^{\varepsilon+1} \leq \frac{\tilde{e}^\varepsilon(\mu_W(\varepsilon+1)-1)+1-\mu_E(\varepsilon)\mu_W(\varepsilon+1)}{(1-\mu_E(\varepsilon))} = RHS(2)$.
Note that $RHS(1)$ is increasing in $\tilde{e}$ since $\frac{(\mu_W(\varepsilon+1)-1)}{(\underline{e}^\varepsilon-\mu_E(\varepsilon))} > 0$ while $RHS(2)$ is decreasing in $\tilde{e}$ since $\frac{(\mu_W(\varepsilon+1)-1)}{(1-\mu_E(\varepsilon))} < 0$. The shaded area of figure 3 depicts for which values of $\tilde{w}$ and $\tilde{e}$, the corresponding $F^{EE,\tilde{w}}(\mathcal{E}^\circ)$ and $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$ fulfill the necessary condition. In part a) the dominance result has to hold for all values of $\tilde{e}$. Consequently, only $F^{EE,\tilde{w}}(\mathcal{E}^\circ)$ with $\tilde{w} \leq (\mu_W(\varepsilon+1))^{\frac{1}{\varepsilon+1}}$ can Lorenz dominate all $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$. In part b) the dominance result has to hold for all values of $\tilde{w}$. Consequently, only $F^{CE,\tilde{e}}(\mathcal{E}^\circ)$ with $\tilde{e} = (\mu_E(\varepsilon))^{\frac{1}{\varepsilon}}$ can be Lorenz dominated by all $F^{EE,\tilde{w}}(\mathcal{E}^\circ)$.

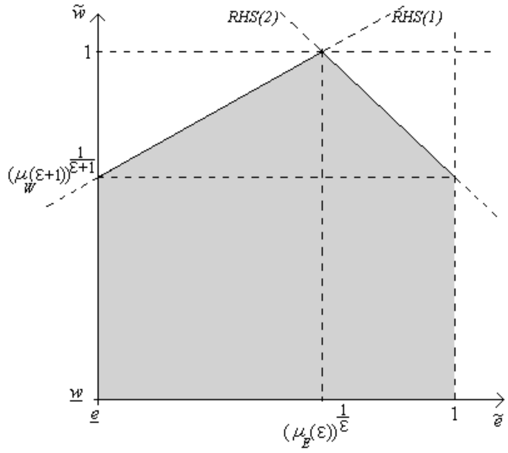


Figure 3: A necessary condition for LD

■

Proposition 5 identifies values for reference characteristics for which Lorenz dominance results are possible. Whether Lorenz dominance results actually occur, depends on the exact density functions $g_E$ and $g_W$. The following numerical example, whose set-up is described in appendix, illustrates:

a) that not all $F^{EE,\tilde{w}}(\mathcal{E}^\circ)$ Lorenz dominate $F^{CE,(\mu_E(\varepsilon))^{\frac{1}{\varepsilon}}}(\mathcal{E}^\circ)$ when $e$ and $w$ are uniformly distributed over the interval $[0.1, 1]$,

b) the unique comparison where a conditionally egalitarian mechanism Lorenz dominates an egalitarian equivalent mechanism and

c) that different density functions lead to different Lorenz dominance results in the comparison of a conditionally egalitarian mechanism and an egalitarian equivalent mechanism.

**Numerical example:** Denote $\mathcal{E}_\gamma \in D^\circ$ the economy where $f(e, w)$ is given by (8), $w$ is uniformly distributed over the interval $[0.1, 1]$ and $e$ is distributed over the interval $[0.1, 1]$ such that $\forall e \in [0.1, 0.55] : g_E(e) = \frac{\gamma}{0.45}$ and $\forall e \in [0.55, 1] : g_E(e) = \frac{1-\gamma}{0.45}$ with $\gamma \in (0, 1)$. When $\gamma = \frac{1}{2}$, $e$ is uniformly distributed.

Consider $\mathcal{E}_{\frac{1}{2}}$. Suppose $\varepsilon = 1$. We test for Lorenz dominance of respectively an egalitarian equivalent mechanism with $\tilde{w} = 0.5$, $\tilde{w} = 0.7$ and $\tilde{w} = 1$ over a conditionally egalitarian mechanism with $\tilde{e} = \mu_E(1) = 0.55$. The incomes of the poorest and the richest under the four mechanisms equal:

| | poorest | richest |
|---|---|---|
| $F^{CE,0.55}(\mathcal{E}_{\frac{1}{2}})$ | $-0.246$ | $0.653$ |
| $F^{EE,0.5}(\mathcal{E}_{\frac{1}{2}})$ | $0.091$ | $0.316$ |
| $F^{EE,0.7}(\mathcal{E}_{\frac{1}{2}})$ | $-0.017$ | $0.424$ |
| $F^{EE,1}(\mathcal{E}_{\frac{1}{2}})$ | $-0.246$ | $0.653$ |

Figure 4 depicts three curves of $S^{EE,\tilde{w};CE,0.55}_{\mathcal{E}_{\frac{1}{2}}}(y) = \int_{-0.246}^{y}(G^{CE,0.55}(x) - G^{EE,\tilde{w}}(x))dx$ for all $y \in [-0.246, 0.653]$. In the upper curve $\tilde{w} = 0.5$, in the middle curve $\tilde{w} = 0.7$ and in the lower curve $\tilde{w} = 1$.

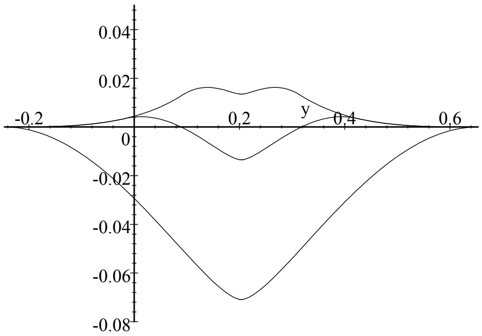


Figure 4: $S^{EE,\tilde{w};CE,0.55}_{\mathcal{E}_{\frac{1}{2}}}(y)$

a) Figure 4 shows that $F^{EE,0.5}(\mathcal{E}_{\frac{1}{2}})$ Lorenz dominates $F^{CE,0.55}(\mathcal{E}_{\frac{1}{2}})$, but $F^{EE,0.7}(\mathcal{E}_{\frac{1}{2}})$ does not Lorenz dominate $F^{CE,0.55}(\mathcal{E}_{\frac{1}{2}})$.

b) Recall the remark in subsection 3.4: the incomes of the poorest and the richest are unchanged under $F^{CE,0.55}(\mathcal{E}_{\frac{1}{2}})$ and $F^{EE,1}(\mathcal{E}_{\frac{1}{2}})$. This is the unique combination of $\tilde{e}$ and $\tilde{w}$ for which the conditionally egalitarian mechanism can Lorenz dominate the egalitarian equivalent mechanism. For $\mathcal{E}_{\frac{1}{2}}$, indeed $F^{CE,0.55}(\mathcal{E}_{\frac{1}{2}})$ Lorenz dominates $F^{EE,1}(\mathcal{E}_{\frac{1}{2}})$.

c) For all $\mathcal{E}_\gamma$ with $\gamma \neq \frac{1}{2}$, $F^{CE,0.55}(\mathcal{E}_\gamma)$ does not Lorenz dominate $F^{EE,1}(\mathcal{E}_\gamma)$. The incomes of the poorest and the richest are unchanged under $F^{CE,\mu_E(1)}(\mathcal{E}_\gamma)$ and $F^{EE,1}(\mathcal{E}_\gamma)$, implying that only for this specific comparison of mechanisms the former mechanism can Lorenz dominate the latter mechanism. Note that $\mu_E(1) = \frac{1.55}{2} - \frac{0.9}{2}\gamma$. However, if $\gamma \neq \frac{1}{2}$, $\mu_E(1) \neq 0.55$.

# 5 Conclusion

The implementation of a non-welfaristic redistribution mechanism not only requires a normative judgement for the choice of reference responsibility or compensation characteristic. In order to trace out the implications of non-welfaristic redistribution, also empirical information is needed. Only with accurate estimations of the pre-tax income function and the distributions of responsibility and compensation characteristics, the income distributions and levels of poverty of different non-welfaristic mechanisms can be thoroughly compared. If one lacks such detailed information, but at least some agreement on the properties of the pre-tax income function is reached, our analysis suggests that:

- Confronted with the choice of which egalitarian equivalent mechanism to implement and depending on the economy concerned, a non-welfaristic social planner concerned about income inequalities implements either an egalitarian equivalent mechanism with a low (if the economy belongs to $D^+$) or with a high (if the economy belongs to $D^-$) reference compensation characteristic.
- Confronted with the choice of which conditionally egalitarian mechanism to implement, none of the mechanisms can be ordered using the Lorenz dominance criterion. If a non-welfaristic social planner has a particular concern for the poorest in society a conditionally egalitarian mechanism with a low reference preference characteristic is implemented.
- If a non-welfaristic social planner faces the full choice of which non-welfaristic mechanism to implement, she should not believe that, since the egalitarian equivalent mechanism satisfies stronger compensation axioms and weaker responsibility axioms than the conditionally egalitarian mechanism, all egalitarian equivalent mechanisms Lorenz dominate all conditionally egalitarian mechanisms. In our specific example only egalitarian equivalent mechanisms with a sufficiently low reference compensation characteristic can Lorenz dominate all conditionally egalitarian mechanisms. Conversely, only one conditionally egalitarian mechanism can be Lorenz dominated by all egalitarian equivalent mechanisms. Whether such Lorenz dominance results occur, depends on the distributions of responsibility and compensation characteristics.

# Appendix (set-up of the numerical example in section 4.3)

Take an arbitrary income level $y \in [F^{CE,\tilde{e}}_{(\underline{e},1)}(\mathcal{E}^\circ), F^{CE,\tilde{e}}_{(1,1)}(\mathcal{E}^\circ)]$.

1) Using (9), $G^{EE,\tilde{w}}_{\mathcal{E}^\circ}(y)$ equals the probability (Pr)

$$\Pr(e^\varepsilon \tilde{w}^{\varepsilon+1} - \mu_E(\varepsilon)\tilde{w}^{\varepsilon+1} + \mu_E(\varepsilon)\mu_W(\varepsilon+1) \leq y)$$

$$= \Pr\left(e \leq \left(\frac{y+\mu_E(\varepsilon)\tilde{w}^{\varepsilon+1}-\mu_E(\varepsilon)\mu_W(\varepsilon+1)}{\tilde{w}^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}\right)$$

$$= \int_{\underline{e}}^{\left(\frac{y+\mu_E(\varepsilon)\tilde{w}^{\varepsilon+1}-\mu_E(\varepsilon)\mu_W(\varepsilon+1)}{\tilde{w}^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}} g_E(e)de.$$

2) Using (10), $G^{CE,\tilde{e}}_{\mathcal{E}^\circ}(y)$ equals:
$\Pr(e^\varepsilon w^{\varepsilon+1} - \tilde{e}^\varepsilon w^{\varepsilon+1} + \tilde{e}^\varepsilon \mu_W(\varepsilon+1) \leq y)$.

Two cases arise, depending on whether $e < \tilde{e}$ or $e > \tilde{e}$.
When $e < \tilde{e}$, $F^{CE,\tilde{e}}_{(e,w)}(\mathcal{E}^\circ) < \tilde{e}^\varepsilon \mu_W(\varepsilon+1)$ such that for $y < \tilde{e}^\varepsilon \mu_W(\varepsilon+1)$:

$$G^{CE,\tilde{e}}_{\mathcal{E}^\circ}(y) = 1 - \left( \int\limits_{e^2(y)}^{1} g_E(e)de \right) - \left( \int\limits_{e^1(y)}^{e^2(y)} \int\limits_{\underline{w}}^{\left(\frac{y-\tilde{e}^\varepsilon \mu_W(\varepsilon+1)}{e^\varepsilon - \tilde{e}^\varepsilon}\right)^{\frac{1}{\varepsilon+1}}} g_W(w)g_E(e)dwde \right)$$

where $e^1(y) = \max\{\underline{e}, \left(\frac{y-\tilde{e}^\varepsilon \mu_W(\varepsilon+1)+\tilde{e}^\varepsilon \underline{w}^{\varepsilon+1}}{\underline{w}^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}\}$ and $e^2(y) = (y - \tilde{e}^\varepsilon \mu_W(\varepsilon+1) + \tilde{e}^\varepsilon)^{\frac{1}{\varepsilon}}$.

When $e > \tilde{e}$, $F^{CE,\tilde{e}}_{(e,w)}(\mathcal{E}^\circ) > \tilde{e}^\varepsilon \mu_W(\varepsilon+1)$ such that for $y > \tilde{e}^\varepsilon \mu_W(\varepsilon+1)$:

$$G^{CE,\tilde{e}}_{\mathcal{E}^\circ}(y) = \left( \int\limits_{\underline{e}}^{e^2(y)} g_E(e)de \right) + \left( \int\limits_{e^2(y)}^{e^3(y)} \int\limits_{\underline{w}}^{\left(\frac{y-\tilde{e}^\varepsilon \mu_W(\varepsilon+1)}{e^\varepsilon - \tilde{e}^\varepsilon}\right)^{\frac{1}{\varepsilon+1}}} g_W(w)g_E(e)dwde \right)$$

where $e^3(y) = \min\{\left(\frac{y-\tilde{e}^\varepsilon \mu_W(\varepsilon+1)+\tilde{e}^\varepsilon \underline{w}^{\varepsilon+1}}{\underline{w}^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}, 1\}$.

If we suppose that $\varepsilon = 1$ and that $e$ and $w$ are uniformly distributed between $[\underline{e}, 1]$ and $[\underline{w}, 1]$ respectively, then:
$g_E(e) = \frac{1}{1-\underline{e}}$, $g_W(w) = \frac{1}{1-\underline{w}}$, $\mu_E(1) = \frac{1+\underline{e}}{2}$ and $\mu_W(2) = \frac{(\underline{w}^2+\underline{w}+1)}{3}$.
Then $S^{EE,\tilde{w};CE,\tilde{e}}_{\mathcal{E}^\circ}(y) = \int^{y}_{F^{CE,\tilde{e}}_{(\underline{e},1)}(\mathcal{E}^\circ)} (G^{CE,\tilde{e}}_{\mathcal{E}^\circ}(x) - G^{EE,\tilde{w}}_{\mathcal{E}^\circ}(x))dx$ becomes a function of $y$, $\underline{e}$, $\underline{w}$, $\tilde{e}$ and $\tilde{w}$ and is simulated for $\underline{e} = 0.1$, $\underline{w} = 0.1$, $\tilde{e} = \mu_E(1) = 0.55$ and $\tilde{w} = 0.5$, $0.7$ and $1$ respectively.

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# Is it fair to 'make work pay'?*

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**Abstract**

The design of the income transfer program for the lower incomes is a hot issue in current public policy debate. Should we stick to a generous welfare state with a sizeable basic income but high marginal tax rates for the lower incomes and thus little incentives to work? Or should we 'make work pay' by subsidizing the work of low earners but possibly at the cost of a smaller safety net? We think it is difficult to answer this question without making clear what individuals are (held) responsible for and what not. First, we present a new fair allocation, coined a Pareto Efficient and Shared resources Equivalent allocation (PESE), which compensates for different productive skills but not for different tastes for working. We also characterize a fair social ordering which rationalizes the PESE allocation. Second, we illustrate the optimal second-best allocation in a discrete Stiglitz (1982, 1987) economy. The question whether we should have subsidies for the low earners or not crucially depends on whether the low-skilled have a strictly positive or zero skill. Third, we simulate fair taxes for a sample of Belgian singles. Our simulation results suggest that 'making work pay' policies can be optimal, according to our fairness criterion, but only in the unreasonable case in which none of the unemployed are ever willing to work.

JEL Classification: D63, H21.
Keywords: make work pay, optimal income taxation, fairness.

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# 1 Motivation

Focussing on the tax-benefit system as a whole, many European countries combine a sizeable basic income with high marginal taxes for the low income earners. These programs are praised for their redistributional appeal, directing large transfers towards the low incomes in society. But, at the same time, critics have held these schemes responsible for large unemployment traps because they do not provide incentives to (start) work(ing). Therefore some continental European countries —such as Belgium, Finland, France, Germany, Italy and the Netherlands— have proposed and/or introduced tax credit schemes recently; see Bernardi and Profeta (2004) for an overview. At the same time, the US and the UK, with a much longer tradition in tax credit schemes, have reinforced the role of their tax credits. The increased policy interest for such 'making work pay' schemes, i.e., policies aiming at subsidizing the low income earners, is its ability to tackle two problems at the same time. It has a positive effect on employment (the number of people working and, to a lesser extent, the aggregate labour hours), while it increases the income of poor households; see Pearson and Scarpetta (2000) for an overview.

While 'making work pay' schemes may attain desirable objectives, it is not clear whether it is also optimal to make work pay for a given budget constraint. The 'welfarist' optimal income tax literature consists of three canonical models, depending on whether labour supply responses are modelled intensively and/or extensively (Heckman, 1993). *First*, in a Mirrlees (1971) economy, individuals respond via the intensive margin, i.e., by varying their labour hours or effort. Marginal taxes should be non-negative everywhere (Mirrlees, 1971), which excludes the possibility of subsidizing work. At the bottom, the marginal tax has to be zero, but only in case everybody works (Seade, 1977). Once there exists an atom of non-workers, the marginal tax rate has to be positive (Ebert, 1992) and, according to some numerical simulations (Tuomala, 1990), rather high. Using the empirical earnings distribution, (i) a U-shaped pattern of positive marginal tax rates and (ii) high marginal tax rates at the bottom turn out to be optimal in many cases (for (i) see Diamond, 1998, Saez, 2001 and Salanié, 1998 and for (ii) see Piketty, 1997, Bourguignon and Spadaro, 2000 and Choné and Laroque, 2005). *Second*, in Diamond's (1980) approach, individuals respond via the extensive margin, i.e., they choose to work or not. Marginal tax rates can be negative, suggesting at least the possibility of subsidizing the work of low earners. *Third*, Saez (2002) presents a unifying framework where individuals can respond via both margins. Support for one of the two income transfer schemes depends on the relative importance of both response margins and on the redistributive tastes of government. He proposes a sizeable basic income (around \$7300/year), but combined with a tax exemption at the bottom (for gross incomes up to \$5000/year).

In the same year of Mirrlees' (1971) seminal contribution, Rawls (1971) criticizes the welfarist approach, in which only utility information is allowed to judge the desirability of different social states. Rawls (1982) presents the following example of expensive preferences. Otherwise identical individuals differ in their preferences towards food: some are happy with a simple meal, some only with an expensive dinner. Rawls asks the question whether we really want to distribute resources in such a way that those with expensive tastes get more resources than others. If we do not like such a distribution, we have to use an argument to decide which tastes are expensive. Such an argument has to be based on non-utility information. Hence, it implies a rejection of the welfarist approach. In the aftermath of Rawls' influential work, many alternative theories of distributive justice were proposed. Although very diverse in equalisandum, they almost all have Dworkin's (1981) cut in common. Dworkin claims that not all individual characteristics can (should) be considered as morally arbitrary. Therefore one has to make a clear cut between endowments (e.g., skills, talents, handicaps, etc.) and ambitions (e.g., preferences, effort, tastes, etc.). He introduces personal responsibility: individuals are responsible for their ambitions —as long as they identify with them— but not for their endowments. As a consequence, a fair distribution scheme should be ambition-sensitive, but endowment-insensitive.

In an optimal income tax setting, fairness could require to compensate for differences in productive skill (endowment) but not for differences in taste for working (ambition).[1] Schokkaert *et al.* (2004) introduce such fairness considerations in different ways and calculate the corresponding optimal linear income tax, which turns out to be positive. Allowing for non-linear tax schemes, results change drastically. Boadway *et al.* (2002) analyze the optimal non-linear income tax according to a weighted utilitarian or maximin social planner where different weights are chosen for different tastes. Negative marginal taxes (for the low income earners) are optimal in case sufficient weight is given to the hard-working individuals. Fleurbaey and Maniquet (2005a,b) characterize fair social orderings to analyze non-linear income taxes. In both theoretical studies, it is optimal to direct the largest subsidies to the hard-working poor (the agents having the lowest skill and choosing the largest labour time), as long as the lowest skill is strictly positive.

In the next section we present a new fair allocation, coined a Pareto Efficient and Shared resources Equivalent (PESE) allocation. As the name suggests, the optimal allocation is Pareto efficient and all individuals are indifferent between their bundle and what they would get if it were physically possible to divide or share all resources, including the productive skills. Section 3 characterizes the shared resources social ordering which rationalizes the PESE allocation. In section 4, we introduce a 'discrete' Stiglitz (1982, 1987) economy with (i) four types of individuals (defined by a low or high productive skill and a low or high taste for working) and (ii) a government who wants to install fair taxes, but cannot observe individuals' type. We show that on the basis of the shared resources social ordering it is recommended to subsidize the low earners as long as the low-skilled individuals have a strictly positive skill. In case the low-skilled

[1] Roemer *et al.* (2001) consider the education level of the parents as the compensating variable.

have a zero skill subsidies can never be optimal. These theoretical results are in line with those found in Fleurbaey and Maniquet (2005a,b). In section 5, we enhance realism by simulating fair taxes for Belgian singles, while carefully paying attention to the calibration of the compensation (hourly wages) and responsibility (taste for working) variable. Similar to the welfarist simulation results often found in Mirrlees economies, our shared resources social ordering also suggests to install a U-shaped pattern of positive marginal tax rates in almost all cases. Negative marginal tax rates, and thus 'making work pay' policies, can only be optimal in the unreasonable case in which all the unemployed are never willing to work.

# 2 Equality of resources revisited

When all resources in society are alienable and divisible, Dworkin proposes to divide resources equally (endowment insensitivity), followed by an auction to reallocate resources according to taste (ambition sensitivity). This leads to a Pareto efficient and envy-free allocation. To study fair income taxation, however, we have to introduce productive resources (skills) which are not alienable and therefore a problem arises. In production economies Pareto efficient and envy-free allocations do not exist in general.

A first class of solutions tries to extend the above Dworkinian auction by assigning property rights over leisure. Varian (1974) analyzes two, rather extreme, solutions. One may divide consumption goods equally and either (i) assign each individual his own leisure, or (ii) give each individual an equal share in each of the agents' (including his own) leisure time. After trade the resulting competitive (and hence Pareto efficient) equilibria are called respectively (i) wealth-fair and (ii) income-fair. In the wealth-fair allocation productive talents are a private good and the resulting allocation does not compensate at all for inabilities. In the income-fair allocation productive talents are a common good. The high-skilled has to buy back his expensive leisure and is therefore punished for being a high skill type, resulting in a slavery of the talented. Intermediate solutions exist where skills are neither purely private, nor purely common (Fleurbaey and Maniquet, 1996, Kolm, 1996, Maniquet, 1998).

A second class of solutions starts from the concept of fair-equivalence. Pazner and Schmeidler (1978) define an allocation to be fair-equivalent if everyone is indifferent between his bundle in this allocation and the bundle he would receive in a 'hypothetical' fair, i.e., envy-free, allocation. It then suffices to define an interesting 'hypothetical' fair allocation and to look whether there exist Pareto efficient ones among all fair-equivalent allocations. The resulting allocation is called a Pareto efficient and fair-equivalent (PEFE) allocation. Pazner and Schmeidler (1978) propose an egalitarian allocation —an allocation where everybody consumes the same consumption-leisure bundle— as the fair one. We propose a different fair allocation which we coin a 'shared resources' allocation. This is the allocation which would result if it were (physically) possible to divide or share all resources, including the productive ones. To make this idea more precise, we introduce some notation.

A fixed number of individuals, denoted $i \in N = \{1, \ldots, n\}$, differ in skills and preferences. Skill $s \in \mathbb{R}_+$ defines production (called gross income in the sequel) in a linear way, or $y = s\ell$ with $\ell \in [0,1]$ the amount of labour. We denote a skill profile by $\mathbf{s} = (s_1, \ldots, s_n) \in \mathbb{R}_+^n$. Taste for working is represented by a continuously differentiable utility function

$$U : \mathbb{R} \times [0,1] \to \mathbb{R} : (c, \ell) \mapsto U(c, \ell),$$

which is strictly increasing (resp. strictly decreasing) in consumption $c$ (resp. labour $\ell$) and strictly quasi-concave. We call $\mathcal{U}$ the corresponding set of utility functions and, normalizing the consumption price to one, we refer to $c$ as the net income in the sequel. A utility profile is denoted by $\mathbf{U} = (U_1, \ldots, U_n) \in \mathcal{U}^n$. An economy $e = (\mathbf{s}, \mathbf{U})$ is completely defined by a skill and a utility profile; all economies are gathered in a set $\mathcal{E} = \mathbb{R}_+^n \times \mathcal{U}^n$.

Individuals are (held) responsible for their tastes, but not for their skills. Therefore, we want to compensate individuals for different outcomes which are only due to different skills but not for different outcomes which are only caused by different tastes for working. In case skills are alienable —think, e.g., of individuals as farmers who receive, as a matter of brute bad luck, either a blunt or a whetted scythe (the skill $s$) to harvest crops (the consumption $c$)— there is a particularly simple and attractive way to obtain a fair allocation:

(a) each individual pays (or receives) the same lump-sum amount of money,

(b) each individual can use each skill (including his own) for a time equal to $\frac{1}{n}$ at most.

Individuals are assumed to be rational: given (b), the budget set which maximizes net incomes (for all possible labour choices) starts using the highest skill for the first $\frac{1}{n}$ time units, followed by the second highest skill for an additional $\frac{1}{n}$ time units, and so on. In the sequel, we call this budget set the 'shared resources' budget set and the resulting allocation (which ultimately depends on the tastes in society) is called the 'shared resources' allocation.

However, sharing productive resources is not technically feasible in many cases. Labour market productivities, due to inborn characteristics such as intelligence, talents, handicaps and so on, are typically inalienable. Still, we could consider the allocation which would arise if it were possible to divide and share all resources equally as an interesting 'hypothetical' case. However, the resulting hypothetical 'shared resources' allocation is not necessarily Pareto efficient in the actual economy. Therefore, we propose to focus on Pareto Efficient and Shared resources Equivalent (PESE) allocations. We first define our well-being concept, which is closely linked to the PESE allocation. Let $\mathcal{Z} = (\mathbb{R} \times [0,1])^n$ be the set of allocations $\mathbf{z} = (z_i)_{i \in N}$, containing one bundle $z_i = (c_i, \ell_i)$ for each individual $i$ in $N$.

WELL-BEING: For each allocation $\mathbf{z} \in \mathcal{Z}$, the vector of well-being levels $\mathbf{w} = (w_i)_{i \in N} \in \mathbb{R}^n$ is defined by the amounts of money $w_i$ which would make individual $i$ indifferent between (i) receiving (or paying) this amount of money $w_i$ and sharing all productive resources equally (in time), and (ii) his actual bundle $z_i$. Because the well-being vector $\mathbf{w}$ depends on the allocation $\mathbf{z}$ and the economy $e = (\mathbf{s}, \mathbf{U})$, we write $\mathbf{w} = \mathbf{W}(\mathbf{z}, e)$, with $w_i = \mathbf{W}_i(\mathbf{z}, e)$.

We now define a PESE allocation.

PESE ALLOCATION: An allocation $\mathbf{z} \in \mathcal{Z}$ is a PESE allocation if and only if $\mathbf{z}$ is (i) Pareto efficient and (ii) all individuals have the same well-being.

To illustrate these concepts, suppose (i) there are only two skill types possible in society, say low ($L$) and high ($H$), which are equally represented in the skill pool $\mathbf{s}$, and (ii) there are only two tastes for working possible, also called low ($L$) and high ($H$). An allocation $\mathbf{z} = (z_{LL}, z_{LH}, z_{HL}, z_{HH})$ contains one bundle $z_{st} = (c_{st}, \ell_{st})$ for each of the four types $st$, with $s$ referring to the skill (low or high) and $t$ referring to the taste (low or high). Figure 1 illustrates the budget sets and (resulting) allocations in case (a) each individual receives the same lump-sum amount of money $a$, but productive resources are not shared, and (a)+(b) each individual receives the same lump-sum amount $a$ and also the productive resources are shared (each individual can work with each of the skills half-time at most).[2]

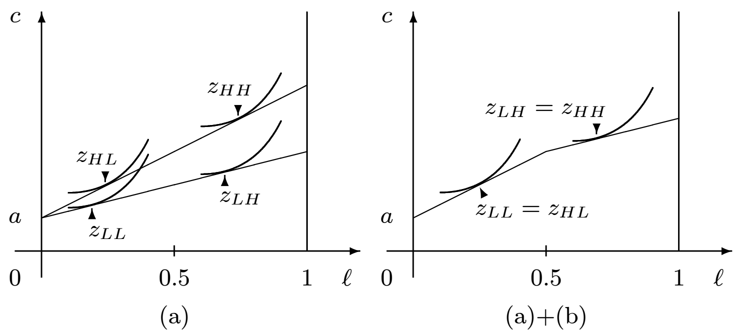



Figure 1: Allocation change when sharing productive resources.

Figure 2 illustrates a PESE allocation for an economy defined by the same assumptions (i) and (ii) as in figure 1. The well-being levels are the same for all individuals and equal to $a'$. Notice that figure 2 is constructed for the sake of clarity in such a way that the allocation is not necessarily feasible. It should be clear, however, that given the skill and preference technology, bundles and

[2] Although fairness is the only thing that matters for a hypothetical allocation, notice that our 'shared resources' allocation is neither efficient in the *hypothetical* economy. In figure 1, case (a)+(b), individuals with types $LL$ and $HL$ have some time left to use the highest skill, while type $LH$ and $HH$ individuals would love to use it, and so there is room for Pareto improvements in the hypothetical economy. Therefore another plausible fair allocation — but more difficult to compute— would be obtained if we allow for trading time slots in the hypothetical economy.

indifference curves could all be translated downwards so that the allocation becomes feasible (Fleurbaey and Maniquet, 2005c).

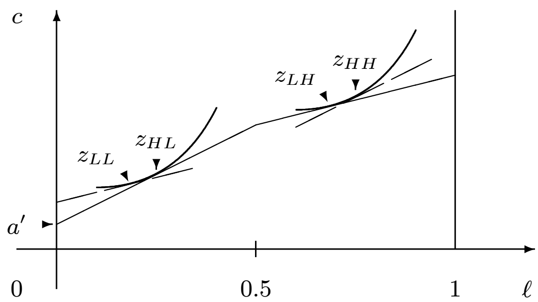



Figure 2: A Pareto efficient and shared resources equivalent allocation.

Two observations need to be stressed. *First*, an allocation which gives an individual a higher utility (puts him on a higher indifference curve) also increases his well-being level. As such, our definition of well-being corresponds with one specific, but, according to us, interesting cardinalization of preferences. *Second*, well-being has to be interpreted as a 'relative' measure of fair treatment in the spirit of the PESE allocation. If two individuals had the same well-being, they would have been treated fairly because both individuals would have been indifferent between their actual bundle and the bundle they would choose if (a) they received the same lump-sum amount of money and (b) all productive resources were shared equally. If one individual had a strictly lower well-being compared to another, he would have been treated unfairly with respect to the other because both individuals would have been indifferent between their actual bundle and the bundle they would choose if (b) all productive resources were shared equally but (a) the former individual received a strictly lower lump-sum amount of money.

# 3 A 'shared resources' social ordering

In case it is possible to recognize the less from the more productive and the lazy from the hard-working individuals, we can choose among all Pareto efficient and 'shared resources' equivalent allocations described in the previous section. However, it is not always possible to observe types. To proceed in such a second-best setting, it is more convenient to characterize a corresponding 'shared resources' social ordering.

A rule $f$ maps economies into orderings, or $f : \mathcal{E} \to \mathcal{R} : e \mapsto R_e = f(e)$, with $\mathcal{R}$ the set of all orderings (complete and transitive binary relations) defined over allocations $\mathbf{z}$ in $\mathcal{Z}$; call $P_e$ and $I_e$ the corresponding asymmetric and symmetric relation. We define some properties for $f$.

Our Pareto principle is equal to Pareto Indifference and the Weak Pareto principle together, i.e., if everyone is indifferent between allocations $\mathbf{z}$ and $\mathbf{z}'$, then $\mathbf{z}$ should also be socially indifferent to $\mathbf{z}'$ and if everyone strictly prefers allocation $\mathbf{z}$ to $\mathbf{z}'$, then $\mathbf{z}$ should also be socially strictly preferred to $\mathbf{z}'$. Anonymity requires that the names of the individuals do not matter. Formally:

Pareto: For each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$: If $U_i(z_i) = U_i(z_i')$ for all $i \in N$, then $\mathbf{z} I_e \mathbf{z}'$. If, $U_i(z_i) > U_i(z_i')$ for all $i \in N$, then $\mathbf{z} P_e \mathbf{z}'$.

Anonymity: For each economy $e \in \mathcal{E}$, for each allocation $\mathbf{z} \in \mathcal{Z}$ and for each permutation $\pi : N \to N$ over individuals: If $U_i = U_j$ for all $i, j \in N$, then $\mathbf{z} I_e \pi(\mathbf{z})$, with $\pi(\mathbf{z}) = \left(z_{\pi(1)}, \ldots, z_{\pi(n)}\right)$.

In line with the idea to compensate for differences in outcomes which are only due to differences in skills, compensation (Fleurbaey and Maniquet, 2005a) requires that a Pigou-Dalton transfer (in terms of net income) from a rich to a poor individual with the same preferences and the same labour should be socially approved:

Compensation: For each economy $e \in \mathcal{E}$, for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ and for all individuals $i, j \in N$: If (i) $\ell_i = \ell_j = \ell_i' = \ell_j'$ and $U_i = U_j$, (ii) $\exists \delta > 0$ such that $c_i = c_i' + \delta < c_j' - \delta = c_j$ and (iii) $z_k = z_k'$ for all $k \neq i, j$, then $\mathbf{z} R_e \mathbf{z}'$.

Finally, in line with (i) our well-being definition and (ii) the idea that individuals are responsible for their tastes, Utility Independence requires the ranking of two allocations to be the same (i) whenever they give rise to the same well-being vector, (ii) irrespective of the utility profile:

Utility Independence: For all economies $e = (\mathbf{s}, \mathbf{U}), e' = \left(\mathbf{s}, \mathbf{U}'\right) \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$: If $\mathbf{W}(\mathbf{z}, e) = \mathbf{W}(\mathbf{z}, e')$ and $\mathbf{W}(\mathbf{z}', e) = \mathbf{W}(\mathbf{z}', e')$, then $\mathbf{z} R_e \mathbf{z}' \Leftrightarrow \mathbf{z} R_{e'} \mathbf{z}'$.

Given these axioms, we should focus on the minimal well-being in society, or:[3]

**Theorem:** *If a rule $f : \mathcal{E} \to \mathcal{R}$ satisfies Pareto, Anonymity, Compensation and Utility Independence, then, for each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$: $\min \mathbf{W}(\mathbf{z}, e) > \min \mathbf{W}(\mathbf{z}', e)$ implies $\mathbf{z} P_e \mathbf{z}'$.*

First, possible candidates are the maximin and the leximin rule applied to well-beings, but in the sequel we do not bother about ranking allocations with equal minimal well-being levels, basically because the above definition is sufficient for optimization purposes. Second, the 'shared resources' social ordering has some formal similarity with Fleurbaey and Maniquet's (2005c) $\tilde{s}$-implicit budget leximin function, where $\tilde{s}$ is a reference skill level. In our case, the reference skill $\tilde{s}$ is piece-wise linear and endogenously defined by the skill pool in society. As such, laisser-faire allocations are selected in case all individuals have the same skill. Third, some readers might find the chosen inequality aversion too extreme.

[3] See appendix A for a proof.

However, the only redistributive condition is Compensation which in this weak form is compatible with any degree of inequality aversion, including zero. It is the combination of these axioms that entails an infinite aversion to inequality; see Fleurbaey and Maniquet (2005a) for a similar result.

# 4 Fair taxes: theory

In the previous section, we characterize a fair social ordering inspired by the PESE allocation. In this section, we analyze what happens when the government uses this fair social ordering to calculate optimal taxes in a discrete Stiglitz economy (1982, 1987) with four types which are not observable to the government.

All individuals in $N = \{1, \ldots, n\}$ can have four types, denoted $(s,t) \in S \times T$, where $s$ is the skill level and $t$ the taste for working; we abbreviate types as $st \in ST$. Each type $st$ is represented by $n_{st} > 0$ individuals, with $n = \sum_{st \in ST} n_{st}$. Skills can be low or high, or $s \in S = \{L, H\}$, with $0 < L < H$; later on, we come back to the issue of zero skills. Tastes for working can also be low or high, or $t \in \{L, H\}$, which correspond with a utility function $U_t$.[4]

As before, utility functions belong to $\mathcal{U}$, but we impose some additional properties. Let $V_{st}$ represent the preferences in the consumption-income space for type $st$, more precisely $V_{st} : \mathbb{R} \times [0, s] \to \mathbb{R} : (c, y) \mapsto V_{st}(c, y) \equiv U_t\left(c, \frac{y}{s}\right)$.[5] We impose two additional properties on the utility functions $U_t$; see Stiglitz (1982, 1987) for the first and Boadway *et al.* (2002) for the second property:

Single-crossingness: A higher taste for working $t$ corresponds with a lower marginal rate of substitution (denoted $MRS_t = -\frac{\partial U_t / \partial \ell}{\partial U_t / \partial c}$), expressing the view that individuals with a higher taste for working require less compensation (in terms of net income $c$) to work a little bit longer. Formally: $MRS_L > MRS_H$ in $\mathbb{R} \times [0, 1]$.

Indistinguishable middle type: The types $LH$ and $HL$ have the same preferences in the consumption-income space. Formally, there exists a continuous and strictly increasing function $\phi : \mathbb{R} \to \mathbb{R}$, such that $V_{LH} = \phi \circ V_{HL}$ in $\mathbb{R} \times [0, L]$.

Both assumptions together, the marginal rates of substitution in consumption-income space (denoted $MRSY_{st} = -\frac{\partial V_{st} / \partial y}{\partial V_{st} / \partial c}$) are also single-crossing, more precisely:

$MRSY_{LL} > MRSY_{LH} = MRSY_{HL} > MRSY_{HH}$, in $\mathbb{R} \times [0, L]$ and

$MRSY_{HL} > MRSY_{HH}$ in $\mathbb{R} \times [L, H]$.

We focus in the sequel on allocations $\mathbf{x} = (x_{LL}, x_{LH}, x_{HL}, x_{HH})$ in consumption-income space, thus $\mathbf{x} \in \mathcal{X} = (\mathbb{R} \times [0, L])^2 \times (\mathbb{R} \times [0, H])^2$, containing one bundle

[4] The exact scalars do not matter here, so we stick to the notation of $L$ to denote low taste for working and/or low-skilled and $H > L$ to denote high taste for working and/or high-skilled.

[5] Whenever $s = 0$, we define $V_{st}(c, y) = c$ for all $(c, y) \in \mathbb{R} \times \{0\}$.

$x_{st} = (c_{st}, y_{st})$ for each type $st \in ST$. The program of the government is to find the best allocation(s) $\mathbf{x}$ —'best' according to the fair social ordering defined in section 3— subject to (i) incentive compatibility constraints (no type envies another type's bundle) and (ii) a feasibility constraint (the sum of all taxes is larger than the government requirement $g \in \mathbb{R}$). With a slight abuse of notation, we write the well-being of type $st$ in allocation $\mathbf{x}$ as $w_{st} = \mathbf{W}_{st}(\mathbf{x})$. We get:

$$\underset{\mathbf{x}\in\mathcal{X}}{\text{MAX}}\ \underset{st\in ST}{\text{MIN}}\ (\mathbf{W}_{st}(\mathbf{x}))_{st\in ST} \text{ subject to} \tag{$*$}$$

INCENTIVE COMPATIBILITY CONSTRAINTS $IC_{st,(st)'}$:

$$\begin{aligned} V_{st}(x_{st}) &\geq V_{st}\left(x_{(st)'}\right), \forall st \in \{H\} \times T, \forall (st)' \in ST, \\ V_{st}(x_{st}) &\geq V_{st}\left(x_{(st)'}\right), \forall st \in \{L\} \times T, \forall (st)' \in ST \text{ with } y_{(st)'} \leq L. \end{aligned}$$

FEASIBILITY CONSTRAINT:

$$\sum_{st\in ST} n_{st}(y_{st} - c_{st}) \geq g.$$

Our first result tells us that the lowest income type, the 'undeserving poor' with type $LL$, must always receive lower subsidies (or pay higher taxes) than the second lowest income type, the 'hard-working poor' with type $LH$.[6] This result suggests that it is optimal —according to our fair social ordering— to 'make work pay' by subsidizing the low earners; this result is in line with Fleurbaey and Maniquet (2005a,b).

**Proposition 1** *: Consider a four type economy with skills $0 < L < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$ we must have $y^*_{LL} - c^*_{LL} \geq y^*_{LH} - c^*_{LH}$.*

We have to put this result in perspective, however. Although it is reasonable to assume that all individuals (with a capacity for work) have strictly positive productive skills, individuals might be constrained in their choice due to labour market frictions. Minimum wage laws, rationing and so on may prevent individuals, in particular the low-skilled, from working. Suppose, in our four type economy, that the low-skilled individuals are willing but cannot work due to such constraints which are beyond their responsibility. In such a case, their skills are nullified and, as shown in appendix B, the $LH$-type individuals always have the lowest level of well-being. Because these individuals will never work, maximizing the minimal well-being boils down to maximizing the basic income

[6] All proofs can be found in appendix B.

in society; see Fleurbaey and Maniquet (2005a,b) for a similar result. This turns proposition 1 round, or, the $LL$- and $LH$-type individuals (with $y^*_{LL} = y^*_{LH} = 0$) must always receive higher subsidies (or pay lower taxes) than the second lowest income type (here the $HL$-type individuals).

**Proposition 2** *: Consider a four type economy with skills $L = 0 < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$ we must have $y^*_{LL} - c^*_{LL} = y^*_{LH} - c^*_{LH} \leq y^*_{HL} - c^*_{HL}$.*

Both proposition 1 and 2 are based on simple fictitious economies. Furthermore, in proposition 2 we consider a rather extreme case in which all unemployed (the low-skilled) are not able to work. In the next section we simulate fair taxes for a sample of Belgian singles. It allows us to focus on (i) more realistic economies with many different types and, more importantly, on (ii) different and more realistic scenarios concerning the ability of the unemployed to work. The different scenarios in (ii) have a crucial impact on the tax-benefit scheme for the low earners.

# 5 Fair taxes: simulation results

## 5.1 Calibration

We use a sample of singles from the 1997 wave of the Panel Study for Belgian Households.[7] We only include singles with a capacity for work (students, pensioners, sick, or handicapped singles are excluded). We observe (i) the pretax yearly labour income $y$, (ii) the amount of labour $\ell$, normalized such that $0 \leq \ell \leq 1$, where $\ell = 1$ corresponds with 2925 hours, i.e., 45 weeks times 65 hours, (iii) the gross hourly wage rate $\sigma$ (only observed for those who worked, i.e., both $y, \ell \neq 0$) which leads to a gross yearly wage rate $s = 2925\sigma$ and (iv) the total net unemployment benefit $\beta$ (only observed for those who were partly or completely unemployed in 1997) from which we derive the net yearly unemployment benefit $b = \frac{\beta}{1-\ell}$, i.e., the net unemployment benefit one would obtain if full-time unemployed ($\ell = 0$).

*First*, we consider all individuals for which we possess all of the above information. We consider quasi-linear preferences (which excludes income effects) represented by utility functions:[8]

$$U_t : \mathbb{R} \times [0,1] \to \mathbb{R} : (c, \ell) \mapsto U_t(c, \ell) = c - \frac{1}{t}\frac{\varepsilon}{1+\varepsilon}\ell^{\frac{1+\varepsilon}{\varepsilon}},$$

[7] Mortelmans, D., Casman, M.-T. (2002) PSBH. Panel Study on Belgian Households (1992-2002) Universiteit Antwerpen, Université de Liège.

[8] Due to quasi-linearity, other non-labour income (e.g., due to rents, gifts, alimony, child allowances) does not matter for the labour choice of an individual. Furthermore, we keep individuals responsible for the other non-labour income and it is therefore excluded from our analysis.

with $t$ the taste parameter (possibly different for different individuals) and $\varepsilon$ the labour supply elasticity (the same for all individuals). Preferences in consumption-income space become:

$$V_{st} : \mathbb{R} \times [0, s] \to \mathbb{R} : (c, y) \mapsto V_{st}(c, y) = c - \frac{1}{s^{\frac{1+\varepsilon}{\varepsilon}} t} \frac{\varepsilon}{1+\varepsilon} y^{\frac{1+\varepsilon}{\varepsilon}}.$$

The net income of a Belgian single equals $y - \tau_{97}(y) + b(1 - \frac{y}{s})$, with $\tau_{97}(\cdot)$ the actual tax system for singles in Belgium in 1997 (reported in appendix C) and $b(1 - \frac{y}{s})$ the benefit when working $\ell = \frac{y}{s}$ units of time. Both tax and benefit parts separately, as well as the resulting budget set (the solid line), are illustrated in figure 3.

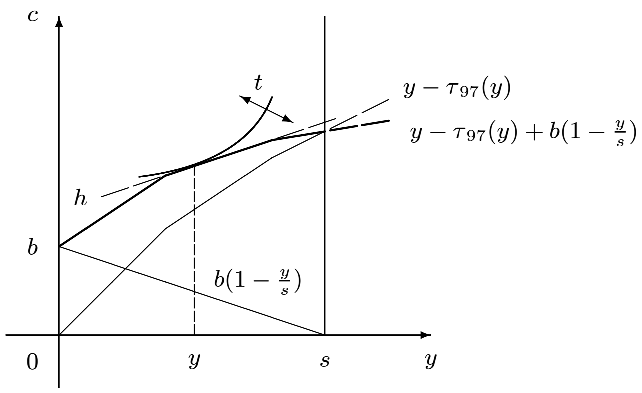



Figure 3: Calibration of the taste parameter $t$.

We calibrate $t$ such that the choice of $y$ is rationalized for each single. More precisely, the slope of the individual's budget set at $y$, denoted $h(y)$, should be equal to the marginal rate of substitution between consumption and gross income for the quasi-linear preferences $V_{st}$; we get[9]

$$t = \frac{1}{s} \frac{1}{h(y)} \left(\frac{y}{s}\right)^{\frac{1}{\varepsilon}}, \text{ with } h(y) = 1 - \tau_{97}'(y) - \frac{b}{s}.$$

*Second*, since we could only observe gross yearly wages $s$ (resp. net yearly unemployment benefits $b$) for individuals who worked in 1997 (resp. individuals who received unemployment benefits in 1997), we complete our dataset by imputing values for $s$ and $b$, whenever unobserved, via a Heckman selection model. Thus, in estimating $s$ and $b$, we correct for a possible sample selection bias due to the fact that we only observe wages $s$ for those who worked and benefits $b$ for those who were (permanently or temporarily) unemployed. The variables used for the imputation as well as the estimation results are described in appendix D.

[9] In order to have preferences that are strictly decreasing in labour, $t$ must be strictly positive. Therefore, we drop all individuals with $h(y) \leq 0$ out of the sample (17 observations).

We end up with a heterogeneous sample of 621 singles who differ in skills $s$ and tastes $t$, which drive their labour market behaviour; appendix E contains some descriptive statistics for our dataset. Three points are worth mentioning. First, propositions 1 and 2 from section 4 are not directly applicable in this section. The reason being that, unlike in the four type economy, we no longer observe individuals for every combination of $s$ and $t$. In other words, the theoretical No Identification assumption of Fleurbaey and Maniquet (2005b) is not fulfilled in our sample. Second, the non-responsibility parameter $s$ and the responsibility parameter $t$ in our dataset are barely correlated: using a low labour supply elasticity $\varepsilon = 0.1$ for singles, the correlation between $s$ and $t$ equals $-0.071$, suggesting independently distributed skills and tastes. With a strong correlation, compensating for skills only (and not for tastes) would be a dubious exercise. Third, given the nature of our quasi-linear preferences, all unemployed receive a taste for working $t = 0$. To put it differently, all unemployed are considered unwilling to work. We relax this crucial assumption later on.

## 5.2 Results

Rather than using allocations as in the government program $(*)$, we use a piecewise linear tax-benefit scheme as our instrument to approximate a non-linear tax scheme. As we are mainly concerned with the bottom incomes, we consider a piecewise linear tax-benefit scheme up to yearly gross earnings of €20000 in steps of €500 and we use a constant marginal tax rate afterwards. Using either a wider range of piecewise linear taxes (up to €80000 in steps of €500) or a finer grid (up to €20000 in steps of €250) does not change our results for the bottom incomes drastically. Remarkably, using a wider range leads to approximately constant marginal tax rates for incomes above €20000 which, except for the very high incomes, approximate $\frac{1}{1+\varepsilon}$, the optimal linear tax rate when maximizing basic income (see Atkinson, 1995). Given such a tax-benefit scheme, individuals choose their best bundle (according to their tastes and skills) and, therefore, incentive constraints are automatically satisfied. For the feasibility constraint, we use the total government requirement ($g$) in the actual system, which is (in per capita terms) equal to € 3851. Finally, to obtain realistic proposals, we add participation constraints to the government program $(*)$, such that no one prefers the bundle $(0, 0)$ to the allocated bundle in the optimum.

### 5.2.1 Three simulations

In this subsection we report and discuss simulation results for three cases; a sensitivity analysis with respect to the main parameters is postponed to the next subsection.

No income effects and a low labour supply elasticity do not seem unrealistic for singles; see, e.g., Blundell and MaCurdy (1999) for an empirical assessment. Using a labour supply elasticity $\varepsilon = 0.1$, figure 4 depicts the chosen bundles in the consumption-gross income space for the following cases.[10]

*First*, we present the Rawlsian optimal allocation (denoted RAWLS in figure 4), i.e., the one which maximizes the basic income, as a benchmark case. It installs a high basic income equal to €9363 and high (and almost constant) positive marginal tax rates for the bottom incomes.

*Second*, we show the optimal allocation according to our fair social ordering in the extreme case that all unemployed are never willing to work (denoted 100% in figure 4). This gives a low (yearly) basic income equal to €518, moderate subsidies fading in around €3000 and fading out around €10500, followed by progressive taxes. Thus, it seems fair to 'make work pay', even if it brings about a very low basic income.

*Third*, the 100%-case is clearly an extreme viewpoint. For example, minimum wage laws in Belgium could keep some individuals (especially those with low skills $s$) from working and, therefore, our calibration might underestimate their true taste parameter $t$, thus overestimating their well-being level $w$. More reasonably, at least some of the observed unemployment must be involuntary, especially in the case of singles. Suppose unemployment is voluntary for $p$% of the unemployed: their taste levels remain equal to zero. The other $(1-p)\,\%$ are constrained, i.e., although they would like to work, they are and will always remain constrained at $y=0$, but we want to use their 'true' taste parameter to calculate well-being levels. Taste levels are unfortunately unknown (and difficult to infer from our data), therefore we make the following assumption: we assign all the constrained unemployed the same taste level in a way to be explained later on. Although simplifying, giving them the same taste level allows us to disregard the exact value of $p$, as long as it differs from 100%. The reason is that all the constrained unemployed end up with the same well-being and our maximin-type criterion is not sensitive to population size. Thus, for example, only one (or all but one) constrained unemployed would lead to the same result. For the moment, we suppose that their tastes equal the average taste of those currently working. Although one might be inclined to choose a lower taste level for the constrained unemployed, we refer to the fact that skills and tastes in our dataset are approximately uncorrelated. The optimal allocation for this case is denoted <100% in figure 4.[11] We get a sizeable basic income of €6318, illustrating that the constrained unemployed are in the <100% case among the worst-off in terms of well-being. In addition, it displays a U-shaped pattern of positive marginal tax rates: high marginal taxes for the bottom incomes, followed by an almost neutral region (for incomes between €4000-€10500) and again high marginal taxes afterwards.

[10]Notice that dotted lines do not represent an optimal tax-benefit schedule, but are only connecting the chosen consumption-gross income bundles.

[11]For reasons of calculation, we choose $p$ equal to 90% and assign the average taste level to the unemployed with the lowest productivities.

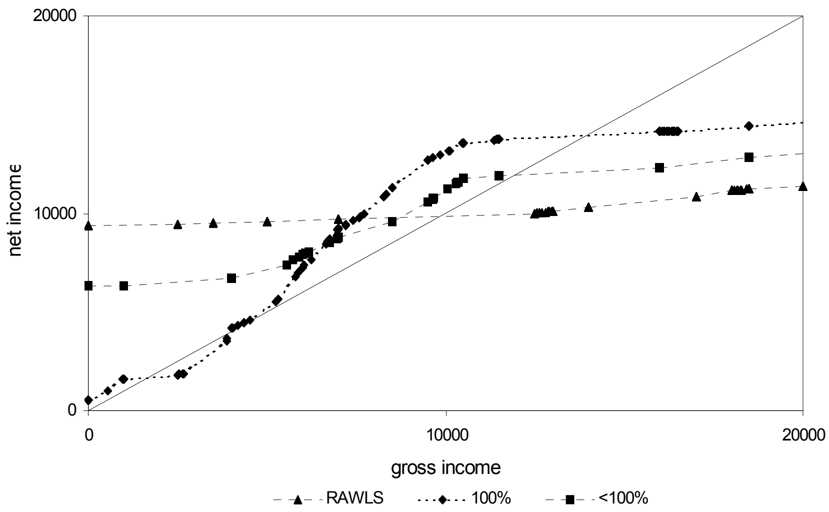



Figure 4: Optimal allocation for the Rawlsian case, the 100%- and <100%-case.

Table 1 summarizes for all three cases (in columns) and for different gross income groups $G$ (in rows) (i) the proportion of individuals $\frac{|G|}{n}$ and (ii) the average tax rate $(\frac{1}{|G|}\sum_{i\in G}(y_i - c_i))$. Average tax rates are monotonically increasing in gross income, except in the 100%-case, which is in line with our previous observation that 'making work pay' can only be optimal in the unreasonable case where none of the unemployed is ever willing to work.

| | RAWLS | | 100% | | <100% | |
|---|---|---|---|---|---|---|
| income group | prop. | avg. tax | prop. | avg. tax | prop. | avg. tax |
| 0 | 0.21 | −9363 | 0.19 | −518 | 0.19 | −6318 |
| $0 < y \leq 5000$ | 0.07 | −5595 | 0.05 | −98 | 0.05 | −4294 |
| $5000 < y \leq 10000$ | 0.06 | −2679 | 0.06 | −2222 | 0.05 | −1610 |
| $10000 < y \leq 15000$ | 0.14 | 3370 | 0.17 | −2424 | 0.18 | −615 |
| $15000 < y \leq 20000$ | 0.26 | 7205 | 0.23 | 2785 | 0.24 | 4972 |
| $20000 < y$ | 0.26 | 15856 | 0.30 | 13104 | 0.29 | 14714 |

Table 1: Some characteristics for the Rawlsian case, the 100%- and <100%-case.

### 5.2.2 Sensitivity analysis

As we believe that no one would be willing to defend the 100%-case, we focus on the <100%-case for our sensitivity analysis. We investigate the impact of (i) the labour supply elasticity and (ii) the taste level assigned to the constrained unemployed.

First, the labour supply elasticity $\varepsilon$ equaled 0.1 in our previous simulations. Figure 5 shows the impact of halving and doubling $\varepsilon$ on our optimal allocation, here denoting the <100%-case of figure 4 by 'eps=0.1'. Changing $\varepsilon$ has a moderate impact on marginal tax rates but the U-shaped pattern of marginal tax rates remains intact; see Saez (2001) for similar results in welfarist Mirrlees economies. As expected, choosing a higher labour supply elasticity leads to lower marginal tax rates everywhere and, for a given budget constraint, it installs a (substantially) lower basic income. For low values of $\varepsilon$, marginal tax rates are everywhere positive, excluding 'making work pay'-type policies. Choosing a higher value of $\varepsilon$ introduces subsidies for gross income earners between €6000 and €10000, but still the largest subsidies are directed towards the unemployed.

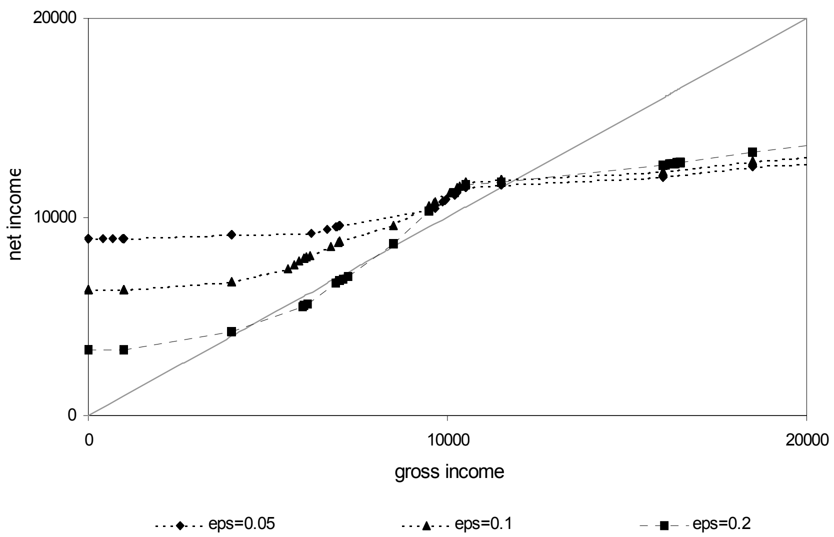



Figure 5: Measuring the impact of varying $\varepsilon$.

Second, the taste level assigned to the constrained unemployed equaled the average taste of the working class in the previous section. Here we consider two additional cases: half of the average taste of those currently working (denoted HALF in figure 6) and the double of the average taste (denoted DOUBLE in figure 6). These variants have a clear interpretation. The HALF-case (resp. DOUBLE-case) corresponds with the assumption that each constrained unemployed needs two times more (resp. half of the) additional consumption for a unit of additional labour compared to the 'average' worker. Figure 6 presents the HALF- and DOUBLE-cases, as well as the <100%-case of figure 4, here denoted by 'ONE'.

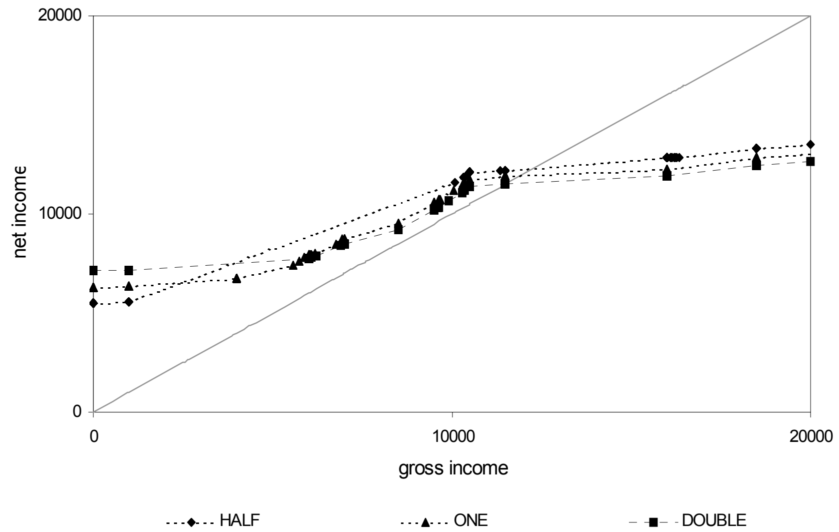



Figure 6: Measuring the impact of varying tastes.

Choosing less or more conservative estimates for the tastes of the constrained unemployed plays a moderate role. As expected, the higher their taste for working, the lower their well-being, which puts upward pressure on the optimal basic income. Furthermore, halving or doubling the assigned taste level does not alter the U-shaped pattern of positive marginal tax rates.

# 6 Conclusion

Given the increased importance many governments attach to 'making work pay' policies, we examine whether subsidizing low earners is optimal according to a specific 'fair' social ordering. Fairness considerations are kept simple in this paper: we want to compensate individuals for differences in productive skills, but we keep them responsible for their tastes for working.

We consider a discrete Stiglitz (1982, 1987) economy with four types, defined by a low or high productive skill and a low or high taste for working, and a government that wants to install fair taxes but cannot observe individuals' type. We show that fairness recommends to subsidize the low earners as long as the low-skilled individuals have a strictly positive skill. In case the low-skilled have a zero skill, subsidies can never be optimal.

To enhance realism, we simulate fair taxes for a sample of Belgian singles. Our fairness criterion suggests a U-shaped pattern of positive marginal tax rates in almost all cases. This strongly suggests that negative marginal tax rates, and thus 'making work pay' policies, cannot be optimal in the reasonable case in which at least some unemployed are willing to work but cannot due to exogenous labour market constraints.

# Appendix A: The 'shared resources' social ordering: proof

*If a rule $f : \mathcal{E} \to \mathcal{R}$ satisfies Pareto, Anonymity, Compensation and Utility Independence, then, for each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$:* $\min \mathbf{W}(\mathbf{z}, e) > \min \mathbf{W}(\mathbf{z}', e)$ *implies* $\mathbf{z} P_e \mathbf{z}'$.

**Proof.**

*First*, we show in three steps that Pareto Indifference (the first part of the Pareto axiom) and Utility Independence for $f$ are equivalent with Neutrality for $f$:

Neutrality: For all economies $e = (\mathbf{s}, \mathbf{U})$, $e' = (\mathbf{s}, \mathbf{U}') \in \mathcal{E}$ and for all allocations $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathcal{Z}$: If $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$, then $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$.

**1.** If $f$ satisfies Neutrality then $f$ also satisfies Pareto Indifference. Suppose the antecedent of Pareto Indifference is true for a certain economy $e \in \mathcal{E}$ and two allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$, i.e., $U_i(z_i) = U_i(z_i')$ for all $i \in N$. As such, $z_i$ lies on the same indifference curve as $z_i'$ for all individuals and, by definition of our well-being concept, $\mathbf{W}(\mathbf{z}, e) = \mathbf{W}(\mathbf{z}', e)$. Let $e' = e$ and define allocations $\mathbf{a} = \mathbf{d} = \mathbf{z}$ and $\mathbf{b} = \mathbf{c} = \mathbf{z}'$. As a consequence, $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$ are true by construction. Using Neutrality, we get $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$, or equivalently, $\mathbf{z} R_e \mathbf{z}' \Leftrightarrow \mathbf{z}' R_e \mathbf{z}$ ($\bullet$). Because of completeness of $R_e$, we must have either $\mathbf{z} R_e \mathbf{z}'$ (and also $\mathbf{z}' R_e \mathbf{z}$ via ($\bullet$)) or $\mathbf{z}' R_e \mathbf{z}$ (and also $\mathbf{z} R_e \mathbf{z}'$ via ($\bullet$)). Both cases lead to $\mathbf{z} I_e \mathbf{z}'$ establishing Pareto Indifference.

**2.** If $f$ satisfies Neutrality then $f$ also satisfies Utility Independence. Suppose the antecedent of Utility Independence is true, i.e., there exist two economies $e = (\mathbf{s}, \mathbf{U}), e' = (\mathbf{s}, \mathbf{U}') \in \mathcal{E}$ and two allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ such that $\mathbf{W}(\mathbf{z}, e) = \mathbf{W}(\mathbf{z}, e')$ and $\mathbf{W}(\mathbf{z}', e) = \mathbf{W}(\mathbf{z}', e')$. Simply choose $\mathbf{a} = \mathbf{b} = \mathbf{z}$ and $\mathbf{c} = \mathbf{d} = \mathbf{z}'$ such that $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$ holds. Using Neutrality, we get $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$, or equivalently, $\mathbf{z} R_e \mathbf{z}' \Leftrightarrow \mathbf{z} R_{e'} \mathbf{z}'$ establishing Utility Independence.

**3.** If $f$ satisfies Pareto Indifference and Utility Independence then $f$ also satisfies Neutrality. Suppose the antecedent of Neutrality holds, i.e., there exist two economies $e = (\mathbf{s}, \mathbf{U})$ and $e' = (\mathbf{s}, \mathbf{U}') \in \mathcal{E}$ and four allocations $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in \mathcal{Z}$ such that $\mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e')$ and $\mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e')$. Let us focus on an arbitrary individual $i \in N$. Because $\mathbf{W}_i(\mathbf{a}, e) = \mathbf{W}_i(\mathbf{b}, e')$, the indifference curve of $U_i$ through $a_i$ and $U_i'$ through $b_i$ are tangent to the same 'shared resources' opportunity set defined by $\mathbf{s}$. Given $U_i, U_i' \in \mathcal{U}$, both indifference curves must cross at least once in $\mathbb{R} \times [0, 1]$. Choose a bundle $\alpha_i$ where both cross. Repeating this construction of $\alpha_i$ for all individuals, we get an allocation $\boldsymbol{\alpha} \in \mathcal{Z}$ such that $\mathbf{W}(\boldsymbol{\alpha}, e) = \mathbf{W}(\mathbf{a}, e) = \mathbf{W}(\mathbf{b}, e') = \mathbf{W}(\boldsymbol{\alpha}, e')$. In the same way, define an allocation $\boldsymbol{\beta} \in \mathcal{Z}$ such that $\mathbf{W}(\boldsymbol{\beta}, e) = \mathbf{W}(\mathbf{c}, e) = \mathbf{W}(\mathbf{d}, e') = \mathbf{W}(\boldsymbol{\beta}, e')$. Using Pareto Indifference and transitivity of $R_e$ and $R_{e'}$, we get:

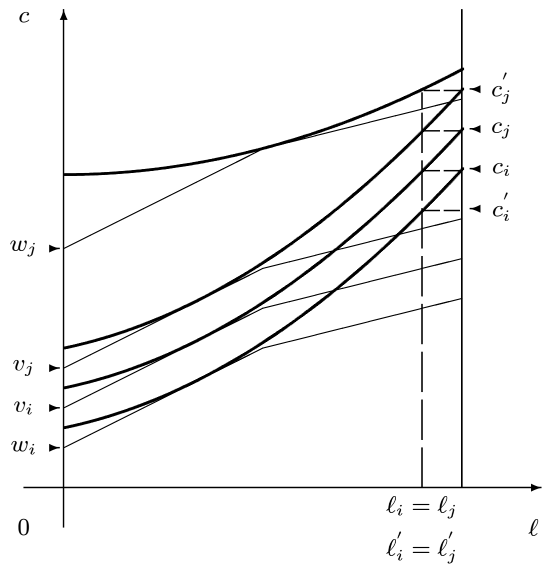
$$(\bullet) \ \ \mathbf{a} R_e \mathbf{c} \Leftrightarrow \boldsymbol{\alpha} R_e \boldsymbol{\beta} \text{ and } \boldsymbol{\alpha} R_{e'} \boldsymbol{\beta} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}.$$

Using Utility Independence, we get $\boldsymbol{\alpha} R_e \boldsymbol{\beta} \Leftrightarrow \boldsymbol{\alpha} R_{e'} \boldsymbol{\beta}$. Using ($\bullet$), we get $\mathbf{a} R_e \mathbf{c} \Leftrightarrow \mathbf{b} R_{e'} \mathbf{d}$, establishing Neutrality.

*Second*, a rule $f$ satisfies Neutrality if and only if there exists a unique ordering $R^*$ defined over $\mathbb{R}^n$, such that for each economy $e \in \mathcal{E}$ and for all allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ we have $\mathbf{z} R_e \mathbf{z}'$ if and only if $\mathbf{W}(\mathbf{z}, e)\ R^*\ \mathbf{W}(\mathbf{z}', e)$. Neutrality has to be interpreted as follows: only well-being levels matter to rank two allocations (given a fixed size $n$ of the population and a fixed skill vector $\mathbf{s}$). It suffices to notice that our set-up is sufficiently rich to obtain Neutrality: for any two well-being vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$, there exist two allocations $\mathbf{z}, \mathbf{z}' \in \mathcal{Z}$ and an economy $e \in \mathcal{E}$ such that $\mathbf{W}(\mathbf{z}, e) = \mathbf{v}$ and $\mathbf{W}(\mathbf{z}', e) = \mathbf{w}$.

*Third*, the unique ordering $R^*$ inherits certain properties from $f$: $R^*$ must satisfy weak Pareto* (if $v_i > w_i$ for all $i \in N$, then $\mathbf{v} P^* \mathbf{w}$) and Anonymity* ($\mathbf{v} I^* \pi(\mathbf{v})$ with $\pi : N \to N$ a permutation of individuals in $N$). This follows from Pareto and Anonymity for $f$ straightforwardly. We show that, given Compensation for $f$, $R^*$ must also satisfy

Hammond Equity* (Hammond, 1976): For all well-being vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and for all individuals $i, j \in N$: If (i) $w_i < v_i < v_j < w_j$ and (ii) $v_k = w_k$ for all $k \neq i, j$, then $\mathbf{v} R^* \mathbf{w}$.




Suppose the antecedent of Hammond Equity* holds, or there exist two well-being vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ and two individuals $i, j \in N$ such that $w_i < v_i < v_j < w_j$ and $v_k = w_k$ for all $k \neq i, j$ hold. The figure illustrates how it is possible to construct bundles $z_i, z_j$ and $z_i', z_j'$ and a utility function $U_i = U_j \in \mathcal{U}$ such that $\mathbf{W}_i(\mathbf{z}, e) = v_i$, $\mathbf{W}_j(\mathbf{z}, e) = v_j$ and $\mathbf{W}_i(\mathbf{z}', e) = w_i$, $\mathbf{W}_j(\mathbf{z}', e) = w_j$ and the antecedents of the Compensation principle are satisfied for $i, j$, i.e., (i) $\ell_i = \ell_j = \ell_i' = \ell_j'$ and $U_i = U_j$ and (ii) $\exists \delta > 0$ such that $c_i = c_i' + \delta < c_j' - \delta = c_j$. The bundles $z_i, z_j$

and $z_i', z_j'$ can be extended with bundles $z_k = z_k'$ for the other individuals $k \neq i, j$ to obtain allocations $\mathbf{z}$ and $\mathbf{z}'$, such that $\mathbf{W}_k(\mathbf{z}, e) = v_k = w_k = \mathbf{W}_k(\mathbf{z}', e)$ holds for all $k \neq i, j$. Using Compensation, we must have $\mathbf{z} R_e \mathbf{z}'$ and hence $\mathbf{v} R^* \mathbf{w}$ must hold.

*Finally*, Given the axioms for $R^*$, Tungodden (2000, theorem 1) shows that $\min \mathbf{v} > \min \mathbf{w}$ implies $\mathbf{v} P^* \mathbf{w}$ for any vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$, which completes our proof.

# Appendix B: Proofs of propositions 1 and 2

To prove propositions 1 and 2, we need two 'tricks' and two lemmas. We start with the tricks.

Consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ as in figure B1. The bundles $x_{LL}$ and $x_{HH}$ lie somewhere in the left and right shaded zone, respectively, to satisfy the incentive constraints. The bundle $x^\circ$ is constructed to satisfy $V_{HL}(x^\circ) = V_{HL}(x_{HL})$ and $MRSY_{HL}(x^\circ) = 1$.

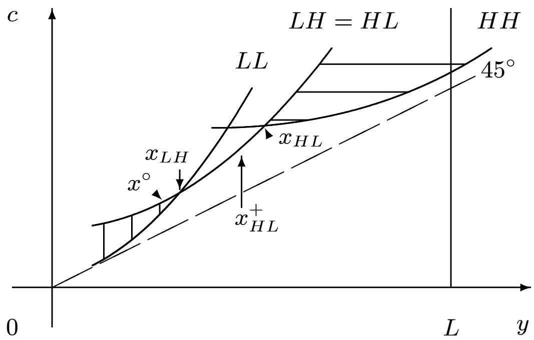



Figure B1: the allocations $\mathbf{x}$ and $\mathbf{x}^+$ illustrating trick 1.

Now, consider the allocation $\mathbf{x}^+ \in \mathcal{X}$ with $x_{st}^+ = x_{st}$ for all types $st \neq HL$ and $x_{HL}^+$ is constructed by moving $x_{HL}$ on his indifference curve towards the bundle $x^\circ$. It is clear that the allocation $\mathbf{x}^+$ is implementable. Furthermore, given the preference technology defined by $\mathcal{U}$, we have $y_{HL}^+ - c_{HL}^+ > y_{HL} - c_{HL}$. Thus, the allocation $\mathbf{x}^+$ is also feasible with $\sum_{st \in ST} n_{st}\left(y_{st}^+ - c_{st}^+\right) - \sum_{st \in ST} n_{st}\left(y_{st} - c_{st}\right) = m > 0$. The amount of money $m$ can now be freely redistributed to the net income of all types (while still satisfying all incentive constraints) resulting in a weak Pareto improvement and thus also an improvement according to the government's program $(*)$. More generally, we obtain:

Trick 1: Consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ and a type $st$ whose bundle $x_{st}$ can be moved along his indifference curve (i) without violating incentive constraints and (ii) making an amount of money $m$ free for redistribution. The allocation $\mathbf{x}$ cannot be optimal according to program $(*)$, because everyone can be made strictly better-off (by redistributing the amount of money $m$ to the net incomes of all types), without violating incentive constraints.

To illustrate the second trick, consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ as in figure B2.

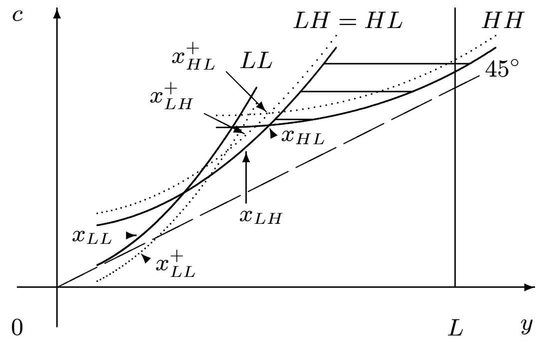



Figure B2: the allocation $\mathbf{x}$ and $\mathbf{x}^+$ illustrating trick 2.

Again, the bundle $x_{HH}$ lies somewhere in the right shaded zone to satisfy the incentive constraints. Now it is possible to construct a feasible and implementable allocation $\mathbf{x}^+ \in \mathcal{X}$, transferring in $\mathbf{x}$ some net income from type $LL$ to the other types $LH$, $HL$ and $HH$. Whether or not the resulting allocation is better according to program $(*)$ ultimately depends on the well-being levels in society: if $LL$ is strictly better off compared to one of the other types, it is always possible to find an allocation $\mathbf{x}^+$ which is better according to program $(*)$. We summarize

Trick 2: Consider an implementable and feasible allocation $\mathbf{x} \in \mathcal{X}$ and one or more types $st$ whose bundle(s) $x_{st}$ can be moved downwards (i) without violating incentive constraints and (ii) making an amount of money $m > 0$ free for redistribution to the other types. The allocation $\mathbf{x}$ cannot be optimal according to program $(*)$, if all donor type(s) $st$ were strictly better off in $\mathbf{x}$ compared to (one of) the other types.

Besides two tricks, we need two lemmas. The first lemma tells us that the program $(*)$ can, loosely speaking, focus on the lower-skilled, because they are always worse-off in terms of well-being, more precisely:

**Lemma 1** *: Consider two types with the same taste for working $t \in T$ (and thus the same utility function $U_t \in \mathcal{U}$), but different skills $0 \leq L < H$. In an implementable allocation $\mathbf{x} \in \mathcal{X}$, with $V_{Ht}(x_{Ht}) \geq V_{Ht}(x_{Lt})$ (resp. $V_{Ht}(x_{Ht}) > V_{Ht}(x_{Lt})$) the lower-skilled type $Lt$ is always worse off (resp. strictly worse off) compared to the higher-skilled type $Ht$, i.e., $\mathbf{W}_{Ht}(\mathbf{x}) \geq \mathbf{W}_{Lt}(\mathbf{x})$ (resp. $\mathbf{W}_{Ht}(\mathbf{x}) > \mathbf{W}_{Lt}(\mathbf{x})$).*

**Proof.** Consider two types with the same taste for working $t \in T$. We prove the case where skills satisfy $0 < L < H$ and $V_{Ht}(x_{Ht}) \geq V_{Ht}(x_{Lt})$; the other cases are analogous. Call $(c_{Lt}, y_{Lt})$ and $(c_{Ht}, y_{Ht})$ their bundles. Individuals with the same taste $t$ have the same utility functions $U_t$ and thus also the same indifference curves and therefore the same well-being level for bundles on the same indifference curve. Because our well-being measure is ordinally equivalent with utility, measured by $U_t$, it suffices to show that $U_t\left(c_{Lt}, \frac{y_{Lt}}{L}\right) \leq U_t\left(c_{Ht}, \frac{y_{Ht}}{H}\right)$. Suppose not, i.e., suppose (i) $U_t\left(c_{Lt}, \frac{y_{Lt}}{L}\right) > U_t\left(c_{Ht}, \frac{y_{Ht}}{H}\right)$. Because $V_{Ht}(x_{Ht}) \geq V_{Ht}(x_{Lt})$ we get, by definition of $V_{Ht}$, that (ii) $U_t\left(c_{Ht}, \frac{y_{Ht}}{H}\right) \geq U_t\left(c_{Lt}, \frac{y_{Lt}}{H}\right)$. Combining (i) and (ii), we obtain $U_t\left(c_{Lt}, \frac{y_{Lt}}{L}\right) > U_t\left(c_{Lt}, \frac{y_{Lt}}{H}\right)$, a contradiction given $U_t \in \mathcal{U}$ and $0 < L < H$. □

Lemma 2 tells us that it cannot be optimal —according to the government's program $(*)$— to treat the indistinguishable middle types $LH$ and $HL$ differently in case $y^*_{HL} \leq L$. Otherwise (if $y^*_{HL} > L$) it might be optimal to treat them differently, but only under certain conditions:

**Lemma 2** *: Consider a four type economy with skills $0 < L < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$, we must have:*

*(a).* $x^*_{LH} = x^*_{HL}$, *if* $y^*_{HL} \leq L$, *or else,*
*(b).* $V_{HL}(x^*_{LH}) = V_{HL}(x^*_{HL})$, *with* $y^*_{LH} = L < y^*_{HL}$ *and* $MRSY_{HL}(x^*_{HL}) \leq 1$.

**Proof of part (a).** Suppose $y^*_{HL} \leq L$ and $x^*_{LH} \neq x^*_{HL}$. We show that it is always possible to construct another allocation $\mathbf{x} \in \mathcal{X}$, which is feasible, implementable and strictly better than $\mathbf{x}^*$ according to the government's program $(*)$. Because $y^*_{HL} \leq L$, the incentive compatibility constraints $IC_{LH,HL}$ and $IC_{HL,LH}$ require

$$\begin{aligned} V_{HL}(x^*_{HL}) &\geq V_{HL}(x^*_{LH}) \text{ and} \\ V_{LH}(x^*_{LH}) &\geq V_{LH}(x^*_{HL}) \Leftrightarrow V_{HL}(x^*_{LH}) \geq V_{HL}(x^*_{HL}), \end{aligned}$$

where the equivalence $\Leftrightarrow$ is due to indistinguishable middle types. We must have $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$, or $x^*_{LH}$ and $x^*_{HL}$ must lie on the same indifference curve.

Given our preference technology $\mathcal{U}$, there are only two cases for $x^*_{LH} \neq x^*_{HL}$. Assume $x^*_{LH} < x^*_{HL}$, i.e., $c^*_{LH} < c^*_{HL}$ and $y^*_{LH} < y^*_{HL}$; for the other case $x^*_{LH} > x^*_{HL}$ simply switch subscripts $HL$ and $LH$ in the sequel. Define a bundle $x^\circ = (c^\circ, y^\circ)$ in $\mathbb{R} \times [0, L]$ such that $x^\circ$ also lies on the same indifference curve through $x^*_{LH}$ and $x^*_{HL}$, i.e., $V_{HL}(x^\circ) = V_{HL}(x^*_{HL})$, and choose (i) $y^\circ = 0$, if $MRSY_{HL} \geq 1$ everywhere in $\mathbb{R} \times [0, L]$, (ii) $y^\circ = L$, if $MRSY_{HL} \leq 1$ everywhere in $\mathbb{R} \times [0, L]$, or else (iii) choose $x^\circ$ such that $MRSY_{HL}(x^\circ) = 1$. Each case leads to any of the following three cases: either $(\alpha)$ $y^\circ \leq y^*_{LH} < y^*_{HL}$, or $(\beta)$ $y^*_{LH} < y^\circ < y^*_{HL}$ or $(\gamma)$ $y^*_{LH} < y^*_{HL} \leq y^\circ$. In each of the three cases $(\alpha)$, $(\beta)$ and $(\gamma)$, it is possible to use TRICK 1, by moving either $x^*_{HL}$ to the left on his indifference curve (in case ($\alpha$) and ($\beta$)) or moving $x^*_{LH}$ to the right on his indifference curve (in case ($\gamma$)), contradicting that $\mathbf{x}^*$ was optimal.

**Proof of part (b)**. Suppose $y^*_{HL} > L$. We show that $x^*_{LH} < x^*_{HL}$, with $y^*_{LH} = L$ and $MRSY_{HL}(x^*_{HL}) \leq 1$, must hold. Recall that, in case $y^*_{HL} > L$, the incentive constraint $IC_{LH,HL}$ does not exist, because type $HL$'s bundle is not attainable for $LH$.

We first show that the incentive constraint $IC_{HL,LH}$ must bind, i.e., $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$. Suppose not, i.e., $V_{HL}(x^*_{HL}) > V_{HL}(x^*_{LH})$. Single-crossingness ensures that $V_{HL}(x^*_{HL}) > V_{HL}(x^*_{LL})$ and thus $LL$ is strictly worse-off compared to $HL$ (lemma 1); for the same reason, $LH$ is strictly worse off than $HH$. Now, it is possible to use TRICK 2, transferring from type $HL$ (and possibly $HH$ as well if $IC_{HL,HH}$ binds) to both other types $LL$ and $LH$, which must improve the lowest well-being, contradicting that $\mathbf{x}^*$ was optimal according to program $(*)$.

Now, we are back in the same situation as in part (a) because both $x^*_{LH}$ and $x^*_{HL}$, with $x^*_{LH} < x^*_{HL}$, lie on the same indifference curve (of type $HL$), i.e., $V_{HL}(x^*_{LH}) = V_{HL}(x^*_{HL})$, but here $y^*_{LH} \leq L < y^*_{HL}$. Now proceed as in part (a). Define the bundle $x^\circ = (c^\circ, y^\circ)$ in $\mathbb{R} \times [0, H]$ such that $x^\circ$ also lies on the same indifference curve through $x^*_{LH}$ and $x^*_{HL}$, i.e., $V_{HL}(x^\circ) = V_{HL}(x^*_{HL})$, and choose (i) $y^\circ = 0$, if $MRSY_{HL} \geq 1$ everywhere in $\mathbb{R} \times [0, H]$, (ii) $y^\circ = H$, if $MRSY_{HL} \leq 1$ everywhere in $\mathbb{R} \times [0, H]$, or else (iii) choose $x^\circ$ such that $MRSY_{HL}(x^\circ) = 1$. Now, $y^\circ < y^*_{HL}$ is not possible (otherwise we can use TRICK 1, moving $x^*_{HL}$ to the left on his indifference curve); thus $MRSY_{HL}(x^*_{HL}) \leq 1$. As a consequence $y^\circ \geq y^*_{HL}$ must hold. Now, $y^*_{LH} < L$ is not possible (because then $y^*_{LH} < L < y^\circ$ and using trick 1 again, we could move $x^*_{LH}$ to the right on his indifference curve). Thus $y^*_{LH} = L$, which completes the proof. □

We are ready to prove propositions 1 and 2, on the basis of lemmas 1 and 2 and tricks 1 and 2.

# Proof of proposition 1

Consider a four type economy with skills $0 < L < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$, we must have $y^*_{LL} - c^*_{LL} \geq y^*_{LH} - c^*_{LH}$.

Our proof consists of two parts, depending on whether (a) $y^*_{HL} \leq L$ in the optimum $\mathbf{x}^*$, or (b) $y^*_{HL} > L$. Given the definition of $\mathcal{X}$, one of both cases must hold. We show, for both cases, that $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$ is not possible.

**Proof of part (a)**. Suppose $y^*_{HL} \leq L$ (thus $x^*_{LH} = x^*_{HL}$ via lemma 2) and $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$. We consider four possible cases, depending on whether $IC_{LL,LH}$ and/or $IC_{LH,LL}$ bind, or not. In an optimum $\mathbf{x}^*$ of the program $(*)$, one of these four cases must hold. For all cases, we show that it is possible to construct a strictly better allocation according to the program $(*)$, which also satisfies the feasibility and incentive compatibility constraints.

**1.** $IC_{LL,LH}$ and $IC_{LH,LL}$ bind. This requires $x^*_{LL} = x^*_{LH}$ which contradicts $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$.

**2.** $IC_{LL,LH}$ binds, $IC_{LH,LL}$ does not bind. **2a.** If $MRSY_{LL}(x^*_{LL}) < 1$, we could use TRICK 1 moving $x^*_{LL}$ somewhat to the right on his indifference curve. **2b.** We must have $MRSY_{LL}(x^*_{LL}) \geq 1$, from (2a). But, given the preference technology defined by $\mathcal{U}$, $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$ is not possible, a contradiction.

**3.** $IC_{LL,LH}$ does not bind, $IC_{LH,LL}$ binds. **3a.** Let us first focus on type $LH$. If $y^*_{LH} = 0$, then incentive constraints and single-crossingness require $x^*_{LL} = x^*_{LH}$, which violates $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$. So $y^*_{LH} > 0$. If $MRSY_{LH}(x^*_{LH}) > 1$, we can use TRICK 1 again by moving both $x^*_{LH} = x^*_{HL}$ to the left on their (common) indifference curve. So $MRSY_{LH}(x^*_{LH}) \leq 1$ must hold. **3b.** We focus now on type $LL$. We must have either (i) $y^*_{LL} = 0$ or (ii) $y^*_{LL} > 0$. In case (ii), we have $MRSY_{LL}(x^*_{LL}) \leq 1$ (otherwise we can use TRICK 1, moving $x^*_{LL}$ somewhat to the left on his indifference curve). **3c.** Due to lemma 1, either type $LH$ or $LL$ has the minimal well-being. Figure B3 illustrates (3a), (3b(ii)) and $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$; type $HH$'s bundle is somewhere in the shaded zone. We measure well-being in $(c, y)$- rather than in $(c, \ell)$-space. Therefore, one has to divide the slopes of the shared resources budget line by the skill level of the individual under consideration, here a low skilled individual. As a consequence, it is easy to verify that type $LH$ is always strictly worse-off compared to type $LL$, irrespective of the proportion of high-skilled $\frac{n_{HL}+n_{HH}}{n}$ (which, multiplied with $L$, defines the kink in the budget set where the slope changes from $\frac{H}{L} > 1$ to 1) and irrespective of whether (3b(i)) or (3b(ii)) applies.

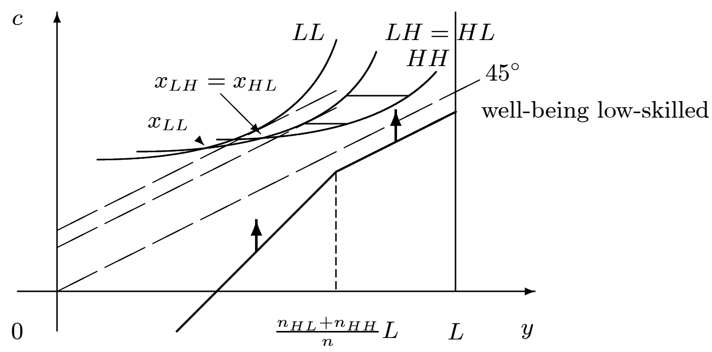



Figure B3: type $LL$ is strictly better off compared to type $LH$.

Since $IC_{LL,LH}$ does not bind, it is always possible to use TRICK 2 transferring a small amount of money from $LL$ to the other types $LH$, $HL$ and $HH$, improving the minimal well-being in society, a contradiction.

**4.** $IC_{LL,LH}$ and $IC_{LH,LL}$ do not bind. Using TRICK 1, it can be verified that only the following cases are possible: (i) $y^*_{LL} = 0 < y^*_{LH}$, $MRSY_{LL}(x^*_{LL}) \geq 1$

and $MRSY_{LH}(x^*_{LH}) \leq 1$, or (ii) $0 < y^*_{LL} < y^*_{LH}$, $MRSY_{LL}(x^*_{LL}) = 1$ and $MRSY_{LH}(x^*_{LH}) \leq 1$. We are back in the same situation as in (3). In both cases (i) and (ii) and given $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$, type $LH$ is strictly worse-off compared to $LL$ irrespective of the proportion of high-skilled. Here again, TRICK 2 can be used to obtain a contradiction.

**Proof of part (b)**. Suppose $y^*_{HL} > L$ (thus $V_{HL}(x^*_{LH}) = V_{HL}(x^*_{HL})$, with $y^*_{LH} = L < y^*_{HL}$ and $MRSY_{HL}(x^*_{HL}) \leq 1$ via lemma 2) and $y^*_{LL} - c^*_{LL} < y^*_{LH} - c^*_{LH}$. It is again possible to consider four cases, depending on whether $IC_{LL,LH}$ and/or $IC_{LH,LL}$ bind or not, and to show, for each case, a contradiction. Actually, the proof is completely analogous as in steps 1-4 of part (a) and therefore omitted.

# Proof of proposition 2

Consider a four type economy with skills $L = 0 < H$ and tastes represented by utility functions $U_L, U_H \in \mathcal{U}$, which satisfy single-crossingness and indistinguishable middle type. Consider a government who optimizes the program defined by $(*)$. In an optimal allocation $\mathbf{x}^* \in \mathcal{X}$ we must have $y^*_{LL} - c^*_{LL} = y^*_{LH} - c^*_{LH} \leq y^*_{HL} - c^*_{HL}$.

**Proof.** Suppose $y^*_{LL} - c^*_{LL} = y^*_{LH} - c^*_{LH} > y^*_{HL} - c^*_{HL}$ holds. **1.** Because $L = 0$ and $\mathbf{x}^* \in \mathcal{X}$, we must have $y^*_{LL} = y^*_{LH} = 0$ and, given the incentive constraints, also $c^*_{LL} = c^*_{LH}$ must hold. **2.** Due to lemma 1, the lowest well-being is either $LH$ or $LL$, thus, given (1), we must maximize the basic income, i.e., maximize $c^*_{LL} = c^*_{LH}$. **3.** $y^*_{HL} = 0$ is excluded, otherwise we must have $c^*_{LH} = c^*_{HL}$ (due to incentive constraints) and $y^*_{LH} - c^*_{LH} > y^*_{HL} - c^*_{HL}$ would be violated. **4.** So, $y^*_{HL} > 0$ holds from (3). Now, $x^*_{LH}$ and $x^*_{HL}$ must lie on the same indifference curve, or $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$. Otherwise (see the proof of lemma 2, part (b)) it would be possible to improve the situation of the worst-off types $LL$ and $LH$, at the cost of the better-off types $HL$ and $HH$ (on the basis of TRICK 2). **5.** If $MRSY_{HL}(x^*_{HL}) > 1$ at $y^*_{HL} > 0$, we can use TRICK 1, moving $x^*_{HL}$ to the left on his indifference curve. **6.** To summarize, we must have $MRSY_{HL}(x^*_{HL}) \leq 1$ and $y^*_{HL} > 0$ while $V_{HL}(x^*_{HL}) = V_{HL}(x^*_{LH})$ and $y^*_{LH} = 0$. But this contradicts $y^*_{LH} - c^*_{LH} > y^*_{HL} - c^*_{HL}$, given our preference technology defined by $\mathcal{U}$.

# Appendix C: The Belgian tax system for singles

| pre-tax income $y$ | marginal tax rate (in %) |
|---|---|
| $\leq$ €5032 | 0 |
| €5033 – €6272 | 25 |
| €6273 – €8304 | 30 |
| €8305 – €11849 | 40 |
| €11850 – €27268 | 45 |
| €27269 – €40902 | 50 |
| €40903 – €59990 | 52.5 |
| $>$ €59990 | 55 |

# Appendix D: Imputation via a sample selection model

First, we present the variables used for imputing gross hourly wages $\sigma$ and net hourly benefits $b_h$; afterwards, we show the estimates for both sample selection models.

## Imputing gross hourly wages $\sigma$

In the wage equation, the independent variables are:

- age and its square (*age*, *agesq*)
- educational dummies indicating the highest achieved education level of the individual, starting from primary education (base case), lower secondary education (*dumeduc2*), higher secondary education (*dumeduc3*), higher education short type (*dumeduc4*), higher education long type (*dumeduc5*)
- a gender dummy (*sex*) taking the value of 1 for females

In the selection equation, the independent variables are:

- physical health dummies indicating the general health situation of the individual, ranging from very good (base case), good (*dumhealth2*), reasonable (*dumhealth3*), to bad (*dumhealth4*)
- mental health dummies indicating how often the individual feels depressed, ranging from never (base case), seldom (*dumdepri2*), at times (*dumdepri3*), regularly (*dumdepri4*), to frequently (*dumdepri5*); and how often the individual longs for death, ranging from never (base case), seldom (*dumdeath2*), at times (*dumdeath3*), regularly (*dumdeath4*), to frequently (*dumdeath5*)
- smoking dummies indicating smoking behaviour, ranging from never (base case), occasionally (*dumsmoke2*), to daily (*dumsmoke3*)
- care dummies indicating whether the individual has to take care for his children (*child*) taking the value of 1 if affirmative; and/or has to take care of others (*depperson*) taking the value of 1 if affirmative
- the independent variables of the wage equation

## Imputing net hourly benefits $b_h$

The dependent variable is the marginal benefit per hour $b_h = \frac{b}{2925}$, with $b$ the net yearly benefit. The independent variables are identical to those in the Heckman selection model imputing $\sigma$. In addition, we add in both the benefit and the selection equation civil status dummies, indicating whether the individual is divorced (*divorce*), taking the value of 1 if affirmative; widowed (*widow*), taking the value of 1 if affirmative; living together (*cohabit*), taking the value of 1 if affirmative.

Number of observations = 644, from which 136 censored and 508 uncensored. Wald's $\chi^2(7) = 265.68$ with $\Pr > \chi^2(7) = 0.00$ and likelihood ratio test of independent wage and selection equations results in $\chi^2(1) = 4.66$ with $\Pr > \chi^2(1) = 0.031$.

| **wage equation** | Coefficient | Standard Error | $\Pr > \lvert z \rvert$ |
|---|---|---|---|
| age | 0.497 | 0.123 | 0.000 |
| agesq | -0.003 | 0.002 | 0.075 |
| dumeduc2 | 1.364 | 1.141 | 0.232 |
| dumeduc3 | 2.834 | 1.103 | 0.010 |
| dumeduc4 | 4.508 | 1.160 | 0.000 |
| dumeduc5 | 5.617 | 1.149 | 0.000 |
| sex | -0.917 | 0.383 | 0.017 |
| cons | -2.680 | 2.568 | 0.297 |
| **selection equation** | Coefficient | Standard Error | $\Pr > \lvert z \rvert$ |
| dumhealth2 | -0.576 | 0.176 | 0.001 |
| dumhealth3 | -0.686 | 0.227 | 0.002 |
| dumhealth4 | -1.221 | 0.476 | 0.010 |
| dumdepri2 | 0.014 | 0.186 | 0.938 |
| dumdepri3 | -0.243 | 0.185 | 0.188 |
| dumdepri4 | -0.486 | 0.249 | 0.051 |
| dumdepri5 | -0.799 | 0.342 | 0.019 |
| dumdeath2 | 0.374 | 0.173 | 0.031 |
| dumdeath3 | 0.113 | 0.196 | 0.563 |
| dumdeath4 | 0.160 | 0.297 | 0.591 |
| dumdeath5 | -0.657 | 0.352 | 0.062 |
| dumsmoke2 | -0.041 | 0.245 | 0.868 |
| dumsmoke3 | -0.219 | 0.138 | 0.114 |
| child | -0.355 | 0.153 | 0.020 |
| depperson | -0.232 | 0.210 | 0.269 |
| age | 0.179 | 0.040 | 0.000 |
| agesq | -0.002 | 0.001 | 0.000 |
| dumeduc2 | 0.587 | 0.254 | 0.021 |
| dumeduc3 | 0.784 | 0.243 | 0.001 |
| dumeduc4 | 1.584 | 0.305 | 0.000 |
| dumeduc5 | 1.685 | 0.291 | 0.000 |
| sex | -0.507 | 0.139 | 0.000 |
| cons | -2.016 | 0.728 | 0.006 |

Number of observations = 638, from which 480 censored and 158 uncensored. Wald's $\chi^2(10) = 50.51$ with $\Pr > \chi^2(7) = 0.00$ and likelihood ratio test of independent benefit and selection equations results in $\chi^2(1) = 7.09$ with $\Pr > \chi^2(1) = 0.008$.

| **benefit equation** | Coefficient | Standard Error | $\Pr > \lvert z \rvert$ |
|---|---|---|---|
| age | 0.115 | 0.051 | 0.025 |
| agesq | -0.001 | 0.001 | 0.120 |
| dumeduc2 | 0.361 | 0.259 | 0.163 |
| dumeduc3 | 0.470 | 0.271 | 0.083 |
| dumeduc4 | 0.181 | 0.358 | 0.613 |
| dumeduc5 | 0.575 | 0.410 | 0.161 |
| sex | -0.317 | 0.187 | 0.091 |
| divorce | 0.376 | 0.197 | 0.057 |
| widow | -0.504 | 0.582 | 0.386 |
| cohabit | -0.035 | 0.196 | 0.858 |
| cons | -0.226 | 0.921 | 0.806 |
| **selection equation** | Coefficient | Standard Error | $\Pr > \lvert z \rvert$ |
| dumhealth2 | 0.304 | 0.154 | 0.048 |
| dumhealth3 | 0.416 | 0.203 | 0.040 |
| dumhealth4 | 0.384 | 0.415 | 0.355 |
| dumdepri2 | 0.137 | 0.163 | 0.400 |
| dumdepri3 | 0.244 | 0.166 | 0.142 |
| dumdepri4 | 0.470 | 0.237 | 0.048 |
| dumdepri5 | 0.334 | 0.329 | 0.311 |
| dumdeath2 | -0.464 | 0.154 | 0.003 |
| dumdeath3 | 0.020 | 0.173 | 0.909 |
| dumdeath4 | 0.369 | 0.254 | 0.145 |
| dumdeath5 | 0.889 | 0.339 | 0.009 |
| dumsmoke2 | 0.114 | 0.214 | 0.596 |
| dumsmoke3 | 0.228 | 0.124 | 0.065 |
| child | 0.304 | 0.143 | 0.033 |
| depperson | 0.158 | 0.192 | 0.410 |
| age | -0.108 | 0.040 | 0.007 |
| agesq | 0.001 | 0.001 | 0.006 |
| dumeduc2 | -0.376 | 0.242 | 0.120 |
| dumeduc3 | -0.707 | 0.232 | 0.002 |
| dumeduc4 | -1.049 | 0.265 | 0.000 |
| dumeduc5 | -1.428 | 0.270 | 0.000 |
| sex | 0.373 | 0.128 | 0.004 |
| divorce | 0.027 | 0.162 | 0.868 |
| widow | 0.036 | 0.490 | 0.941 |
| cohabit | -0.329 | 0.144 | 0.022 |
| cons | 1.247 | 0.717 | 0.082 |

## Appendix E: Some descriptive statistics

We report the number of observations ($n$), the minimum (min), the 25$^{\text{th}}$ percentile (p25), the median (p50), the 75$^{\text{th}}$ percentile (p75) and the maximum (max) for the following variables: age, normalized labour $\ell$, observed gross hourly wages $\sigma$, imputed gross hourly wages $\hat{\sigma}$, observed and imputed gross hourly wages $(\sigma, \hat{\sigma})$, observed net hourly benefits $b_h$, imputed net hourly benefits $\hat{b}_h$ and observed and imputed net hourly benefits $(b_h, \hat{b}_h)$.

| | $n$ | min | p25 | p50 | p75 | max |
|---|---|---|---|---|---|---|
| age | 621 | 16 | 25 | 33 | 42 | 70 |
| $\ell$ | 621 | 0.000 | 0.231 | 0.554 | 0.615 | 1.000 |
| $\sigma$ | 502 | 1.495 | 9.585 | 11.864 | 15.700 | 38.271 |
| $\hat{\sigma}$ | 119 | 1.262 | 4.796 | 6.665 | 9.421 | 19.214 |
| $\sigma, \hat{\sigma}$ | 621 | 1.262 | 8.594 | 11.065 | 14.404 | 38.271 |
| $b_h$ | 143 | 0.166 | 0.915 | 1.849 | 3.051 | 3.661 |
| $\hat{b}_h$ | 478 | 0.002 | 0.072 | 0.214 | 0.510 | 2.089 |
| $b_h, \hat{b}_h$ | 621 | 0.002 | 0.115 | 0.345 | 0.928 | 3.661 |

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# Minimal rights based solidarity

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**Abstract**

In a model where individuals with different levels of skills exert different levels of effort, we propose to use individuals' minimal rights to divide an extra amount of income generated by a change in the skill profile. Priority is given to individuals with a positive minimal right. We characterize two families of Minimal Rights based Egalitarian mechanisms that implement this solidarity idea. One family guarantees each individual her claim when claims are feasible. The other family guarantees a non-negative income after redistribution for all individuals.

JEL Classification: D63.
Keywords: minimal rights, solidarity, compensation, claims.

## 1 Motivation

Suppose income inequalities are determined by unequal exerted effort levels and different innate skills. The goal of fair income redistribution is to guarantee an equal income for individuals exerting the same effort and to perform equal income transfers to individuals with equal skills. However, in many contexts, there does not exist a redistribution mechanism that simultaneously satisfies both requirements. We refer to Fleurbaey and Maniquet (2004) for an extensive survey of this compensation problem. Weakening one of both requirements leads to the proposition of different (families of) redistribution mechanisms. Many of these redistribution mechanisms rely on so called 'reference' income levels. Typically, these reference income levels are computed by replacing either skill levels or effort levels by some reference value. Within one family of redistribution mechanisms, different mechanisms are distinguished by different choices of the reference value. The literature on fair income redistribution has some strong similarities with the literature on bankruptcy problems and surplus sharing problems. This becomes clear when reference income levels are interpreted as

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claims. In a bankruptcy problem, a fixed amount of money must be allocated on the basis of monetary claims that sum up to more than can be divided. The objective is to design allocation mechanisms that associate with each claims problem a division of the amount available over the claimants. We refer to Thomson (2003) for an extensive survey of the literature on competing claims problems. In a surplus sharing problem, an amount of money that exceeds the total sum of claims must be divided over all claimants. An extensive survey on surplus sharing problems is Moulin (2002). Fair income redistribution problems can be interpreted as competing claims (surplus sharing) problems where the total sum of income before redistribution has to be divided over a population of claimants that have reference incomes as claims.

In this paper we focus on two families of fair income redistribution mechanisms: the family of Egalitarian Equivalent mechanisms due to Pazner and Schmeidler (1978) and Fleurbaey (1995) and the family of Proportionally Adjusted Equivalent mechanisms due to Iturbe (1997). These families are characterized in Fleurbaey and Maniquet (2004) using different strenghtenings of a compensation axiom called 'Solidarity', in combination with a weak responsibility axiom called 'Equal Transfer for Reference Skill'. The solidarity axioms describe solutions to 'the solidarity problem'. The solidarity problem is a competing claims/surplus sharing problem where a change in the skill profile generates an extra amount (or a loss) of pre-tax income that has to be divided over (taken away from) the population of claimants. Under an Egalitarian Equivalent mechanism these extra resources are divided equally and hence differences in exerted effort levels are not taken into account. Under a Proportionally Adjusted Equivalent mechanism differences in exerted effort levels are taken into account by dividing extra resources proportionally to claims. We propose to divide extra resources on the basis of the information contained in individuals' minimal rights. The minimal right of an individual equals the amount that remains from the total sum of pre-tax income when all other individuals receive their claim. Priority is given to individuals with a higher claim, which is due to higher exerted effort, once the total sum of pre-tax income exceeds threshold levels where individuals' minimal rights become positive. We characterize two families of Minimal Rights based Egalitarian mechanisms that implement this solidarity idea. One family also satisfies 'Equal Transfer for Reference Skill', while the other family guarantees a non-negative income after redistribution for all individuals.

In the next section we present the model, state the compensation problem, define the different families of fair income redistribution mechanisms, discuss their characterizations and illustrate the income distributions that result from these families. In section 3 we introduce the notion of minimal rights and propose to use minimal rights to solve the solidarity problem. We state the solidarity axioms of 'Symmetry' and 'Priority' that together constitute the central solidarity idea of this paper, Minimal Rights based Solidarity. We then characterize two families of Minimal Rights based Egalitarian mechanisms. For a given reference skill, we show that the family that guarantees all individuals a non-negative income redistributes income more equally than the family that satisfies 'Equal

Transfer for Reference Skill'. This latter family is equivalent with the family of Egalitarian Equivalent mechanisms in rich economies where all minimal rights are positive, but redistributes income more equally in poorer economies where some or all minimal rights are zero. We show that the axioms used in the characterizations are independent. Section 4 summarizes our main conclusions.

# 2 Fair monetary compensation

## 2.1 The model

The fair monetary compensation model used in this paper is a one-dimensional version of the quasi-linear model in Fleurbaey and Maniquet (2004) which is due to Bossert (1995). Denote $N = \{1, \ldots, n\}$ the finite population of size $n \geq 2$. Let $x \in \mathbb{R}$ be an amount of transferable resource. The characteristic which elicits compensation, hereafter called 'skill', is $y \in Y$ and $Y$ is an interval of $\mathbb{R}$. Denote $y_N = (y_1, \ldots, y_n)$ the skill profile in the population. The characteristic which does not elicit compensation, hereafter called 'effort', is $z \in Z$ and $Z$ is an interval of $\mathbb{R}$. Effort is not influenced by redistribution as incentive issues are not taken up in the model. Denote $z_N = (z_1, \ldots, z_n)$ the effort profile in the population. Without loss of generality, we assume that individuals are ranked such that $z_1 \geq z_2 \geq \ldots \geq z_n$. An economy $e = (y_N, z_N)$ is the pair of characteristics' profiles. Denote $\mathcal{E}$ the set of economies.

We assume that utility functions are quasi-linear as follows:

$$u_i(x_i, y_i) = x_i + v(y_i, z_i).$$

In the context of this paper $u_i(x_i, y_i)$ measures a monetary outcome, namely final income after redistribution. The function $v : Y \times Z \to \mathbb{R}_{++} : (y_i, z_i) \to v(y_i, z_i)$ describes the pre-tax income function. We assume that $v$ is continuous and strictly increasing in $y$ and $z$. Furthermore, we assume that $v$ is not additively separable in $y$ and $z$, i.e. $v(y_i, z_i)$ cannot be written as $v_1(y_i) + v_2(z_i)$. Denote $R = \sum_{i \in N} v(y_i, z_i)$ the total sum of pre-tax income.

Let the transferable resource $x_i$ be an element of an allocation $x_N = (x_1, \ldots, x_n) \in \mathbb{R}^n$. We assume that the total amount to be distributed is 0, such that we are looking at a pure redistribution problem. An allocation for the economy $e \in \mathcal{E}$ is feasible when $\sum_{i \in N} x_i = 0$. The set of feasible allocations is denoted $F$. Notice that all feasible allocations are Pareto efficient since we ruled out free disposal in the definition of feasibility. The function $S : \mathcal{E} \to F : e \to S(e)$ is an allocation mechanism. Denote $\mathcal{S}$ the set of all allocation mechanisms.

Let $\tilde{y} \in Y$ be the reference skill. We assume throughout the paper that this constant parameter is exogenously determined by the social planner. Denote $v(\tilde{y}, z_i)$ the claim of individual $i$. It equals the pre-tax income that an individual would receive when exerting her effort level $z_i$ but having skill $\tilde{y}$ instead of her own skill $y_i$. We bring all claims together in a vector $\nu$ and denote $C =$

$\sum_{i \in N} v(\tilde{y}, z_i)$ the total sum of claims . Define the interval $Y_{cc} = \{\tilde{y} \in Y : C \geq R\}$. For parameter values of $\tilde{y}$ in $Y_{cc}$, the total sum of claims is at least as high as the total sum of pre-tax income (which can be redistributed) and a competing claims problem arises. A competing claims problem is a pair $(\nu, R) \in \mathbb{R}^n_{++} \times \mathbb{R}_{++}$, such that $C \geq R$. Define the interval $Y_{ss} = \{\tilde{y} \in Y : C < R\}$. For parameter values of $\tilde{y}$ in $Y_{ss}$, the total sum of claims is not as high as the total sum of pre-tax income and a surplus sharing problem arises. A surplus sharing problem is a pair $(\nu, R) \in \mathbb{R}^n_{++} \times \mathbb{R}_{++}$, such that $C < R$.

## 2.2 The compensation problem

We state the two key axioms that express the ethical goal of fair income redistribution, namely neutralizing income inequalities due to $y$ while preserving income inequalities due to $z$.

The first axiom states that when two individuals only differ in skill levels, both should receive the same income after redistribution. An allocation mechanism $S$ satisfies '**Equal Income for Equal Effort**' ($EIEE$, Fleurbaey (1994)) if:

$\forall e \in \mathcal{E}, x_N \in S(e), \forall i, j \in N,$

$$z_i = z_j \Rightarrow x_i + v(y_i, z_i) = x_j + v(y_j, z_j).$$

Denote $\mathcal{S}_{EIEE}$ the set of all allocation mechanisms that satisfy $EIEE$.

The second axiom states that when two individuals only differ in exerted effort levels, they should be equally affected by the performed redistribution. An allocation mechanism $S$ satisfies '**Equal Transfer for Equal Skill**' ($ETES$, Fleurbaey (1994)) if:

$\forall e \in \mathcal{E}, x_N \in S(e), \forall i, j \in N,$

$$y_i = y_j \Rightarrow x_i = x_j.$$

Denote $\mathcal{S}_{ETES}$ the set of all allocation mechanisms that satisfy $ETES$.

The compensation problem states that there does not exist an allocation mechanism that satisfies $EIEE$ and $ETES$, i.e. $\mathcal{S}_{EIEE} \cap \mathcal{S}_{ETES} = \emptyset$. We refer to Fleurbaey and Maniquet (2004) for a proof.

## 2.3 Allocation mechanisms

Weakening $ETES$ or $EIEE$ leads to the proposal of a number of interesting allocation mechanisms. Two mechanisms that belong to $\mathcal{S}_{EIEE}$ (and thus weaken $ETES$) play an important role in this paper. We define them here. It concerns a) the family of Egalitarian Equivalent mechanisms due to Pazner and Schmeidler (1978) and Fleurbaey (1995) and b) the family of Proportionally Adjusted Equivalent mechanisms due to Iturbe (1997). Denote $(x_i)_S$ the income transfer for an individual $i$ from an allocation $(x_N)_S \in S(e)$ of an allocation mechanism $S$.

a) An $\tilde{y}$-Egalitarian Equivalent mechanism ($S_{\tilde{y}EE}$) allocates resources as follows:

$\forall e \in \mathcal{E}, \forall i \in N,$

$$(x_i)_{S_{\tilde{y}EE}} = -v(y_i, z_i) + v(\tilde{y}, z_i) + \tfrac{1}{n}(R - C).$$

b) An $\tilde{y}$-Proportionally Adjusted Equivalent mechanism ($S_{\tilde{y}PAE}$) allocates resources as follows:

$\forall e \in \mathcal{E}, \forall i \in N,$

$$(x_i)_{S_{\tilde{y}PAE}} = -v(y_i, z_i) + \tfrac{R}{C} v(\tilde{y}, z_i).$$

## 2.4 The characterizations of $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$

### 2.4.1 ETRS

The axiom of $ETES$ is weakened to only apply to economies where all skills are equal to the reference skill. In these economies no redistribution is performed. An allocation mechanism $S$ satisfies '**Equal Transfer for Reference Skill**' ($ETRS$, Fleurbaey (1995)) if:

$\forall e \in \mathcal{E}, x_N \in S(e),$
$[y_i = \tilde{y} \;\; \forall i \in N] \Rightarrow [x_i = 0 \;\; \forall i \in N].$

Denote $\mathcal{S}_{ETRS}$ the set of all allocation mechanisms that satisfy $ETRS$. It is easy to check that $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$ belong to $\mathcal{S}_{ETRS}$.

### 2.4.2 Solidarity axioms

Solidarity axioms consider the effect of a change in one individual's skill on the allocation. Consider two skill profiles $y_N = (y_1, \ldots, y_k, \ldots, y_n)$ and $y'_N = (y'_1, \ldots, y'_k, \ldots, y'_n)$, where, for all $j$ in $N \backslash \{k\}$, $y_j$ equals $y'_j$. Let $e' = (y'_N, z_N)$ and $R' = \sum_{i \in N} v(y'_i, z_i)$. Denote the change in total pre-tax income $\Delta_R = R' - R$. Without loss of generality, we assume throughout the paper that $e'$ yields more pre-tax income than $e$ and hence $\Delta_R \in \mathbb{R}_{++}$. The solidarity problem is how $\Delta_R$ should be divided over the population. Note that, as $\tilde{y}$ is constant, a change in the skill profile does not alter individuals' claims, i.e. $\nu$ equals $\nu'$.

The axiom of $EIEE$ is strengthened to an axiom that says that a change in the skill profile should affect all agents' final incomes in the same direction.[1] An allocation mechanism $S$ satisfies '**Solidarity**' (Fleurbaey and Maniquet (2004)) if:

[1] We refer to Fleurbaey and Maniquet (2004) for a proof.

$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e')$,

$$[x'_i + v(y'_i, z_i) \geq x_i + v(y_i, z_i) \ \ \forall i \in N].$$

The axiom of Solidarity is easily defensible. As differences in the skill profile elicit compensation, it is clear that changes in the skill profile should not make some individuals gain income while others lose income.

### 2.4.3 $S_{\tilde{y}EE}$

Solidarity can be strengthened by stating that all incomes should change equally due to a change in the skill profile. An allocation mechanism $S$ satisfies '**Additive Solidarity**' ($AS$, Bossert (1995)) if:

$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e')$,

$$\left[x'_i + v(y'_i, z_i) - (x_i + v(y_i, z_i)) = x'_j + v(y'_j, z_j) - (x_j + v(y_j, z_j)) \ \ \forall i, j \in N\right].$$

An allocation mechanism $S$ satisfies $ETRS$ and $AS$ if and only if it is an $S_{\tilde{y}EE}$. We refer to Bossert and Fleurbaey (1996) for a proof.

When an $S_{\tilde{y}EE}$ is implemented and the skill profile changes, $\Delta_R$ is divided equally over all individuals. However, effort also determines $\Delta_R$. Therefore, $S_{\tilde{y}EE}$ can be criticized for not taking differences in exerted effort into account when dividing $\Delta_R$.

### 2.4.4 $S_{\tilde{y}PAE}$

Alternatively, Solidarity can be strengthened by requiring that all individuals' outcomes change proportionally. An allocation mechanism $S$ satisfies '**Multiplicative Solidarity**' ($MS$, Iturbe (1997)) if:

$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e')$,

$$\left[(x'_i + v(y'_i, z_i))(x_j + v(y_j, z_j)) = (x_i + v(y_i, z_i))(x'_j + v(y'_j, z_j)) \ \ \forall i, j \in N\right].$$

An allocation mechanism $S$ satisfies $ETRS$ and $MS$ if and only if it is an $S_{\tilde{y}PAE}$. We refer to Iturbe (1997) for a proof.

When an $S_{\tilde{y}PAE}$ is implemented and the skill profile changes, differences in effort are taken into account by dividing $\Delta_R$ proportionally to individuals' claims. Indeed, the ratio of changes in individuals' incomes equals the ratio of their respective claims, i.e. $\frac{x'_i + v(y'_i, z_i) - (x_i + v(y_i, z_i))}{x'_j + v(y'_j, z_j) - (x_j + v(y_j, z_j))} = \frac{v(\tilde{y}, z_i)}{v(\tilde{y}, z_j)}$ for all $i, j$ in $N$.[2]

In this paper we propose to reward effort in a different way by giving, in the division of $\Delta_R$, priority to the highest claims once the total sum of pre-tax income exceeds a particular threshold level to be explained in section 3.

[2] We suppose that the denominator at the left hand side is different from zero. As such $MS$ is a strengthening of a strict version of Solidarity with a strict inequality sign in the definition.

## 2.5 Income distributions under $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$

Figure 1 illustrates for every value of $R$ the income distributions under an $S_{\tilde{y}EE}$ and an $S_{\tilde{y}PAE}$ for an economy with four individuals whose claims are in a ratio of 6:4:2:1. As both redistribution mechanisms satisfy the axiom of $ETRS$, income is redistributed such that every individual receives her claim when $R$ equals $C$. As a consequence of their respective solidarity axioms, when $R$ changes, the absolute income inequality remains constant under $S_{\tilde{y}EE}$ (full line), while the relative income inequality remains constant under $S_{\tilde{y}PAE}$ (dotted line).

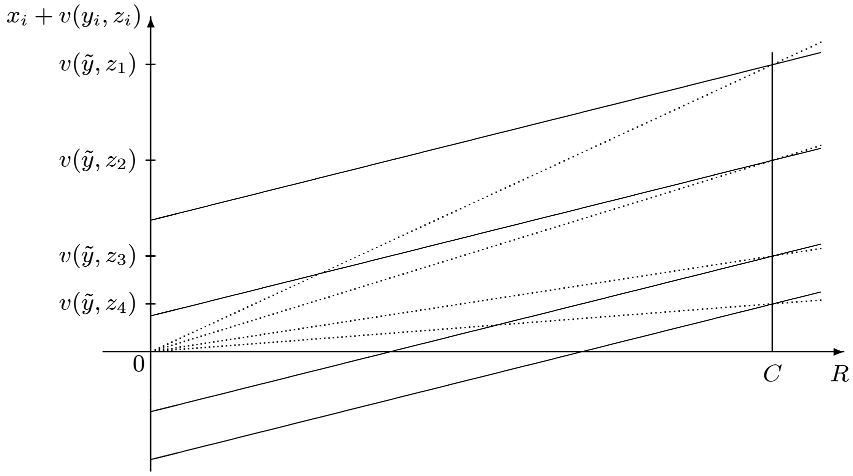


Figure 1: Income distributions under $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$

# 3 Minimal rights based solidarity

## 3.1 Minimal rights

Rather than to base the division of $\Delta_R$ on individuals' claims, we propose to use individuals' *minimal rights*, a concept often used in the competing claims literature originating from seminal contributions of O'Neill (1982) and Aumann and Maschler (1985).[3]

The minimal right of an individual equals the amount that remains from the total sum of pre-tax income when all other claimants have received their claim. However, the minimal right is not allowed to be negative or to exceed the individual's own claim. Formally, we define the minimal right of an individual $i$ as follows:

[3]Minimal rights should not be confused with the concept of *equal rights* introduced in Maniquet (1998). In a model with production, an allocation mechanism guarantees an equal right when every individual weakly prefers her bundle over her best choice from a common opportunity set.

$$m_i\,(\nu, R) = \min\,(v(\tilde{y}, z_i), \max\,(R - C_{-i}, 0)),$$

where

$$C_{-i} = \textstyle\sum_{j \in N \setminus \{i\}} v(\tilde{y}, z_j) \text{ for all } i \text{ in } N.$$

Figure 2 shows that as long as $R$ is smaller than $C_{-1}$, all minimal rights are zero. As soon as $R$ exceeds $C_{-1}$, the minimal right of the individual with effort level $z_1$ becomes positive. As soon as $R$ exceeds $C_{-2}$, the minimal right of the individual with effort level $z_1$ exceeds $C_{-2} - C_{-1}$ and the minimal right of the individual with effort level $z_2$ becomes positive. As $R$ increases, more and more individuals start to get a positive minimal right and as soon as $R$ exceeds $C_{-n}$ all minimal rights are positive. When $R$ equals $C$ (and thus claims are feasible) every individual has a minimal right equal to her claim. When $R$ exceeds $C$ minimal rights do not change anymore, so in all surplus sharing problems $m_i\,(\nu, R)$ equals $v(\tilde{y}, z_i)$ for all $i$ in $N$.

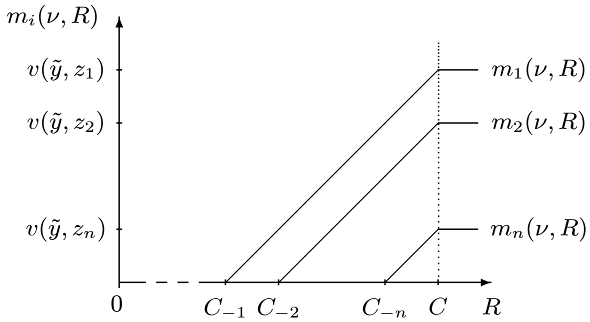


Figure 2: Aggregate resources, minimal rights and claims

Minimal rights could be given a 'democratic' interpretation. For a given $\tilde{y}$, suppose that $R$ equals $C_{-2}$. Then all individuals in society agree that the individual with effort level $z_1$ deserves at least her minimal right, i.e. $C_{-2} - C_{-1}$, of $R$. Suppose that $R$ equals $C_{-3}$. Then there is agreement that the amount $C_{-2} - C_{-1}$ should only go to the individual with effort level $z_1$, while the amount $C_{-3} - C_{-2}$ should be divided only over the individual with effort level $z_1$ and the individual with effort level $z_2$. Since both individuals deserve the amount $C_{-3} - C_{-2}$, it seems natural to divide the amount $C_{-3} - C_{-2}$ equally between both individuals.

## 3.2 Solidarity axioms

We exploit these ideas in the statement of our solidarity axioms. When the skill profile changes, it is clear that none, some or all minimal rights may change.

Denote $\Delta m_i(\nu, R, \Delta_R) = m_i(\nu, R + \Delta_R) - m_i(\nu, R)$ the change in the minimal right of individual $i$ due to a change of total pre-tax income equal to $\Delta_R$. These changes in minimal rights could be informative for the division of $\Delta_R$. Suppose that $R$ and $R'$ are smaller than $C_{-1}$. Since all minimal rights are zero before and after the change in the skill profile, it seems reasonable to divide $\Delta_R$ equally over all individuals. Suppose now that $R'$ further increases to $C_{-2}$. Then priority could be given to the individual with effort level $z_1$ who receives $C_{-2} - C_{-1}$ —an amount equal to the change in her minimal right— while the rest, $C_{-1} - R$, is divided equally over all individuals. Suppose now that $R'$ further increases to $C_{-3}$. Then the amount $C_{-1} - R$ could be divided equally over all individuals, because nobody has a positive minimal right up to income level $C_{-1}$. Then priority could be given to the individual with effort level $z_1$ who additionally receives $C_{-2} - C_{-1}$ because there is no other individual with a positive minimal right from income level $C_{-1}$ up to income level $C_{-2}$. Finally, the amount $C_{-3} - C_{-2}$ could be divided equally between the individual with effort level $z_1$ and the individual with effort level $z_2$ because they are the only individuals with a positive minimal right from income level $C_{-2}$ up to income level $C_{-3}$. This iterative process can be continued as $R'$ increases until $R'$ exceeds $C_{-n}$. As all individuals have a positive minimal right after the income level $C_{-n}$ and their minimal rights change equally between $C_{-n}$ and $R'$, it seems reasonable to divide $R' - C_{-n}$ again equally over all individuals. The following axioms express these ideas.

The first axiom weakens Additive Solidarity by requiring an equal treatment in the allocation of $\Delta_R$ only when minimal rights change equally. A redistribution mechanism $S$ satisfies '**Symmetry**' if:[4]

$$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e'), \forall i, j \in N,$$
$$\Delta m_i(\nu, R, \Delta_R) = \Delta m_j(\nu, R, \Delta_R)$$
$$\Rightarrow x'_i + v(y'_i, z_i) - (x_i + v(y_i, z_i)) = x'_j + v(y'_j, z_j) - (x_j + v(y_j, z_j)).$$

The second axiom is inspired by the idea that individuals whose minimal rights change should be given priority over individuals whose minimal rights do not change in the allocation of $\Delta_R$ *once* not all minimal rights are equal to zero. Denote $N_1 = \{i \in N | \Delta m_i(\nu, R, \Delta_R) > 0\}$ the set of individuals whose minimal rights change due to a change of $R$. A redistribution mechanism $S$ satisfies '**Priority**' if:

$$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e'),$$
$$\text{if there exists } i \in N_1,\ \Delta m_i(\nu, R, \Delta_R) = \Delta_R :$$
$$\textstyle\sum_{i \in N_1} (x'_i + v(y'_i, z_i) - x_i - v(y_i, z_i)) = \Delta_R.$$

[4]Note that, as minimal rights do not change anymore in surplus sharing problems, $ETRS$ and Symmetry characterize the $\tilde{y}$-Egalitarian Equivalent mechanism for $\tilde{y} \in Y_{ss}$ (see 2.4.3). In the surplus sharing literature, this mechanism is better known as the Equal Surplus Sharing mechanism: every claimant receives her claim plus an equal share of the surplus.

Symmetry and Priority together express the paper's central solidarity idea of *Minimal Rights based Solidarity*, i.e. it explains how $\Delta_R$ should be divided over the population. When all minimal rights change equally, Symmetry implies that $\Delta_R$ is divided equally. As the pre-tax income function $v$ is continuous and strictly increasing in $y$, there exist unique intermediary skill levels such that, when changes in minimal rights differ, $\Delta_R$ can be divided in specific subchanges. For each of these subchanges of $\Delta_R$, the population is partitioned in two groups: $(i)$ a group whose change in minimal rights equals the subchange of $\Delta_R$ and $(ii)$ a group whose minimal rights do not change. Symmetry implies that within each group all individuals are treated in the same way, whereas Priority requires that the first group receives the subchange of $\Delta_R$. Hence, the subchange of $\Delta_R$ is equally divided among the individuals of the first group. We state Minimal Rights based Solidarity formally in appendix.

## 3.3 Minimal Rights based Egalitarian mechanisms

In the previous subsection Minimal Rights based Solidarity described how to divide $\Delta_R$. We call fair income redistribution mechanisms that satisfy Symmetry and Priority *Minimal Rights based Egalitarian mechanisms.* In order to characterize one particular family of Minimal Rights based Egalitarian mechanisms, it suffices to combine the solidarity axioms of Symmetry and Priority with an axiom that for one specific $R$ implies one specific income distribution.

### 3.3.1 $S_{\tilde{y}MRE/E}$

The axiom of $ETRS$ states that when $R$ equals $C$ every individual should receive her claim. Combining Symmetry, Priority and $ETRS$ characterizes the following mechanism.

An $\tilde{y}$-Minimal Rights based Egalitarian mechanism $S_{\tilde{y}MRE/E}$ allocates resources as follows:

$\forall e \in \mathcal{E}$,

$(\mathbf{1})$ when $C_{-n} \leq R$ :

$$(x_i)_{S_{\tilde{y}MRE/E}} = -v(y_i, z_i) + v(\tilde{y}, z_i) + \tfrac{R-C}{n} \text{ for all } i \text{ in } N,$$

$(\mathbf{2})$ when, for $k \leq n-1$, $C_{-k} \leq R < C_{-(k+1)}$ :

$$(x_i)_{S_{\tilde{y}MRE/E}} = -v(y_i, z_i)+v(\tilde{y}, z_i)+\tfrac{C_{-n}-C}{n}-\sum_{h=k+1}^{n-1}\left(\tfrac{C_{-(h+1)}-C_{-h}}{h}\right)+\tfrac{R-C_{-(k+1)}}{k}$$

for all $i \in \{1, \ldots, k\}$ and

$$(x_j)_{S_{\tilde{y}MRE/E}} = -v(y_j, z_j) + v(\tilde{y}, z_j) + \tfrac{C_{-n}-C}{n} - \sum_{h=j}^{n-1}\left(\tfrac{C_{-(h+1)}-C_{-h}}{h}\right)$$

for all $j \in \{k+1, \ldots, n-1\}$ and

$(x_n)_{S_{\tilde{y}MRE/E}} = -v(y_n, z_n) + v(\tilde{y}, z_n) + \frac{C_{-n}-C}{n}$,

(**3**) when $R < C_{-1}$ :

$(x_i)_{S_{\tilde{y}MRE/E}} = -v(y_i, z_i) + v(\tilde{y}, z_i) + \frac{C_{-n}-C}{n} - \sum_{h=i}^{n-1} \left( \frac{C_{-(h+1)}-C_{-h}}{h} \right) + \frac{R-C_{-1}}{n}$

for all $i \in \{1, \ldots, n-1\}$ and

$(x_n)_{S_{\tilde{y}MRE/E}} = -v(y_n, z_n) + v(\tilde{y}, z_n) + \frac{C_{-n}-C}{n} + \frac{R-C_{-1}}{n}$.

**Proposition 1** *: $\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/E} \Leftrightarrow S$ satisfies Symmetry, Priority and ETRS.*

The proof of proposition 1 can be found in appendix.

Figure 3 depicts for every value of $R$ the income distributions under an $S_{\tilde{y}MRE/E}$ for the same economy as in figure 1.

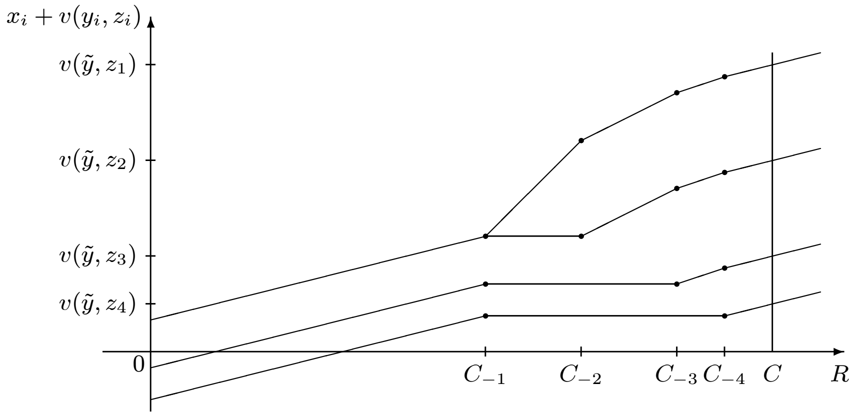


Figure 3: Income distributions under $S_{\tilde{y}MRE/E}$

As $S_{\tilde{y}MRE/E}$ also satisfies $ETRS$, income is redistributed such that every individual receives her claim when $R$ equals $C$. When, due to a change in the skill profile, $R$ becomes higher than $C$, every individual receives her claim plus an equal part of $R - C$. When $R$ becomes lower than $C$, every individual receives an income that is lower than her claim, but the shortfall from the claim is never lower for individuals with a higher claim. As long as $R$ is higher than $C_{-4}$, the loss of total pre-tax income is equally borne by all individuals. But, when $R$ falls below $C_{-4}$, the income of the poorest individual is left constant, which brings about an extra loss of income for all other individuals. When $R$ becomes smaller than $C_{-3}$, the incomes of the poorest and second poorest individuals remain constant and, when $R$ falls below $C_{-2}$, the richest individual alone is saddled with the entire cost of keeping the incomes of all other individuals constant. When $R$ becomes smaller than $C_{-1}$, the loss of total pre-tax income is again borne equally by all individuals.

Some general conclusions can be drawn from comparing figures 1 and 3. We say that, in the comparison of two income distributions $A$ and $B$ with the same mean, $A$ is more equal than $B$ when $A$ is obtained from $B$ by performing a series of (Pigou-Dalton) rich-to-poor transfers that do not entail rank reversals.

- In economies where every individual has a strictly positive minimal right ($C_{-n} < R$), an $S_{\tilde{y}MRE/E}$ redistributes income just like an $S_{\tilde{y}EE}$. Both mechanisms redistribute income more equally than an $S_{\tilde{y}PAE}$ as soon as $R$ is larger than $C$.

- In economies where some but not all minimal rights are strictly positive ($C_{-1} < R \leq C_{-n}$), the income distribution under an $S_{\tilde{y}MRE/E}$ is more equal than the income distribution under an $S_{\tilde{y}EE}$.

- In economies where all minimal rights are zero ($R \leq C_{-1}$), absolute income inequalities remain constant under an $S_{\tilde{y}MRE/E}$ and under an $S_{\tilde{y}EE}$ when $R$ changes, but incomes are more equally distributed under the former mechanism. Note that the incomes of the individuals with the highest and second highest effort level coincide under an $S_{\tilde{y}MRE/E}$ in these economies.

### 3.3.2 $S_{\tilde{y}MRE/P}$

Figures 1 and 3 show a debatable property of $S_{\tilde{y}EE}$ and $S_{\tilde{y}MRE/E}$: the poorest individuals might end up with a negative income after redistribution in poor societies (i.e. when $R$ is sufficiently low). Our ethical intuition may lead us to consider a minimal amount of redistribution that we at least want to perform. Suppose that the poorest in society could not satisfy their basic needs when they receive a negative income after redistribution. Society wants to exclude this possibility in every situation by incorporating the requirement of a non-negative income after redistribution for all individuals in the construction of the redistribution mechanism. A redistribution mechanism $S$ satisfies '**Participation**' (Maniquet (1998)) if:

$\forall e \in \mathcal{E}, x_N \in S(e),$

$$x_i + v(y_i, z_i) \geq 0 \quad \forall i \in N.$$

An implication of Participation is that, when $R$ converges to zero, all incomes should also converge to zero. Combining Participation with the solidarity axioms of Symmetry and Priority characterizes the following mechanism.

An $\tilde{y}$-Minimal Rights based Egalitarian mechanism $S_{\tilde{y}MRE/P}$ allocates resources as follows:

$\forall e \in \mathcal{E}$,

(**1**) when $R < C_{-1}$ :

$$(x_i)_{S_{\tilde{y}MRE/P}} = -v(y_i, z_i) + \frac{R}{n} \text{ for all } i \text{ in } N,$$

(**2**) when, for $k \leq n-1$, $C_{-k} \leq R < C_{-(k+1)}$ :

$$(x_i)_{S_{\tilde{y}MRE/P}} = -v(y_i, z_i) + \frac{C_{-1}}{n} + \sum_{h=i}^{k} \left( \frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R - C_{-(k+1)}}{k}$$

for all $i \in \{1, \ldots, k\}$ and

$$(x_j)_{S_{\tilde{y}MRE/P}} = -v(y_j, z_j) + \frac{C_{-1}}{n} \text{ for all } j \in \{k+1, \ldots, n\},$$

(**3**) when $C_{-n} \leq R$ :

$$(x_i)_{S_{\tilde{y}MRE/P}} = -v(y_i, z_i) + \frac{C_{-1}}{n} + \sum_{h=i}^{n-1} \left( \frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R - C_{-n}}{n}$$

for all $i \in \{1, \ldots, n-1\}$ and

$$(x_n)_{S_{\tilde{y}MRE}} = -v(y_n, z_n) + \frac{C_{-1}}{n} + \frac{R - C_{-n}}{n}.$$

**Proposition 2** *: $\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/P} \Leftrightarrow S$ satisfies Symmetry, Priority and Participation.*

The proof of proposition 2 can be found in appendix.

Figure 4 illustrates, for the same economy as in figures 1 and 3, the income distributions under $S_{\tilde{y}MRE/P}$.

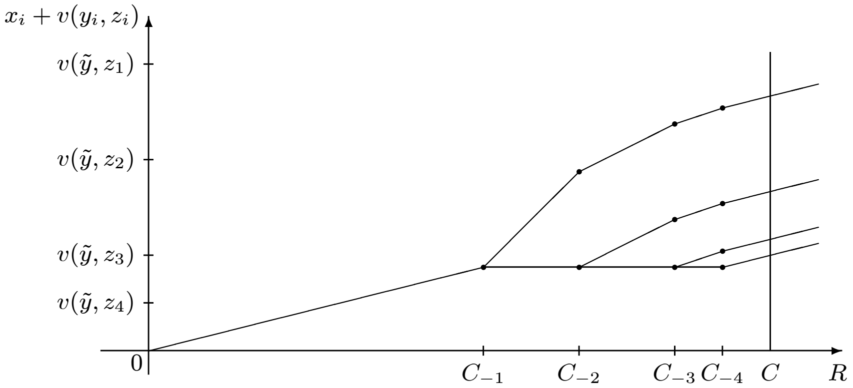


Figure 4: Income distributions under $S_{\tilde{y}MRE/P}$

An $S_{\tilde{y}MRE/P}$ redistributes incomes very equally. It is easy to check that, for every value of $R$, the income distribution under $S_{\tilde{y}MRE/P}$ is more equal than the income distribution under $S_{\tilde{y}MRE/E}$. An equal distribution of income prevails as long as all minimal rights are zero. More generally, when $R$ is lower than or equal to $C_{-i}$ for some $i$ in $N$, all individuals with $z$ lower than or equal to $z_i$ receive the same income. However, one could argue that too much redistribution is performed. Figure 4 shows that when all individuals have $y$ equal to $\tilde{y}$ and hence only differ with respect to their responsibility characteristic $z$ (such that no redistribution is needed), Pigou-Dalton transfers are still performed. As $S_{\tilde{y}MRE/E}$ and $S_{\tilde{y}MRE/P}$ are different mechanisms, Participation is incompatible with $ETRS$ when Symmetry and Priority are imposed.

### 3.3.3 Discussion

We end this section by discussing that the incompatibility between Participation and $ETRS$ when Symmetry and Priority are imposed, is due to Symmetry rather than Priority. We also show that the axioms used in propositions 1 and 2 are independent.

When Priority is dropped, imposing Participation, Symmetry and $ETRS$ still leads to an incompatibility, except when the lowest $n-1$ responsibility characteristics are equal. This is most easily explained as follows. Start from an income distribution $(0, 0, \ldots, 0)$ when $R$ converges to zero (Participation). Now suppose $R'$ equals $C$. Then $ETRS$ requires that every individual receives her claim. Hence, the individual with effort level $z_n$ should in that case receive $v(\tilde{y}, z_n)$. Symmetry requires that the subchanges of $\Delta_R$ for which all minimal rights change equally, i.e. $C_{-1}$ and $C - C_{-n}$ $(= v(\tilde{y}, z_n))$, are divided equally over the entire population. Given our assumptions about the pre-tax income function $v$, the condition $\frac{1}{n}C_{-1} + \frac{1}{n}v(\tilde{y}, z_n) = v(\tilde{y}, z_n)$ can only hold when $z_2 = z_3 = \ldots = z_n$. Note the ineffectiveness of restricting the range of choices of $\tilde{y}$ the social planner can make, as the incompatibility between Symmetry, Participation and $ETRS$ holds for every value of $\tilde{y}$.

When Symmetry is dropped, Participation, Priority and $ETRS$ are compatible but do not characterize a unique redistribution mechanism. Figure 5 illustrates two different mechanisms that satisfy Participation, Priority and $ETRS$. When $R$ falls below $C$ both mechanisms first reduce the income of the poorest individual to zero. But, as soon as $R$ falls below $C_{-1}$, the first mechanism (full line) in turn reduces the income of the second poorest, second richest and richest individual respectively to zero, whereas the second mechanism (dotted line) first equalizes the incomes of the three individuals and afterwards reduces their incomes by equal amounts.

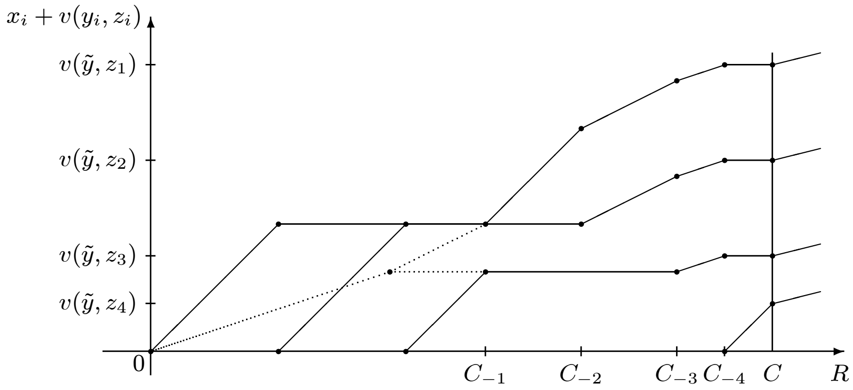


Figure 5: Two different mechanisms satisfying Participation, Priority and ETRS

The axioms used in the characterization of $S_{\tilde{y}MRE/E}$ are independent. For example, $S_{\tilde{y}EE}$ satisfies $ETRS$ and Symmetry but violates Priority. The two mechanisms of figure 5 satisfy $ETRS$ and Priority but violate Symmetry. $S_{\tilde{y}MRE/P}$ satisfies Symmetry and Priority but violates $ETRS$. The axioms used in the characterization of $S_{\tilde{y}MRE/P}$ are also independent. A straightforward example of a mechanism that satisfies Participation and Symmetry but violates Priority is the equal division of income. The two mechanisms of figure 5 satisfy Participation and Priority but violate Symmetry. Finally, $S_{\tilde{y}MRE/E}$ satisfies Symmetry and Priority but violates Participation.

# 4 Conclusion

The choice of a particular mechanism to divide the sum of total pre-tax income over a population of claimants inextricably brings about a particular way to solve the solidarity problem. We propose to use the information of individuals' minimal rights to divide an extra amount of income generated by a change in the skill profile. The idea is to give priority to individuals with a positive minimal right. We present Minimal Rights based Egalitarian mechanisms that implement this solidarity idea. We contrast Egalitarian Equivalent mechanisms ($S_{\tilde{y}EE}$) and Proportionally Adjusted Equivalent mechanisms ($S_{\tilde{y}PAE}$), two well known families of fair redistribution mechanisms presented in the literature, with two families of Minimal Rights based Egalitarian mechanisms ($S_{\tilde{y}MRE/E}$ and $S_{\tilde{y}MRE/P}$). The following table summarizes our axiomatic analysis into the properties of these mechanisms.

| | $S_{\tilde{y}EE}$ | $S_{\tilde{y}PAE}$ | $S_{\tilde{y}MRE/E}$ | $S_{\tilde{y}MRE/P}$ |
|---|---|---|---|---|
| *EIEE* | + | + | + | + |
| *ETES* | − | − | − | − |
| *ETRS* | ★ | ★ | ★ | − |
| *Solidarity* | + | + | + | + |
| *AS* | ★ | − | − | − |
| *MS* | − | ★ | − | − |
| *Symmetry* | + | − | ★ | ★ |
| *Priority* | − | − | ★ | ★ |
| *Participation* | − | + | − | ★ |

★: used in characterization of $S$
+: $S$ satisfies the axiom (but not used in characterization)
−: $S$ violates the axiom

The study into the income inequalities resulting from these mechanisms learns that, given $\tilde{y}$, an $S_{\tilde{y}MRE/P}$ always redistributes income more equally than an $S_{\tilde{y}MRE/E}$. The latter mechanism redistributes income more equally than an $S_{\tilde{y}EE}$ when none or some (but not all) minimal rights are positive. When all minimal rights are positive $S_{\tilde{y}EE}$ and $S_{\tilde{y}MRE/E}$ are equivalent. All three mechanisms redistribute income more equally than an $S_{\tilde{y}PAE}$ in surplus sharing problems.

# Appendix: Proofs

Lemma 1 states formally the solidarity idea of Minimal Rights based Solidarity, i.e. how a change in the total sum of pre-tax income is divided over the population such that the axioms of Symmetry and Priority are satisfied.

**Lemma 1 (Minimal Rights based Solidarity):** Consider $\Delta_R > 0$ due to a skill change from $y_i$ to $y_i'$ of an individual $i$ in $N$. Denote $d_i = x_i' + v(y_i', z_i) - (x_i + v(y_i, z_i))$; $\sum_{i \in N} d_i = \Delta_R$. One of five possible situations occurs:

(**1**) when $R' \leq C_{-1}$ or $C_{-n} \leq R$ :

$d_i = \frac{\Delta_R}{n}$ for all $i$ in $N$,

(**2**) when, for $k \leq n-1$, $C_{-k} \leq R < R' < C_{-(k+1)}$ :

$d_i = \frac{\Delta_R}{k}$ for all $i \in \{1, \dots, k\}$ and

$d_j = 0$ for all $j \in \{k+1, \dots, n\}$,

(**3**) when, for $k \leq n-1$, $C_{-k} \leq R < C_{-(k+1)}$ and, for $2 \leq l \leq n$, $C_{-l} \leq R' < C_{-(l+1)}$[5] and $k < l$ :

$d_i = \frac{C_{-(k+1)} - R}{k} + \sum_{h=k+1}^{l-1} \left( \frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-l}}{l}$ for all $i \in \{1, \dots, k\}$ and

[5] When $l = n$, define $C_{-(l+1)} = +\infty$.

$d_j = \sum_{h=j}^{l-1} \left( \frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-l}}{l}$ for all $j \in \{k+1, \dots, l-1\}$ and

$d_l = \frac{R' - C_{-l}}{l}$ and

$d_q = 0$ for all $q \in \{l+1, \dots, n\}$,

(**4**) when $R \leq C_{-1}$ and, for $l \leq n-1$, $C_{-l} \leq R' < C_{-(l+1)}$ :

$d_i = \frac{C_{-1} - R}{n} + \sum_{h=i}^{l} \left( \frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-(l+1)}}{l}$ for all $i \in \{1, \dots, l\}$ and

$d_j = \frac{C_{-1} - R}{n}$ for all $j \in \{l+1, \dots, n\}$,

(**5**) when $R \leq C_{-1}$ and $C_{-n} \leq R'$ :

$d_i = \frac{C_{-1} - R}{n} + \sum_{h=i}^{n-1} \left( \frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-n}}{n}$ for all $i \in \{1, \dots, n-1\}$ and

$d_n = \frac{C_{-1} - R}{n} + \frac{R' - C_{-n}}{n}$.

**Proof.** Suppose the antecedent of (1) is true. None of the minimal rights change and the division of $\Delta_R$ is obtained by Symmetry. Suppose the antecedent of (2) is true. There are two groups of individuals. For individuals 1 to $k$ minimal rights change equally. For individuals $k+1$ to $n$ minimal rights do not change. Symmetry implies that, within each group, all individuals are treated in the same way. Priority requires that the first group receives $\Delta_R$. The division of $\Delta_R$ then follows straightforwardly. Suppose the antecedent of (3) is true. As $v$ is continuous and strictly increasing in $y$, there exist for individual $i$ unique intermediary skill levels $\hat{y}_i^k, \hat{y}_i^{k+1}, \dots, \hat{y}_i^l$ such that, ceteris paribus, the total sum of pre-tax income equals $C_{-k}, C_{-(k+1)}, \dots, C_{-l}$ respectively. Now consider skill changes from $y_i$ to $\hat{y}_i^k$, $\hat{y}_i^k$ to $\hat{y}_i^{k+1}$,..., $\hat{y}_i^l$ to $y_i'$ such that the total change of pre-tax income equals $C_{-k} - R, C_{-(k+1)} - C_{-k}, \dots, R' - C_{-l}$ respectively. For each of these subchanges there are two groups of individuals: $(i)$ a group whose change in minimal rights is equal to the subchange and $(ii)$ a group whose minimal rights do not change. Symmetry implies that within each group all individuals are treated in the same way, whereas Priority requires that the first group receives the subchange. Hence, the subchange is equally divided among the individuals of the first group. The division of $\Delta_R$ is obtained from applying Symmetry and Priority to the division of these subchanges. The division of $\Delta_R$ under (4) and (5) is obtained by similar reasoning as in (3). ■

**Proof of proposition 1**

$\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/E} \Leftrightarrow S$ satisfies Symmetry, Priority and $ETRS$.

**Proof.** We only proof ($\Leftarrow$). Define an economy $\tilde{e} = ((\tilde{y}, \dots, \tilde{y}), (z_1, \dots, z_n))$. By $ETRS$, $\tilde{x}_i = 0$ for all $i$ in $N$ and individuals' final incomes are equal to their claims. Call this the 'initial income distribution'. Rather than successively considering $n$ changes from $\tilde{y}$ to $y_i$ for every $i$ in $N$ and using Lemma 1 successively to divide the intermediate subchanges in total pre-tax income (a process where in many cases previous subchanges in total pre-tax income would cancel out), we immediately use Lemma 1 to divide $\Delta_R = C - R$.[6] The transfers of (1) in the definition of $S_{\tilde{y}MRE/E}$ then follow from adding to the initial income distribution the transfers described in case (1) in lemma 1. The transfers of (2) then follow from subtracting of the initial income distribution the transfers described in case (3) with $l = n$ in lemma 1. The transfers of (3) then follow from subtracting of the initial income distribution the transfers described in case (5) in lemma 1. ■

**Proof of proposition 2**

$\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/P} \Leftrightarrow S$ satisfies Symmetry, Priority and Participation.

**Proof.** We only proof ($\Leftarrow$). Participation requires that, when $R$ converges to zero, all incomes also converge to zero. Let $(0, 0, \ldots, 0)$ be the initial income distribution. Now, use Lemma 1 to divide $\Delta_R = R - 0$. The transfers of (1) in the definition of $S_{\tilde{y}MRE/P}$ are then described in case (1) in lemma 1. The transfers of (2) are then described in case (4) in lemma 1. The transfers of (3) are then described in case (5) in lemma 1. ■

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