Data-Driven Model Order Reduction Using Orthonormal Vector Fitting

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Accurate frequency-domain macromodels are becoming increasingly important for the design, study and optimization of complex physical systems. These macromodels approximate the complex frequency-dependent input-output behaviour of broadband multi-port systems in the frequency domain by rational functions [28]. Unfortunately, due to the complexity of the physical systems under study and the dense discretization required for accurately modelling their behaviour, the rational or state-space macromodels may lead to unmanageable levels of storage and computational requirements. Therefore, Model Order Reduction (MOR) methods can be applied to build a model of reduced size, which captures the dynamics of the larger model as closely as possible.

Orthonormal Vector Fitting (OVF) [5, 9] is an identification method, which is typically used to approximate simulated or measured frequency responses by an analytic function. In this chapter, it is shown that the OVF method can also be seen as a *data-driven* MOR method. Rather than reducing the dimensions of the state-space matrices of a model directly (*model-based* MOR), this technique is used to build a new state-space model with a reduced model complexity based on input-output data. The goal of this algorithm is to parameterize a rational transfer function, such that its spectral behaviour matches the response of the larger model in a least-squares sense.

Most available identification methods suffer poor numerical conditioning for large state-space dimensions or broad frequency ranges. The OVF method tackles these issues by combining the benefits of a Sanathanan-Koerner iteration [32] and a well-chosen set of orthonormal rational basis functions. It is shown that the method is applicable to reduce systems with a large amount of poles. The method does not preserve passivity by default, however several techniques are available to enforce a desired physical behaviour in a post-processing step [12, 14].

1 Identification Problem

1.1 Goal

The major goal of the algorithm is to identify the mapping between the inputs and outputs of a complex system by an analytic model of reduced size. For continuous-time linear-time-invariant (LTI) systems in the frequency domain, this reduces to finding a rational transfer function

$$R(s) = \frac{N(s)}{D(s)} = \frac{\sum_{n=0}^{N} N_n \phi_n(s)}{\sum_{d=0}^{D} D_d \phi_d(s)} \quad s = i2\pi f \tag{1}$$

which approximates the spectral response of a system over some predefined frequency range of interest $[f_{min}, f_{max}]$. The spectral behaviour is characterized by a set of frequency-domain data samples $(s_k, H(s_k))$, $\forall k = 0, ..., K$, which are obtained by evaluating the state-space matrices of the large model. N_n and D_d are the real-valued system parameters which need to be estimated, and N and D represent the order of numerator and denominator respectively. In practice, N and D are chosen to be much smaller than the order of the large model. A dense frequency sweep is required in many situations, so the amount of available data samples can be quite numerous. Therefore, numerically stable fitting techniques are required which estimate the model coefficients in a least-squares sense [10].

1.2 Non-linearity of the Estimator

Rational least-squares approximation is essentially a non-linear problem, and corresponds to minimizing the following cost function [29]

$$\arg \min_{N_n, D_d} \sum_{k=0}^{K} \left| H(s_k) - \frac{N(s_k)}{D(s_k)} \right|^2 = \arg \min_{N_n, D_d} \sum_{k=0}^{K} \frac{|D(s_k)H(s_k) - N(s_k)|^2}{|D(s_k)|^2} . \tag{2}$$

Due to its non-linear nature, it can be hard to estimate the system parameters in a fast and accurate way. In many papers, e.g. [34], this difficulty is avoided by assuming that a-priori knowledge about the poles is available. In this case, the non-linear problem reduces to a linear problem since the denominator parameters are assumed to be known. In practice, however, this situation is often not a realistic one. Another possible option is the use of non-linear optimization techniques, such as Newton-Gauss type algorithms, in order to minimize (2). A known drawback of these methods, is that the solutions may converge to local minima, even when Levenberg-Marquardt algorithms are used to extend the region of convergence [22, 25].

In [2], it was proposed to minimize Levi's linearized cost function [19, 23]

$$\arg \min_{N_n, D_d} \sum_{k=0}^{K} |D(s_k)H(s_k) - N(s_k)|^2 . \tag{3}$$

This formulation basically reduces to (2), if the weighting factor $1/\left|D(s_k)\right|^2$ is set equal to one for all frequencies s_k. Clearly, this weighting will bias the fitted transfer function, and this often results in poor low-frequency fits, due to an undesired overemphasis of high-frequency errors.

In this chapter, the use of a Sanathanan-Koerner iteration is advocated [32]. First, an estimate of the poles is obtained by minimizing Levi's linearized cost function. Given this initial (iteration step 0) or previous (iteration step $t-1$) estimate of the poles, the model parameters of the next iteration step are calculated by minimizing the weighted linear cost function

$$\arg \min_{N_n^{(t)}, D_d^{(t)}} \left(\sum_{k=0}^{K} \frac{\left| D^{(t)}(s_k) H(s_k) - N^{(t)}(s_k) \right|^2}{\left| D^{(t-1)}(s_k) \right|^2} \right). \tag{4}$$

By analyzing the gradients of the error criterion, it is straightforward to show that this method generates solutions that don't converge asymptotically to the solution of (2) either, even though the error criterion itself tends asymptotically to the fundamental least squares criterion [35]. In practice, however, this approach often gives favorable results for sufficiently small modelling errors. The interested reader is hereby referred to an excellent survey [29].

1.3 Choice of Basisfunctions

To solve the identification problem, (4) reduces naturally to a linear set of least-squares equations, which needs to be solved with sufficient accuracy.

Suppose that $\mathbf{H} = diag(H(s_0), ..., H(s_K))$, $w_k = [D^{(t-1)}(s_k)]^{-1}$, and $\Phi_{0:X}$ is defined as

$$\Phi_{0:X} = \begin{pmatrix} w_0\phi_0(s_0) & \dots & w_0\phi_X(s_0) \\ \dots & \dots & \dots \\ w_K\phi_0(s_K) & \dots & w_K\phi_X(s_K) \end{pmatrix}, \tag{5}$$

then the least-squares solution of $\mathbf{V}\mathbf{x} = \mathbf{b}$ can be calculated to estimate the parameter vector $\mathbf{x}$, provided that $\mathbf{V}$, $\mathbf{x}$ and $\mathbf{b}$ are defined as ($D_0 = 1$)

$$\mathbf{V} = \begin{pmatrix} \Re e\left(\Phi_{0:N} \; -\mathbf{H}\Phi_{1:D}\right) \\ \Im m\left(\Phi_{0:N} \; -\mathbf{H}\Phi_{1:D}\right) \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} \Re e(\mathbf{H}\Phi_0) \\ \Im m(\mathbf{H}\Phi_0) \end{pmatrix} \tag{6}$$

$$\mathbf{x} = (N_0^{(t)} \dots N_N^{(t)} \; D_1^{(t)} \dots D_D^{(t)})^T. \tag{7}$$

Each equation is split in its real and imaginary part to enforce the poles and zeros to be real, or to occur in complex conjugate pairs (under the assumption that the basis functions $\phi(s)$ are real-valued as well). This ensures that the coefficients of the transfer function are real, and that no imaginary terms occur in the time-domain.

It becomes clear that the accuracy of the parameter vector $\mathbf{x}$, and the numerical conditioning of this problem is highly dependent on the structure of the system equations. If the basisfunctions $\phi(s)$ are chosen to be a monomial power series

basis $(1,s,s^2,...)$, the matrix Φ will be a Vandermonde matrix which is notoriously ill-conditioned. Adcock and Potter [1] suggested the use of polynomials which are orthogonal with respect to a continuous inner product, such as Chebyshev polynomials, as basis functions. The large variation of the Chebyshev polynomials with increase in order makes it possible to downsize the effects of ill-conditioning. On the other hand, Richardson and Formenti [30] proposed the use of Forsythe polynomials which are orthonormal with respect to a discrete inner product, defined by the normal equations of the estimator. This implies that a different set of basis functions is used for numerator and denominator. Rolain et al. [31] have shown that a basis transformation from the Forsythe polynomials to a different, arbitrary polynomial basis results in an inferior conditioning of $\mathbf{V}^T\mathbf{V}$. Hence, the Forsythe polynomial basis is optimal in a sense that there doesn't exist any other polynomial basis resulting in a better conditioned form of the normal equations.

2 Vector Fitting

2.1 Model Representation

Quite recently, Gustavsen and Semlyen [13] proposed the use of partial fractions as basis functions for the numerator and denominator

$$R(s) = \frac{N(s)}{D(s)} = \frac{\sum_{p=1}^{P} c_p \phi_p(s)}{1 + \sum_{p=1}^{P} \tilde{c}_p \phi_p(s)} = \frac{\sum_{p=1}^{P} \frac{c_p}{s+a_p}}{1 + \sum_{p=1}^{P} \frac{\tilde{c}_p}{s+a_p}}, \tag{8}$$

provided that c_p and $\tilde{c}_p$ represent the residues, and $-a_p$ are a set of prescribed poles. The denominator has an additional basisfunction which equals the constant value 1. Its coefficient can be fixed to one, since numerator and denominator can be divided by the same constant value without loss of generality. Other non-triviality constraints are also possible [16]. Given the constraint that the poles of the numerator and denominator expression of (8) are the same, it's easy to see that these basis functions are complete, in a sense that they can approximate any strictly proper transfer function with distinct poles arbitrarily well. To approximate systems which require a proper or improper transfer function, an optional constant and linear term can be added to the numerator expression.

2.2 Parameterization of the Transfer Function

In the first iteration, Levi's cost function is applied, which results that $\mathbf{V}$ becomes

$$\mathbf{V} = \begin{pmatrix} \Re e\left(\Phi_{1:P} - H\Phi_{1:P}\right) \\ \Im m\left(\Phi_{1:P} - H\Phi_{1:P}\right) \end{pmatrix} \tag{9}$$

and that the parameter vector consist of unknown residues

$$\mathbf{x} = (c_1 \; ... \; c_P \; \widetilde{c}_1 \; ... \; \widetilde{c}_P)^T \; . \tag{10}$$

The Φ matrix is then a Cauchy matrix, which makes the system equations often well-conditioned if the prescribed poles are well-chosen. To make sure that the transfer function has real-valued coefficients, a linear combination of $\phi_p(s)$ and $\phi_{p+1}(s)$ is formed to make the residues $c_{p+1} = c_p^*$ complex conjugate if the poles $-a_{p+1} = -a_p^*$. This way, two basis functions of the following form are obtained

$$\phi_p(s) = \frac{1}{(s + a_p)} + \frac{1}{(s + a_{p+1})} \tag{11}$$

$$\phi_{p+1}(s) = \frac{i}{(s + a_p)} - \frac{i}{(s + a_{p+1})} \; . \tag{12}$$

This causes the corresponding elements in the solution vector to become equal to $\Re e(c_p)$, $\Im m(c_p)$ and $\Re e(\widetilde{c}_p)$, $\Im m(\widetilde{c}_p)$.

In successive iterations, a Sanathanan-Koerner iteration can be applied. In theory, one could use the denominator of the previous iteration as an inverse weighting to the system equations. The Vector Fitting technique is different, in a sense that weighting is performed implicitly by pole-relocation without weighting. The implicit weighting was found to be more robust if poles need to be relocated over long distances [8]. More details about this procedure are described in Appendix A.

As suggested in [13] and [17], the poles of the basis functions are optimally selected as complex conjugate pairs on a vertical or skew line, close to the imaginary axis. Due to the iterative behaviour of the SK-iteration, the prescribed poles are relocated until the poles converge in such way that the minimization of the SK cost function is converged. In general, this happens quite fast (i.e. <3 iterations). When poles are chosen too far to the left in the complex plane, the real part of the poles dominates the matrix entries, which deteriorates the numerical conditioning. However, even when the initial poles are inappropriately chosen, the algorithm succeeds in minimizing (4), at the expense of additional iterations.

After parameterization of $\mathbf{x}$, (8) can be simplified by cancelling out common poles. This means that the zeros of the denominator expression become the poles of the final transfer function. Calculating the zeros can easily be done, as shown in the following section.

2.3 Calculation of Transfer Function Poles

The minimal LTI state-space realization

$$\begin{aligned} sX(s) &= \mathbf{A}X(s) + \mathbf{B}U(s) \\ Y(s) &= \mathbf{C}X(s) + \mathbf{D}U(s) \end{aligned} \tag{13}$$

of the denominator

$$D(s) = \sum_{p=1}^{P} \frac{\widetilde{c}_p}{s + a_p} + 1 \tag{14}$$

can be obtained by a parallel connection (initially $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C} = \emptyset$ and $\mathbf{D} = 0$)

$$\mathbf{A} = \begin{pmatrix} \mathbf{A} & 0 \\ 0 & \mathbf{A}_p \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{B} \\ \mathbf{B}_p \end{pmatrix} \tag{15}$$

$$\mathbf{C} = \begin{pmatrix} \mathbf{C} & \mathbf{C}_p \end{pmatrix}, \mathbf{D} = \mathbf{D} + \mathbf{D}_p \tag{16}$$

of the minimal state space realizations ($\mathbf{A}_p$,$\mathbf{B}_p$,$\mathbf{C}_p$,$\mathbf{D}_p$) of each simple fraction, with

$$\mathbf{A}_p = -a_p,\ \mathbf{B}_p = 1,\ \mathbf{C}_p = \widetilde{c}_p,\ \mathbf{D}_p = 0 \tag{17}$$

provided that $-a_p$ is real. If $-a_p$ and $-a_{p+1}$ constitute a complex conjugate pair of poles (i.e. $-a_{p+1} = -a_p^*$), the corresponding state space realization of the linear combination is given as

$$\begin{aligned} \mathbf{A}_p &= \begin{pmatrix} \Re e(-a_p) & \Im m(-a_p) \\ -\Im m(-a_p) & \Re e(-a_p) \end{pmatrix}, \mathbf{B}_p = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \\ \mathbf{C}_p &= \begin{pmatrix} \Re e(\widetilde{c}_p) & \Im m(\widetilde{c}_p) \end{pmatrix}, \mathbf{D}_p = 0\ . \end{aligned} \tag{18}$$

Afterwards, the constant term 1 of (14) can simply be added to the scalar $\mathbf{D}$. This transformation makes the state-space realization of $D(s)$

$$D(s) = \mathbf{C}(sI - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \tag{19}$$

real-valued, such that the poles and zeros occur as complex conjugate pairs. The zeros of (19) can then be solved by calculating the eigenvalues of $\mathbf{A}$-$\mathbf{BC}$. After simplification of (8), these eigenvalues will become the relocated poles of the transfer function

$$-a_p = eig(\mathbf{A} - \mathbf{BC}) \tag{20}$$

and this procedure can be repeated iteratively ($t = 1, ..., T$) until the minimization of the SK-cost function is converged.

2.4 Identification of the Residues

Once the final poles $-a_p^{(T)}$ are identified, the corresponding residues θ_p can be solved as a linear problem

$$\arg\min_{\theta_p} \sum_{k=0}^{K} \left| H(s_k) - \left(\sum_{p=1}^{P} \frac{\theta_p}{s_k + a_p^{(T)}} \right) \right|^2 . \tag{21}$$

This technique was called "Vector Fitting" [13], and it has been widely applied to many modelling problems within power systems, high-speed interconnection structures, electronic packages and microwave systems.

3 Orthonormal Vector Fitting

3.1 Orthonormalization of Partial Fraction Basis

Instead of using the partial fractions as rational basis functions, it was shown that orthonormal rational basis functions can lead to significant improvements in numerical conditioning [6, 7, 26]. A straightforward way to calculate an orthonormal basis, is to apply a Gram-Schmidt procedure on the partial fractions [2, 21, 27]. Hence, orthonormal rational functions $\phi_p(s)$ are obtained, which are in fact linear combinations of the partial fractions, of the form

$$\phi_p(s) = \frac{Q_p(s)}{\prod_{j=1}^{p}(s+a_j)} \tag{22}$$

for $p = 1, ..., P$ and $Q_p(s)$ an arbitrary polynomial of order $p-1$, such that

$$\langle \phi_m(s), \phi_n(s) \rangle = \delta_{mn} \tag{23}$$

with $1 \leq m, n \leq P$. If the inner product is defined as

$$\langle \phi_m(s), \phi_n(s) \rangle = \frac{1}{2\pi i} \int_{i\mathbb{R}} \phi_m(s) \phi_n^*(s) ds \tag{24}$$

then the $Q_p(s)$ polynomial can be determined by imposing the orthonormality conditions on the basis functions. As an example, consider the construction of the first function $\phi_1(s)$.

$$\langle \phi_1(s), \phi_1(s) \rangle = \frac{1}{2\pi i} \int_{i\mathbb{R}} \phi_1(s) \phi_1^*(s) ds \tag{25}$$

$$= \frac{1}{2\pi i} \int_{i\mathbb{R}} \frac{|\gamma_1|^2}{(s+a_1)(-s+a_1^*)} ds \tag{26}$$

$$= \frac{|\gamma_1|^2}{a_1 + a_1^*} \tag{27}$$

To normalize $\phi_1(s)$, $Q_1(s) = \gamma_1$ must equal $\kappa_1 \sqrt{2\Re e(a_1)}$, where κ_1 is an arbitrary unimodular complex number. $\phi_1(s)$ is then obtained as

$$\phi_1(s) = \kappa_1 \sqrt{2\Re e(a_1)} \frac{1}{s+a_1}. \tag{28}$$

Now consider the construction of the second function $\phi_2(s)$. First of all, $\phi_2(s)$ must be orthogonal to $\phi_1(s)$

$$\langle \phi_1(s), \phi_2(s) \rangle = \frac{1}{2\pi i} \int_{i\mathbb{R}} \phi_1(s) \phi_2^*(s) ds = 0 \tag{29}$$

which implies that $\phi_2^*(s)$ must vanish for $s = -a_1$. Therefore $Q_2(s) = \gamma_2(s - a_1^*)$. This constant γ_2 is determined by imposing the normalization condition

$$\begin{aligned}
&\langle \phi_2(s), \phi_2(s) \rangle \\
&= \frac{1}{2\pi i} \int_{i\mathbb{R}} \frac{\gamma_2(s - a_1^*)}{(s + a_1)(s + a_2)} \frac{\gamma_2^*(-s - a_1)}{(-s + a_1^*)(-s + a_2^*)} ds \qquad (30) \\
&= \frac{1}{2\pi i} \int_{i\mathbb{R}} \frac{|\gamma_2|^2}{(s + a_2)(-s + a_2^*)} ds = \frac{|\gamma_2|^2}{a_2 + a_2^*} . \qquad (31)
\end{aligned}$$

Clearly, it follows that $\gamma_2 = \kappa_2 \sqrt{2\Re e(a_2)}$, where κ_2 is an arbitrary unimodular complex number. So, $\phi_2(s)$ is then given by

$$\phi_2(s) = \kappa_2 \sqrt{2\Re e(a_2)} \frac{s - a_1^*}{(s + a_1)(s + a_2)} . \qquad (32)$$

Similarly continuing this approach, the general polynomials are obtained

$$\phi_p(s) = \kappa_p \sqrt{2\Re e(a_p)} \left(\prod_{j=1}^{p-1} \frac{s - a_j^*}{s + a_j} \right) \frac{1}{s + a_p} . \qquad (33)$$

This basis originates from the discrete-time Takenaka-Malmquist basis [24, 33], and has later been transformed to the continuous time domain. It is a generalization of the Laguerre basis [4], where all poles $\{-a_p\}$ are the same real number, and the 2-parameter Kautz bases [20] where all poles $\{-a_p, -a_{p+1}\}$ are the same complex conjugate pair with $-a_p^* = -a_{p+1}$. A theoretical analysis of these basis functions is well-described in literature. The interested reader is referred to [18] which gives an excellent survey.

To make sure that the transfer function has real-valued coefficients, a linear combination of $\phi_p(s)$ and $\phi_{p+1}(s)$ is formed which can be made real-valued if the poles are real or occur in a complex conjugate pair. This way, two orthonormal functions of the following form are obtained

$$\varphi_p(s) = \gamma_p \left(\prod_{j=1}^{p-1} \frac{s - a_j^*}{s + a_j} \right) \frac{s - x}{(s + a_p)(s + a_{p+1})} \qquad (34)$$

$$\varphi_{p+1}(s) = \gamma_{p+1} \left(\prod_{j=1}^{p-1} \frac{s - a_j^*}{s + a_j} \right) \frac{s - y}{(s + a_p)(s + a_{p+1})} \qquad (35)$$

with real $\gamma_p, \gamma_{p+1}, x$ and y. To impose the orthogonality,

$$\langle \varphi_p(s), \varphi_{p+1}(s) \rangle = \gamma_p \gamma_{p+1} \frac{xy + a_p a_{p+1}}{2(a_p + a_{p+1}) a_p a_{p+1}} = 0 \qquad (36)$$

x and y are set to be $\sqrt{a_p a_{p+1}} = |a_p|$ and $-\sqrt{a_p a_{p+1}} = -|a_p|$ respectively. Similarly, γ_p and γ_{p+1} are set to $\sqrt{a_p + a_{p+1}} = \sqrt{2\Re e(a_p)}$. Note that this choice is not unique, and that other possibilities exist. Note also that the orthonormalization of the basis functions is done analytically instead of numerically, so it doesn't require any additional computation time.

3.2 Calculation of Transfer Function Poles

The minimal continuous-time LTI state-space realization

$$sX(s) = \mathbf{A}X(s) + \mathbf{B}U(s) \tag{37}$$
$$Y(s) = \mathbf{C}X(s) + \mathbf{D}U(s) \tag{38}$$

of the denominator

$$D(s) = 1 + \sum_{p=1}^{P} \tilde{c}_p \phi_p(s) \tag{39}$$

can then be calculated, by cascading the minimal state-space realization of smaller, first and second order sections [11]

$$\frac{s - a_1^*}{s + a_1} \rightarrow \frac{s - a_2^*}{s + a_2} \rightarrow \ldots \rightarrow \frac{s - a_{P-1}^*}{s + a_{P-1}} \rightarrow \frac{1}{s + a_P}. \tag{40}$$

The minimal state-space realization $(\mathbf{A}_p,\mathbf{B}_p,\mathbf{C}_p,\mathbf{D}_p)$ of the all-pass function

$$\frac{Y_p(s)}{U_p(s)} = \frac{s - a_p^*}{s + a_p} \tag{41}$$

for $p = 1, \ldots, P-1$ is given as

$$\mathbf{A}_p = -a_p,\ \mathbf{B}_p = 1,\ \mathbf{C}_p = 2\Re e(-a_p),\ \mathbf{D}_p = 1 \tag{42}$$

and the minimal state-space realization $(\mathbf{A}_p,\mathbf{B}_p,\mathbf{C}_p,\mathbf{D}_p)$ of the low-pass function

$$\frac{Y_p(s)}{U_p(s)} = \frac{1}{s + a_p} \tag{43}$$

is given as

$$\mathbf{A}_p = -a_p,\ \mathbf{B}_p = 1,\ \mathbf{C}_p = 1,\ \mathbf{D}_p = 0 \tag{44}$$

for $p = P$. Then the minimal state-space realization of the compound system (40) is obtained as the cascade construction

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & 0 & \ldots & 0 \\ \mathbf{B}_2\mathbf{C}_1 & \mathbf{A}_2 & \ldots & 0 \\ \mathbf{B}_3\mathbf{D}_2\mathbf{C}_1 & \mathbf{B}_3\mathbf{C}_2 & \ldots & 0 \\ \mathbf{B}_4\mathbf{D}_3\mathbf{D}_2\mathbf{C}_1 & \mathbf{B}_4\mathbf{D}_3\mathbf{C}_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \mathbf{B}_P\mathbf{D}_{P-1}\ldots\mathbf{D}_2\mathbf{C}_1 & \mathbf{B}_P\mathbf{D}_{P-1}\ldots\mathbf{D}_3\mathbf{C}_2 & \ldots & \mathbf{A}_P \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2\mathbf{D}_1 \\ \mathbf{B}_3\mathbf{D}_2\mathbf{D}_1 \\ \mathbf{B}_4\mathbf{D}_3\mathbf{D}_2\mathbf{D}_1 \\ \ldots \\ \mathbf{B}_P\mathbf{D}_{P-1}\ldots\mathbf{D}_1 \end{bmatrix}, C = \begin{bmatrix} \mathbf{D}_P\ldots\mathbf{D}_2\mathbf{C}_1 \\ \mathbf{D}_P\ldots\mathbf{D}_3\mathbf{C}_2 \\ \ldots \\ \mathbf{C}_P \end{bmatrix}^T \tag{45}$$

$$\mathbf{D} = \mathbf{D}_P\ldots\mathbf{D}_1$$

of the smaller state space models, with $y_p(t) = u_{p+1}(t)$.

The state matrix $\mathbf{A}$ and the input vector $\mathbf{B}$ are build such that the states contain exactly the unnormalized basis functions. The output vector $\mathbf{C}$ and scalar $\mathbf{D}$ are chosen to obtain the denominator expression (39), by compensating for the coefficients $\widetilde{c}_p$ and normalization constant $\sqrt{2\Re e(a_p)}$ in the vector $\mathbf{C}$, and setting the scalar $\mathbf{D}$ equal to the constant value 1. The following real-valued state space realization is obtained

$$\mathbf{A}_{P\times P} = \begin{bmatrix} -a_1 & 0 & 0 & \ldots & 0 \\ 2\Re e(-a_1) & -a_2 & 0 & \ldots & 0 \\ 2\Re e(-a_1) & 2\Re e(-a_2) & -a_3 & \ldots & 0 \\ 2\Re e(-a_1) & 2\Re e(-a_2) & 2\Re e(-a_3) & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 2\Re e(-a_1) & 2\Re e(-a_2) & 2\Re e(-a_3) & \ldots & -a_P \end{bmatrix}$$

$$\mathbf{B}_{1\times P} = \begin{bmatrix} 1 \\ 1 \\ \ldots \\ 1 \end{bmatrix}, \mathbf{C}_{P\times 1} = \begin{bmatrix} \widetilde{c}_1\sqrt{2\Re e(a_1)} \\ \widetilde{c}_2\sqrt{2\Re e(a_2)} \\ \ldots \\ \widetilde{c}_P\sqrt{2\Re e(a_P)} \end{bmatrix}^T \tag{46}$$

$$\mathbf{D}_{1\times 1} = 1$$

provided that the poles $-a_p$ are real.

If $-a_p$ and $-a_{p+1}$ constitute a complex conjugate pair of poles (i.e. $-a_{p+1} = -a_p^*$), a real-valued state-space realization is obtained by replacing

$$\frac{s-a_p^*}{s+a_p} \longrightarrow \frac{s-a_{p+1}^*}{s+a_{p+1}} \tag{47}$$

in the cascade scheme (40) by

$$\frac{(s-a_p^*)(s-a_{p+1}^*)}{(s+a_p)(s+a_{p+1})} = 1 + \frac{4\Re e(-a_p)s}{(s+a_p)(s+a_p^*)} \,. \tag{48}$$

This corresponds to replacing

$$\begin{pmatrix} -a_p & 0 \\ 2\Re e(-a_p) & -a_{p+1} \end{pmatrix} \tag{49}$$

in the state matrix A, by

$$\begin{pmatrix} \Re e(-a_p) & \Re e(-a_p) - |a_p| \\ \Re e(-a_p) + |a_p| & \Re e(-a_p) \end{pmatrix} \,. \tag{50}$$

The other state space matrices remain unchanged. Appendix B describes this transformation in more detail. Again, the zeros of the denominator are calculated by solving the eigenvalues of $\mathbf{A}\text{-}\mathbf{BC}$. These eigenvalues replace the set of prescribed poles, and the procedure is repeated iteratively ($t = 1, ..., T$) until the minimization of the SK-cost function is converged.

Once the final poles are identified, the residues can be solved as a linear problem using the partial fraction basis (21). The orthonormal basis functions can also be applied if stability of the poles is enforced. Both representations can easily be realized to state-space as was shown before.

4 Example

As an example, the technique is illustrated on a dense model of an atmospheric storm track (eady), which is obtained from the NICONET benchmark dataset collection [3]. Based on the state-space matrices of the large model (598×598), the frequency response is densely calculated over the frequency range of interest $[10^{-1},10^{2}]$ and shown in Fig. 1.

First, a prescribed set of complex conjugate starting poles is chosen as was proposed by [13]

$$-a_p = -\alpha + \beta i, -a_{p+1} = -\alpha - \beta i \tag{51}$$

$$\alpha = \beta/100 \tag{52}$$

with imaginary parts β logarithmically spaced over the frequency range of interest. The amount of poles is chosen in terms of the desired reduction. In this example, it was chosen to be 54, in order to have an RMS error which corresponds to the order of 10^{-5}.

The weighted linear cost function (4) is solved using the orthonormal rational basis functions (33), (34), (35) and an estimate for the residues c_p and $\tilde{c}_p$ is obtained. Using the residues $\tilde{c}_p$ and the poles $-a_p$, the minimal state-space realization (A,B,C,D) of the denominator $D(s)$ (39) is calculated. From this state-space

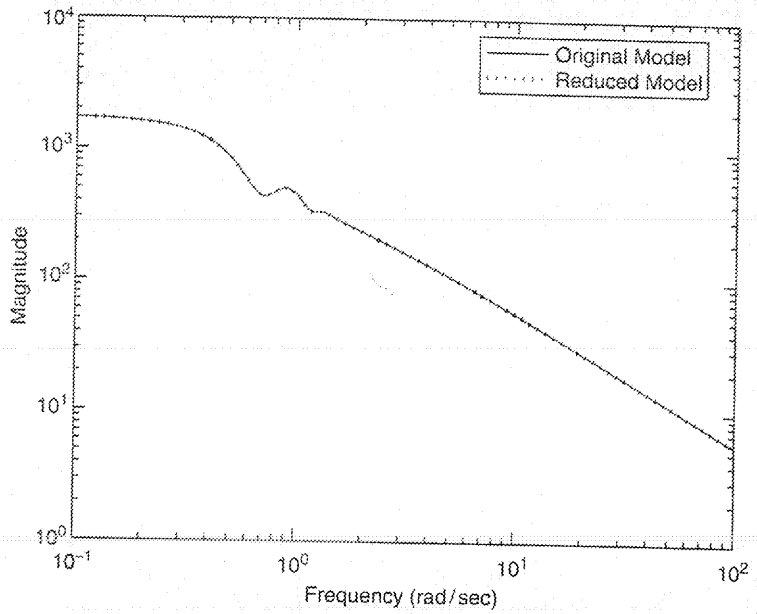

Fig. 1. Frequency response : original and reduced model

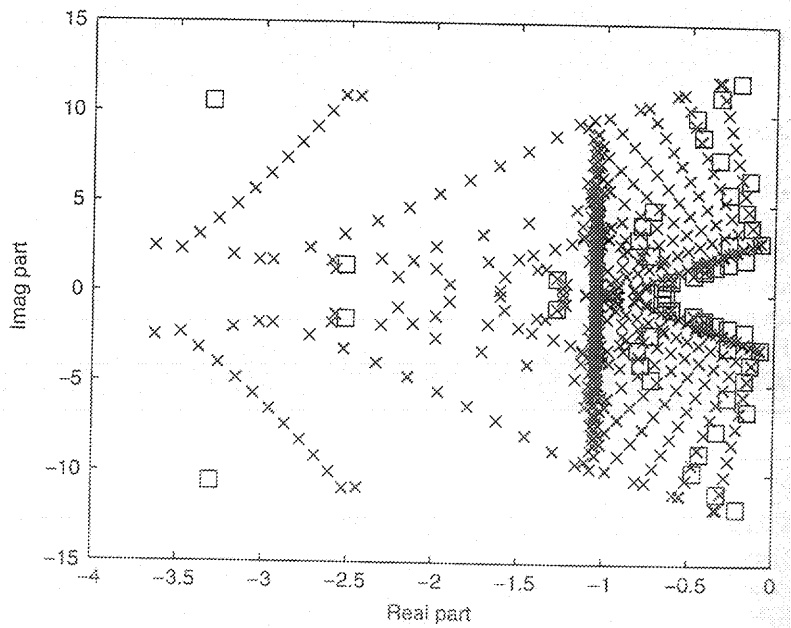

Fig. 2. Poles of original (×) and reduced (□) model

model, the poles of the transfer function are calculated by solving the eigenvalues of A-BC. These poles are chosen as new starting poles, and the method iterates until the poles are converged to their optimal location. Once the poles are known, the residues of the transfer function can be estimated as a linear problem. The final accuracy of the model in terms of RMS error is 5.61×10^{-8}.

As is shown in Fig. 1, no visual difference can be observed between the frequency response of the original and reduced system. The poles of the original model and the reduced model are shown in Fig. 2. The OVF method can also be extended to fit systems with multiple ports. It is noted that the extension is completely analogous to the Matrix Fitting algorithm [15].

5 Conclusion

This paper shows that Orthonormal Vector Fitting can be useful to reduce the state space dimensions of large circuit models. First, the spectral response of the large model is calculated, and then the OVF algorithm is used to approximate the data with a model of reduced size. It was shown that the method is quite robust, even when the original system has a large amount of poles. The method extends in a natural way to multi-port systems (not shown in this paper). The reduced model is represented as state-space realization, which can easily be converted e.g. to an RLCG circuit.

6 Acknowledgements

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A Sanathanan-Koerner Iteration

The least-squares SK-cost function is defined as

$$\arg\min_{N_0^{(i)},D_0^{(i)}}\left(\sum_{k=0}^{K}\left|\frac{1}{D^{(i-1)}(s_k)}\right|^2\left|D^{(i)}(s_k)H(s_k)-N^{(i)}(s_k)\right|^2\right). \tag{53}$$

If the basis functions are chosen as partial fractions, based on a prescribed set of poles $-a_1,\ldots,-a_P$, then it follows that

$$N^{(i)}(s)=\sum_{p=1}^{P}\frac{c_p^{(i)}}{s+a_p}=\frac{\prod_{p=1}^{P-1}(s+z_{p,n}^{(i)})}{\prod_{p=1}^{P}(s+a_p)} \tag{54}$$

$$D^{(i)}(s)=\sum_{p=1}^{P}\frac{\tilde{c}_p^{(i)}}{s+a_p}+\tilde{c}_0^{(i)}=\frac{\prod_{p=1}^{P}(s+z_{p,d}^{(i)})}{\prod_{p=1}^{P}(s+a_p)}. \tag{55}$$

The denominator has an additional basisfunction, which equals the constant value 1. In the first iteration step ($t = 0$), Levi's linearization is applied to obtain a first guess of the denominator ($D^{(-1)}(s) = 1$)

$$\arg \min_{N_n^{(t)}, D_d^{(t)}} \left(\sum_{k=0}^{K} \left| \frac{1}{D^{(t-1)}(s_k)} \right|^2 \left| D^{(t)}(s_k) H(s_k) - N^{(t)}(s_k) \right|^2 \right) \tag{56}$$

$$= \arg \min_{\tilde{c}_p^{(0)}, c_p^{(0)}} \left(\sum_{k=0}^{K} \left| \left(\sum_{p=1}^{P} \frac{\tilde{c}_p^{(0)}}{s_k + a_p} + \tilde{c}_0^{(0)} \right) H(s_k) - \sum_{p=1}^{P} \frac{c_p^{(0)}}{s_k + a_p} \right|^2 \right) . \tag{57}$$

This reduces to solving the following set of least-squares equations, for all complex frequencies s

$$\left(\sum_{p=1}^{P} \frac{\tilde{c}_p^{(0)}}{s + a_p} + \tilde{c}_0^{(0)} \right) H(s) - \sum_{p=1}^{P} \frac{c_p^{(0)}}{s + a_p} = 0 . \tag{58}$$

One coefficient of the rational function, e.g. $\tilde{c}_0^{(0)}$, can be fixed to unity, since numerator and denominator can be divided by the same complex value without loss of generality. So, (58) is equivalent to

$$\sum_{p=1}^{P} \frac{c_p^{(0)}}{s + a_p} - \sum_{p=1}^{P} \frac{\tilde{c}_p^{(0)}}{s + a_p} H(s) = H(s) . \tag{59}$$

Once the parameters $c_p^{(0)}$ and $\tilde{c}_p^{(0)}$ are estimated, $N^{(0)}(s)$ and $D^{(0)}(s)$ are known (54-55). It's straightforward to calculate $z_{p,n}^{(0)}$ and $z_{p,d}^{(0)}$ in a robust way, by solving the eigenvalue problem (20). In practice, only $z_{p,d}^{(0)}$ is needed.

Now, the Sanathanan-Koerner linearization can be applied for iteration step $t = 1, ..., T$

$$\arg \min_{N_n^{(t)}, D_d^{(t)}} \left(\sum_{k=0}^{K} \left| \frac{1}{D^{(t-1)}(s_k)} \right|^2 \left| D^{(t)}(s_k) H(s_k) - N^{(t)}(s_k) \right|^2 \right) \tag{60}$$

$$= \arg \min_{\tilde{c}_p^{(t)}, c_p^{(t)}} \left(\sum_{k=0}^{K} \left| \frac{\prod_{p=1}^{P} (s_k + a_p)}{\prod_{p=1}^{P} (s_k + z_{p,d}^{(t-1)})} \right|^2 \left| \left(\sum_{p=1}^{P} \frac{\tilde{c}_p^{(t)}}{s_k + a_p} + \tilde{c}_0^{(t)} \right) \right. \right.$$

$$\left. \left. \times H(s_k) - \sum_{p=1}^{P} \frac{c_p^{(t)}}{s_k + a_p} \right|^2 \right) \tag{61}$$

$$= \arg \min_{z_{p,d}^{(t)}, z_{p,n}^{(t)}} \left(\sum_{k=0}^{K} \left| \frac{\prod_{p=1}^{P}(s_k + a_p)}{\prod_{p=1}^{P}(s_k + z_{p,d}^{(t-1)})} \right|^2 \left| \left(\frac{\prod_{p=1}^{P}(s_k + z_{p,d}^{(t)})}{\prod_{p=1}^{P}(s_k + a_p)} \right) \right. \right.$$
$$\left. \left. \times H(s_k) - \frac{\prod_{p=1}^{P-1}(s_k + z_{p,n}^{(t)})}{\prod_{p=1}^{P}(s_k + a_p)} \right|^2 \right) \tag{62}$$

$$= \arg \min_{z_{p,d}^{(t)}, z_{p,n}^{(t)}} \left(\sum_{k=0}^{K} \left| \left(\frac{\prod_{p=1}^{P}(s_k + z_{p,d}^{(t)})}{\prod_{p=1}^{P}(s_k + z_{p,d}^{(t-1)})} \right) \right. \right.$$
$$\left. \left. \times H(s_k) - \frac{\prod_{p=1}^{P-1}(s_k + z_{p,n}^{(t)})}{\prod_{p=1}^{P}(s_k + z_{p,d}^{(t-1)})} \right|^2 \right) \tag{63}$$

$$= \arg \min_{\tilde{d}_p^{(t)}, d_p^{(t)}} \left(\sum_{k=0}^{K} \left| \left(\sum_{p=1}^{P} \frac{\tilde{d}_p^{(t)}}{s_k + z_{p,d}^{(t-1)}} + \tilde{d}_0^{(t)} \right) \right. \right.$$
$$\left. \left. \times H(s_k) - \sum_{p=1}^{P} \frac{d_p^{(t)}}{s_k + z_{p,d}^{(t-1)}} \right|^2 \right) . \tag{64}$$

When the classical SK-iteration is used, one can solve the coefficients $c_p^{(t)}$ and $\tilde{c}_p^{(t)}$ of $N^{(t)}$ and $D^{(t)}$ if a weighting is applied to each row of the system equations (explicit weighting). The Vector Fitting performs this weighting implicitly, by calculating the coefficients $d_p^{(t)}$ and $\tilde{d}_p^{(t)}$ of $N^{(t)}/D^{(t-1)}$ and $D^{(t)}(s)/D^{(t-1)}$ instead (without an explicit weighting). In successive iterations ($t > 0$), the coefficients $\tilde{d}_p^{(t)}$ of $D^{(t)}/D^{(t-1)}$ are then used to calculate the poles, which does not pose a problem, as the zeros of $D^{(t)}$ and $D^{(t)}/D^{(t-1)}$ are the same. It is noted, however, that the poles of the basis functions of $N^{(t)}(s)$ and $D^{(t)}(s)$ remain unchanged, and cancel out in each iteration (62).

B Real-Valued State Space

This appendix describes how the real-valued state-space realization of

$$\frac{(s - a_p^*)(s - a_{p+1}^*)}{(s + a_p)(s + a_{p+1})} = 1 + \frac{4\Re e(-a_p)s}{(s + a_p)(s + a_p^*)} \tag{65}$$

can be obtained.

Define the state matrix $\mathbf{A}$ and input vector $\mathbf{B}$ as

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{pmatrix} \tag{66}$$

where $\mathbf{A}_{ij}$ and $\mathbf{B}_i$ are the scalar elements of the matrix and the vector respectively. Capitals are used to avoid notational confusion between the poles and the entries of the state matrix.

A first constraint on the entries, is that the poles of (65), $\{-a_p,-a_p^*\}$, must equal the eigenvalues of $\mathbf{A}$. More specifically, the transfer function from the input $U_1(s)$ and $U_2(s)$ to the states $X_1(s)$ and $X_2(s)$ respectively, must satisfy

$$\frac{X_1(s)}{U_1(s)} = \frac{s-|a_p|}{(s+a_p)(s+a_p^*)} \tag{67}$$

$$\frac{X_2(s)}{U_2(s)} = \frac{s+|a_p|}{(s+a_p)(s+a_p^*)}\,, \tag{68}$$

The input-to-state transfer function is given by

$$\frac{X(s)}{U(s)} = (sI-\mathbf{A})^{-1}\mathbf{B} \tag{69}$$

$$= \frac{\begin{pmatrix} s-\mathbf{A}_{22} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & s-\mathbf{A}_{11} \end{pmatrix}}{(s-\mathbf{A}_{11})(s-\mathbf{A}_{22})-\mathbf{A}_{12}\mathbf{A}_{21}}\mathbf{B}\,, \tag{70}$$

so

$$\frac{X_1(s)}{U_1(s)} = \frac{(s-\mathbf{A}_{22})\mathbf{B}_1+\mathbf{A}_{12}\mathbf{B}_2}{(s-\mathbf{A}_{11})(s-\mathbf{A}_{22})-\mathbf{A}_{12}\mathbf{A}_{21}} \tag{71}$$

and

$$\frac{X_2(s)}{U_2(s)} = \frac{\mathbf{A}_{21}\mathbf{B}_1+(s-\mathbf{A}_{11})\mathbf{B}_2}{(s-\mathbf{A}_{11})(s-\mathbf{A}_{22})-\mathbf{A}_{12}\mathbf{A}_{21}} \tag{72}$$

By equating the numerators of (67) to (71), (68) to (72), and applying some basic linear algebra, the following constraints are easily obtained

$$\mathbf{B}_1 = 1 \tag{73}$$

$$\mathbf{B}_2 = 1 \tag{74}$$

$$-\mathbf{A}_{22}+\mathbf{A}_{12} = -|a_p| \tag{75}$$

$$\mathbf{A}_{21}-\mathbf{A}_{11} = |a_p| \tag{76}$$

which determine the input vector $\mathbf{B}$ completely. Unfortunately, the elements of the state matrix $\mathbf{A}$ are still ambiguous.

By equating the denominators, it follows that

$$s^2-(\mathbf{A}_{11}+\mathbf{A}_{22})s+(\mathbf{A}_{11}\mathbf{A}_{22}+\mathbf{A}_{12}\mathbf{A}_{21}) \tag{77}$$

$$= s^2+(a_p+a_p^*)s+a_pa_p^* \tag{78}$$

so

$$\mathbf{A}_{11}+\mathbf{A}_{22} = -2\mathrm{Re}(a_p) \tag{79}$$

$$\mathbf{A}_{11}\mathbf{A}_{22}-\mathbf{A}_{12}\mathbf{A}_{21} = |a_p|^2\,, \tag{80}$$

Combining (80) with (75) and (76) gives

$$\mathbf{A}_{11} - \mathbf{A}_{12} = |a_p| \,. \tag{81}$$

Using (75) and (81)

$$\mathbf{A}_{11} - \mathbf{A}_{22} = 0 \tag{82}$$

$$\mathbf{A}_{11} = \mathbf{A}_{22} \,. \tag{83}$$

Obviously from (79) and (83), it results that

$$\mathbf{A}_{11} = \Re e(-a_p) \tag{84}$$

$$\mathbf{A}_{22} = \Re e(-a_p) \,. \tag{85}$$

Combining this with (75) and (76), it follows that

$$\mathbf{A}_{12} = \Re e(-a_p) - |a_p| \tag{86}$$

$$\mathbf{A}_{21} = \Re e(-a_p) + |a_p| \tag{87}$$

which determines $\mathbf{A}$ uniquely.

Verifying that the eigenvalues of $\mathbf{A}$ are actually equal to $-a_p$ and $-a_p^*$ is trivial. Now, $\mathbf{C}$ and $\mathbf{D}$ can easily be formed to obtain (65)

$$\mathbf{C} = \left(2\Re e(-a_p)\ 2\Re e(-a_p)\right), \mathbf{D} = 1 \,. \tag{88}$$

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Model Order Reduction

The goal of this book is three-fold: it describes the basics of model order reduction and related aspects in numerical linear algebra, it covers both general and more specialized model order reduction techniques for linear and nonlinear systems, and it discusses the use of model order reduction techniques in a variety of practical applications. The book contains many recent advances in model order reduction, and presents several open problems for which techniques are still in development. It will serve as a source of inspiration for its readers, who will discover that model order reduction is a very exciting and lively field.

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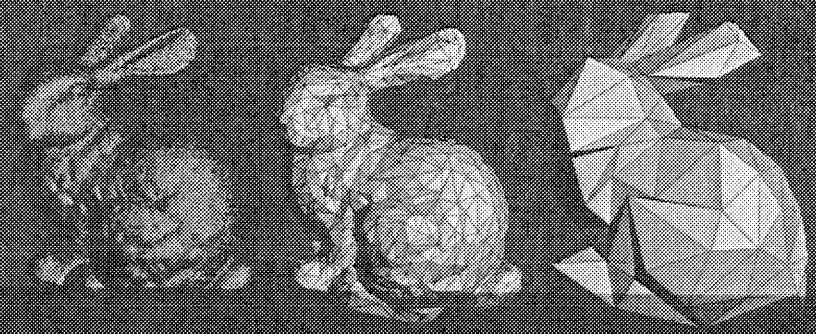

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