

Neutrino versus antineutrino cross sections and CP violation

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We discuss the nuclear interactions of neutrinos versus those of antineutrinos, a relevant comparison for CP violation experiments in the neutrino sector. We consider the MiniBooNE quasielastic-like double-differential neutrino and antineutrino cross sections that are dependent on the energy profiles of the neutrino fluxes and hence specific to the MiniBooNE setup. We combine them introducing their sum ($\nu + \bar{\nu}$) and their difference ($\nu - \bar{\nu}$). We show that the last combination gives general information on the multinucleon content of the axial-vector interference term. Our theoretical model reproduces well the two cross-section combinations, confirming the need for a sizable multinucleon component.

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I. INTRODUCTION

One of the challenging goals of neutrino experiments is the detection of CP violation in the neutrino sector. Convincing evidence would be the detection of an asymmetry between the oscillation rates of muon neutrinos and antineutrinos into electron neutrinos because, in the absence of CP violation, these rates are the same. For this test one needs a muonic neutrino beam and an antineutrino one. The electrons (positrons) produced in a far detector by the charged current interaction of the electron neutrinos (antineutrinos) are the signature for the $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$) oscillation process, once the direct ν_e ($\bar{\nu}_e$) background is eliminated. Several obstacles can stand on the way of the detection of CP violation through the ν $\bar{\nu}$ asymmetry. One is that the interactions of neutrinos and antineutrinos with any nucleus are not identical but they differ by the sign of the axial-vector interference term, creating an asymmetry unrelated to CP violation and which must be fully mastered. This is not a trivial task due to the complexity of the nuclear dynamics. It is reflected in the neutrino interactions and may obscure the message that one wants to extract on the oscillation mechanism. One example concerns the role of the multinucleon emission process, which in a Cherenkov detector is misidentified as a quasielastic one [1,2]. This error produces an apparent increase of the neutrino quasielastic cross section, at the origin of the so-called axial mass anomaly found in the MiniBooNE experiments [3] as now widely accepted [2,4–15]. The detection of CP violation, which involves a comparison between neutrino and antineutrino events, needs an even more detailed understanding of the multinucleon processes because it concerns the difference between neutrino and antineutrino

cross sections. This understanding is not trivial and it is the objective of the present article.

II. ANALYSIS AND RESULTS

To illustrate the respective roles of the multinucleon components in the ν and $\bar{\nu}$ cross sections, as we have introduced in a previous work [4], we start by giving below the following simplified expression (we remind the reader however that for the actual evaluation we use a more complete one) for the double-differential neutrino or antineutrino cross sections on a nuclear target such as ^{12}C :

$$\begin{aligned} \frac{d^2\sigma}{d\cos\theta d\omega} = & \frac{G_F^2 \cos^2\theta_c}{\pi} k_l E_l \cos^2\frac{\theta}{2} \left[\frac{(\mathbf{q}^2 - \omega^2)^2}{\mathbf{q}^4} G_E^2 R_\tau \right. \\ & + \frac{\omega^2}{\mathbf{q}^2} G_A^2 R_{\sigma\tau(L)} + 2 \left(\tan^2\frac{\theta}{2} + \frac{\mathbf{q}^2 - \omega^2}{2\mathbf{q}^2} \right) \\ & \times \left(G_M^2 \frac{\omega^2}{\mathbf{q}^2} + G_A^2 \right) R_{\sigma\tau(T)} \\ & \left. \pm 2 \frac{E_v + E_l}{M_N} \tan^2\frac{\theta}{2} G_A G_M R_{\sigma\tau(T)} \right], \quad (1) \end{aligned}$$

where the plus (minus) sign applies to neutrinos (antineutrinos). In Eq. (1), G_F is the weak coupling constant; θ_c is the Cabibbo angle; G_E , G_M , and G_A are the nucleon electric, magnetic, and axial form factors; E_v and E_l are the initial and final lepton energies; k_l is the modulus of the final lepton momentum; ω and \mathbf{q} are the energy and the momentum transferred to the nucleus; and θ is the lepton scattering angle. The cross section on the nuclear target is expressed here in terms of the nuclear responses R to probes with various couplings to the nucleon, isovector (index τ), or isovector with isospin and spin coupling (index $\sigma\tau$). For the last responses the isovector spin coupling can be spin transverse, $\sigma\tau(T)$, or spin longitudinal, $\sigma\tau(L)$. The responses are \mathbf{q} and ω dependent. The last term of Eq. (1), which changes sign between ν and $\bar{\nu}$, is the axial-vector interference term, the basic asymmetry which

follows from the weak interaction theory. It is expressed here in terms of the isospin spin-transverse nuclear response. Its evaluation for complex nuclei is not trivial due to the presence of many-body effects. This is why it is important to obtain experimental information on this term, the object of the present work.

For our description of the multinucleon component of the responses, we followed the experimental indications provided by the electron scattering data. In the transverse, or magnetic, data the dip between the quasielastic and the Δ part of the response is filled, which we interpret as an indication of the presence of two-nucleon emission [16]. In the charge response instead this component is absent. With these indications, for neutrinos we have introduced the multinucleon component only in the spin isospin response, the pure isovector one keeping its quasielastic character.

We have previously tested our model on the MiniBooNE data for the differential cross sections [3,17], independently for neutrinos [8] and antineutrinos [13] reaching a good fit of the data. However in these works, the test was performed separately for the neutrinos and the antineutrinos and we did not specifically address the detailed comparison between the two. Because this comparison is essential for the CP violation detection we want to investigate in the present work this question in more detail. Are we able to describe quantitatively the difference between neutrino and antineutrino nuclear interactions? The wealth of experimental data that has accumulated in the last years by the MiniBooNE experiment allows an experimental test for this comparison and we explore the message that can be drawn from these data. The quantity best suited for this exploration is the measured double-differential quasielastic-like (i.e., which incorporates also multinucleon states) cross section $\frac{d^2\sigma}{d\cos\theta dE_\mu}$ with respect to the muon energy E_μ and the muon emission angle θ , which is not affected by the reconstruction problem of the neutrino energy [11,18–21]. However it depends on the flux energy profile, with

$$\frac{d^2\sigma}{d\cos\theta dE_\mu} = \int \frac{d^2\sigma}{d\cos\theta d\omega} \Big|_{\omega=E_\nu-E_\mu} \Phi(E_\nu) dE_\nu, \quad (2)$$

where ω is the energy transferred to the nucleus and $\Phi(E_\nu)$ is the neutrino (or antineutrino) normalized flux energy distribution. Neutrino experiments measure $\frac{d^2\sigma}{d\cos\theta dE_\mu}$ while nuclear physics evaluations calculate the quantity $\frac{d^2\sigma}{d\cos\theta d\omega}$, which is the basic ingredient for all analysis on neutrino data. The flux dependence in $\frac{d^2\sigma}{d\cos\theta dE_\mu}$ is *a priori* an obstacle to extracting a universal comparison between the two cross sections for neutrinos and antineutrinos, applicable to any CP violation experiments. For each set of flux profiles the measured differential cross sections are different. In particular the ν and $\bar{\nu}$ asymmetry for $\frac{d^2\sigma}{d\cos\theta dE_\mu}$ has two sources, one arises from the basic weak interaction theory, as given in Eq. (1). The second one, which arises from the flux asymmetry, has no universal character and is specific for each experiment. Is there nevertheless something general and informative in the MiniBooNE data is the question that we address.

Let us consider the following combinations: the sum, *sum*, and the difference, *dif*, of the double-differential cross sections for neutrinos and antineutrinos with respect to the lepton emission angle θ and to the energy ω transferred to the nucleus,

$$sum(\cos\theta, \omega) = \frac{d^2\sigma_\nu}{d\cos\theta d\omega} + \frac{d^2\sigma_{\bar{\nu}}}{d\cos\theta d\omega}, \quad (3)$$

while for the difference *dif* the plus sign is changed to a minus sign. Notice from Eq. (1) that the difference, *dif*, contains only one term, the axial-vector interference one. It is this quantity that governs the difference in cross sections of ν and $\bar{\nu}$ but it is not accessible directly from neutrino data because only derivatives with respect to the emitted lepton energy are measurable. We also introduce S and D , the corresponding flux integrated quantities, which are instead experimentally accessible in the MiniBooNE data,

$$S(\cos\theta, E_\mu) = \frac{d^2\sigma_\nu}{d\cos\theta dE_\mu} + \frac{d^2\sigma_{\bar{\nu}}}{d\cos\theta dE_\mu}, \quad (4)$$

$$D(\cos\theta, E_\mu) = \frac{d^2\sigma_\nu}{d\cos\theta dE_\mu} - \frac{d^2\sigma_{\bar{\nu}}}{d\cos\theta dE_\mu},$$

in which quantities such as $\frac{d^2\sigma_\nu}{d\cos\theta dE_\mu}$ have been defined previously in Eq. (2). The MiniBooNE experimental distribution of $D(\cos\theta, E_\mu)$ has been given by Grange and Katori [22] in a tridimensional plot.

For identical normalized neutrino and antineutrino flux profiles, $\Phi_\nu(E_\nu) \equiv \Phi_{\bar{\nu}}(E_\nu)$, this common value can be factorized in the integrals over the neutrino energies implicitly contained in the above Eqs. (4). In this case only the axial-vector interference term survives in the difference D , while the sum S totally eliminates this term, which would allow a direct experimental evaluation of the interference part. However this is not quite realized as is shown in Fig. 1, which compares the two normalized MiniBooNE fluxes. To assess the influence of the flux difference, which is mild, we express the two cross-section combinations in terms of the average flux, $\Phi_+ = 1/2[\Phi_\nu + \Phi_{\bar{\nu}}]$, and of the flux difference, $\Phi_- = 1/2[\Phi_\nu - \Phi_{\bar{\nu}}]$, as follows:

$$S(\cos\theta, E_\mu) = \int dE_\nu [sum(\cos\theta, \omega)|_{\omega=E_\nu-E_\mu} \Phi_+(E_\nu) + dif(\cos\theta, \omega)|_{\omega=E_\nu-E_\mu} \Phi_-(E_\nu)] \quad (5)$$

and

$$D(\cos\theta, E_\mu) = \int dE_\nu [sum(\cos\theta, \omega)|_{\omega=E_\nu-E_\mu} \Phi_-(E_\nu) + dif(\cos\theta, \omega)|_{\omega=E_\nu-E_\mu} \Phi_+(E_\nu)]. \quad (6)$$

Again, for identical fluxes, $\Phi_-(E_\nu) = 0$, D would only probe the quantity *dif*, i.e., the axial-vector interference term.

The fact that the flux difference is moderate raises the following question: what remains in the MiniBooNE data of the purity of the difference D with respect to the axial-vector interference term and accordingly of its elimination in the sum S ? For this we have evaluated the sum S and the difference D with the real MiniBooNE fluxes on the one hand and with only the mean flux $\Phi_+(E_\nu)$ on the other hand. These evaluations are

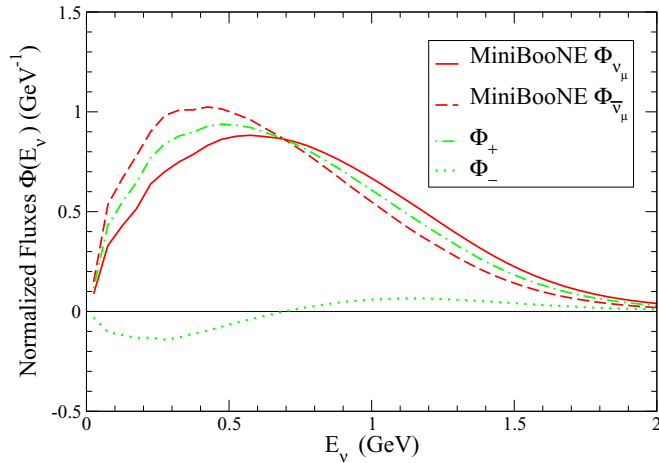


FIG. 1. (Color online) Normalized MiniBooNE ν_μ and $\bar{\nu}_\mu$ fluxes derived from Refs. [3] and [17], respectively. Their half sum (Φ_+) and half difference (Φ_-) are also shown.

performed with our theoretical model described in Refs. [2,8]. It incorporates the multinucleon component of the cross section and the responses are treated in the random-phase approximation (RPA).

For the sum S , it turns out that for all angles and in the full range of muon energies, the two sets of curves are extremely close, such that there is very little sensitivity in the sum on the axial-vector interference term. For the difference D , the two

sets of curves, displayed in Fig. 2, are quite close up to a muon kinetic energy of $T_\mu \simeq 0.6$ to 0.7 GeV. Beyond this energy they progressively depart. Despite the moderate deviations at large muon energies, for all T_μ values the contribution from the mean flux remains dominant also in the difference D .

This is fortunate because it implies that the axial-vector interference term is experimentally accessible in the MiniBooNE data through this neutrino and antineutrino cross-section difference. We can in particular test if our multinucleon component, which in our model is maximum in the axial-vector interference term, i.e., in the difference D , is compatible, or not, with the data. While with the sum of the neutrino and antineutrino cross sections instead we can explore its role in the remaining part of the cross section. This result is general: when the ν and $\bar{\nu}$ normalized flux profiles are close, the difference in the measurable differential cross sections between neutrinos and antineutrinos is dominated by the axial-vector interference term. This is the case for the T2K beams [23] and also for the NuMI [24] beams, the ones used in the MINOS, MINER ν A, and NO ν A experiments.

We have seen the message carried by the comparison with the mean flux curves, namely, that the axial-vector interference term dominates the difference and has very little influence on the sum. For the test of our theoretical model on these combinations, to avoid unnecessary errors in the following we calculate them with the *real* neutrinos and antineutrinos fluxes. Our present results are then the sum and the difference of our previously published neutrino [8] and antineutrino [13] results. In Figs. 3 and 4 we display our calculated values of $S(\cos\theta, E_\mu)$

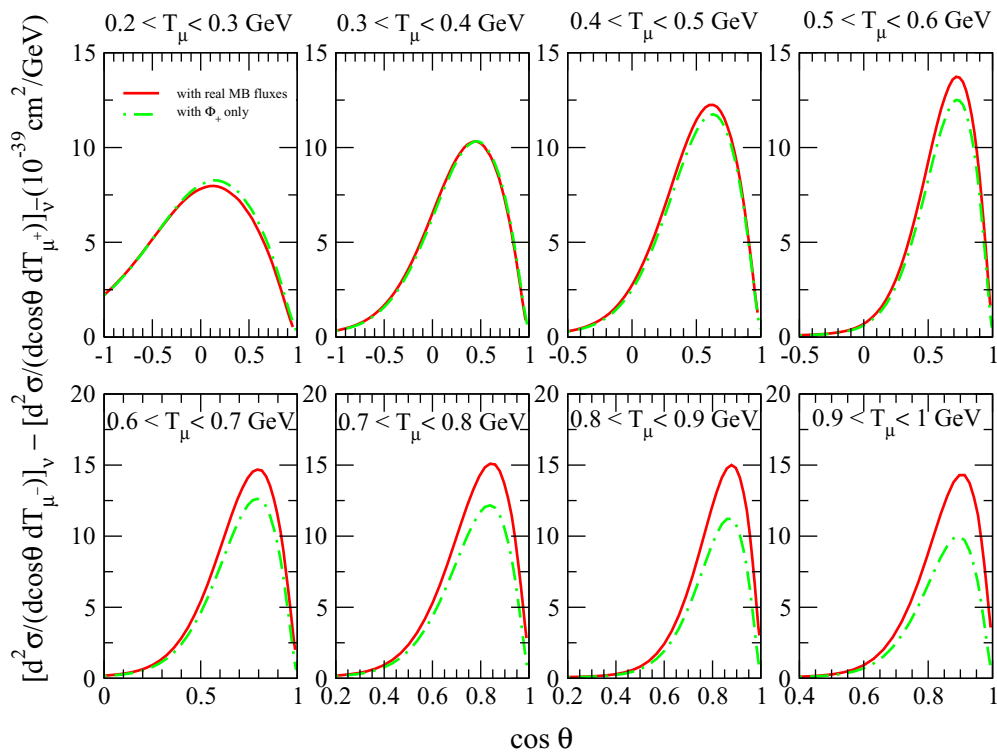


FIG. 2. (Color online) Difference of the ν and $\bar{\nu}$ flux-folded double-differential cross sections on carbon per active nucleon plotted as a function of the $\cos\theta$ for different values of the emitted muon kinetic energies. Continuous line: evaluation (RPA + np-nh) with the real ν and $\bar{\nu}$ MiniBooNE fluxes; dot-dashed line: evaluation (RPA + np-nh) with the mean flux Φ_+ .

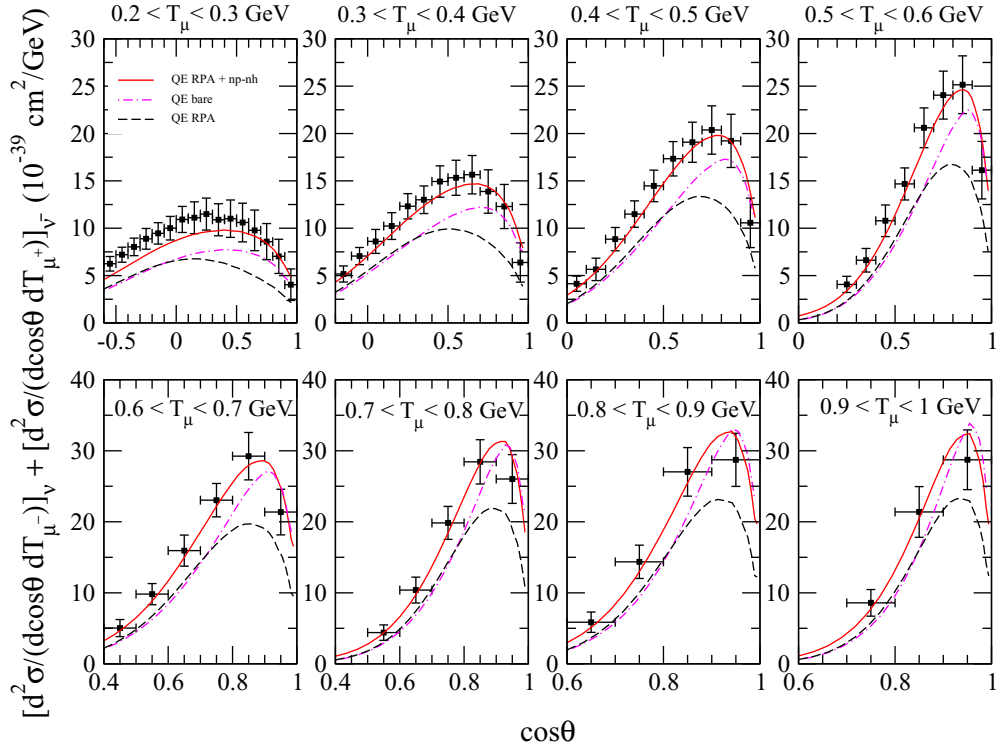


FIG. 3. (Color online) Sum S of the ν and $\bar{\nu}$ MiniBooNE flux-folded double-differential cross sections on carbon per active nucleon plotted as a function of the $\cos \theta$ for different values of the emitted muon kinetic energies. Continuous line: our complete RPA evaluation including the multinucleon emission channel; dashed line: genuine quasielastic contribution calculated in RPA; dot-dashed line: quasielastic contribution in the bare case. The points are the combination of the MiniBooNE experimental results [3,17]. For the neutrino and antineutrino data there are additional normalization uncertainties of 10% and 17.2%, respectively, not taken into account here.

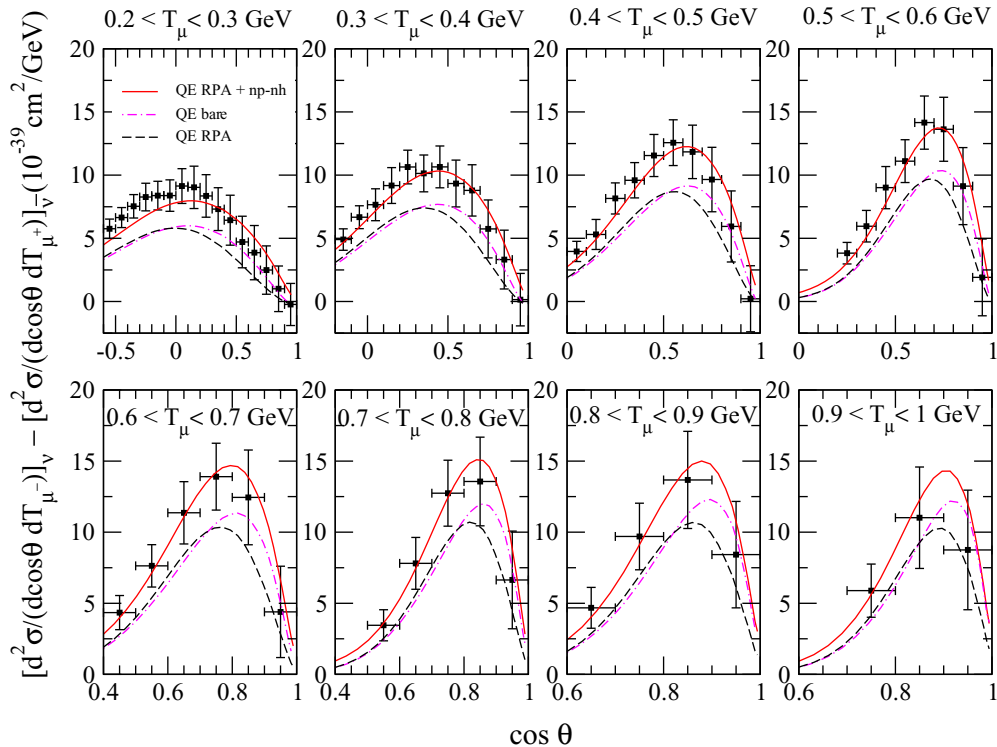


FIG. 4. (Color online) The same as Fig. 3 but for the difference D .

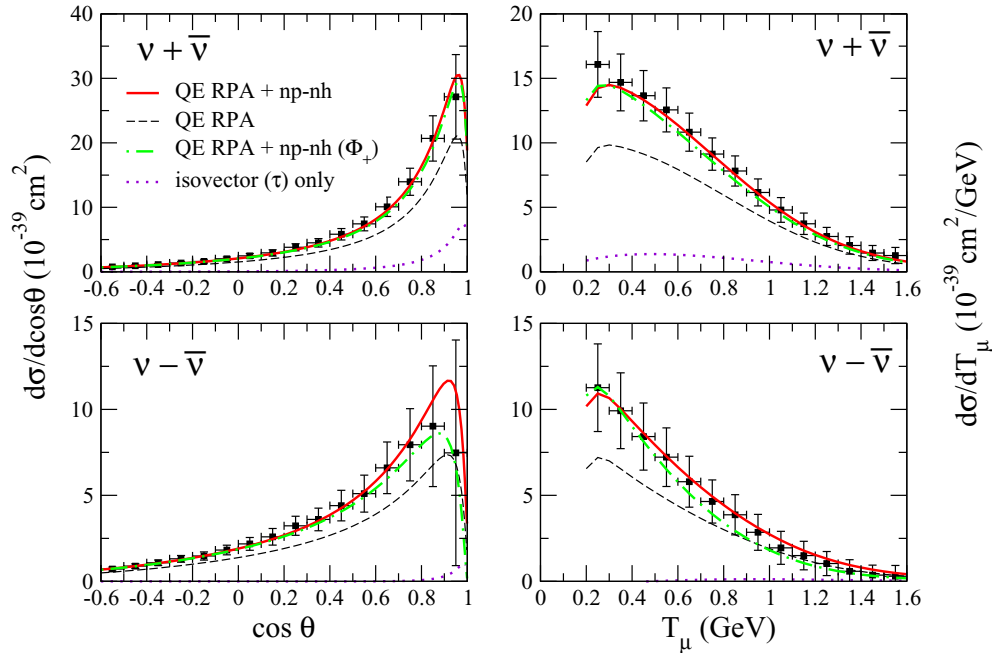


FIG. 5. (Color online) Sum and difference of the ν and $\bar{\nu}$ flux-folded differential cross sections $\frac{d\sigma}{d\cos\theta}$ and $\frac{d\sigma}{dT_\mu}$ on carbon per active nucleon. Continuous line: complete RPA evaluation including the multinucleon emission channel with the real ν and $\bar{\nu}$ MiniBooNE fluxes; dashed line: genuine quasielastic contribution calculated using the RPA with the real ν and $\bar{\nu}$ MiniBooNE fluxes; dot-dashed line: complete RPA evaluation including the multinucleon emission channel with the mean flux Φ_+ ; dotted line: contribution of the isovector response only.

and $D(\cos\theta, E_\mu)$ as a function of the muon emission angle, for various values of the muon kinetic energy, together with the experimental data points. Our predictions, which incorporate the multinucleon component account quite well for the data for all angles and in the full range of muon energies, both for the sum and for the difference. Only for the smallest T_μ bin can some small deviations be observed.

This test of the validity of our model for these combinations is important because it addresses directly the question of our understanding of the neutrino versus antineutrino interactions, in particular for what concerns the crucial and more debated role of the multinucleon component. Our predicted cross sections without the multinucleon part are also shown in Figs. 3 and 4; they definitely fail to account for the data both for the sum and the difference. Another check remains to be performed to fully confirm the necessity of the multinucleon piece, in particular, in the axial-vector interference term: in our description the collectivity of the quasielastic responses has been included in the form of the RPA treatment. It produces a suppression effect of the quasielastic part due to the repulsive character of the particle-hole force [25]. For the spin-isospin transverse response it is the Ericson-Ericson-Lorentz-Lorenz quenching arising from the mixture of Δ -hole states into nucleon-hole ones [26]. The question is then the following: if a good fit is obtained with the combined and opposite effects of the RPA and of the multinucleon component, could it be that a similar good fit would be achieved by omitting both effects? Would the simplest quasielastic description also account for the data? In Figs. 3 and 4 the effect of the RPA is suppressed in the quasielastic cross section, which indeed has some enhancement effect. For the cross-section difference this enhancement is moderate and not enough to account

by itself for the data. We can safely conclude that a large multinucleon component is needed to describe the data for the axial-vector interference term that governs difference of the cross sections. The same conclusion applies to the sum of the cross sections, i.e., to the remaining part of the interaction; it is also appreciably influenced by the multinucleon component. Our model for the neutrino nucleus interaction is able to describe both components. As an additional illustration we report in Fig. 5 the single-differential cross sections, with respect to the muon kinetic energy or to the muon emission angle. As previously we deal with the sum and the averages fluxes. We can observe that the sum shows practically no sensitivity to the flux difference, while the difference of the cross sections displays a mild sensitivity, in particular, in the forward direction or at large T_μ values. Our predictions, which include the multinucleon component, reproduce well the data. In our model the relative importance of the multinucleon term in the cross-section combinations depends on the role of the isovector response, which is shown in Fig. 5; the smaller this role the larger the multinucleon contribution is. This last contribution is the largest in the difference of the cross sections. Similarly the isovector response weight is larger in antineutrino cross sections than in neutrino ones; hence the smaller multinucleon contribution for antineutrinos. It is, however, not a large difference and the multinucleon influence remains important also for antineutrinos.

Finally for completeness we combine in Fig. 6 our previous evaluations of the neutrino and antineutrino Q^2 distributions, published in Refs. [8] and [13], respectively, to evaluate their sum and their difference. We also display the result obtained with the averaged flux Φ_+ . As previously the difference

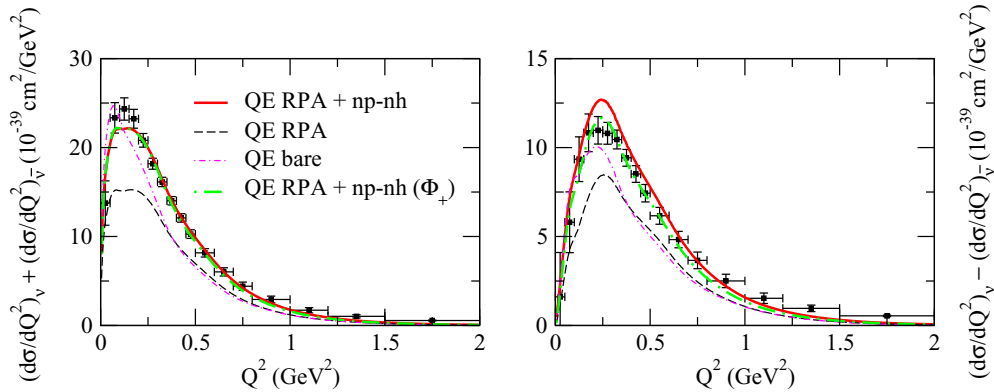


FIG. 6. (Color online) Sum and difference of the ν and $\bar{\nu}$ flux-folded Q^2 distributions on carbon per active nucleon. Continuous line: complete RPA evaluation including the multinucleon emission channel with the real ν and $\bar{\nu}$ MiniBooNE fluxes; dashed line: genuine quasielastic contribution calculated in RPA with the real ν and $\bar{\nu}$ MiniBooNE fluxes; thin dot-dashed line: quasielastic contribution calculated in the bare case with the real ν and $\bar{\nu}$ MiniBooNE fluxes; thick dot-dashed line: complete RPA evaluation including the multinucleon emission channel with the mean flux Φ_+ .

is more sensitive to the mean flux approximation, which, however, gives the bulk of this difference. As a consequence the difference of the experimental MiniBooNE points $(\frac{d\sigma}{dQ^2})_\nu - (\frac{d\sigma}{dQ^2})_{\bar{\nu}}$ is directly related to the Q^2 distribution of the axial-vector interference term. This conclusion would apply as well to the MINER ν A neutrino [27] and antineutrino [28] Q^2 distributions, due to the closeness of the neutrino and antineutrino normalized fluxes. It will be the object of a future investigation.

Notice that the present study is done for the muonic neutrinos of the MiniBooNE experiment, for which data are available, while CP violation experiments through the asymmetry of the oscillations rates of muonic neutrinos and antineutrinos into electron ones involve the detection of electrons or positrons produced in a detector by the charged current interactions of these electron neutrinos. We have already addressed the question of electron neutrino cross sections versus the muon neutrino ones [21]. The effect of the small change in kinematics due to the smaller electron mass will not affect the present conclusions: the electron neutrino nuclear interactions produce by themselves an important $\nu_e \bar{\nu}_e$ asymmetry. The present study indicates that the multinucleon role is essential in this problem and to what precision this asymmetry can be mastered.

III. SUMMARY AND CONCLUSIONS

In summary we have investigated two combinations of the neutrino and antineutrino MiniBooNE flux-folded double-differential cross sections on ^{12}C , their sum and their difference, which probe different pieces of the neutrino or antineutrino interactions with the nucleus. These quantities depend on the neutrino or antineutrino normalized energy flux profiles. In the case of identical ones, the difference provides

a direct access to the axial-vector interference term, while the sum eliminates it. For the MiniBooNE fluxes we have tested how much the flux difference influences the two combinations. We have shown that this influence is small both on the sum and on the difference of the cross sections. These combinations remain rather pure with respect to the axial-vector interference term, which is either dominant (difference) or nearly absent (sum). This allows more specific tests of our theoretical model on the axial-vector interference term, important for the CP violation data. Our model gives a good fit for the MiniBooNE data for the sum and the difference of the cross sections reproducing well the data in the full range of muon energy and emission angle. The introduction of the multinucleon component is necessary for a good fit, an important test for its presence in the axial-vector interference term. The success of our description indicates that we can reach a good understanding of the nuclear effects in neutrino interactions, also for what concerns the comparison between neutrino and antineutrino cross sections. We have concentrated in this work on the interactions of muonic neutrinos where a complete set of data is available. Our predictions can easily be extended to electron neutrinos, relevant for CP violation data. The nuclear cross-section difference for neutrinos and antineutrinos stands as a potential obstacle in the interpretation of experiments aimed at the measurement of the CP violation angle, δ . The present analysis, performed on the MiniBooNE data, shows the importance, for Cherenkov detectors, of the inclusion of the multinucleon contribution in the quasielastic-like cross section for mastering this difference.

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