Monte-Carlo Simulation of Chess Tournament Classification Systems

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Abstract

What makes you win a chess tournament? You should be talented, but the choice of opponents will influence your final ranking as well. This article compares different ranking systems from mathematical point of view, such as the Swiss system and the elimination method and discusses how to handle tiebreaks. In order to judge which system is fair, a theoretical analysis by means of an appropriate error function is made, as well as simulations with the Monte-Carlo method.

Keywords: Monte-Carlo simulation, ranking system, estimation, error function

1 Introduction

In section 1 we describe different ranking systems used in chess tournaments, as well as in other sports events. To make a mathematical comparison of the different methods in section 2, an error function is described to evaluate the Swiss system and the elimination method for appointing a winner. The statistics from the Monte-Carlo method that was used as simulation technique, confirm the theoretical results.

2 Tournament Classification systems

2.1 Swiss system versus elimination system

Many games (chess, bridge, tennis, ...) confront two players. When tournaments take place, many players have to face each other. The way of pairing should lead to a correct classification. A ”Round Robin” is a format of chess tournaments where each opponent plays all of the other opponents. This is the best way of determining playing strength; however, the number of rounds needed are prohibitive for a large number of entrants. For example, for 16 players, there would be 15 rounds using the Round Robin format. To obtain a representative classification without the inpractical option of creating all possible C_m^2 pairs if there are m players, there exist two main systems: **the elimination system** and **the Swiss system** [2][4]. The basic rule for the elimination system is once you lose a round, you fall out and will no longer play. This reduces the number of confrontations between players and is often used in tennis tournaments with infrastructural limitations. Chess tournaments require much less infrastructure. Here the Swiss tournament system is chosen more often. It was invented by J. Muller and first used in a chess tournament at Zurich (Switzerland) in 1895 (hence "Swiss" system). We assume that players are paired ad random for the starting round. There exist systems where this assumption is replaced by an initial classification based on results of the tournament of the previous year, e.g. the McMahon system. As only the winning players continue, the last round is the most spectacular one where the two best players duel.

In the Swiss system the players are classified by the number of games they have won so far. When the number of players is a power of 2, i.e. $m = 2^k$, it is possible to select pairs of players who both won the same number of games, which makes this case the most preferable. We assume one may not play the same player twice within the same tournament.

2.2 Tiebreaks

The final classification in the Swiss system is based firstly on the total number of games won by the player. But this doesn't make the players unique and tiebreaks will occur. To make a further ranking withing the sets S_j of players who won the same number j of games, a refinement of the Swiss system is required. Two important tiebreak methods are **the Solkoff method** and **the cumulative method** [5]. The Solkoff method is based on the strength of opponents a player has beaten and is quantified by a weight quantity w_i. This weight quantity w_i of player A can be defined as the sum of the number of games won by the players beaten by player A. Within a set S_j ($j = 0, 1, \ldots, 4$) players with higher weights get smaller ranking numbers. For those players with equal weights, the mean of the concerned weight is attributed to each of them. For example: when the ranking numbers 10 and 11 are to be attributed to two players within the same set with equal weights, the are both ranked by number 10.5. The cumulative method is based on the sum sc_i of the cumulative (running) scores for each round. So if you won your first 3 games, lost the fourth game, and won the fifth, your cumulative score is: 1 2 3 3 4. This makes $sc_i = 1+2+3+3+4 = 13$, which would be your cumulative tiebreak. A player who lost his first game, then won the last four would have a cumulative tiebreak as follows: $sc_i = 0 + 1 + 2 + 3 + 4 = 10$ tiebreak points. The reasoning behind this method is based on the Swiss system of playing an opponent with the same score as oneself. The assumption is that if you win early, you're playing tougher opponents (opponents who also won early and probably finished higher). If you lost in the early rounds, you played weaker opponents (who also lost early and probably didn't finish as high). The higher sc_i, the better the player i.

3 Mathematical comparison

3.1 Initial conditions

We consider 16 participants numbered from 1 to 16 with the assumption that the smaller the number, the better the player. This means that when player i and player j are confronted, player i will win if $i < j$. As the goal of a tournament is to create a classification of players that represents their level, we expect as outcome of the tournament: player i ends up at ranking i.

The number of active players in each round is shown in Table 1 for both systems when the tournament starts with 16 players. As $m = 2^4$, a winner can be indicated after four rounds.

method	round 1	round 2	round 3	round 4
elimination	16	8	4	2
Swiss	16	16	16	16

Table 1: number of players during the different rounds

After i rounds $\frac{m}{2^i} C_i^j$ players were able to win j games with the Swiss system, as is shown in Table 4 where the cardinality of the sets S_i is given at the different stages of the tournament. Here S_i is the set of players who have won i games so far. Table 3 gives analoguous cardinalities with the elimination system.

Set	round 1	round 2	round 3	round 4
S_0	8	8	8	8
S_1	8	4	4	4
S_2	0	4	2	2
S_3	0	0	2	1
S_4	0	0	0	1

Table 3: Cardinality of S_i ($i \in \{0, 1, 2, 3, 4\}$) after several rounds with the elimination system

Set	round 1	round 2	round 3	round 4
S_0	8	4	2	1
S_1	8	8	6	4
S_2	0	4	6	6
S_3	0	0	2	4
S_4	0	0	0	1

Table 4: Cardinality of S_i ($i \in \{0, 1, 2, 3, 4\}$) after several rounds with the Swiss system

To compare and evaluate the two basic systems of tournament classification, we simulate the Swiss method without secondary ranking within the sets S_i. They all receive the same mean ranking of the set they are in (see Table 5 based on the cardinalities of Table 3 and 4 to determine the mean values).

System	S_4	S_3	S_2	S_1	S_0
elimination	1	2	3.5	6.5	12.5
Swiss	1	3.5	8.5	13.5	16

Table 5: Awarded ranking for elements of S_i ($i \in \{0, 1, 2, 3, 4\}$) at the end of the tournament

3.2 Elimination method

We discuss an example with 16 players in detail where a random generator determines the initial pairing:

$$(9, 8), (13, 5), (10, 1), (4, 16), (2, 3), (14, 6), (11, 12), (7, 15) \tag{1}$$

Applying the elimination method to this initial situation leads to

$$S_0 = \{9, 13, 16, 10, 3, 14, 12, 15\} \tag{2}$$

$$S_1 = \{8, 5, 1, 4, 2, 6, 11, 7\} \tag{3}$$

After randomization in S_1 the following pairing is proposed:

$$(5, 6), (4, 2), (11, 8), (1, 7) \tag{4}$$

After two rounds, the players are partitioned into the sets

$$S_0 = \{9, 13, 16, 10, 3, 14, 12, 15\} \tag{5}$$

$$S_1 = \{6, 4, 11, 7\} \tag{6}$$

$$S_2 = \{5, 2, 8, 1\} \tag{7}$$

After randomization the following pairing is proposed:

$$(2, 8), (1, 5) \tag{8}$$

After three rounds, the players are partitioned into the sets

$$S_0 = \{9, 13, 16, 10, 3, 14, 12, 15\} \tag{9}$$

$$S_1 = \{6, 4, 11, 7\} \tag{10}$$

$$S_2 = \{5, 8\} \tag{11}$$

$$S_3 = \{2, 1\} \tag{12}$$

A final fourth round will indicate player 1 as the winner.

3.3 Swiss system

We start by pairing the 16 players at random as in (1) with the elimination method. The first round with the Swiss system gives the same sets S_0 and S_1 as in respectively (2) and (3). After randomization in the two sets the following pairing is proposed:

$$(3,\ 9), (10,\ 15), (14,\ 16), (12,\ 13), (5,\ 6), (4,\ 2), (11,\ 8), (1,\ 7) \tag{13}$$

After two rounds, the players are partitioned into the sets

$$S_0 = \{9,\ 16,\ 15,\ 13\} \tag{14}$$

$$S_1 = \{3,\ 10,\ 14,\ 12,\ 6,\ 4,\ 11,\ 7\} \tag{15}$$

$$S_2 = \{5,\ 2,\ 8,\ 1\} \tag{16}$$

After randomization the following pairing is proposed:

$$(13,\ 9), (16,\ 15), (14,\ 11), (7,\ 3), (4,\ 10), (6,\ 12), (2,\ 8), (1,\ 5) \tag{17}$$

After two rounds, the players are partitioned into the sets

$$S_0 = \{13,\ 16\} \tag{18}$$

$$S_1 = \{9,\ 15,\ 14,\ 7,\ 10,\ 12\} \tag{19}$$

$$S_2 = \{11,\ 3,\ 4,\ 6,\ 8,\ 5\} \tag{20}$$

$$S_3 = \{2,\ 1\} \tag{21}$$

Player i	r_i	w_i	r_i^s	sc_i	r_i^c	Player i	r_i	w_i	r_i^s	sc_i	r_i^c
1	1	10	1	10	1	9	13.5	1	12.5	2	13.5
2	3.5	8	2	9	2	10	8.5	2	9.5	4	10.5
3	3.5	5	4.5	6	5	11	8.5	3	7	6	7.5
4	3.5	4	3	7	4	12	13.5	1	12.5	3	12
5	3.5	5	4.5	8	3	13	13.5	0	14.5	1	15
6	8.5	3	7	6	7.5	14	8.5	1	11	4	10.5
7	8.5	2	9.5	5	9	15	13.5	0	14.5	2	13.5
8	8.5	3	7	7	6	16	16	0	16	0	16

Table 6: Ranking of the different players with the Swiss system (basic and refined version with Solkoff and cumulative tiebreak methods)

After a last randomization of the sets, the last round with pairs (22) will determine the final ranking given in Table 6. This table also mentions the refined ranking r_i^s and r_i^c heading tiebreaks within the sets S_i ($i = 0,\ 1 2,\ 3$) with respectively the Solkoff and the cumulative tiebreak methods.

$$(13,\ 16), (7,\ 9), (15,\ 14), (10,\ 12), (3,\ 6), (11,\ 4), (8,\ 5), (1,\ 2) \tag{22}$$

3.4 Error measuring of tournament classification methods

In order to evaluate the different tournament classification systems, we need to measure the error of the created ranking. Therefore we consider the error (23) where r_i is the ranking for player i.

$$e = \sum_{i=1}^{m} (i - r_i)^2 \tag{23}$$

When internal ranking is applied within the sets, the error e takes the value 77.5 in the basic version of the Swiss system and can be reduced to $\sum_{i=1}^{16}(i - r_i^s)^2 = 52$ and $\sum_{i=1}^{16}(i - r_i^c)^2 = 69.5$ for the Solkoff method and the cumulative method respectively.

3.5 Monte Carlo simulation of tournaments

The Monte Carlo simulation [3], [1] technique uses multiple trial runs to discover statistical characteristic features. We apply this method to the tournament case and make several runs of a tournament starting from a random pairing of the players.

We consider $e_j = \sum_{i=1}^{m}(i - r_i^j)^2$, where r_i^j is the ranking for player i after run number j of the simulation, with $\bar{e}$ its mean after n runs, defined by (24). It represents the summed square of the deviation of the expected ranking and the obtained value by each simulation run of the tournament with m players.

$$\bar{e} = \frac{1}{n} \sum_{j=1}^{n} \sum_{i=1}^{m} (i - r_i^j)^2 \tag{24}$$

Figure 1 illustrates this error: the mean ranking $\bar{r_i}$ of a player i as in (25) after $n = 200$ runs in the simulation of the tournament is plotted as function of the number i of the player. The expected value is added by the straight line. Figure 2 shows the same variables but the ranking is made here with the Swiss system. For example: in the case of the elimination method $\bar{r_5} = 184.225$, but is reduced to $\bar{r_5} = 59.70$ in the case of the Swiss system.

$$\bar{r_i} = \frac{1}{n} \sum_{j=1}^{n} r_i^j \tag{25}$$

Further statistics of e including its 95% confidence interval can be compared by means of Table 7.

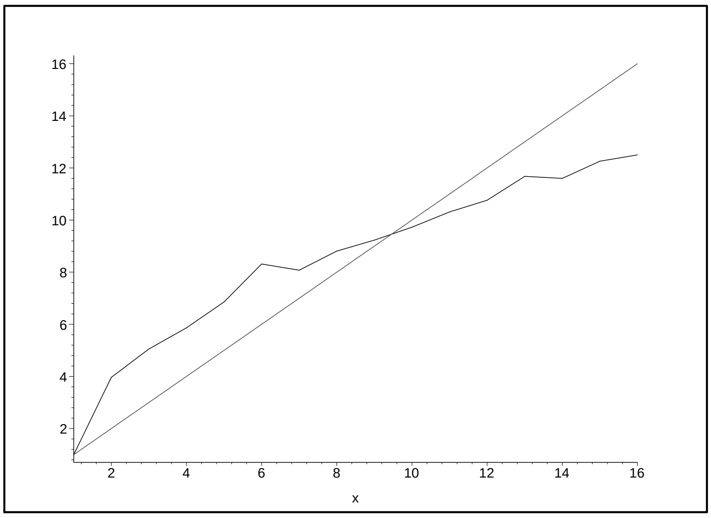

Figure 1: the mean score after 200 runs with the elimination method versus expected value for the different players.

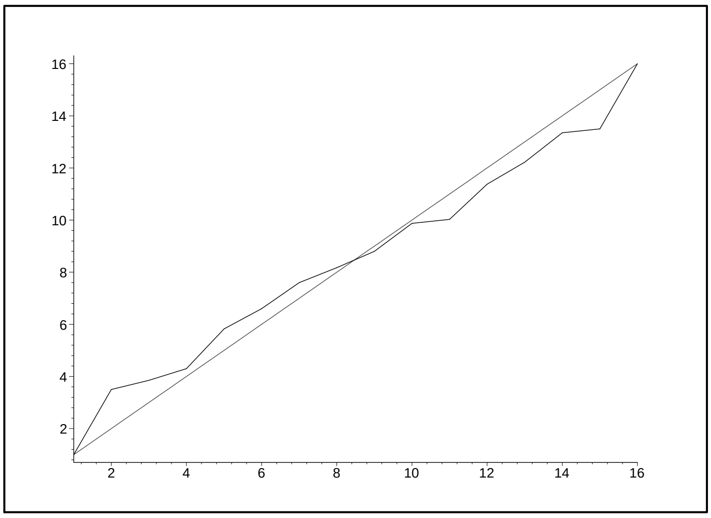

Figure 2: the mean score after 200 runs with the Swiss system versus expected value for the different players.

System	$\bar{e}$	s_e	95% C.I. of e
Elimination	184.225	936.50	$[54.432, 314.018]$
Swiss	59.7	297.039	$[18.533, 100.867]$

Table 7: Statistics of the error e

4 Conclusion

By means of the error function (23) we were able to quantify the superior classification feature of the Swiss system compared to the elimination system when ranking players at a chess tournament. The Swiss system appears to be the ranking method with the highest probability that the best player will end up as the winner of the tournament.

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