Compact and Accurate Models of Large Single Wall Carbon Nanotube Interconnects

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***Abstract*—Single wall carbon nanotubes (SWCNTs) have been proposed for VLSI interconnect applications and their modeling is carried out using the multiconductor transmission line (MTL) formulation. Their time-domain analysis has some simulation issues related to the high number of SWCNTs within each bundle, which results in a highly complex model and loss of accuracy in the case of long interconnects. In recent years, several techniques have been proposed to reduce the complexity of the model whose accuracy decreases as the interconnection length increases. This paper presents a rigorous new technique to generate accurate reduced order models of large SWCNT interconnects. The frequency response of the MTL is computed by using the spectral form of the dyadic Green's function of the 1-D propagation problem and the model complexity is reduced using rational model identification techniques. The proposed approach is validated by numerical results involving hundreds of SWCNTs, which confirm its capability of reducing the complexity of the model, while preserving accuracy over a wide frequency range.**

***Index Terms*—Single wall carbon nanotubes (SWCNTs), model order reduction, nano-interconnects, transient analysis, transmission line modeling.**

I. Introduction

Carbon nanotubes (CNTs) have been proposed for VLSI interconnect applications [1]–[4] due to their outstanding electrical, thermal properties and and large-current carrying capability [5]–[9].

This novel interconnect technology has the potentiality to replace copper in the future and, hence, a realistic analysis of these interconnects is necessary to evaluate their performance in the domain of on-chip interconnections.

It is known that isolated SWCNTs are characterized by an intrinsic ballistic resistance of approximately 6.5 $k\Omega$ [5] that is not dependent on the length of the nanotube [10], causing excessive delay for realistic interconnect applications. To alleviate the intrinsic interconnect problem, bundles of SWCNTs connected in parallel have been proposed and physically demonstrated as possible interconnect for local, intermediate and global interconnects [3], [4], [9], [11], [12]. Hence, SWCNTs have been modeled as transmission lines (TL) in the pioneering works [13]–[15] and as multiconductor transmission lines (MTLs) in more recent papers [16]–[20], where the transmission line theory has been extended to bundles of SWCNTs in the frequency domain. The crosstalk coupling in SWCNTs bundles has been investigated in [21], [22]. In [21], the inverse fast Fourier transform (IFFT) is used to obtain time-domain results, while in [22], equivalent distributed circuits are adopted to perform an accurate crosstalk analysis. When the number of SWCNTs in each bundle increases and the length of the interconnect exceeds some tens of microns, the standard techniques based on the MTL theory may not be suitable for several reasons: 1) the complexity of the models is extremely high due to the high number of nanotubes in each bundle; 2) the accuracy of the two port parameters of the interconnection may be lost due to the need to solve high-order eigenvalues problems at extremely high frequencies [23]; 3) if a distributed equivalent circuit is used (e.g. [22]) and, thus, no eigenvalue problem has to be solved, the necessity to discretize each nanotube along its length, may result in an extremely large number of unknowns.

To mitigate the problem of the complexity of the overall MTL model, which increases with the number of nanotubes, approximate formulations have been proposed based on the equivalent single conductor (ESC) distributed circuit in the frequency-domain [21], [24], [25]. Although effective in guaranteeing significant speed-up, this approach is affected by an uncontrolled error which may result in loss of accuracy, especially in the case of long interconnects for frequencies exceeding some tens of GHz [25]. Furthermore, its use in conjunction with non-linear terminations is quite difficult, as it is a frequency-domain technique.

A fluid model description of carbon nanotubes has also been investigated in [26] leading to an equivalent transmission line model. Such a technique has been extended to incorporate diameter-dependent parameters in [27].

More recently, in [28], general equations are presented for the spatially dispersive conductivity, distributed impedance, Ohm's law relation, and transmission line model of both carbon nanotubes (CNTs) and solid material nanowires.

Both the techniques [27] and [28], although physically rigorous, have been applied only to simple interconnects.

In [29], a spectral model of nano-interconnects has been proposed which is able to generate accurate macromodels well suited for time-domain simulations incorporating non-linear terminations. This approach avoids any longitudinal space

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discretization [23] of the interconnect as it adopts continuous basis functions. Furthermore, since the macromodel of the interconnect is computed by means of orthogonal modes, the poles of the system can be identified independently for each mode, thus reducing the computational complexity. Nevertheless, the analysis of interconnects including bundles with hundreds of nanotubes can still be a challenge because, when the number of SWCNTs exceed few hundreds, the explicit computation of poles as described in [29] can be affected by numerical inaccuracies, leading to a not enough accurate macromodel.

Hence, it can be claimed that, although several techniques have already been developed for the analysis of SWCNT interconnects, none of them has been found really effective to generate time-domain macromodels of large SWCNT interconnects.

The aim of this paper is to address a systematic modeling of large SWCNTs interconnects leading to compact and accurate time-domain macromodels, allowing to overcome all the limits exhibited by the other techniques. This task is achieved by a three-step approach. First, the frequency response is computed using the spectral approach in [29]. Since the proposed technique finally provides a state-space macromodel, the IFFT is avoided and the frequency sampling can be relaxed. Second, the complexity of the interconnect model is reduced by enforcing Kirchhoff voltage and current laws at the electrical ports. Third, using the frequency samples obtained in the first and second steps, a state-space macromodel is obtained by means of the standard Vector Fitting (VF) technique [30]. Hence, the spectral approach is used to overcome the drawbacks related to the computation of the eigenvalues of large interconnects. Finally, the VF technique is applied to the system reduced to the electrical ports.

The paper is organized as follows: Section II briefly reviews the Telegrapher's equations for nano-interconnects constituted by SWCNTs providing the spectral form of the $\boldsymbol{Z}$ matrix. Section III shows how the overall model is reduced to its electrical ports and the macromodel is generated. A comparison of the proposed method with existing frequency- and time-domain techniques is presented in Section IV. Numerical results validating the proposed approach are reported in Section V. The conclusions are drawn in Section VI.

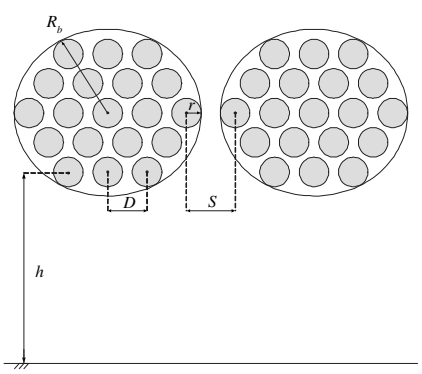

Fig. 1. Geometric configuration of two SWCNT bundles above a ground plane.

II. Spectral Model of the MTL

This section briefly describes the spectral approach which is used to compute the frequency response of the SWCNT interconnect. A more detailed description of the theoretical derivation can be found in [29] and not repeated here. Consider a nano-interconnect constituted by N_c SWCNTs. At the generic abscissa z, the Telegrapher's equations, in the Laplace domain read [23], [31]:

$$\frac{d}{dz}\boldsymbol{V}(z,s) = -\boldsymbol{Z}_{pul}(s)\boldsymbol{I}(z,s), \tag{1a}$$

$$\frac{d}{dz}\boldsymbol{I}(z,s) = -\boldsymbol{Y}_{pul}(s)\boldsymbol{V}(z,s) + \boldsymbol{I}_S(z,s), \tag{1b}$$

where port currents are regarded as external current sources

$$\boldsymbol{I}_S(z,s) = \boldsymbol{I}_0(z,s)\delta(z) + \boldsymbol{I}_\ell(z,s)\delta(z-\ell). \tag{2}$$

The per-unit-length impedance matrix $\boldsymbol{Z}_{pul}(s)$ of the CNTs can be written as:

$$\boldsymbol{Z}_{pul}(s) = \boldsymbol{R}_S + s\left(\boldsymbol{L}_K + \boldsymbol{L}_M\right), \tag{3}$$

where $\boldsymbol{R}_S$ and $\boldsymbol{L}_K$ are diagonal matrices containing the scattering resistance and kinetic inductance of each CNT, respectively, and $\boldsymbol{L}_M$ is a full-rank matrix accounting for the magnetic field coupling.
The per-unit-length admittance matrix $\boldsymbol{Y}_{pul}(s)$ is given by

$$\boldsymbol{Y}_{pul}(s) = s\left(\boldsymbol{C}_Q^{-1} + \boldsymbol{C}_E^{-1}\right)^{-1}, \tag{4}$$

where $\boldsymbol{C}_Q$ is a diagonal matrix accounting for the per-unit-length quantum capacitance and $\boldsymbol{C}_E$ is a full-rank matrix modeling the electric field coupling. In the following, it is assumed that:

$$R_S = \frac{h}{2Ne^2}\frac{1}{\lambda_{mfp}}, \tag{5a}$$

$$L_K = \frac{h}{4Ne^2\nu_F}, \tag{5b}$$

$$R_Q = \frac{h}{2Ne^2}. \tag{5c}$$

where N is the number of conducting channels, λ_{mfp} is the electron mean free path and R_Q is the intrinsic resistance accounting for the ballistic transport phenomena [29]. For low-biases (i.e. for applied axial electric field less than 0.1 V/μm) and low temperatures, the main scattering mechanism is due to acoustic phonons having λ_{mfp} of the order of a few micrometers. Temperature dependent models for resistance of SWCNTs can also be used [32], [33]. At higher biases, optical phonon and zone boundary phonon scattering become dominant, thus reducing the effective mean free path to the range of nanometers [34]. Interconnects are usually long enough to ignore high-bias resistance [7], [22].

At both ends, each tube of the bundle, is terminated on a lumped resistance

$$R_t = \frac{R_Q}{2} + R_{mc} \tag{6}$$

where R_{mc} is the imperfect metal-CNT contact resistance which ranges from few ohms up to hundreds of kilo-ohms, depending on the fabrication process. Hence, each SWCNT in

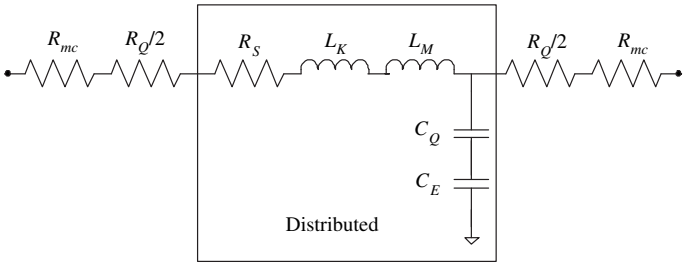

Fig. 2. Equivalent circuit model of each SWCNT, where R_{mc} is the imperfect metal-CNT contact resistance, R_Q is the quantum resistance, R_S is the scattering resistance, L_K is the kinetic inductance, L_M is the magnetic inductance, C_Q is the quantum capacitance, and C_E is the electrostatic capacitance.

the bundle can be represented by the equivalent circuit shown in Fig. 2.

In the following, metallic nanotubes will be considered resulting in $N = 2$.

A non-homogeneous second-order differential equation is obtained from (1) as:

$$\frac{d^2}{dz^2}\boldsymbol{V}(z,s) - \boldsymbol{\gamma}^2(s)\boldsymbol{V}(z,s) = -\boldsymbol{Z}_{pul}(s)\boldsymbol{I}_S(z,s), \tag{7}$$

where $\boldsymbol{\gamma}^2 = \boldsymbol{Z}_{pul}(s)\boldsymbol{Y}_{pul}(s)$.

In [31] it has been proven that equations (7), along with homogeneous boundary conditions of the Neumann type on the port voltages, represent a Sturm-Liouville problem whose formal solution can be obtained through the use of the dyadic Green's function [35]. The general solution for the port voltages of the MTL as a function of the port currents is given by:

$$\begin{bmatrix}\boldsymbol{V}_0(s)\\ \boldsymbol{V}_\ell(s)\end{bmatrix} = \begin{bmatrix}\boldsymbol{Z}_{11} & \boldsymbol{Z}_{12}\\ \boldsymbol{Z}_{21} & \boldsymbol{Z}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{I}_0(s)\\ \boldsymbol{I}_\ell(s)\end{bmatrix}, \tag{8}$$

where

$$\boldsymbol{Z}_{11} = \boldsymbol{Z}_{22} = \sum_{n=0}^{\infty}\boldsymbol{\Psi}_n(s)^{-1}\cdot A_n^2\boldsymbol{Z}(s), \tag{9a}$$

$$\boldsymbol{Z}_{12} = \boldsymbol{Z}_{21} = \sum_{n=0}^{\infty}\boldsymbol{\Psi}_n(s)^{-1}\cdot A_n^2\boldsymbol{Z}(s)\left(-1\right)^n, \tag{9b}$$

with $\boldsymbol{\Psi}_n(s) = \left[\boldsymbol{\gamma}^2(s) + \left(\frac{n\pi}{\ell}\right)^2\boldsymbol{U}\right], A_n = \sqrt{1/\ell}$ for $n = 0, A_n = \sqrt{2/\ell}$ for $n > 0$, and $\boldsymbol{U}$ the identity matrix.

It is worth mentioning that each rational function in the series (9) is characterized by different poles and therefore captures different resonances of the system. This feature is crucial for interconnects with a large number of nanotubes since it allows to avoid the cumbersome similarity transformation used to diagonalize the propagation problem [23].

III. Port reduction of the impedance matrix $\boldsymbol{Z}$

Assume that each bundle is constituted by N_c^b SWCNTs and let N_b be the number of bundles so that the global number of nanotubes is $N_c = N_b N_c^b$. Once the impedance matrix $\boldsymbol{Z}$ is obtained through the summation (9) truncated to $N_{mod} + 1$ modes, the additional resistance R_t of a metallic SWCNT can be incorporated:

$$\boldsymbol{Z}_t(s) = \boldsymbol{Z}(s) + [\text{diag}(R_t)]_{2N_c\times 2N_c} \tag{10}$$

The global impedance matrix $\boldsymbol{Z}_t(s)$ can be converted into the admittance form $\boldsymbol{Y}_t(s)$ in the frequency domain as:

$$\boldsymbol{Y}_t(j\omega_k) = \boldsymbol{Z}_t^{-1}(j\omega_k), \quad \omega_k = 2\pi f_k, k = 1,\cdots,N_f \tag{11}$$

where N_f denotes the number of frequency samples. Two conditions can be enforced based on the observation that, within each bundle, the nanotubes are connected in parallel:

1) the port voltage is common to all nanotubes within each bundle

$$\boldsymbol{V}_0(j\omega_k) = \boldsymbol{V}_{0,i}(j\omega_k) \quad i = 1,\cdots,N_c^b \tag{12a}$$

$$\boldsymbol{V}_\ell(j\omega_k) = \boldsymbol{V}_{\ell,i}(j\omega_k) \quad i = 1,\cdots,N_c^b \tag{12b}$$

2) the global current at the input and output port of each bundle is the sum of the currents through each nanotubes within each bundle

$$\boldsymbol{I}_0(j\omega_k) = \sum_{i=1}^{N_c^b}\boldsymbol{I}_{0,i}(j\omega_k) \quad i = 1,\cdots,N_c^b \tag{13a}$$

$$\boldsymbol{I}_\ell(j\omega_k) = \sum_{i=1}^{N_c^b}\boldsymbol{I}_{\ell,i}(j\omega_k) \quad i = 1,\cdots,N_c^b \tag{13b}$$

for each frequency sample $\omega_k, k = 1,\cdots,N_f$.

Hence, the N_b bundles can be represented as a $2N_b$ port system whose admittance matrix representation, for each frequency sample, reads:

$$\begin{bmatrix}\boldsymbol{I}_0(j\omega_k)\\ \boldsymbol{I}_\ell(j\omega_k)\end{bmatrix} = \begin{bmatrix}\boldsymbol{Y}_{t,11}(j\omega_k) & \boldsymbol{Y}_{t,12}(j\omega_k)\\ \boldsymbol{Y}_{t,21}(j\omega_k) & \boldsymbol{Y}_{t,22}(j\omega_k)\end{bmatrix}\begin{bmatrix}\boldsymbol{V}_0(j\omega_k)\\ \boldsymbol{V}_\ell(j\omega_k)\end{bmatrix}, \tag{14}$$

where each sub-block $\boldsymbol{Y}_{t,mn}(j\omega_k), m,n = 1,2$ is obtained summing the entries of a order-N_c^b sub-block of matrix $\boldsymbol{Y}_t(j\omega_k)$ (11). It is to be pointed out that the only approximation which is done so far is the truncation of the infinite series of modes (9). The number of modes can be adaptively selected as described in [36].

The time-domain macromodel can easily be obtained by applying the VF technique [30] to the admittance matrix (14):

$$\boldsymbol{Y}_t(s) = \sum_{k=1}^{p}\frac{\boldsymbol{R}_k}{s - p_k}. \tag{15}$$

The stability of a rational macromodel obtained by means of the VF method is ensured by a simple pole-flipping scheme [30], while its passivity [37], [38] is not guaranteed and must be checked and enforced by means of standard techniques (see e.g. [39]–[41]).

Although the frequencies of interest may range from dc to thousands of GHz, the small length of the bundles limits the number of resonances falling in the spectrum of interest and, thus, a reduced number of poles is sufficient to provide a very good approximation. Furthermore, it is to be pointed out that the fitting process is extremely fast since it is applied

to a reduced number of ports. The corresponding state-space equivalent form can be written as:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{\mathcal{A}}\boldsymbol{x}(t) + \boldsymbol{\mathcal{B}}\boldsymbol{v}(t), \quad (16a)$$

$$\boldsymbol{i}(t) = \boldsymbol{\mathcal{C}}\boldsymbol{x}(t) + \boldsymbol{\mathcal{D}}\boldsymbol{v}(t), \quad (16b)$$

where $\boldsymbol{\mathcal{A}} \in \Re^{p\times p}$, $\boldsymbol{\mathcal{B}} \in \Re^{p\times q}$, $\boldsymbol{\mathcal{C}} \in \Re^{q\times p}$, $\boldsymbol{\mathcal{D}} \in \Re^{q\times q}$, p is the number of states and q is the number of ports. In the case under analysis, the input and output vectors correspond to port voltages $\boldsymbol{v}(t)$ and currents $\boldsymbol{i}(t)$, respectively.

Each bundle is assumed to be terminated on current sources and voltage-driven lumped linear and nonlinear elements, which can be described by the following equation:

$$\boldsymbol{i}(t) = \boldsymbol{i}_s(t) - \boldsymbol{G}_{term}\boldsymbol{v}(t) - \boldsymbol{C}_{term}\frac{d\boldsymbol{v}(t)}{dt} - \boldsymbol{f}(\boldsymbol{v}(t)), \quad (17)$$

where $\boldsymbol{v}(t)$ and $\boldsymbol{i}(t)$ are the port voltages and currents, matrices $\boldsymbol{G}_{term}, \boldsymbol{C}_{term}$ model linear resistive and capacitive terminations, respectively, and $\boldsymbol{f}(\boldsymbol{v}(t))$ describes lumped nonlinear components. The global nonlinear system (16)-(17) can be solved by using standard techniques [42].

The most time-consuming step in the presented approach is the computation of the admittance matrix $\boldsymbol{Y}_t$, for the entire set of nanotubes, for the frequency samples $\omega_k, k = 1, \cdots, N_f$. It is worth noticing that the proposed procedure totally avoids the inverse fast Fourier transform to recover time-domain results. Using the IFFT would require a very fine time-step to preserve accuracy. Since the proposed approach generates the macromodel through the use of the VF algorithm, only a limited number of samples is required, making the overall procedure well suited for large bundles of nanotubes.

IV. Comparison with Existing Frequency- and Time-Domain Based Methods

The use of standard frequency-domain based methods [23] is prone to several drawbacks:

- the IFFT is to be adopted to recover time-domain results; the necessity to properly sample a broadband frequency range turns into a large number of frequency samples which can make the frequency-domain analysis extremely time-consuming;
- the presence of non-linear terminations is a problem, it can be circumvented by using mixed frequency-time domain methods which are not commonly efficient, nor accurate;
- the evaluation of the impulsive response of electrically long interconnects is ordinarily an ill conditioned problem which is affected by numerical inaccuracies when carried out by standard techniques which compute the eigenvalues of the propagation matrix $\boldsymbol{\gamma}$ at extremely high frequencies [43]. Fig. 3 shows the magnitude spectrum of the $Z_{12}(1,1)$ impedance of a 100 μm long SWCNT interconnect constituted by two bundles of 416 SWCNTs. It is clearly seen that the standard transmission line theory (TLT), although rigorous, is inaccurate for frequencies exceeding 40 GHz. On the contrary, the proposed technique based on the orthogonal decomposition of the impedance matrix remains accurate even at very high frequencies.

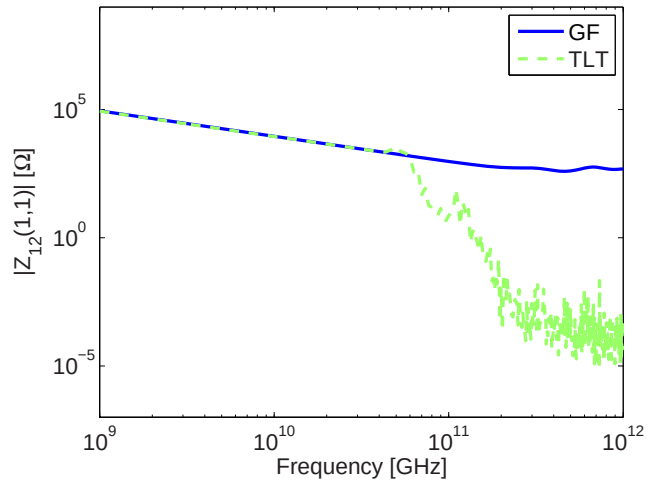

Fig. 3. Magnitude spectrum of impedance $Z_{12}(1,1)$.

Standard time-domain techniques usually require the longitudinal space discretization [23], or compute the dominant poles all at once, resulting in heavy computations. On the other hand, the use of the Green's function-based approach presents the following positive aspects:

- it does not require any similarity transformation [23];
- the numerical results are well conditioned as a consequence of the fact that each mode is orthogonal to the others and captures a different set of poles;
- the frequency sampling can be significantly relaxed since no inverse fast Fourier transform needs to be performed;
- furthermore, the frequency sampling can be non-uniform and adaptive sampling techniques can be adopted as well [44], keeping minimal the number of samples;
- non-linear terminations can be incorporated in a straightforward way.

V. Numerical Results

In the following, two case studies will be considered focusing on both signal integrity and crosstalk analysis. The SWCNTs are assumed to be all conductive within each bundle. As regards the semiconducting SWCNTs, it is still an issue if their presence affects the electric field distribution. It is reported in [45] that for a quasi-1-D structure such as an SWCNT, the relative dielectric constant is always 1. Hence, in the following, it is assumed that, according to [45], semiconducting tubes can be neglected [33].

The results of the proposed time-domain macromodel (Macromodel) are compared with those obtained by applying the IFFT transform to the frequency spectra obtained using the standard transmission line theory (TLT IFFT) and the equivalent single conductor method (ESC IFFT). The time-domain integration is carried out by means of the Gear-Shichman algorithm [46]. In all the numerical simulations, it has been assumed that each bundle is terminated on a resistance $R_s = 25\ \Omega$ at the input port and a capacitance $C_\ell = 0.1$ fF at the output port.

A. Two-coupled bundles of SWCNTs

In the first example, two parallel bundles, each of 120 nanotubes are considered. Each nanotube has a radius of $r = 0.5$ nm, the inter-tube distance $\delta = 0.34$ nm, and the lattice constant is $D = 1.34$ nm. The length of the bundles is $\ell = 20$ μm. The spacing between the bundles is $S = 9.54$ nm and the height above the reference plane is $h = 103.1$ nm. The first bundle is excited by a voltage step with amplitude 100 mV and rise time $\tau_r = 2$ ps. Its frequency spectrum extends to 2000 GHz. The macromodel has been built to be accurate up to this frequency. Hence, the spectral model of the interconnect has been generated using 60 modes. A rational model has been computed by means of the VF method [30]. Figure 4 shows the magnitude and phase spectra of impedance $Z_{12}(1,2)$ obtained by using the proposed method (GF), the transmission line theory (TLT) and the equivalent single conductor method (ESC). It is clearly seen that the Green's function-based results are in a good agreement with the transmission line theory, while the ESC method remains accurate below 500 GHz. Similar results are obtained for all the other impedances.

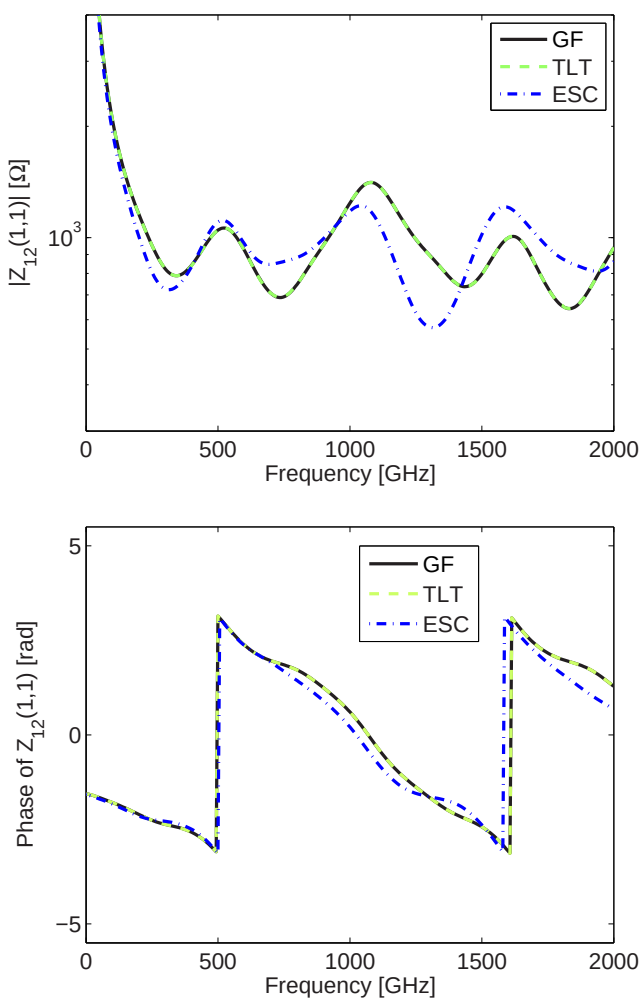

Fig. 4. Magnitude and phase spectra of impedance $Z_{12}(1,2)$ of two-coupled bundles of SWCNTs (example V-A).

Then, a time-domain simulation has been run using the proposed macromodel. Time-domain results have been recovered by the TLT and ESC methods, via IFFT. Fig. 5 shows the transient port voltages as obtained by using the three methods. As before, while the Green's function-based results are in a good agreement with those of the TLT method, some discrepancies are found with respect to the ESC method.

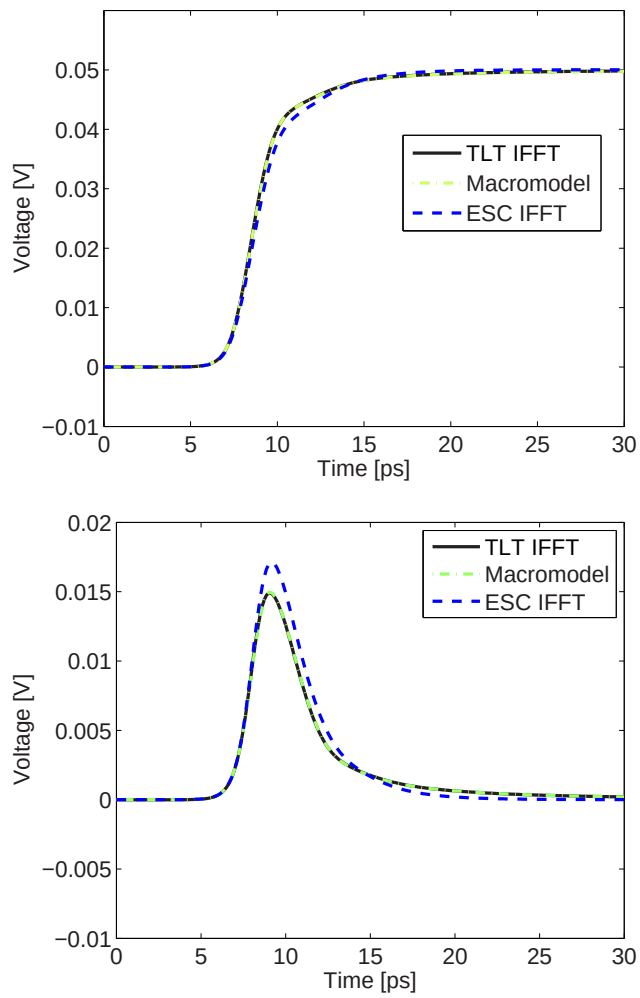

Fig. 5. Output port voltages of two-coupled bundles of SWCNTs (example V-A).

B. Four-coupled bundles of SWCNTs

In the second example, four bundles of nanotubes have been considered. Each bundle contains 45 nanotubes. The radius is $r = 0.5$ nm, the inter-tube distance $\delta = 0.34$ nm, and the lattice constant is $D = 1.34$ nm. The length of the bundles is $\ell = 50$ μm. The spacing between the bundles is $S = 9.54$ nm and the height above the reference plane is $h = 103.1$ nm.

Fig. 6 shows the magnitude and phase spectra of impedance $Z_{12}(1,2)$. It is seen that, as before, the proposed Green's function-based results are in a good agreement with the TLT method, while the ESC method remains accurate below 250 GHz. Similar results are obtained for all the other impedances.

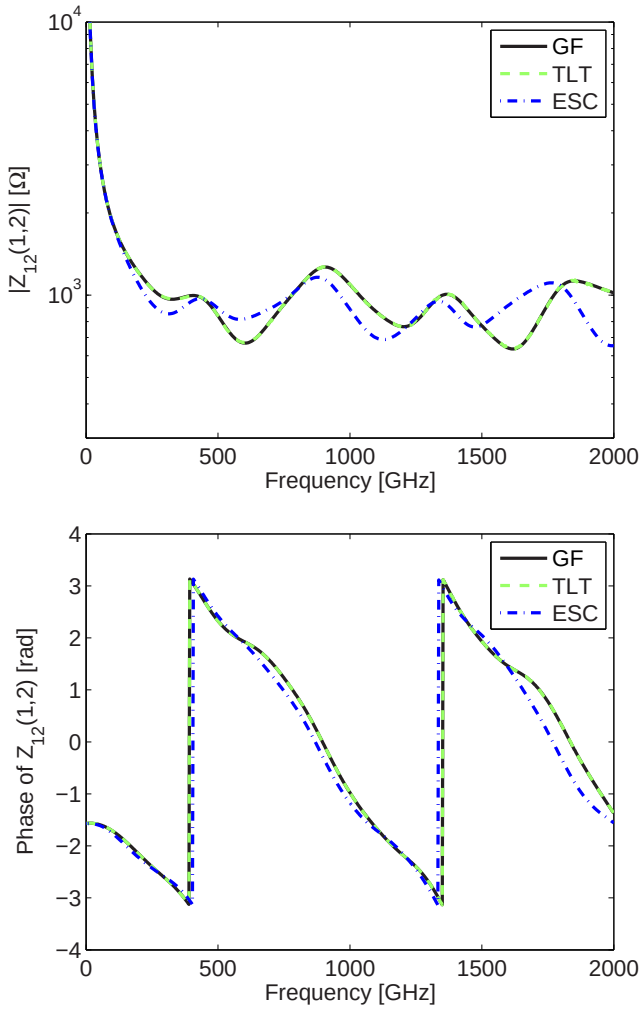

Fig. 6. Magnitude and phase spectra of impedance $Z_{12}(1,2)$ of four-coupled bundles of SWCNTs (example V-B).

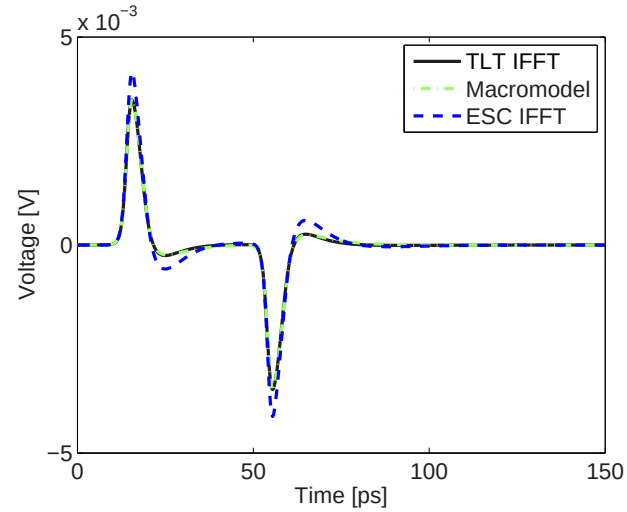

Fig. 7. Far-end port voltage of the fourth of four-coupled bundles of SWCNTs (example V-B).

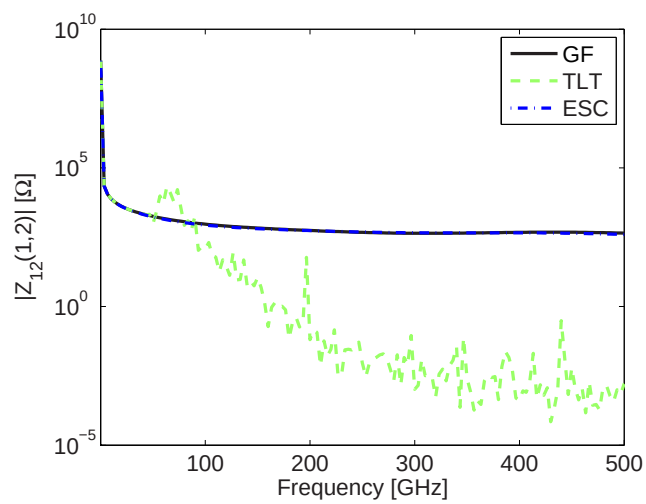

Fig. 8. Magnitude spectrum of impedance $Z_{12}(1,2)$ of four-coupled bundles of SWCNTs for a length $\ell = 100$ μm (example V-B).

The first bundle is excited by the same voltage source of the previous example. Figure 7 shows the far-end port voltage of the fourth line in the bundle: the results confirm that a satisfactory accuracy is achieved by the proposed macromodel approach when compared to the IFFT recovered results of the TLT method, while the ESC model is not able to properly capture the peaks of the induced far-end voltage.

If the length of the bundles is increased to $\ell = 100$ μm, the similarity transformation adopted to evaluate the chain parameters [47] is affected by numerical errors. Figure 8 shows the magnitude spectrum of impedance $Z_{12}(1,2)$. It is clearly seen that the GF and ESC approaches are able to capture the frequency content of the impedance, while the TLT method suffers from numerical problems above 50 GHz.

This numerical noise prevents the results obtained via IFFT to be physically meaningful. Fig. 9 shows the transient far-end voltage on the fourth bundle. As expected, the similarity transformation-based approach (TLT) is affected by unphysical oscillations; the ESC method is not affected by spurious oscillations, but it is less accurate than the proposed Macromodel approach which is able to capture the voltage peaks, while remaining physically consistent.

C. Eight-coupled bundles of nanotubes

In the last example, eight-coupled bundles of nanotubes are considered with the same setting of the previous example. The length of the interconnect is $\ell = 100$ μm. Each bundle contains 32 nanotubes. One of the bundles is excited by the same voltage source of the previous examples. Fig. 10 shows the output voltage of the driven bundle and the far-end voltage of the eighth bundle. It is seen that, again, numerical oscillation due to ill conditioning problems in the computation of the chain parameters appear when using the standard similarity transformation. They are less relevant in the driven bundle, but unacceptable on the victim lines where the crosstalk is to be modeled. The ESC method is not affected by oscillations, but it is not able to capture correctly the voltage peaks either on the driven bundle or the victim one. It is clearly seen that the proposed Macromodel approach is accurate over the entire

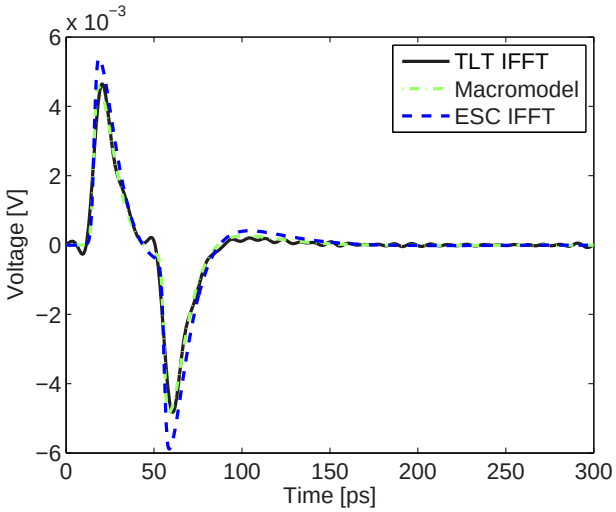

Fig. 9. Far-end port voltage on the fourth of four-coupled bundles of SWCNTs (example V-B).

frequency range of interest and, as a consequence, the time-domain results are not affected by oscillations.

VI. Conclusions

In this paper a systematic data-driven model order reduction method for carbon nanotube interconnects is presented. The dyadic Green's function of the 1-D propagation along the interconnect is used to compute the input/output frequency response of the system, thus avoiding any similarity transformation. The global interconnect is reduced by enforcing equivalence conditions at the electrical ports of each bundle and a state-space macromodel is generated by means of the VF algorithm applied to the reduced model. The number of frequency samples used in the modeling process is much smaller than that needed when the IFFT is adopted. Spurious oscillations appearing in the IFFT-based time-domain results due to inaccuracies in the similarity transform-based approach are avoided. The overall macromodel is accurate, stable and passive and well suited for time-domain analysis of SWCNT interconnects.

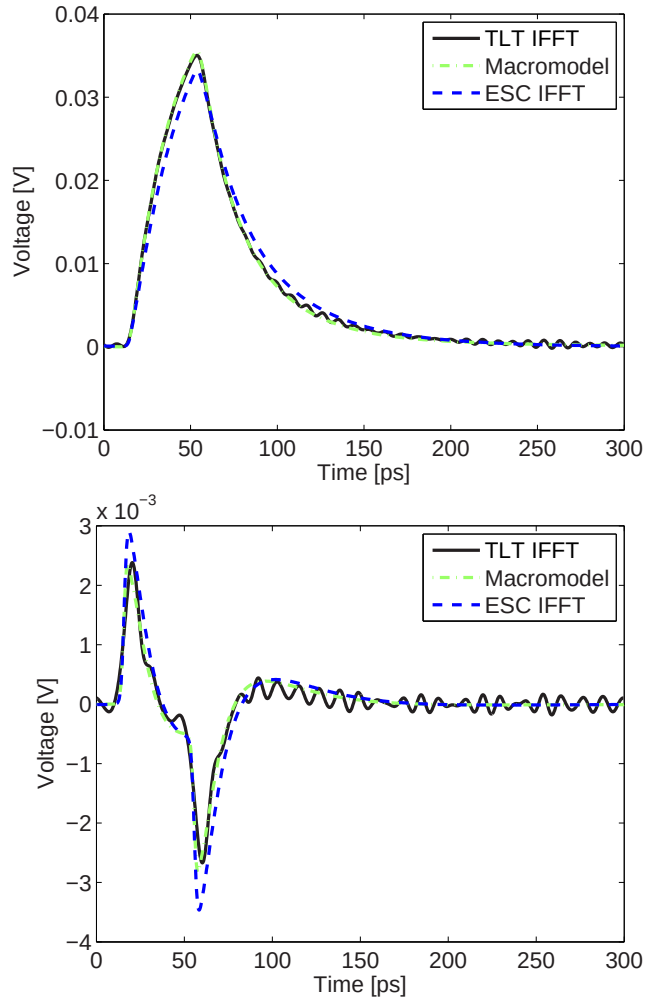

Fig. 10. Output voltages on the driven (top) and the eighth bundle of eight-coupled bundles of SWCNTs (example V-C).

References

[1] F. Kreupl, et al., "Carbon nanotubes in interconnect applications," *Microelectronic Engineering*, vol. 64, pp. 399–408, 2002.

[2] F. Kreupl, P. Graham, M. Liebau, G. S. Duesberg, R. Seidel and E. Unger, "Carbon nanotubes for interconnect applications," in *IEDM Tech. Dig.*, 2004.

[3] N. Srivastava and K. Banerjee, "Performance analysis of carbon nanotube interconnects for VLSI applications," in *Proc. IEEE/ACM ICCAD*, 2005, pp. 383–390.

[4] K. N. Srivastava, H. Li, F. Kreupl, and K. Banerjee, "On the applicability of single-walled carbon nanotubes as VLSI interconnects," *IEEE Transactions on Nanotechnology*, vol. 8, no. 4, pp. 542–559, Jul. 2009.

[5] P. L. McEuen, M. S. Fuhrer and H. Park, "Single-walled carbon nanotube electronics," *IEEE Transactions on Nanotechnology*, vol. 1, no. 1, pp. 78–85, Mar. 2002.

[6] A. Naeemi, R. Sarvari and J. D. Meindl, "Performance comparison between carbon nanotube and copper interconnects for gigascale integration (gsi)," *IEEE Electron Device Letters*, vol. 26, no. 2, pp. 84–86, Feb 2005.

[7] A. Naeemi and J. D. Meindl, "Impact of electron-phonon scattering on the performance of carbon nanotube interconnects for gigascale integration (GSI)," *IEEE Transactions on Electron Devices*, vol. 27, no. 5, pp. 338–440, May 2006.

[8] A. Raychowdhury and K. Roy, "Modelling of metallic carbon-nanotube interconnects for circuit simulations and a comparison with cu interconnects for scaled technologies," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 25, no. 1, pp. 58–65, Jan 2006.

[9] A. Naeemi and J. D. Meindl, "Design and performance modelling for single-walled carbon nanotubes as local, semiglobal and global interconnects in gigascale integrated systems," *IEEE Transactions on Electron Devices*, vol. 54, no. 1, pp. 26–37, Jan. 2007.

[10] A. Nieuwoudt, Y. Massoud, "Evaluating the impact of resistance in carbon nanotube bundles for VLSI interconnects using diameter-dependent modeling techniques," *IEEE Transactions on Electron Devices*, vol. 53, no. 10, pp. 2460–2466, Oct. 2006.

[11] J. Li et al., "Bottom-up approach for carbon nanotubes interconnects," *Appl. Phys. Lett.*, vol. 82, pp. 2491–2493, Apr. 2003.

[12] H. Li, W. Y. Yin, K. Banerjee, and J. F. Mao, "Circuit modeling and performance analysis of multi-walled carbon nanotube interconnects," *IEEE Transactions on Electron Devices*, vol. 55, no. 6, pp. 1328–1337, Jun. 2008.

[13] P. J. Burke, "Lüttinger liquid theory as a model of the gigahertz electrical properties of carbon nanotubes," *IEEE Transactions on Nanotechnology*, vol. 1, no. 3, pp. 129–144, September 2002.

[14] ——, "An RF circuit model for carbon nanotubes," *IEEE Transactions on Nanotechnology*, vol. 2, no. 1, pp. 55–58, Mar. 2003.

[15] S. Salahuddin, M. Lundstrom and S. Datta, "Transport effects on signal propagation in quantum wires," *IEEE Transactions on Electron Devices*, vol. 52, no. 8, pp. 1734–1742, Aug. 2005.

[16] M. S. Sarto and A. Tamburrano, "Electromagnetic analysis of radio-frequency signal propagation along SWCNT bundles," in *6th IEEE Conference on Nanotechnology*, Cincinnati, OH, Aug. 2006, pp. 201–204.

[17] M. D'Amore, M. S. Sarto, A. Tamburrano, "Sensitivity analysis of wave propagation in a single-walled carbon nanotube," in *6th IEEE Conference on Nanotechnology*, Cincinnati, OH, Aug. 2006, pp. 242 – 245.

[18] M. S. Sarto and A. Tamburrano, "Multiconductor transmission line modeling of SWCNT bundles in common-mode excitation," in *IEEE International Symposium on Electromagnetic Compatibility*, Portland, OR, Aug. 2006, pp. 466–471.

[19] M. D'Amore, M. S. Sarto and A. Tamburrano, "Signal integrity of carbon nanotube bundles," in *IEEE International Symposium on Electromagnetic Compatibility*, Honolulu, Hawaii, Jul. 2007.

[20] M. S. Sarto, A. Tamburrano and M. D'Amore, "New electron-waveguide-based modeling for carbon nanotube interconnects," *IEEE Transactions on Nanotechnology*, vol. 8, no. 2, pp. 214 – 225, Mar. 2009.

[21] M. D'Amore, M. S. Sarto and A. Tamburrano, "Transient analysis of crosstalk coupling between high-speed carbon nanotube interconnects," in *IEEE International Symposium on Electromagnetic Compatibility*, Detroit, MI, Aug. 2008, pp. 1–6.

[22] S. N. Pu, W. Y. Yin, J. F. Mao and Q. H. Liu, "Crosstalk prediction of single- and double-walled carbon-nanotube (SWCNT/DWCNT) bundle interconnects," *IEEE Transactions on Electron Devices*, vol. 56, no. 4, pp. 560–568, Apr. 2009.

[23] C. R. Paul, *Analysis of Multiconductor Transmission Lines*, 2nd ed. New York, NY: John Wiley & Sons, 2008.

[24] M. D'Amore, M. Ricci and A. Tamburrano, "Equivalent single-conductor modeling of carbon nanotube bundles for transient analysis of high-speed interconnects," in *Proc. of Int. Symp. IEEE-NANO*, Arlington, VA, Aug. 2008.

[25] M. S. Sarto, A. Tamburrano and A. D'Aloia, "Equivalent effective p.u.l. parameters for reduced circuit of SWCNT interconnects," in *Proc. of Int. Symp. IEEE-NANO*, Genoa, Jul. 2009.

[26] A. Maffucci, G. Miano and F. Villone, "A transmission line model fr carbon nanotube interconnects," *Int. J. Circuit Theory Appl.*, vol. 36, no. 1, pp. 31–51, Jan. 2008.

[27] ——, "A new circuit model for carbon nanotube interconnects with diameter-dependent parameters," *IEEE Transactions on Nanotechnology*, vol. 8, no. 3, pp. 345–354, Aug. 2009.

[28] G. Hanson, "A common electromagnetic framework for carbon nanotubes and solid nanowires-spatially dispersive conductivity, generalized Ohm's law, distributed impedance, and transmission line model," *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 1, pp. 9–20, Jan. 2011.

[29] G. Antonini, A. Orlandi and L. Raimondo, "Advanced models for signal integrity and electromagnetic compatibility-oriented analysis of nano-interconnects," *IEEE Transactions on Electromagnetic Compatibility*, vol. 52, no. 2, pp. 447 – 454, May 2010.

[30] B. Gustavsen and A. Semlyen, "Rational approximation of frequency domain responses by vector fitting," *IEEE Transactions on Power Apparatus and Systems*, vol. 14, no. 3, pp. 1052–1061, Jul. 1999.

[31] G. Antonini, "A dyadic Green's function based method for the transient analysis of lossy and dispersive multiconductor transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, vol. 56, no. 4, pp. 880–895, Apr. 2008.

[32] A. Naeemi and J. D. Meindl, "Performance modeling for single- and multiwall carbon nanotubes as signal and power interconnects in gigascale systems," *IEEE Transactions on Electron Devices*, vol. 55, no. 10, pp. 2574–2582, Oct. 2008.

[33] M. D'Amore, A. Tamburrano and M. S. Sarto, "Fast transient analysis of next-generation interconnects based on carbon nanotubes," *IEEE Transactions on Electromagnetic Compatibility*, vol. 52, no. 2, pp. 496 – 503, May 2010.

[34] A. Naeemi and J. D. Meindl, "Physical modeling of temperature coefficient of resistance for single- and multi-wall carbon nanotube interconnects," *IEEE Electron Device Letters*, vol. 28, no. 2, pp. 135–138, Feb. 2007.

[35] G. W. Hanson and A. B. Yakovlev, *Operator Theory for Electromagnetics*. Springer, 2002.

[36] G. Antonini, F. Ferranti, T. Dhaene, L. Knockaert, "Parameterized models of multiconductor transmission lines," *IEEE Transactions on Advanced Packaging*, vol. 33, no. 2, pp. 481 – 491, May 2010.

[37] B. D. Anderson, S. Vongpanitlerd, *Network Analysis and Synthesis*. Englewood Cliffs, NJ, 1973.

[38] P. Triverio, S. Grivet-Talocia, M. S. Nakhla, F. G. Canavero and R. Achar, "Stability, causality and passivity in electrical interconnect models," *IEEE Transactions on Advanced Packaging*, vol. 30, no. 4, pp. 795–808, 2007.

[39] D. Saraswat, R. Achar and M. S. Nakhla, "Global passivity enforcement algorithm for macromodels of interconnect subnetworks characterized by tabultaed data," *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol. 13, no. 7, pp. 819 – 832, Jul. 2005.

[40] B. Gustavsen, "Fast passivity enforcement for pole-residue models by perturbation of residue matrix eigenvalues," *IEEE Trans. Power Delivery*, vol. 23, no. 4, pp. 2278–2285, Oct. 2008.

[41] T. Dhaene, D. Deschrijver, and N. Stevens, "Efficient algorithm for passivity enforcement of S-parameter based macromodels," *IEEE Transactions on Microwave Theory and Techniques*, vol. 57, no. 2, pp. 415–420, Feb. 2009.

[42] J. M. Ortega and W. C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables*. Boston: SIAM, 2000.

[43] C. R. Paul, "Decoupling the multiconductor transmission line equations," *IEEE Transactions on Microwave Theory and Techniques*, vol. 44, no. 8, pp. 1429–1440, Aug. 1996.

[44] T. Dhaene, J. Ureel, N. Faché, D. De Zutter, "Adaptive frequency sampling algorithm for fast and accurate S-parameter modeling of general planar structures," in *IEEE International Microwave Symposium 1995, MTT-S 1995*, Orlando (FL), 1995, pp. 1427–1430.

[45] F. Leonard and J. Tersoff, "Dielectric response of semicondunting carbon nanotubes," *Appl. Phys. Lett.*, vol. 81, no. 25, pp. 4835–4837, 2002.

[46] H. Shichman, "Integration system of a nonlinear network analysis program," *IEEE Transactions on Circuits and Systems*, vol. 17, pp. 378–386, Aug. 1970.

[47] C. R. Paul, *Introduction to Electromagnetic Compatibility*. Hoboken, NJ: John Wiley Interscience, 2006.

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