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Gribov no-pole condition, Zwanziger horizon function, Kugo-Ojima confinement criterion, boundary conditions, BRST breaking and all that

D. Dudal[a], S.P. Sorella[b], N. Vandersickel[a], and H. Verschelde[a*]

[a] *Ghent University, Department of Mathematical Physics and Astronomy, Krijgslaan 281-S9, B-9000 Gent, Belgium*

[b] *Departamento de Física Teórica, Instituto de Física,UERJ - Universidade do Estado do Rio de Janeiro, Rua São Francisco Xavier 524, 20550-013 Maracanã, Rio de Janeiro, Brasil*

We aim to offer a kind of unifying view on two popular topics in the studies of nonperturbative aspects of Yang-Mills theories in the Landau gauge: the so-called Gribov-Zwanziger approach and the Kugo-Ojima confinement criterion. Borrowing results from statistical thermodynamics, we show that imposing the Kugo-Ojima confinement criterion as a boundary condition leads to a modified yet renormalizable partition function. We verify that the resulting partition function is equivalent with the one obtained by Gribov and Zwanziger, which restricts the domain of integration in the path integral within the first Gribov horizon. The construction of an action implementing a boundary condition allows one to discuss the symmetries of the system in the presence of the boundary. In particular, the conventional BRST symmetry is softly broken.

I. INTRODUCTION

The Gribov-Zwanziger (GZ) approach focuses on the issue of gauge copies in the Landau gauge. Gribov signalled in his seminal work [1] that the Landau gauge condition, $\partial_\mu A_\mu = 0$ is ambiguous: there exist gauge equivalent configurations A'_μ which also obey $\partial_\mu A'_\mu = 0$. Examples of gauge copies are provided by the zero modes of the Faddeev-Popov (FP) operator, which enters the quantization formula of Yang-Mills theories. Indeed, given an infinitesimal gauge transformation connecting A_μ with A'_μ, i.e. $A^{a'}_\mu = A^a_\mu - D^{ab}_\mu \omega^b$, it is clear that $\partial_\mu A'_\mu = \partial_\mu A_\mu = 0$ is fulfilled when $M^{ab}\omega^b = 0$, with $M^{ab} = -\partial_\mu D^{ab}_\mu = -\partial_\mu \left(\partial_\mu \delta^{ab} + g f^{acb} A^c_\mu\right)$ being the FP operator. We recall that the FP action in the Landau gauge for a d-dimensional Euclidean gauge theory, with $d \leq 4$, reads

$$S_{\rm YM+gf} = S_{\rm YM} + \int d^d x \, \left(b^a \partial_\mu A^a_\mu + \overline{c}^a \partial_\mu D^{ab}_\mu c^b \right) , \qquad (1)$$

with $S_{\rm YM} = \frac{1}{4}\int d^d x F^a_{\mu\nu} F^a_{\mu\nu}$ the classical Yang-Mills action. Expression (1) enjoys the well-known BRST symmetry, generated by the nilpotent operator s, $s^2 = 0$, i.e.

$$sA^a_\mu = -D^{ab}_\mu c^b \, , sc^a = \frac{1}{2} g f^{abc} c^b c^c \, , s\overline{c}^a = b^a \, , sb^a = 0 \, . \qquad (2)$$

For the partition function, we can write

$$Z = \int d\mu_{\rm FP} = \int d\Phi e^{-S_{\rm YM+gf}} = \int dA \det M^{ab} \delta(\partial A) e^{-S_{\rm YM}} \, . \qquad (3)$$

We introduced the notational shorthand Φ denoting all the fields present in the action, with $d\mu_{\rm FP}$ the usual FP measure.

Gribov proposed to restrict the domain of integration to the subspace Ω, where the Hermitian operator M^{ab} is positive definite. More precisely, we define the Gribov region as $\Omega \equiv \{A^a_\mu, \, \partial_\mu A^a_\mu = 0, \, M^{ab} > 0\}$. We recognize that configurations $A^a_\mu \in \Omega$ are relative minima of the functional $\int d^d x (A^u_\mu)^2$, $u \in SU(N)$. The boundary, $\partial\Omega$, of Ω is called the (first) Gribov horizon. It was shown with increasing rigor that Ω is convex, bounded in all directions in field space, and that every gauge field has at least one gauge equivalent representant in Ω (see [2, 3] and references therein). The inverse of the FP operator, or equivalently the ghost propagator with external gauge field, $G^{ab}(k,A)$, can be used to implement the restriction to Ω, as done semiclassically by Gribov. Following [1], we can write

$$G^{ab}(k,A) = \frac{\delta^{ab}}{k^2} \frac{1}{1+\sigma(k,A)} = (M^{-1})^{ab}(k,A) \, . \qquad (4)$$

At lowest order, it can be shown that $1+\sigma(k,A)$ is a decreasing function of k [1], hence one can impose

$$1 + \sigma(0,A) \geq 0 \, . \qquad (5)$$

Condition (5), known as the Gribov no-pole condition, implies that the ghost propagator $G^{ab}(k,A)$ has no poles at finite non-vanishing k. Moreover, positivity of $G^{ab}(k,A)$ ensures that the Gribov horizon $\partial\Omega$ is not crossed. As done by Gribov [1], the no-pole condition can be embodied into the partition function using a δ-function[1],

$$Z' = \int d\Phi \delta(1+\sigma(0,A)) e^{-S_{\rm YM+gf}} \, . \qquad (6)$$

Later on, Zwanziger [2] was able to implement the no-pole condition to all orders. Relying on the equivalence between the microcanonical and the canonical Boltzmann ensemble (see also Section II), he was able to show that the partition function (6) has to be replaced by

$$Z'' = \int dA \delta(\partial_\mu A_\mu) \det M^{ab} e^{-S_{\rm YM} + \gamma^4 \int d^d x h(x)} \, , \qquad (7)$$

[1] Condition (5) can be implemented by inserting a step function factor $\theta(1+\sigma(k,A))$. However, in the thermodynamic limit, the θ-function can be exponentially replaced by a δ-function, see [1, 2, 15].

*Electronic address: david.dudal@ugent.be, sorella@uerj.br, nele.vandersickel@ugent.be, henri.verschelde@ugent.be

where the Zwanziger horizon function reads

$$\int d^dx h(x,A) = g^2 \int d^dx d^dy f^{abc} A^b_\mu(x) (M^{-1})^{ad}(x,y) f^{dec} A^e_\mu(y) \tag{8}$$

The mass parameter γ is determined by a gap equation, commonly called the horizon condition

$$\langle h(x) \rangle = d(N^2-1)\,, \tag{9}$$

where $\langle h \rangle$ is calculated with Z'', i.e. with the measure $d\mu_{\rm FP} e^{\gamma^4 \int d^dx h(x)}$. The factor $d(N^2-1)$ in the r.h.s. was obtained [2] by determining the lowest eigenvalue of the FP operator. Working out the condition (9) at lowest order reproduces the Gribov result [1]. The action corresponding to the partition function (7) contains the *nonlocal* horizon term (8). To arrive at a workable quantum model, it was shown [3] that (7) can be put in an equivalent *local* form by introducing a set of complex conjugate commuting variables, $(\varphi^{ac}_\mu, \overline{\varphi}^{ac}_\mu)$, and anticommuting ones, $(\omega^{ac}_\mu, \overline{\omega}^{ac}_\mu)$, so that we finally obtain the Gribov-Zwanziger action,

$$\begin{aligned} S_{\rm GZ} &= S_{\rm YM+gf} + \int d^dx \left(\overline{\varphi}^{ac}_\mu \partial_\nu D^{ab}_\nu \varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu \partial_\nu D^{ab}_\nu \omega^{ac}_\mu \right. \\ &\quad -g \left(\partial_\nu \overline{\omega}^{ac}_\mu\right) f^{abm} (D_\nu c)^b \varphi^{mc}_\mu \\ &\quad \left. -\gamma^2 g f^{abc} A^a_\mu \left(\varphi^{bc}_\mu + \overline{\varphi}^{bc}_\mu \right) + d\left(N^2-1\right)\gamma^4 \right)\,. \end{aligned} \tag{10}$$

The horizon condition (9) is translated as $\frac{\partial \Gamma}{\partial \gamma} = 0$, with $\Gamma(\gamma)$ the effective action, defined as $e^{-\Gamma} = \int d\Phi e^{-S_{\rm GZ}}$. This can be easily checked, given that we take $\gamma \neq 0$. The mass parameter γ turns out to be proportional to $\Lambda_{\overline{\rm MS}}$, and as such it can give rise to nonperturbative corrections. This is not unexpected, as the restriction to the region Ω is a highly nontrivial operation, which goes beyond perturbation theory. At the perturbative level, the ghost propagator stays positive. We are thus far from the horizon and nothing happens. It is only at lower momenta, where normal perturbation theory starts to fail, that the fields begin to feel the restriction to Ω. Having brought the action in standard local form, we have all the usual concepts and machinery of local quantum field theory to our disposal. A first important property of (10) is its renormalizability to all orders of perturbation theory. Hence, the restriction to Ω makes perfect sense at the quantum level, and finite results are found, consistent with the renormalization group [3, 4]. We stress here that the action (10), with the horizon condition (9) implemented, is nothing else than the correct extension to all orders of the usual Yang-Mills action, supplemented with the Landau gauge fixing, in the presence of a nontrivial boundary condition, being the no-pole condition (5). In this fashion, it is assured that we have taken care of a certain amount of gauge copies, including those related to the zero modes of M^{ab}. Notice that this does not mean that the Gribov issue has been completely solved. It is known that Ω still contains copies, related to the fact that $\int d^dx (A^u_\mu)^2$ can have many relative minima starting from the same A_μ. A further restriction is needed, keeping only gauge configurations that are absolute minima of $\int d^dx (A^u_\mu)^2$; the latter define the fundamental modular region (FMR) Λ.

Evidently, the extra fields, $(\overline{\varphi}^{ac}_\mu, \varphi^{ac}_\mu, \overline{\omega}^{ac}_\mu, \omega^{ac}_\mu)$, can influence the dynamics of the theory in a nontrivial fashion [5]. These fields arise as a consequence of the presence of the Gribov horizon. As such, they can give rise to additional nonperturbative effects. For example, in [6], we have provided evidence of the existence of a dimension 2 condensate, $\langle \overline{\varphi}^{ac}_\mu \varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu \omega^{ac}_\mu \rangle$, in $d=4$. A posteriori, this is not that surprising, given that the restriction to Ω introduces the mass scale γ into the theory, and that the horizon condition (9) can be reexpressed at the local level as $\langle g f^{abc} A^a_\mu (\varphi^{ac}_\mu + \overline{\varphi}^{ac}_\mu) \rangle = -2\gamma^2 d(N^2-1)$, i.e. a dimension 2 condensate for $d=4$. Nontrivial condensates are an important source of nonperturbative effects in gauge theories, hence the general interest in their study. In particular, dimension 2 condensates attracted a lot of attention in recent years, see e.g. [4, 7] and references therein. In the current case, the operator $(\overline{\varphi}^{ac}_\mu \varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu \omega^{ac}_\mu)$ can be added to the theory in a way that preserves renormalizability [6], which is already a remarkable feature, indicative of its possible relevance. We studied the effects of this condensate using variational perturbation theory, and found that the gluon propagator[2] does not vanish at zero momentum ($D(0) \neq 0$), that the ghost propagator behaves like $\sim \frac{1}{k^2}$ at small momenta, and that there is a violation of positivity in the gluon propagator [6]. Any of these findings is in good agreement with *all* most recent lattice data, obtained at previously unseen large volumes [8, 9]. Also certain results based on Schwinger-Dyson (SD) and/or Functional Renormalization Group (FRG) equations are consistent with these data, see e.g. [10, 11]. Without taking into account the effects related to $\langle \overline{\varphi}^{ac}_\mu \varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu \omega^{ac}_\mu \rangle$, the GZ action (10) also leads to the positivity violation of the gluon propagator, however with $D(0)=0$, and an infrared enhanced ghost. These latter two results are no longer supported by lattice data. Hence, it seems crucial to take into account additional nonperturbative effects related to the restriction to the region Ω (i.e. the boundary condition) to allow for consistency between the analytical GZ results and most recent lattice predictions. The interpretation of the analytical [5, 6] and the lattice results of [8] was challenged in papers like [9, 10, 12]. It was argued that the ghost propagator must be infrared enhanced to ensure confinement, whereby only colorless states are physical. These statements are based on the Kugo-Ojima (KO) analysis of gauge theories [13, 14]. This analysis relies on the operator formalism, and it has been shown that, given a globally well-defined BRST charge Q_B, the color charge Q^a is a BRST exact variation, $Q^a = Q_B(\ldots)$, if the gluon propagator contains no massless poles. The color charge Q^a is then well-defined only if the KO confinement criterion holds

$$u(0) = -1\,, \tag{11}$$

with $u(k^2)$ defined through the following Green function

$$\int d^dx e^{ikx} \left\langle D^{ad}_\mu c^d(x) D^{\nu\, be}_\nu \overline{c}^e(0) \right\rangle_{\rm FP} = \delta^{ab} \left(P_{\mu\nu}(k) u(k^2) - \frac{k_\mu k_\nu}{k^2} \right) \tag{12}$$

[2] The Landau gluon propagator can be parametrized in terms of the form factor $D(k^2)$ as $\langle A^a_\mu(k) A^b_\nu(-k) \rangle = \delta^{ab}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) D(k^2)$.

in Minkowski space. $\langle\ldots\rangle_{\rm FP}$ stands for the expectation value taken with the FP action (3), while $P_{\mu\nu}(k)=g_{\mu\nu}-k_\mu k_\nu/k^2$ for the transverse projector. Using the nilpotent BRST charge Q_B, one can invoke its cohomology to define the physical subspace, and by means of $Q^a=Q_B(\ldots)$, conclude that physical states cannot carry color. A few comments are in order. First of all, in the KO framework [13, 14], the existence of a globally well-defined BRST charge is *assumed*. Thus, the issue of the (non)existence of a nonperturbatively valid BRST symmetry is not explicitly faced. Secondly, the link between the BRST charge Q_B and global color charge Q^a is made using the action (1), i.e. by employing the usual FP gauge fixed action. As such, the Gribov problem is simply not addressed. It is worth noticing that Kugo and Ojima did not impose the criterion (11), but they derived it as a condition to be checked/calculated. Though, nowadays, in functional formalisms as in [10], the criterion is used as *input*. Kugo showed in [13] that, in the Landau gauge, one can rewrite the ghost propagator

$$G^{ab}(k)=\frac{\delta^{ab}}{k^2}\frac{1}{1+u(k^2)+k^2v(k^2)}\,, \tag{13}$$

meaning that the criterion (11) is equivalent to an infrared enhanced ghost. The ghost enhancement is then imposed as a boundary condition in order to favor the so-called scaling type solution of the SD and/or FRG equations [10]. Let us already draw attention to the close similarity existing between the no-pole condition (5) and the criterion (11). Imposing[3] $\sigma(0)=-1$ exactly corresponds to $u(0)=-1$.

II. $u(0)=-1$ AS A BOUNDARY CONDITION

We want to show that the constraint $u(0)=-1$ can be implemented directly into the theory, by appropriately modifying the measure one starts from. We shall see that the resulting action will be exactly the same as the GZ action. This has several interesting consequences which we will discuss in Section III. We shall first give an overview of some results from thermodynamics we intend to employ.

A. Microcanonical ensemble and equivalence with the canonical Boltzmann ensemble in the thermodynamic limit

We consider a discrete system, whose Hamiltonian is $H(q,p)$, with $3N$ degrees of freedom. The averages in the microcanonical ensemble are constructed out of

$$\Sigma(E)=\int_{H=E}d\mu\;=\int d\mu\,\delta(E-H)\,,$$

where $d\mu=d^{3N}qd^{3N}p$ represents the classical phase space and E stands for the constant energy of the system. Averages in the microcanonical ensemble are defined by $\langle O\rangle_{\rm Micr}=\frac{\int_{H=E}d\mu\;O}{\int_{H=E}d\mu}$. In order to establish the equivalence between the microcanonical and the (Boltzmann) canonical ensemble we rewrite the quantity $\Sigma(E)$ in the following form

$$\begin{aligned}\Sigma(E)&=\int d\mu\,\delta(E-H)=\int d\mu\int_{-i\infty+\varepsilon}^{i\infty+\varepsilon}\frac{d\beta}{2\pi i}e^{\beta(E-H)}\\&=\int\frac{d\beta}{2\pi i}f(\beta)\;=\int\frac{d\beta}{2\pi i}e^{-\omega(\beta)}\,,\end{aligned}\tag{14}$$

$$f(\beta)=\int d\mu\;e^{(\beta(E-H))}\,,\qquad\omega(\beta)=-\log f(\beta)\,.\tag{15}$$

It can be shown that, in the thermodynamic limit, $N,V\to\infty$, with N/V fixed, the saddle point approximation becomes exact. We refer to [15] for an overview of the proof. So,

$$\Sigma(E)=\frac{1}{2\pi i}f(\beta^\star)\,,\text{with }\omega'(\beta^\star)=\frac{f'(\beta^\star)}{f(\beta^\star)}\;=\;0\,.\tag{16}$$

From eq.(16) it follows that

$$E=\langle H\rangle_{\rm Boltz}=\frac{\int d\mu\;He^{-\beta^\star H}}{\int d\mu\;e^{-\beta^\star H}}\,.\tag{17}$$

This is the gap equation determining the critical parameter $\beta^\star$. Analogously, it can also be shown that [15] $\langle O\rangle_{\rm Micr}=\langle O\rangle_{\rm Boltz}=\frac{\int d\mu\;O\;e^{-\beta^\star H}}{\int d\mu\;e^{-\beta^\star H}}$ for the average of any quantity $O(q,p)$.

B. Imposing the KO criterion yields the GZ framework

Starting from (12) and performing Lorentz and color contractions and taking the $p\to0$ limit, we can write

$$\begin{aligned}&-(VT)^{-1}\int d^dy\int d^dx\,\langle D^{ad}_\mu(x)D^{ae}_\mu(y)(M^{-1})^{de}(x,y)\rangle_{\rm FP}\\&=\;(N^2-1)((d-1)u(0)-1)\,,\end{aligned}\tag{18}$$

after passing to Euclidean space, as in any functional or lattice approach. VT denotes the spacetime volume. The identification between $\langle\ldots c^d(x)\overline{c}^e(y)\rangle_{\rm FP}$ and $\langle\ldots(M^{-1})^{de}(x,y)\rangle_{\rm FP}$ can be easily proven using the path integral (3). After discarding terms which are total derivatives, one easily sees that the quantity in the l.h.s. of (18) is, up to the sign, the Zwanziger horizon function $h(x)$. More precisely, we have

$$\begin{aligned}(18)&=\int\frac{d^dy}{VT}d^dx\,\langle gf^{akd}A^k_\mu(x)(M^{-1})^{ed}(x,y)gf^{ame}A^m_\mu(y)\rangle_{\rm FP}\\&=\;-(VT)^{-1}\int d^dx\,\langle h(x)\rangle_{\rm FP}=-\langle h\rangle_{\rm FP}\,.\end{aligned}\tag{19}$$

We observe that the KO condition cannot be realized with the standard FP measure $d\mu_{\rm FP}$, otherwise we would have

$$\langle h(x)\rangle_{\rm FP}=d(N^2-1)\,,\tag{20}$$

which would contradict Zwanziger's result (9), obtained by restricting the path integral to the Gribov region Ω. We now

[3] $\sigma(0)$ is related to $\sigma(0,A)$ by making the gauge field dynamical and performing the corresponding path integration.

implement the KO criterion $u(0) = -1$ as a boundary condition, amounting to start from the modified measure

$$d\mu_{\rm FP} \;\to\; d\mu' \equiv d\mu_{\rm FP}\;\delta\left(VTd(N^2-1) - \int d^dx h(x)\right)\,, \quad (21)$$

which clearly implements $\langle h(x)\rangle = d(N^2-1)$, or equivalently $u(0) = -1$. We are thus led to consider the partition function

$$\begin{aligned}\int d\mu' \;&=\; \int d\mu_{\rm FP}\;\delta\left(VTd(N^2-1) - \int d^dx h(x)\right)\\ &= \int dA\;\delta(\partial A)\;\det\mathcal{M}\;\; e^{-S_{\rm YM}}\;\delta\left(VTd(N^2-1) - \int d^dx h(x)\right)\\ &=\; \int d\Phi\delta\left(VTd(N^2-1) - \int d^dx h(x)\right) e^{-S_{\rm YM+gf}}\,. \end{aligned} \quad (22)$$

Expression (22) defines a microcanonical ensemble. Since we are working in a continuum field theory, we are working in the thermodynamic limit, hence we have an equivalence with a Boltzmann canonical ensemble as outlined in the previous section. Using analogous arguments as there, we arrive at

$$\int d\mu' = \int d\mu_{\rm FP}\; e^{\gamma^4\int d^dx h(x)} \equiv \int d\mu_{\rm FP}\; e^{-S_{\rm H}}\,, \quad (23)$$

where the mass parameter γ follows from the gap equation

$$d\left(N^2-1\right) = \langle h(x)\rangle_{\rm Boltz} = \frac{\int d\mu_{\rm FP}\; e^{-S_{\rm H}}\; h(x)}{\int d\mu_{\rm FP}\; e^{-S_{\rm H}}}\,, \quad (24)$$

which is the analogue of (17). We conclude that we can consistently encode the boundary condition (11) at the level of the action, which turns out to be identical to the GZ action, eq.(7). Of course, we can localize it into the form (10), with corresponding local formulation of the gap equation.

III. DISCUSSION

Naively, one might already expect that the introduction of a nontrivial boundary condition can seriously influence the dynamics of the theory. One of our main points is to stress that one should introduce the boundary condition into the theory from the beginning, to fully grasp all its nontrivial aspects. Having at our disposal an action automatically implementing the boundary condition, we can study an important aspect: the symmetries of the theory in the presence of the boundary. In principle, imposing a boundary could jeopardize certain symmetries of the original action. We have already shown in [6] that placing a boundary in field space at the first Gribov horizon breaks the conventional BRST symmetry (2). The practical implementation of the horizon by means of the GZ formulation confirms this, as $sS_{\rm GZ} = g\gamma^2\int d^dx f^{abc}\left(A^a_\mu\omega^{bc}_\mu - \left(D^{am}_\mu c^m\right)\left(\overline{\varphi}^{bc}_\mu + \varphi^{bc}_\mu\right)\right) \neq 0$. We notice that the BRST generator (2) has a natural extension to the extra fields $(\overline{\varphi}^{ac}_\mu, \varphi^{ac}_\mu, \overline{\omega}^{ac}_\mu, \omega^{ac}_\mu)$, given by

$$s\varphi^{ac}_\mu = \omega^{ac}_\mu\;, s\omega^{ac}_\mu = 0\;, s\overline{\omega}^{ac}_\mu = \overline{\varphi}^{ac}_\mu\;, s\overline{\varphi}^{ac}_\mu = 0\,, \quad (25)$$

forming 2 pairs of BRST doublets. In the absence of the GZ restriction or equivalently of the KO criterion, i.e. when $\gamma \equiv 0$, we are then assured that these fields are trivial in the BRST cohomology, thus completely decoupling from the physical subspace [16]. Let us come to another important statement. If Gribov copies are taken into account à la GZ, which is equivalent to imposing the KO criterion as we have verified in the previous section, the precise meaning of the KO confinement criterion becomes unclear. Since the BRST symmetry is broken, one can no longer simply use it to define the physical subspace. It is sometimes mentioned in the literature that there might be a nonperturbative, globally well-defined BRST charge Q'_B, and it is this Q'_B KO is referring to [10, 12]. We cannot exclude this possibility, but this is a highly nontrivial statement and, obviously, it asks for a proof. At present, we are unaware of any such proof. Even if the charge Q'_B would be known, the KO analysis would need to be reworked from the start, as it explicitly relies on the FP action (1) and conventional BRST symmetry (2). Simply stating that Q'_B must exist in order to define the physical subspace analogously to what is done at perturbative level, does, in our opinion, not solve the problem. Also, the relation between a new BRST charge Q'_B and the global color charge would need to be reestablished, if any relation exists to begin with. One can speculate that it might be possible to modify s into s_γ, such that $\lim_{\gamma\to 0} s_\gamma = s$ and $s_\gamma S_{\rm GZ} = 0$. However, such a possibility can be easily disproved. Indeed, as γ has mass dimension 1, and by keeping in mind that the BRST generator s does not affect the dimension of the fields[4], it is impossible to introduce extra γ-dependent terms in the BRST transformation of the fields while preserving locality, Lorentz covariance and global $SU(N)$ structure. Let us briefly return to the functional SD (FRG) approaches. Now that a renormalizable action has been constructed, which implements the desired boundary condition explicitly, one can write down the corresponding SD (FRG) equations and try to solve them, given that the gap equation (24) must be solved simultaneously. We expect that different kinds of solutions, similar to those found in [10], will emerge. A way to distinguish between them could be based on selecting the most stable solution, i.e. the one with the lowest corresponding vacuum energy. We notice that there is still a lot of information available about the action (10), e.g. nonrenormalization properties typical of the Landau gauge, a renormalizable softly broken Slavnov-Taylor identity, etc. [6]. We conclude that in the current spirit of using the KO condition (11) as in [10, 12], there is no clear connection between the KO criterion $u(0) = -1$ and the highly nontrivial issue of confinement. All that one can say is that there is a violation of positivity in the gluon propagator, which is indicative of confinement, but certainly not a proof of it. Also, in the light of our previous results, we disagree with the statement made in [9, 10, 12] about the fact that the SD(FRG) solution with an infrared enhanced ghost propagator would refer to the absolute Landau gauge, i.e. to the restriction to the FMR Λ. Unfortunately, at present,

[4] The usual canonical dimensions are assigned to the fields [16].

a way to implement the restriction to the FMR Λ remains completely unknown. Moreover, we remind that the recent lattice data have given quite clear evidence about the fact that the ghost propagator is not enhanced in the infrared, within the current accuracy of implementing the Landau gauge as the minimum of the functional $\int d^dx (A^u_\mu)^2$, $u \in SU(N)$ [8]. Even if in the future more powerful algorithms would bring the simulations closer to the FMR Λ, the ghost propagator will not get more enhanced than before, on the contrary [17]. Moreover, we have shown in this letter that the KO boundary condition is equivalent with the GZ framework, which explicitly refers to the restriction to the Gribov region Ω. This is irrespective of the fact that the ghost is enhanced or not, implementing KO breaks the conventional BRST symmetry, and refers to Ω, not to Λ. We wish to underline that implementing the boundary as in eqs.(23) and (24) will not necessarily give rise to an infrared enhanced ghost. Additional nontrivial quantum effects can combine with the boundary effect, we refer for instance to the effects of the operator $\overline{\varphi}^{ac}_\mu \varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu \omega^{ac}_\mu$ in the case under study. We can make the analogy with spontaneous symmetry breaking: although the starting action in that case enjoys a certain symmetry, nonperturbative quantum effects can induce a shift from the symmetric, but unstable, vacuum[5], causing qualitative and quantitative effects in the theory.

In conclusion, we hope that this letter has clarified the relation between the KO and GZ framework. Our main result is expressed by eqs.(23) and (24), which show that the KO and GZ frameworks are equivalent, provided the KO boundary condition is properly taken into account from the beginning. The conventional BRST operator (2) suffers from a soft breaking, which relies precisely on the implementation of the boundary condition. Some ingredients in certain formalisms, which we have tried to outline, have thus to be considered as assumptions rather than as proofs. In particular, the precise relation between implementing the KO criterion and confinement remains to be clarified. One of the challenges lying ahead is how to define what the relevant physical operators are in the KOGZ framework, if there is no (local) nilpotent BRST symmetry generator found.

[5] We refer to condensates which are dynamically favored by lowering the vacuum energy.

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