On the curious historical coincidence of algebra and double-entry bookkeeping

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1 Introduction

The emergence of symbolic algebra is probably the most important methodological innovation in mathematics since the Euclidean axiomatic method in geometry. Symbolic algebra accomplished much more than the introduction of symbols in mathematics. It allowed for the abstraction and generalization of the concepts of number, quantity and magnitude. It led to the acceptance of negative numbers and imaginary numbers. It gave rise to new mathematical objects and concepts such as a symbolic equation and an aggregate of linear equations, and revealed the relation between coefficients and roots. It allowed for an algebraic approach to ancient geometrical construction problems and gave birth to analytical geometry. Why did this important methodological revolution happen? Why did it happen in Europe and not in Asia while Indian and Chinese algebra were more advanced before the fourteenth century? Why did it happen in the European Renaissance?

We can only touch the surface of possible answers to these fundamental questions within the scope of this paper. However, we would like to argue that the answers will involve multiple disciplines and will go beyond the boundaries of the history of mathematics. Most historians have taken for granted that symbolic algebra was an inevitable step within the logical development of mathematics. But can we speak of a logic of historical necessity? The history of mathematics at least teaches us that there have been developments within mathematics that were not in logical sequence.

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A full notion of the function concept was developed only after the calculus, while textbooks on calculus first introduce functions and then move to differentiation and integration. Some concepts emerged within a historical context where no sensible interpretation could be given to their meaning. A notorious example is imaginary numbers. Still, in historical accounts these anomalies and anachronisms are considered exceptions, and *exceptio probat regulam*. What if there is no such logic of historical development? Then all historical questions must be addressed within their cultural and social-economical context. Answers cannot be found by appealing to the next logical step in the development, but only in the relation between its practices and their meaning within the society. In other words, philosophy, history and sociology of science all contribute to possible answers to the questions we have raised.

In this text, we shall first give a short overview of the internalistic approach to the emergence of symbolic algebra which, as we shall show, is present in most historical accounts on the history of algebra. We shall then present some studies which take a contextual approach to developments in mathematics during the period we are addressing. We then present our own position that symbolic algebra was made possible by the central idea of value as an objective quantity in mercantilism. As an illustration of how important developments in mathematics can be matched with macroeconomical changes in society we draw the parallel between symbolic algebra and double-entry bookkeeping. These two developments of the fourteenth and fifteenth century were both instrumental in the objectivation of value and they supported the reciprocal relations of exchange on which mercantilism depended. To demonstrate our proposition, we shall present a case study to show how symbolic algebra and double-entry bookkeeping function in our understanding of a special class of bartering problems. It would be wrong to understand a socio-economical account of the history of mathematics as the right one or the only one. At the contrary, we believe that a pluralism of explanations leads to a better understanding. However, concerning the history of European algebra too much emphasis has been put on internal mechanisms and we present our account as complementary to these approaches.

2   Internalist accounts of the history of algebra

Algebra was introduced in medieval Europe through the Latin translations of Arabic texts between 1145 and 1250 and Fibonacci's *Liber Abbaci* (1202) (Boncompagni, 1857; Sigler, 2002). Algebraic problem solving was further practiced within the so-called abbaco tradition in cities of fourteenth- and

fifteenth century Italy and the south of France.[1] From the sixteenth century, under the influence of the humanist program to provide new foundations to this *ars magna*, abbaco algebra evolved to a new logistics of species with François Viète (1591) as the key figure. With Descartes's *Geometry*, this new kind of algebra progressed into our current symbolic algebra. This is a brief characterization of the current view of scholars on the history of European symbolic algebra.

Most of the studies on the history of algebra provide an internalistic account. They accept implicitly or more explicitly that the development towards symbolic algebra was inevitable and depended on some internal mechanisms and intrinsic processes. Moritz Cantor whose *Vorlesungen* (1880; 1892) had an important influence on twentieth-century historians of mathematics, attributes, for the early period, high importance to the Latin works of Fibonacci and Jordanus. He believed that the vernacular tradition of practical arithmetic and algebra did not produce any men capable of understanding the works of these two giants. Cantor assumes this to be true for most of the 14th and 15th century.[2] When dealing with the sixteenth century, extraordinary importance is attributed to the *Arithmetica* of Diophantus (Heath, 1885; Sesiano, 1982; Rashed, 1984). The idea that algebra originated with Diophantus was fabricated by humanist mathematicians after Regiomontanus's Padua lecture of 1464 (Regiomontanus, 1537). As a consequence of their reform of mathematics, humanist writers distanced themselves from "barbaric" influences and created the myth that all mathematics, including algebra descended from the ancient Greeks (Høyrup, 1996). Later writers such as Ramus (1560; 1567); Peletier (1554); Viète (1591) and Clavius (1608) participated in a systematic program to set up sixteenth-century mathematics on Greek foundations. The late discovered *Arithmetica* of Diophantus was taken as an opportunity by Viète to restore algebra "which was spoiled and defiled by the barbarians" to a fictitious pure form. To that purpose he devised a new vocabulary of Greek terms to cover up the Arab roots of algebra "lest it should retain its filth and continue to stink in the old way" (Klein, 1968, p. 318). The reality was that, with some exceptions, ancient Greek mathematics was more foreign to European mathematics than Indian and Arabic influences which were well digested within the vernacular tradition (Heeffer, 2007).

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[1]We follow the convention to name the abbaco or abbacus tradition after Fibonacci's *Liber abbaci*, spelled with double b to distinguish it from the material abbacus. It refers to the method of counting and calculating with hindu-arabic numerals.

[2]"Aber die Zeitgenossen der beiden grossen Männer waren nicht reif, deren Schriften vollständig zu verstehen, geschweige denn sie fortzubilden, und besonders für die eigentlichen Gelehrtenkreise gilt dieses harte Urtheil auch noch im XIV. Jahrhunderte, während damals italienische Kaufleute der Algebra so viel Verständniss entgegenbrachten, dass wenigstens versucht wurde, Aufgaben zu lösen, welchen die früheren Schriftsteller ohnmächtig gegenüberstanden." (Cantor, 1892)

Contemporary scholars, such as Rafaella Franci and Laura Toti Rigatelli (1979; 1985) or van der Waerden (1985) narrate the story of the history of algebra from their internal dynamics. Early European algebra, as inherited from the Arabs, recognized six types of equations. Instead of dealing with the general form of a quadratic equation $ax^2 + bx + c = 0$, the first Latin translations distinguish three cases depending on the sign of the coefficients and three cases with one or two terms missing. Each case had its own solution method. Double solutions were not recognized except for the case of two positive roots. Early abbacus masters extended the list of six to include higher degree cases most of which could be reduced to the original six. During the fourteenth century Maestro Dardi of Pisa expands this to no less than 198 cases (van Egmond, 1983; Franci, 2001)! Already from the fourteenth century abbacus masters started experimenting with irreducible cases of higher degree. False rules were given for special cases of the cubic equation (Høyrup, 2009). However, Maestro Dardi gives some examples of cubic equations with a correct solution derived from numerical examples. These histories of algebra then focus on the sixteenth-century breakthrough of Scipione del Ferro in solving the depressed cubic and the feud between Tartaglia and Cardano for publishing a general solution for the cubic equation. It then moves to symbolism introduced by Viète and the general approach to problems. With a mention of Girard, these developments culminate into the quest for the fundamental theorem of algebra moving well into the eighteenth century. All this is presented as a continuous flow of necessary logical development. Each step is the necessary next move in the logical puzzle of the history of symbolic algebra. Van der Waerden goes to great lengths of demonstrating this continuity back to the earliest Greek mathematics. In a earlier publication, van der Waerden (1988, p. 116), uses the so-called *Bloom of Thymaridas* to connect algebra with the Pythagoreans. He goes as far as to claim that "we see from this that the Pythagoreans, like the Babylonians, occupied themselves with the solution of systems of equations with more than one unknown". We have elsewhere demonstrated that these claims cannot be sustained and that these should be understood as a by "humanist education deeply inculcated prejudice that all higher intellectual culture, in particular all science, is risen from Greek soil" (Heeffer, 2009).

Jacob Klein, a student of Heidegger and interpreter of Plato, wrote a long treatise in 1936 on the number concept starting with Plato and the development of algebra from Diophantus to Viète (Klein, 1968). It became very influential for the history of mathematics after its translation into English in 1968. Klein goes even further than Van der Waerden or Franci and Rigatelli in their internalistic account of history. For Klein it is not the evolution of solution methods for solving equations which follows some logical

path but the ontological transformation of the underlying concepts within an ideal Platonic realm. He restricts all other possible understandings of the emergence of symbolic algebra by formulating his research question as follows: "What transformation did a concept like that of *arithmos* have to undergo in order that a 'symbolic' calculating technique might grow out of the Diophantine tradition?" (Klein, 1968, p. 147). According to Klein it is ultimately Viète who "by means of the introduction of a general mathematical symbolism actually realizes the fundamental transformation of conceptual foundations" (Klein, 1968, p. 149). Klein places the historical move towards the use of symbols with Viète and thus ignores important contributions by the abbaco masters, by Michael Stifel (1545, 1553), Girolamo Cardano (1539, 1545) and the French algebraists Jacques Peletier (1554), Johannes Buteo (1559) and Guillaume Gosselin (1577). The new environment of symbolic representation provides the opportunity to "the ancient concept of arithmos" to "transfer into a new conceptual dimension" (Klein, 1968, p. 185). As soon as this happens, symbolic algebra is born: "A soon as 'general number' is conceived and represented in the medium of species as an 'object' in itself, that is, symbolically, the modern concept of 'number' is born" (Klein, 1968, p. 175). It is hard to understand why a philosophy like this, rooted in German idealism, where concepts realize themselves with the purpose to advance mathematics, is so appealing to modern historians looking for an explanation for the emergence of symbolic algebra.

The three different approaches to the history of algebra are exemplified by three historians of mathematics. Cantor believes in a continuous development from ancient Greek notions of number and proof to modern algebra, only obscured during the medieval period in which the old masters were not fully understood. Van der Waerden and Franci see a historic realization of the logical development from quadratic equations to cubic and higher degree ones towards a theory of the structure of equations. Klein discerns a realization of symbolic algebra in a necessary ontological transformation of the underlying number concept. All three share the idea that there is some internal necessity and logic in the historical move towards symbolic algebra. But all pass by at the fundamental historical changes that took place in the context in which medieval algebra matured: the mercantile centers of northern Italy and the French Provençe region. We shall now look at some contextual explanations and further demonstrate that the emergence of symbolic algebra cannot be understood without accounting for the socio-economical context of that time. We shall illustrate this with a specific class of bartering problems which were discussed in arithmetic and algebra books during several centuries. We do not want to present such socio-economical interpretation as 'the right one', but at least as a complementary one to the one-sided internatistic interpretation so dominant in the historiography of algebra.

3   Contextual approaches

From the 1920's, history of science began to account for contextual aspects
of the society in which science develops and is practiced. As a reaction to the
romantic narratives of Great Men making Great Discoveries in science com-
munist historians of science pointed out the role of social and economic con-
ditions in the emergence and development of science. Gary Werskey (1978)
describes how Soviet historians irritated Charles Singer, the chairman of
the Second International Congress in the History of Science and Technology
in London in 1931, by repeatedly asking questions about socio-economical
influences on the evolution of science. But this conference was a historical
one making an impact on the thought of many young scholars with social-
ist sympathies such as Joseph Needham and Lancelot Hogben. Beginning
with Boris Hessen's *The Social and Economic Roots of Newton's Principia*
(1931), several papers and books were published, placing the achievements
of individual scientists within the context of social superstructures. Specific
histories of mathematics based on an analysis of socio-economical condi-
tions appeared much later. Dirk Struik was a convinced Marxist who wrote
a widely read *A Concise History of Mathematics* (1987). Although the
book cannot be considered a Marxist analysis, his vision of mathematics as
a product of culture and evolving within a dialectic process was having an
impact on other historians.

Only a limited number of authors focused on the mathematical sciences
during the period that symbolic algebra developed in Europe (between 1300
and 1600). Michael Wolff (1978) in a comprehensive study of the concept of
impetus argues that the "new physics" of the fourteenth century developed
from contemporary social thought. The scientific revolution basically was
a revolution in socio-economical ideas. Drawing upon the theories of Marx
and Borkenau, Richard Hadden (1994) develops the idea that practitioners
of commercial arithmetic, as a consequence of their social relations, deliv-
ered the new concept of "general algebraic magnitude" to the new mechanics.
Joel Kaye (1998) argues that the transformation of the model of the nat-
ural world of the Oxford and Paris scholars such as Thomas Bradwardine,
John Buridan, and Nicole Oresme during the fourteenth century follows the
rapid monetization of the European society. This transformation happened
beyond the university and outside the culture of the book.

We would like to argue that symbolic algebra functioned together with
double-entry bookkeeping as the main instruments for the determination of
objective value, the basic idea of the mercantile society. The foundations for
symbolic algebra were laid within the abbaco tradition. While scholars on
this tradition, such as Jens Høyrup (2007) maintain that the problem solving
treatises written by abbaco masters served no practical purpose whatsoever,
we argue that their activities and writings delivered an essential contribu-

tion to Renaissance mercantilism in the creation of objective, computable value. According to Foucault (1966, p. 188) the essential aspect for the process of exchange in the Renaissance is the representation of value. "In order that one thing can represent another in exchange, they must both exist as bearers of value; and yet value exists only within the representation (actual or possible), that is, within the exchange or the exchangeability". The act of exchanging, i.e., the basic operation of merchant activity, both determines and represents the value of goods. To be able to exchange goods, merchants have to create a symbolic representation of the value of their goods. All merchants involved must agree about this common model to complete a successful transaction. As such, commercial trade can be considered a model-based activity. Given the current global financial market and the universal commensurability of money we pass over the common symbolic representation as an essential aspect of trade. However, during the early Renaissance, the value of money depended on the coinage, viz. the precious metals contained in the coins which differed between cities, and varied in time. As the actions and reciprocal relations of merchants, such as exchange, allegation of metals and bookkeeping became the basis for the symbolic and abstract function of money, so did the operations and the act of equating polynomials lead to the abstract concept of the symbolic equation. Both processes are model-based and use the symbolism as the model. Therefore, we have to understand the emergence of symbolic algebra within the same social context as the emergence of double-entry bookkeeping.

Now consider the following statement: The emergence of double-entry bookkeeping by the end of the fifteenth century was a consequence of the transformation from the traveling to the sedentary merchant, primarily in the wool trade situated in Italy and Flanders (de Roover, 1948; van Egmond, 1976). Given the vast body of evidence from Renaissance economic history and the evident causal relationship, not many will contest the relevance of merchant activities on the emergence of bookkeeping. What about the mitigated statement: "The emergence of symbolic algebra in the sixteenth century is to be situated and understood within the socio-economic context of mercantilism"?. Philosophers of mathematics who believe in an internal dynamics of mathematics will not accept decisive social influences as an explanation for the emergence of something as fundamental as symbolic algebra. At best, they will accept social factors in the acceleration or impediment of what they consider to be a necessary step in the development of mathematics. Also it seems difficult to pinpoint direct causal factors within economic history for explaining new developments in mathematics. However, the relationship between bookkeeping and symbolic algebra is quite remarkable. Many authors who have published about bookkeeping also wrote on algebra. The most notorious example is Pacioli's *Summa*, which

deals with algebra as well as bookkeeping, and the book had an important influence in both domains. But there are several more coincidences during the sixteenth century. Grammateus (1518) gives an early treatment of algebra together with bookkeeping. The Flemish reckoning master Mennher published books on both subjects including one treating both in the same volume (Mennher, 1565). So did Petri in Dutch (Petri, 1583). Simon Stevin wrote an influential book on algebra (Stevin, 1585) and was a practicing bookkeeper who wrote a manual on the subject (Stevin, 1608). In Antwerp, Mellema published a book on algebra (Mellema, 1586) as well as on bookkeeping (Mellema, 1590). While there is no direct relationship between algebra and bookkeeping, the teaching of the subjects and the books published often addressed the same social groups. Children of merchants were sent to reckoning schools (in Flanders and Germany) or abbacus schools (in Italy) where they learned the skills useful for trade and commerce. There is probably no need for algebra in performing bookkeeping operations but for complex bartering operations or the calculation of compound interest, basic knowledge of arithmetic was mandatory and knowledge of algebra was very useful.

4 Case study: Bartering with cash values

In an interesting article in the *Journal of the British Society for the History of Mathematics*, John Mason (2007) expresses his surprise at the solution method adopted for bartering problems which involve cash. He cites a problem from Piero della Francesca in a translation by Judith Field (2005, p. 17):

> Two men want to barter. One has cloth, the other wool. The piece of cloth is worth 15 ducats. He puts it up for barter at 20 and $\frac{1}{3}$ in ready money. A cento of wool is worth 7 ducats. What price for barter so that neither is cheated?

Mason originally expected the solution to be based on the proportion of the barter value to the original value with the barter value being "either 20 + 20/3 ducats or to 20 + 15/3 ducats, depending on which value the $\frac{1}{3}$ is intended to act upon" However Piero's solution appears to be different (Mason, 2007, p. 161):

> This computation intrigued me because I was astonished at the sequence of calculations: first reduce by the ready money paid (as a fraction of the barter price), and only then compare barter prices. It seemed to me that in a modern economy it would be more natural to carry out one of the calculations I considered, since the ready money to be paid is a cash value, and the bartering inflation refers to the noncash-traded amounts.

Thus Piero subtracts one third of 20 from 20, which leaves 13 $\frac{1}{3}$ and the same value from 15 which becomes 8 $\frac{1}{3}$. The proportion of these two values is hence the fair barter profit to be applied by both parties. Though Mason lists several other examples which follow the same solution method, he does not provide an explanation why this particular method is adopted in abbaco treatises and in later printed books. Given that this way of calculating was in use for over two centuries, not only in Italy but in several European countries, this particular bartering practice needs an explanation. We shall demonstrate that his astonishment is based on a wrong interpretation and even more so, a wrong translation of the original problem. We shall provide an explanation by placing these early bartering problems within the specific context of Medieval Italian merchant practices.

The original problem by Piero, in Gino Arrighi's transcription from the manuscript, is formulated as follows (f. 8r; Arrighi, 1970, p. 49):

> Sono doi che voglano baractare, l'uno à panno e l'altro à lana. La peçça del panno vale 15 ducati et mectela a baracto 20 et sì ne vole $\frac{1}{3}$ de contanti; et il cento de la lana vale 7 ducati a contanti. Che la dèi mectere a baracto a ciò che nisuno non sia ingannato?

A literal translation of the medieval Italian would be as follows:[3]

> There are two [men] that want to barter. One has cloth, the other has wool. The piece of cloth is worth 15 ducats. And he puts this to barter [at] 20 and of this he wants $\frac{1}{3}$ in cash. And a hundred of wool is worth 7 ducats in cash. What shall they put for barter so that not one of them is being cheated?

Formulated this way, there is little room for doubt. The one with the cloth wants 20 ducats per piece of cloth, *of which one third in cash*. Obviously then, a third of the value refers to the barter value of 20. The amount of cash per piece is thus 20/3. To know the barter value of the cloth without the cash one has to subtract the cash from it, being 13 $\frac{1}{3}$. That Mason wants to add one third of the value rather than subtracting it stems from the wrong translation of "et sì ne vole $\frac{1}{3}$ de contanti".

Is this interpretation the correct one for all bartering problems of this type in abbaco treatises? Let us look for further clues. Mason provides pointers to several abbaco treatises in which bartering problem appear with a cash value. The earliest he discusses are problems 33, 86, and 87 of Paolo Dagomari's *Trattato d'aritmetica*, written in 1339. He describes problem 86 as a problem which "involves grain to be bartered at 15s but valued at 12, with one-third in ready money, in exchange for *orzo* (?) at 10s.". The word

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[3]My translation. For a discussion on the translation of abbaco texts, cf. a recent critical edition of a fifteenth-century treatise on algebra (Heeffer, 2008, p. 132)

orzo should pose no problems as it is the modern Italian word for barley. In our translation:[4]

> There are two that want to barter together. The one has grain and the other has barley. And the one with the grain which is valued at 12 s. puts it in barter at 15 s. per bushel. And he wants from the one with the barley one third of the value in cash. And from what remains he will get barley. And a bushel of barley values 10 s. Asked is what they arrive at in this barter so that none is left cheated.

Here also, the meaning of the problem is different from the one paraphrased by Mason. It is not the person with the grain who puts in the cash, but the other one. Furthermore, the enunciation clearly specifies that the second person should deliver one third of the value in cash and the rest in barley and this conforms with our interpretation.

4.1 First occurrences of bartering with cash

Was Paolo the first to deal with cash values in bartering problems? We checked all available transcriptions of abbaco treatises before Paolo's *Trattato*. The earliest one is probably the *Columbia algorismus* (Columbia, X 511, A l3) published by Vogel (1977). Vogel himself dated the manuscript in the second half of the 14th century. However, a recent study of the coin list contained in the manuscript is dated between 1278 and 1284, which makes it the earliest extant treatise within the abbaco tradition (Travaini, 2003, pp. 88-92). Høyrup suspects it "likely to be a copy of a still earlier treatise" (Høyrup, 2007, p. 31). It contains two barter problems (19 and 20) but none involves money. The anonymous *Livero del l'abbecho* is dated c. 1289–1290 and has also two bartering problems without money (Arrighi, 1989, p. 24, 28). The *Tractatus Algorismi* by Jacopo da Firenze is extant in an earliest version of 1307. It is the subject of a recent comprehensive study of the abbaco tradition by Jens Høyrup (2007). However this extensive treatise does not contain any bartering problems. The next available transcription is the *Liber habaci*, dated by van Egmond (1980) to 1310, and is the first to involve cash in a bartering transaction. The enunciation of the single bartering problem is more elaborate and functions as a prototype for later reformulations by Paolo and Piero:[5]

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[4]From Arrighi's transcription (Arrighi, 1964, p. 75): "E' xono due che barattano insieme, l'uno àe grano e Il'altro àe orzo; e quello che à grano gli mette in baratto lo staio del grano 15 s., che vale 12 s., e vuole il terzo da quello dell'orzo di ciò che monta il suo grano di chontantj; e dell'avanzo se ne togle orzo. Ello staio dell'orzo vale 10 s., adornando quanto glele chonterà in questo baratto acciò che no' rrimangha inghannato".

[5]From Liber habaci, Biblioteca Nazionale Centrale Firenze, Magl. Cl. XI, 88, transcription by Arrighi (1987, p. 147): "Sono due merchatanti che volglono barattare insieme, l'uno si à lana e l'altro si à pannj; dice quellj ch'à lla lana a quellj del pannj: che vuo'

> There are two merchants who want to barter together. The one has
> wool and the other has cloth. The one with the wool tells the one
> with the cloth: "how much do you want for the *channa* of your cloth".
> And he says: "I want 8 lb. (and he knows well that it values not more
> than 6 lb.) and I want one quarter in cash and I want three quarters
> in wool. And the wool is valued at 20 lb. per hundred. Asked is what
> suits him to sell the wool per hundred so that he is not being cheated.

We find here all the elements of the later bartering problems. The prob-
lem clearly specifies that one party will deliver one quarter of the value in
cash and three quarters in merchandize. The reference to a fair deal be-
comes a standard formulation in abbaco bartering problems. The solution
recipe is the standard formula adopted in later treatises as discussed by
Mason:[6]

> You shall do as such, one quarter is asked in cash, say as such: one
> quarter of eight is 2. The rest until eight is 6. From 2 until 6 is 4,
> therefore say as such: for every 4 lb. I get 6 lb., how much do I get
> for 20 lb.? Multiply 20 lb. against 6 lb. this makes 120 lb. Divide by
> 4 and 30 lb. results from it. This is how much it suits him to get per
> hundred for this wool.

We have now found an adequate interpretation for the subtraction of the
cash value from the barter price, but why is this cash value also subtracted
from the original value? This example from the *Liber habaci* already gives
us an insight. Obviously, if one takes into account the barter value minus the
cash value (here 6 lb.) something also has to be done with the original value
of the merchandize (also 6 lb.). In this example these values are the same
and there would be no profit ratio. However, adapting Mason's original
reasoning to the new interpretation, one could still compare the total barter
value (here 8 lb.) with the original value (6 lb.) and use this as a profit
ratio. Why is it not done this way?

4.2   Early Italian merchant practices

To answer that question we must look at Italian merchant practices at the
beginning of the fourteenth century. One important breakthrough took
place around that time: the introduction of double-entry bookkeeping.

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tu della channa del panno? E que' dice: io ne volglo lb. viij (e sa bene che non vale più
di lb. vj) e volglo il quarto i' d. chontanti e tre quarti volglo in lana. El centinaio della
lana vale lb. xx, adomando che lgli chonviene vendere il centinaio di questa lana acciò
che non sia inghannato". A channa is a unit of length of about 2 m.

[6]Ibid: "De' chosì fare. E' domanda il quarto in danari, diray chosì: il quarto d'otto si
è ij. Insino inn otto si à vj, da ij insino vj si iiij or diray chosì: ongnj iiij lb. mi mette lb.
vj, che mi metterà lb. xx? Multipricha lb. xx via lb. vj farà lb. Cxx, dividi per iiij ne
viene xxx lb.: chotanto gli chonviene mettere il centinaio di questa lana".

Records of stewards of authorities of Genoa in 1278 show no trace of this kind of bookkeeping while by 1340 a complete system of double-entry book-keeping was established (Littleton, 1927, p. 147). While archival evidence suggests the emergence of bookkeeping practices during the course of the thirteenth century, the earliest extant evidence of full double-entry book-keeping is the Farolfi ledger of 1299–1300 (Lee, 1977). So the appearance of cash in bartering problems during the first decades of the fourteenth century coincides with the emergence of double-entry bookkeeping practices. Bar-tering was the dominant practice for traveling merchants during the Middle Ages. When medieval Europe moved to a monetary economy in the thir-teenth century, sedentary merchants depended on bookkeeping to oversee multiple simultaneous transactions financed by bank loans. While standard bartering required no elaborate administration, double-entry bookkeeping supported more complex bartering operations involving cash and time. Cal-culating practices taught in *bottega d'abbaco*, supported the new economy in the same way as double-entry bookkeeping did. If we want to understand these problems we should therefore look at bookkeeping practices.

As is well known, the first printed text on double-entry bookkeeping is Pacioli's *Particularis de Computis Et Scripturis*, treatise XI of distinction nine of his *Summa de arithmetica et geometria* of 1494. Mason cites from the *Scripturis* but oddly not from the chapter 20 on bartering. Pacioli was well aware about the old bartering practices. His until recently unpublished Perugia manuscript (Vat. Lat. 3129, 1478) contains a chapter on barter-ing with no less than 56 problems (folios 61$^{\mathrm{r}}$–83$^{\mathrm{v}}$). Many of them involve cash (Calzoni and Cavazzoni, 1996). In the *Scripturis* he writes "Bartering is commonly of three kinds: Simple, Complex, and Time" (*Semplice, Com-posta, a Tempo*) and he explains how to account for bartering in the Journal and Ledger (Crivelli, 1924, p. 46):

> After you have so described it, you can then reduce it to cash value, and as you wish to see the value in cash of such and such goods you will make out the entry in the Memorandum in whatever kind of money you desire; as it does not matter, providing that the book-keeper afterwards transfers the entry to the Journal and Ledger and reduces the amount to the standard money which you have adopted.

Our bartering problems involving cash are thus of the complex type and Pacioli provides an example of how to note down the value of bartered merchandise for a transaction which involves one third in cash. In Pacioli's terminology *Per* stands for debit and *A* for credit (Crivelli, 1924, p. 47):

> *Per Bellidi* ginger in bulk or packages. *A* sugar of such and such a kind, so many packages, weighing so much. Received ginger from so-and-so in exchange for sugar carried out in this manner: viz., I valued

the sugar at 24 ducats per hundred, on condition that I should receive $1/3$ in cash, and the ginger to be valued at so many ducats per hundred, for which ginger I should give so many loaves of sugar, weighing so much, which if paid for in cash are worth 20 ducats per hundred, and for said ginger he received sugar, so many loaves, each

valued at L[ire] S[oldi] G[rossi] P[icioli]

Unfortunately Pacioli gives no numerical entries but explains that one should debit the cash (you receive) and credit the sugar (you barter). Furthermore "that which is more in the cash entry will nevertheless be missing per contra in the sugar, and this you are to correct". So let us reconstruct the book-keeping transactions for the original example by Piero given the balance sheet equation: Assets = Liabilities + Owners Equity, and using Pacioli's [Debit // Credit] notation system:[7]

Assets	Liabilities	Owners Equity

1) Spend 1/3 cash of barter price

[0 // 20/3]   [20/3 // 0]

2) Deliver products for barter from stock

[15 // 0]

3) Receive barter goods at barter value

[0 // 20]

4) Book profit

[0 // 20 − 15]

One third of the barter value is paid in ready money and therefore credited from cash assets and debited from OE. The products we deliver have a booking value of 15 (multiplied by the number of items) while we receive goods valued at 20. The difference has to be booked as profit to maintain the balance and therefore we credit OE with the difference, being 20 − 15. We now can see that the profit or difference between the booked value and the barter value has to be the same as the difference between the two values

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[7]As the balance sheet equation was introduced after Pacioli this may seem an anachronism. It is here only shown to demonstrate the necessity of step 4.

used to determine the fair profit ratio. Thus the calculation of the barter value of the second party, x depends on the ratio:

$$\frac{20 - \frac{1}{3}(20)}{15 - \frac{1}{3}(20)} = \frac{x}{7}$$

Subtracting the two values $20 - \frac{1}{3}(20)$ and $15 - \frac{1}{3}(20)$ results in the profit 5.

That a seemingly basic problem from the abbaco tradition, which follows practices that were in use for over two centuries, gives rise to a feeling of astonishment for modern scholars on the history of mathematics is rather interesting. That we have to base ourselves on the socio-economical context of mercantilism to understand the solution of the problem is even more so. The case demonstrates that starting from modern conceptions and looking for corresponding ones in a historical context is often not the best way to study history. Ideas, methods and practices, even mathematical ones, are best understood in their historical socio-economical context.

4.3   Concluding remarks

The earliest written evidence of double-entry bookkeeping is the Farolfi ledger of 1299–1300. The earliest extant vernacular text dealing with algebra was written in 1307. Both algebra and double-entry bookkeeping were practiced throughout the fourteenth and fifteenth centuries in the mercantile centers of Northern Italy. The first appearance of these two disciplines in print was in the very same book, the *Summa* by Luca Pacioli (1494). Why was bookkeeping treated in the same book with arithmetic and algebra? Because they both were important instruments in the establishing objective and exact value, the basic principle of reciprocal relations of exchange in a mercantile society. There has been speculation about the purpose and intended audience of a book like the *Summa*. While Pacioli himself received an abbaco education and he taught some years to sons of merchants in Venice, he is often wrongly considered an abbacist (e.g., Biagioli, 1989). In fact, he enjoyed the social status of a well-paid university professor. Between 1477 and 1514, he taught mathematics at the universities of Perugia, Zadar (Croatia), Florence, Pisa, Naples and Rome (Taylor, 1942). This Franciscan friar and university professor saw a way to bring his lengthy treatise in Tuscan vernacular to a larger public by means of book printing. Later he would publish a Latin book on Euclidean geometry but as an educator he recognized the real needs of the mercantile society. His *Summa* literally brings together all important aspects of knowledge in such a mercantile society, including algebra and double-entry bookkeeping. In a recent study on the target audience (Sangster et al., 2008, p. 129), Sangster is

> ... led to the conclusion that the bookkeeping treatise was not only intended to be read and used by merchants and their sons, it was

designed specifically for them. Further analysis of the content and sequencing of *Summa Arithmetica* indicated that the entire book was written primarily as a reference text for merchants and as a school text for their sons. It was sourced mainly from *abbaco* texts and mirrored much of the curriculum of the *abbaco* schools attended by the sons of merchants; and extended it to include all extant material known to Pacioli that was of direct relevance to merchants. No abbaco school or tutor would previously have had access to such a wide range of relevant material in a single source.

From the sixteenth century, under the influence of the humanist program to reform mathematics, algebra changed considerably and by the end of the sixteenth century developed into a symbolic algebra. On the other hand, double-entry bookkeeping did not and is in use today in the smallest coffee shops to the largest multi-national enterprizes, in a way which is not fundamentally different from Pacioli's description. This is remarkable as the structure of the ledgers is designed in a way to avoid negative numbers (Peters and Emery, 1978, wrongly critized by Scorgie, 1989). The balance sheet formula used above can equally be expressed as Assets$-$Liabilities$-$Owners Equity $= 0$, in accordance with our common way of writing down equations. However, this involves the use of negative quantities, a concept which was gradually introduced only from the sixteenth century onwards. However strange this may sound, our modern way of doing bookkeeping thus conserves the medieval concept of number.

As far as we know it was Maestro Antonio de' Mazzinghi who was the first to apply the rules of algebra to bartering problems in the 1380's. In his *Trattato di fioretti* he solves a simple problem of two men bartering wool against cloth (Arrighi, 1967, pp. 31-2). The wool is worth 20 and the barter value is 22. The cloth is worth 6 and bartered with the same profit margin adjusted by 10%. Instead of doing the numerical calculation de' Mazzinghi solves this by taking x for the barter profit so that $\frac{1}{1x} + \frac{1}{1x+1}$ equals 10%. This leads him to the equation

$$\frac{2x+1}{x^2+x} = \frac{1}{10}$$

By solving the quadratic equation he arrives at a barter price for the wool as $\sqrt{401}-1$ *fiorini* against the price for the cloth at $\sqrt{43\frac{6689}{10000}} + \frac{33}{100}$ *fiorini*.

We can find surds as values for goods in almost every abbaco treatise. Høyrup (2009, p. 51) appropriately remarks that "Being satisfied with exactly expressed but irrational solutions remained the habit of abbacus algebra". In contrast with geometry treatises which served the purpose of practical surveying and construction and had its values approximated. You do not find any approximations within the context of abbaco algebra. This

leads him to the conclusion that "abbacus algebra, at least beyond the first degree, must in some sense have been a purely theoretical discipline without intended practical application". However, as we see it, in their perseverance on using exact values for merchant type problems, abbacus masters established the objective true value of goods within a transaction. If this had to be expressed in surds, so be it. The underlying idea was that there always exists a just and true value in exchange. Earlier in medieval society the marketplace was already recognized as a guide to the determination of value. But this value was a fuzzy concept, a value subject to many approximations in calculating the exchange rates of currencies and unit conversions which could be different from city to city.

The earliest extant abbaco text dealing with algebra, Jacoba da Firenzi's *Tractatus Algorismi* of 1307, includes a peculiar discounting problem (Høyrup, 2007, pp. 252-3):

> A merchant shall have from another *libre* 200 within two months and a half from now. This merchant says, give me this money today, and I discount your money at the rate of *denarij* ij [i.e., 2] per *libra* a month. Say me, how much shall he give him in advance for the said *libre* 200.

The problem is followed by an approximate solution based on iterative subtractions. Høyrup (2007, p. 70, note 176) reports a similar approximation in the fifteenth-century *Libro di conti e mercantanzie* (Gregori and Grugnetti, 1998, p. 95). Expressed in modern symbolism this amounts to the development of $\frac{1}{1+p}$ as $1 - p(1 - p(1 - p(1 - p(1 - p(1-p))))))$. One could see these two occurrences as counter examples in the development towards exact value within the abbaco tradition. However, it is important to note that such old merchant calculations are quite rare and that they became gradually replaced by algebra. It is precisely within the algebraic context that we see no approximate solutions. One of the major contributions of the maestri d'abbaco is that they have shown that practically all merchant problems can be approached by algebra. We find a good illustration of this in cumulative interest problems which could be solved by an iterative procedure similar to the one described. However, we see that these problems were successfully solved by algebra even if they lead to fifth-degree equations as in Piero della Francesca's *Trattato d'abacho* (Arrighi, 1970, pp 421–2).

5   Conclusion

Abbaco arithmetic and algebra as well as double-entry bookkeeping gave support to the idea of a value which can be determined in principle as the just value, not only in the mathematical sense but also in the moral sense. The frequent references to the *fair* barter value demonstrate the

moral obligation to account for exact values. The importance of dealing
with exact values should not be underestimated. Margolis's barrier theory
places the concept of an exact quantifiable number for probabilities as the
breakthrough for the probability theory of Pascal and Fermat (Margolis,
1993). All intuitions for a probability theory were present for centuries, but
the habit of mind was to perceive probability as a result of a bargaining
process, like a fair price for the risks involved. Probability theory was made
possible by attaching a single quantifiable number to the concept of proba-
bility which was not considered to be countable. Once this missing concept
was introduced of usefully attaching numbers to comparative values even
if there is nothing immediately to count, a theory of probability could be
established. The idea of an exact value, which could be expressed within the
language of algebra and maintained its exactness through the operations of
algebra, became the basic concept of mercantilism.

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