Scalable Macromodeling of Microwave Systems Responses using Sequential Sampling with Path-Simplexes

K. Chemmangat, T. Dhaene and L. Knockaert

A scattered sequential sampling algorithm for the automatic construction of stable and passive scalable macromodels of parameterized system responses with a well-conditioned refinement strategy using path-simplexes is proposed. The method is tailored towards the local scalable macromodeling schemes on scattered grids. Pertinent numerical example validates the proposed approach.

Introduction: Efficient design of electromagnetic (EM) systems often requires expensive simulations using EM solvers which normally provide high accuracy at a significant cost in terms of memory storage and computing time. Alternatively, scalable macromodels can be used, which approximate the complex behavior of EM systems, characterized by frequency and additional design parameters, such as geometrical or substrate features. Scalable macromodeling of EM systems has attracted a lot of attention during recent years [1, 2]. However, one of the key issues in these modeling approaches is that they select the number of modeling samples *a priori* which might result in under sampling or over sampling at the cost of computational resources.

In this paper the state-of-the art scalable macromodeling schemes are automated with the help of a scattered sampling scheme which works on local refinement of well-conditioned simplexes such that optimum number of data samples are selected [**?**]. The refinement on the simplexes can be done in many ways such as dividing along in-center. However, this might lead to the creation of ill-conditioned simplexes called *slivers*. Generation of slivers can be avoided by refining either locally [3, 4] or globally [5, 6]. The local refinement scheme [3, 4] starts from the corner points of an N-cube and then refines it into smaller simplexes in a tree fashion like the sequential sampling method of [**?**], whereas the global refinement schemes [5, 6] work on a primary Delaunay tessellation and then refine it to improve the condition of simplexes. Hence the local path-simplex method [3, 4] assures good condition number from the beginning of the sampling process and is suitable for the application of different passivity-preserving scalable macromodeling algorithms on scattered grids [1, 2]. In case if the global refinement schemes [5, 6] were used, the existing mesh has to undergo global refinement indicating that the local interpolated models change significantly with a consequent computational burden. Moreover, the tessellation generated by the method of path-simplexes is proved to be Delaunay by construction [4], and for the above mentioned reasons it is used in this paper.

Passivity Preserving Scalable Macromodeling: In this letter, we use one of the local scalable macromodeling schemes which use the Vector Fitting (VF) technique [7] to build frequency-dependent rational models called *root macromodels* at the selected design space samples and then parameterize them, see [1, 2]. These methods preserve stability and passivity over the complete design space, and therefore are suitable for time-domain simulations. The scalable macromodeling process starts with a set of multivariate data samples $\{(s,\vec{g})_k, \mathbf{H}(s,\vec{g})_k\}_{k=1}^{K_{tot}}$ which depends on frequency and additional design variables. From these data samples, a set of *root macromodels* in pole-residue form are built for a set of design space samples $\vec{g}_k$ by means of VF yielding a set of *root macromodels* $\mathbf{R}(s,\vec{g}_k)$. Stability and passivity are enforced using robust standard techniques [8, 7], resulting in a set of stable and passive *root macromodels*. The next step of these scalable macromodeling algorithms is the parameterization of the set of *root macromodels* $\mathbf{R}(s,\vec{g}_k)$. In [1], a scalable macromodel is built by interpolating a set of *root macromodels* at an input-output level, while in [2], a novel enhanced interpolation of *root macromodels* is described, which results in high modeling capability and robustness in comparison to [1].

Refinement using Well Conditioned Path-Simplexes: A path-simplex in $\mathbf{R}^N$ is defined as an N-Simplex having N mutually orthogonal edges which, in the sense of graph theory, form a path [4]. The property of a path-simplex which makes it attractive for the proposed sequential sampling is the fact that it is a non-obtuse simplex. This ensures that, slivers are never created during the local refinement of a simplex, ensuring convergence of the algorithm. The proposed sequential sampling algorithm starts from a single N-box region of the design space which is then normalized to a N-cube and divided into $N!$ path-simplexes using the result of [4].

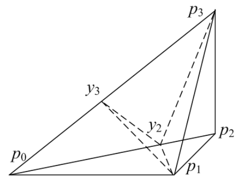

Fig. 1. Coexter's trisection of the path-simplex in $\mathbf{R}^3$ (as in [3]).

In [3], Brandts et al. prove that given a path-simplex in $\mathbf{R}^N$, it can be divided into N path-subsimplexes using Coxeter's trisection method generating $N-1$ new sample points. Fig. 1 shows such a division for a path-simplex in $\mathbf{R}^3$. The corners of the path-simplex are represented by the position vectors p_0, p_1, p_2, and p_3 with respect to any arbitrary origin, and the edges p_0-p_1, p_1-p_2, and p_2-p_3 forming a path. Three new path-simplexes are formed using the points y_2 and y_3 calculated as

$$y_j = p_j(\|p_1\|^2/\|p_j\|^2),\ j=2,3,...N., \tag{1}$$

where, $\|.\|$ is the Euclidean norm [3]. Generation of slivers during the local refinement can be monitored by calculating the aspect ratio $R_{\text{asp}} = N\frac{d}{D}$ where $\frac{d}{D}$ is the ratio of the diameters of the inscribing and circumscribing N-spheres of the N-Simplex. *Root macromodels* are created at the corner points of these simplexes and using the scalable macromodeling method of [2], passive interpolated models are created for the parameterized frequency responses.

Proposed Sequential Sampling Algorithm: The sequential sampling algorithm consists of the following steps:

I) *Initialization*: Define a N-box design space with N design variables $\vec{g} = (g^{(1)},\dots,g^{(N)})$ and generate $Q = N!$ path-simplexes.

II) Update the scalable macromodel $\mathbf{R}(s,\vec{g})$ for the entire design space with Q path-simplexes using the method of [2].

III) For each path-simplex $q = 1,\dots,Q$, check the error criteria at its in-center,

   i. IF: ($Err_q > \Delta$): Divide q^{th}path-simplex into N path-subsimplexes [3], update $Q = Q+N-1$, $q = q+1$ and go to Step II.

   ii. ELSE: increment $q = q+1$. IF ($q \le Q$): Not all subspaces are checked for the error criteria, go to Step III; ELSE: *Termination*

Numerical Example: The S-Parameter response of a Hairpin bandpass filter generated with the help of ADS Momentum[1] on a substrate with relative permittivity $\epsilon_r = 9.9$ and a thickness of 0.0635 mm is modeled (Fig. 2). Two spacings $S_1 \in [0.25, 0.35]$ mm and $S_2 \in [0.65, 0.75]$ mm and two lengths $L_1 \in [12.0, 12.5]$ mm and $L_2 \in [2.75, 3.25]$ mm are chosen as design variables (see Fig. 2) in addition to frequency $\in [1.5, 3.5]$ GHz. The parametric behavior of the filter is shown in Fig. 3.

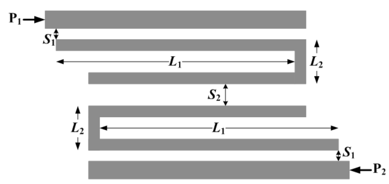

Fig. 2. Layout of the microwave hairpin bandpass filter.

For the sequential sampling the Mean Absolute Error (MAE) is used

$$E^{\text{MAE}}(\vec{g}) = \sum_{i=1}^{P_{in}}\sum_{j=1}^{P_{out}}\sum_{k=1}^{N_s}\frac{|R_{i,j}(s_k,\vec{g}) - H_{i,j}(s_k,\vec{g})|}{P_{in}P_{out}N_s}, \tag{2}$$

to assess the accuracy of the model at the in-center of each simplex. The method compares the actual EM simulation response $H_{i,j}(s,\vec{g})$ to the scalable macromodel response $R_{i,j}(s,\vec{g})$, with P_{in} input ports, P_{out}

[1] Momentum EEsof EDA, Agilent Technologies, Santa Rosa, CA.

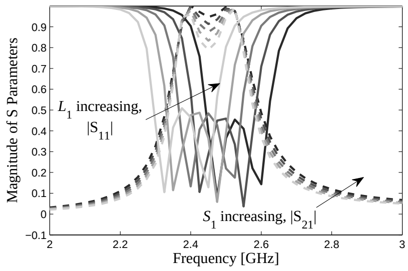

Fig. 3. Parameterization: $|\mathbf{S}_{11}|$ and $|\mathbf{S}_{21}|$ as a function of L_1 and S_1 resp.

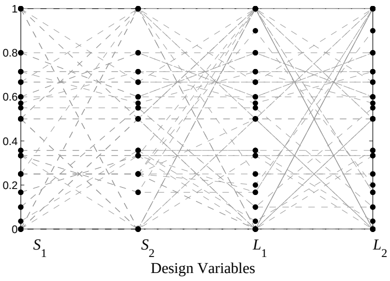

Fig. 4. Design space generated for Hairpin Filter

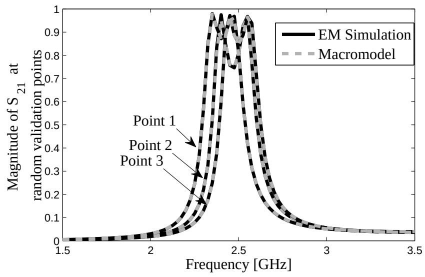

Fig. 5. Magnitude of $\mathbf{S}_{21}$ at three random validation points.

output ports and N_s frequency samples. The proposed sequential sampling algorithm is used along with the scalable macromodeling method of [9]. The MAE measure of (2) is used to assess the accuracy of the models generated with a target accuracy of -50 dB. This resulted in the selection of 68 design space points, with an achieved accuracy of -50.21 dB.

Fig. 4 shows the normalized values of the 68 design space points selected, using a parallel coordinate plot [10]. In Fig. 4, the black dots represent the sample points selected for each design variables with the gray lines representing different samples points in four dimension. The minimum aspect ratio was found to be equal to 0.0514, meaning no slivers were created. Fig. 5 compares the magnitude of S-parameter matrix entry $\mathbf{S}_{21}$ between the actual momentum simulation with the macromodel for three random validation points in the design space and the responses overlap showing the accuracy. In order to check the passivity, the $\mathbf{H}_\infty$ norm $||\mathbf{R}(s,(S_1,S_2,L_1,L_2)||_\infty$ of the scalable macromodel for a dense grid of $5\times5\times5\times5$ (S_1,S_2,L_1,L_2) was calculated and was found to be within the passivity bound, $||\mathbf{R}(s,S_1,S_2,L_1,L_2)||_\infty \leq 1$.

Conclusion: We have presented a scattered sampling algorithm for the automatic construction of stable and passive macromodels of parameterized system responses. The proposed method avoids the generation of slivers by using path-simplex based refinement. The proposed technique is validated on a pertinent numerical example.

Acknowledgment: This work was supported by the Research Foundation Flanders (FWO) and by the Interuniversity Attraction Poles Programme BESTCOM initiated by the Belgian Science Policy Office.

K. Chemmangat, T. Dhaene and L. Knockaert (*Ghent University - IBBT, Gaston Crommenlaan 8 Bus 201, B-9050 Gent, Belgium*)

E-mail: krishnan.cmc@intec.ugent.be

References

1 F. Ferranti, L. Knockaert, and T. Dhaene, “Passivity-preserving interpolation-based parameterized macromodeling of scattered s-data,” *IEEE Microwave and Wireless Components Letters,*, vol. 20, no. 3, pp. 133 –135, March 2010.

2 F. Ferranti, L. Knockaert, T. Dhaene, and G. Antonini, “Parametric macromodeling based on amplitude and frequency scaled systems with guaranteed passivity,” *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 25, no. 2, pp. 139–151, 2012.

3 K. Chemmangat, F. Ferranti, T. Dhaene, and L. Knockaert, “Scalable models of microwave system responses using sequential sampling on unstructured grids,” *Int. J. of Numerical Modelling: Electronic Networks, Devices and Fields*, 2013. [Online]. Available: http://dx.doi.org/10.1002/jnm.1903

4 J. Brandts, S. Korotov, and M. Krizek, “Dissection of the R^n path-simplex in into n path-subsimplices,” *Linear Algebra and its Applications*, vol. 421, no. 23, pp. 382 – 393, 2007.

5 J. Brandts, S. Korotov, M. Krizek, and J. Solc, “On nonobtuse simplicial partitions,” *SIAM Review*, vol. 51, no. 2, pp. 317–335, 2009.

6 X. Li, “Generating well-shaped d-dimensional Delaunay meshes,” *Computing and Combinatorics*, pp. 91–100, 2001.

7 A. Constantiniu, P. Steinmann, T. Bobach, G. Farin, and G. Umlauf, “The adaptive delaunay tessellation: a neighborhood covering meshing technique,” *Computational Mechanics*, vol. 42, no. 5, pp. 655–669, 2008.

8 B. Gustavsen and A. Semlyen, “Rational approximation of frequency domain responses by vector fitting,” *IEEE Transactions on Power Delivery*, vol. 14, no. 3, pp. 1052–1061, July 1999.

9 B. Gustavsen, “Fast passivity enforcement for S-parameter models by perturbation of residue matrix eigenvalues,” *IEEE Transactions on Advanced Packaging*, vol. 33, no. 1, pp. 257–265, Feb. 2010.

10 F. Ferranti, L. Knockaert, and T. Dhaene, “Passivity-preserving parametric macromodeling by means of scaled and shifted state-space systems,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 10, pp. 2394 –2403, Oct. 2011.

11 A. Inselberg, “FT-1 the plane with parallel coordinates,” in *Parallel Coordinates*. Springer New York, 2009, pp. 49–61.