Dispersive wave emission and supercontinuum generation in a silicon wire waveguide pumped around the 1550 nm telecommunication wavelength

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We demonstrate dispersive wave generation, soliton fission and supercontinuum generation in a silicon wire at telecom wavelengths. Despite the strong nonlinear absorption inherent to silicon at telecom wavelengths, we experimentally demonstrate that the compression and subsequent splitting of higher order solitons remains possible. Moreover we observe the emission of resonant radiation from the solitons, leading to the generation a broad supercontinuum.

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Supercontinuum generation has been the subject of many studies, particularly since the advent of photonic crystal fibers (PCFs) [1]. The low losses and high confinement, leading to high nonlinearities, as well as the possibility to tailor the zero-dispersion wavelength (ZDW) has led to the generation of supercontinua spanning over more than an octave [2]. Such wide spectra benefit many applications such as high-precision frequency metrology [3], optical coherence tomography [4], or telecommunication [5].

As on-chip generation of ultrashort pulses is becoming a reality [6, 7], the full integration of supercontinuum-based applications can be envisioned. On-chip supercontinuum generation was performed in chalcogenide [8], Si_3N_4 [9], amorphous silicon [10] and silicon waveguides [11]. On the silicon platform, previous experiments have reported a relatively limited spectral broadening in the 1550 nm telecom band. This is most likely due to the use of waveguides with ZDWs far from that spectral band. In [11], the ZDW was around 1300 nm where two photon absorption (TPA) is very strong. Other studies have focused instead on mid-IR pumping, and most of the broadening, up to 3.5 μm wavelength, was in that wavelength range [12, 13]. At the 1550 nm telecom wavelength in a silicon wire, losses are not dominated by TPA but by the subsequent free carrier absorption (FCA) and these losses typically prevent the observation of nonlinear effects [14]. Note that picosecond soliton compression has nevertheless been very recently demonstrated in silicon photonic crystals [15]. One way to circumvent the free carrier induced losses is to use very short femtosecond pulses such that the carrier density remains negligible [16]. In that regime, the dynamics of supercontinuum generation is known to occur through fission of higher-order solitons and the subsequent emission of resonant dispersive waves (DWs) [1, 17–19]. This process is well described by the well known generalized nonlinear Schrödinger equation (GNLSE) and excellent agreement between this model and experiments has been reported in PCFs [20]. Using a similar model, Yin et al predicted that the same mechanisms should give rise to a broad fs supercontinuum in a silicon wire at telecom wavelengths [16] but an experimental study is still missing. Here we report what is, to the best of our knowledge, the first experimental observation of dispersive wave emission and associated supercontinuum generation in a silicon wire waveguide pumped in the C-band. Our study provides unequivocal evidence of high-order soliton dynamics in silicon in that wavelength range.

We consider 7 mm-long silicon-on-insulator (SOI) waveguides with a standard 220 nm silicon thickness. To achieve anomalous group velocity dispersion at the pump wavelength, which is required for efficient fs supercontinuum generation [1], such waveguides must be no wider than 800 nm. Here we report results for waveguides with two different widths, respectively, 700 and 750 nm. The dispersive properties of these waveguides have been calculated with a full vectorial mode solver, and the wavelength dependence of their second-order dispersion coefficient β_2 is shown in Fig. 1. As can be seen, the dispersion is small and anomalous at the 1565 nm pump wavelength used in our experiments. The pump pulses of 150 fs full-width at half-maximum (FWHM) duration at an 82 MHz repetition rate are generated with an

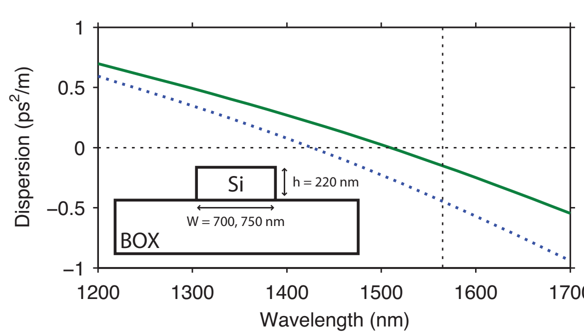

Figure 1. Simulated wavelength dependence of the second-order dispersion coefficient β_2 of 220 nm-thick SOI waveguides with 700 nm (dotted) and 750 nm (solid) width. The vertical dotted line indicates the pump wavelength.

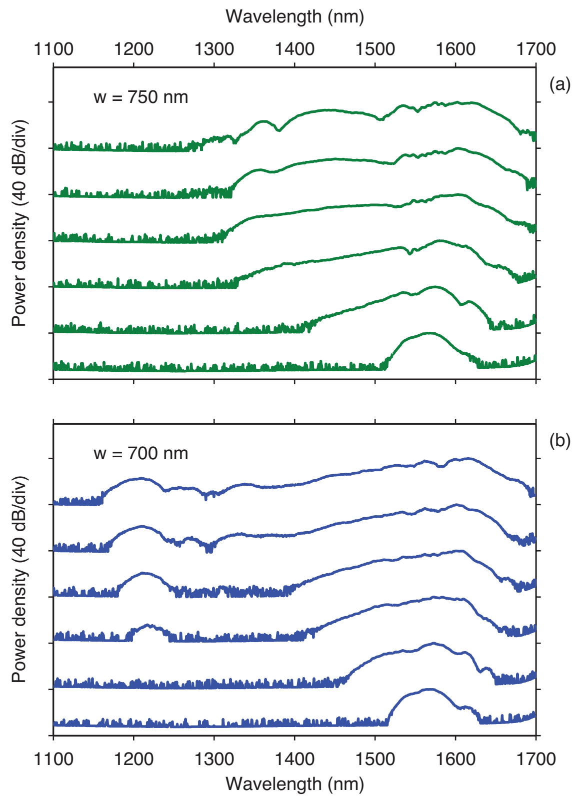

Figure 2. Experimental spectra measured at the output of (a) the 750 nm-wide and (b) the 700 nm-wide waveguide for on-chip peak powers of 0.15 W, 1.5 W, 3.5 W, 7 W, 14 W and 32 W. The spectra have each been shifted by 40 dB for clarity.

OPO (Spectra physics OPAL) pumped by a Ti-Saphire laser (Spectra physics Tsunami) running at 722 nm wavelength. We use the horizontally-polarized idler output, exciting only the quasi-TE mode of the waveguide. The light is coupled into the waveguide with a x60 microscope objective (NA $= 0.65$) and coupled out with a lensed fiber (NA $= 0.4$). The propagation losses are estimated to be 2 dB/cm by cutback measurements on similar waveguides.

The optical spectra measured at the output of the two waveguides for increasing pump peak power (up to 32 W) are shown in Figs. 2(a) and (b). We can readily observe a clear difference between the two sets of measurements, which reveals the strong influence of the waveguide width, hence the dispersion, on spectral broadening in silicon wires. Resonant DWs are observed with pump peak powers as low as 3.5 W. They appear in the normal dispersion regime around 1350 nm and 1200 nm, respectively for the 750 nm and 700 nm-wide guide. In the latter, the DWs allow for the generation of a supercontinuum spanning from 1150 nm to 1700 nm, almost twice what was previously reported in silicon at telecom wavelength [11]. We can also notice a clear saturation of the spectral broadening when increasing the pump peak power beyond 14 W, which can be explained by increased nonlinear losses.

In order to gain further insights in the observed spectral broadening, we have performed numerical simulations based on the GNLSE describing the propagation of the temporal envelope $E(z,t)$ of the electric field of short pulses along the length z of a nonlinear medium. The equation reads [16, 21],

$$\frac{\partial E(z,t)}{\partial z} = i\sum_{k\geq 2} i^k \frac{\beta_k}{k!}\frac{\partial^k E}{\partial t^k} - \frac{\alpha_l}{2}E - \frac{\alpha_c}{2}(1+i\mu)E + i\gamma\left(1+\frac{i}{\omega_0}\frac{\partial}{\partial t}\right)E\int_{-\infty}^{t} R(t-t')|E(z,t')|^2 dt'. \quad (1)$$

Here the β_k are the Taylor series expansion coefficients that fit the chromatic dispersion curves of Fig. 1 in terms of angular frequency around the pump central angular frequency ω_0. α_l and α_c account, respectively, for linear and free carrier absorption losses. We have $\alpha_c = \sigma N_c$ where N_c is the free carrier density and $\sigma = 1.45\times 10^{-21}\,\mathrm{m}^2$ for silicon [22]. The parameter μ accounts for the free carrier dispersion and is taken as $\mu = 2k_c\omega_0/(\sigma c)$ with $k_c = 1.35\times 10^{-27}\,\mathrm{m}^3$ [23] and c the speed of light in vacuum. The nonlinear parameter γ is estimated from experiments on similar waveguides [24] and scaled through the effective mode area $A_{\mathrm{eff}} = 0.2\,\mu\mathrm{m}^2$. We find $\gamma = (234+44i)\,\mathrm{W}^{-1}\mathrm{m}^{-1}$. $R(t)$ is the nonlinearity response function defined as in fibers, $R(t) = (1-f_R)\delta(t) + f_R h_R(t)$ where f_R is the fractional Raman contribution and $h_R(t)$ is the Raman response function. Both can be deduced from the known spectral Lorentzian shape of the Raman response of silicon [21]. We use the relation $f_R = g_R(\omega_0)\Gamma_R/[\Omega_R A_{\mathrm{eff}}\mathrm{Re}(\gamma)]$ with $g_R(\omega_0) = 3.7\times 10^{-10}\,\mathrm{m/W}$ [25], $\Omega_R/(2\pi) = 15.6\,\mathrm{THz}$ and $\Gamma_R/\pi = 105\,\mathrm{GHz}$ [21] which yields $f_R = 0.026$. Finally, the carrier density can be calculated by solving

$$\frac{\partial N_c(z,t)}{\partial t} = \frac{2\pi \mathrm{Im}(\gamma)}{h\omega_0 A_{\mathrm{eff}}}|E(z,t)|^4 - \frac{N_c(z,t)}{\tau_c}, \quad (2)$$

where h is Planck's contant and τ_c is the carrier lifetime, estimated to be 1 ns [26].

Equations (1) and (2) have been solved with a split-step Fourier algorithm [19, 27]. Results for a hyperbolic secant input pulse with 150 fs duration (FWHM) and a peak power of 32 W corresponding to the experimental parameters are shown in Figs. 3 and 4 for our two different waveguides. The top-left panel of each figure reveals a good agreement between measured and simulated output spectra. In particular the position of the DWs and the overall spectral width are well predicted by simulations. The DWs relate to Čerenkov radiation emitted by solitons perturbed by higher-order dispersion and their spectral position can be analytically predicted with the phase matching relation [17]

$$\beta(\omega_{\mathrm{DW}}) - \frac{\omega_{\mathrm{DW}}}{v_{g,s}} = \beta(\omega_s) - \frac{\omega_s}{v_{g,s}} + (1-f_R)\gamma P_s, \quad (3)$$

where $\beta(\omega)$ designates the frequency-dependent wavenumber of the waveguide, ω_s and ω_{DW} are the frequencies of the soliton and the emitted DW respectively, while P_s and $v_{g,s}$ are the soliton peak power and group velocity. By using an average value $P_s = 10\,\mathrm{W}$ extracted from the simulations, we find

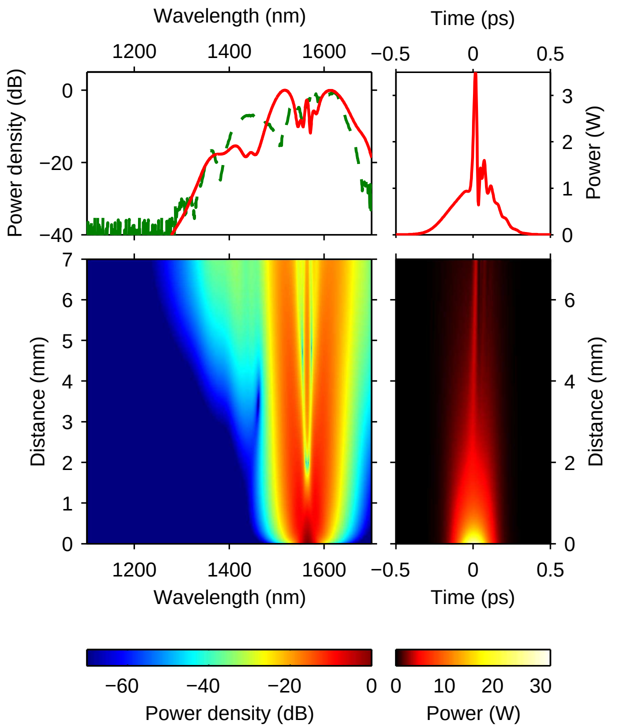

Figure 3. Pseudocolor plots of the simulated spectral (left) and temporal (right) evolution along the 750 nm-wide silicon waveguide for a 150 fs (FWHM) 32 W sech input pulse. Top plots (red) highlight the waveguide output at $z = 7$ mm. The dashed curve is the measured output spectrum for comparison.

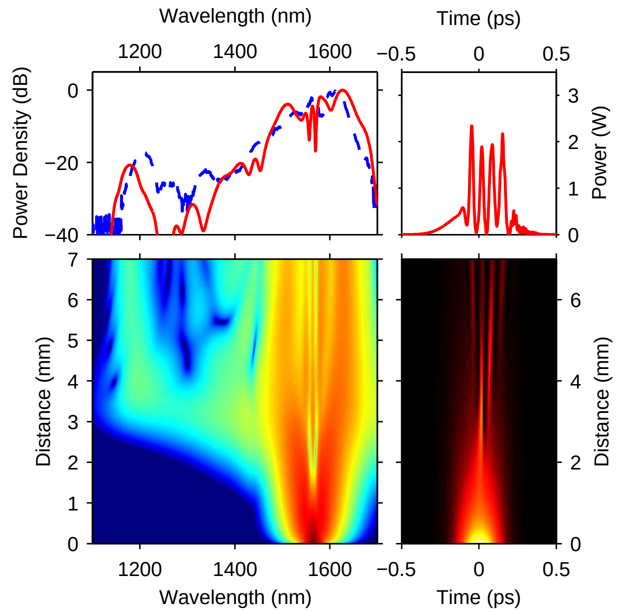

Figure 4. Same as Fig. 3 but for the 700 nm-wide waveguide.

that for the 750 nm-wide (700 nm-wide) waveguide the DWs should be emitted at 1300 nm (1150 nm), which is in good agreement with our experimental and numerical results. This confirms the origin of these spectral peaks.

Additional information can be gained by examining the simulated evolution of the temporal intensity profiles of the pulses along the waveguide, which are plotted in the right panels of Figs. 3 and 4. These figures reveal the temporal compression and subsequent splitting (or fission) of the input pulse that is typical of supercontinuum generation in PCFs [1]. In agreement with theory [1], the fission length is longer for the waveguide with the smaller (in absolute value) pump group-velocity-dispersion coefficient, i.e., for the 750 nm-wide waveguide (see Fig. 1). This can be seen from the fact that the temporal profile of the pulse at the end of the 750 nm-wide waveguide is strongly compressed but not yet splitted [Fig. 3] while we observe four subpulses at the end of the 700 nm-wide waveguide [Fig. 4]. We can note however some differences with the soliton fission dynamic reported in PCFs. Fission of a soliton of order N in PCF leads to N fundamental solitons with different peak powers and temporal widths (and these characteristics are well predicted analytically [1]). In contrast, while our input soliton number is $N = 18$ for the 700 nm-wide waveguide, we only observe four subpulses at the output of that waveguide, and they all have roughly the same temporal duration and peak power (note that $N = 32$ for the 750 nm-wide waveguide). We believe that the TPA-induced peak-power limitation during the very strong temporal compression is responsible for these differences. Previous theoretical results broadly agree with this hypothesis [28, 29]. In particular, those works have highlighted that TPA can induce soliton fission even in the absence of Raman scattering or higher-order dispersion, and that the splitted pulses have very close characteristics. Only the cases of $N = 2$ and $N = 3$ have been considered in details however, and more theoretical work is needed to fully understand the soliton fission dynamics of the GNLSE in presence of TPA in the case of high soliton orders.

In conclusion we have experimentally and numerically studied high-order soliton fission, dispersive wave generation, and supercontinuum generation in a silicon photonic wire. We have reported a supercontinuum spanning from 1200 nm to 1700 nm. It is obtained from 150 fs input pulses at 1565 nm wavelength, i.e., in the C-band of telecommunications, and to the best of our knowledge it constitutes the widest reported supercontinuum in silicon at telecom wavelength. Our work also highlights that the high-order soliton dynamics, and in particular the soliton fission process, of the nonlinear Schrödinger equation is still mostly preserved in these conditions, despite the strong influence of TPA. Let us note that we have checked that both Raman scattering and free carrier effects play very little role in our simulations and can in practice be neglected. This results from operating in the femtosecond regime and confirms the findings of Refs. [16]. We therefore believe that the good agreement between our numerical and experimental results clearly shows that the simple NLSE with a complex nonlinear parameter is sufficient to describe short pulse propagation in a silicon wire at telecom wavelength.

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