Design of invisibility cloaks with an open tunnel

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Abstract: In this paper we apply the methodology of transformation optics for design of a novel invisibility cloak which can possess an open tunnel. Such a cloak facilitates the insertion (retrieval) of matter into (from) the cloak's interior without significantly affecting the cloak's performance, overcoming the matter exchange bottleneck inherent to most previously proposed cloak designs. We achieve this by applying a transformation which expands a point at the origin in electromagnetic space to a finite area in physical space in a highly anisotropic manner. The invisibility performance of the proposed cloak is verified by using full-wave finite-element simulations.

OCIS codes: (160.1190) Anisotropic optical materials; (160.3918) Metamaterials; (230.3205) Invisibility cloaks.

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1. Introduction

Transformation optics is an elegant and straightforward formalism used to design optical devices that can control light in an unprecedented manner. In recent years, transformation optics has been applied to the design of various electromagnetic devices such as invisibility cloaks [1, 2], concentrators [3], cylindrical superlenses [4] and field-rotating cloaks [5] etc. Invisibility cloaks are arguably the most interesting optical devices which are designed using this design methodology. To design an invisibility cloak, one first chooses a geometrical transformation which maps a point, line or sheet to a finite region of space (void space), this is followed by an interpretation of the geometrical transformation as a set of material parameters of an effective medium; the resultant cloaks correspond to a point-transformed, line-transformed and surface-transformed (or carpet) cloaks respectively [6]. The material obtained would guide light around the void created by the aforementioned transformation. Any object placed in this void would not scatter incident electromagnetic waves.

There have been several different types of cloaks put forward in the literature. In a 2D scenario, most cloaks have a circular annular cross-section [1, 2, 5], or an elliptical cross-section [7, 8]. These cloaking designs have either been verified by geometrical ray tracing [9] or numerical full-wave simulations [10]. For all these cloaking designs, it is not possible to insert (retrieve) objects into (from) the cloaking domain due to their closed annular structures; we call this the *matter-exchange bottleneck*. In many practical cloaking applications it is desirable for objects to be able to freely enter or leave the cloaking device. Ref [11] reports the design of an open cloak, which allows for material exchange between the cloak's exterior and interior. The open cloak was designed by positioning the null point before transformation very close to the cloak's outer boundary, such that there exists only a trivial (close to an identity) coordinate transformation along a certain angular direction. Such a cloak was realized by breaking it down into quadrilaterals; therefore although the approach can easily generate cloaks with different shapes, its design procedure is tedious and the final cloak lacks of closed-form descriptions. In this paper, we propose an open cloak design with a less complicated procedure. Its geometry and in turn the material parameters have analytical expressions. Such a global view of the cloak is especially advantageous for optimization purposes. The open cloak is realized with a transformation function which deforms space in a highly anisotropic manner. We verify the designed cloak, both with and without an opening, using full-wave finite-element simulations.

2. Cardioid cloak

The anisotropic expansion of space is achieved by expanding a point, for our case, at the origin in cylindrical coordinates (r, θ) in electromagnetic (EM) space to a cardioid in cylindrical coordinates (r', θ') in physical space. Suppose we have a mapping between the EM and physical spaces in cylindrical coordinates specified in the form $r = r(r', \theta')$, $\theta = \theta'$ and that in EM space we have $\varepsilon = \mu = I$, where I is the identity matrix and ε and μ are the dielectric permittivity and magnetic permeability tensors respectively. Our choice of mapping function between EM and physical space is given by,

$$r = \frac{r' - a(1+\cos\theta')}{b - a(1+\cos\theta')}b, \tag{1}$$

where b denotes the radius of the cloak's outer boundary and a is a constant determining the size of the cloak's inner boundary. This mapping function can equivalently be written as

$$r' = \frac{b - a(1+\cos\theta)}{b}r + a(1+\cos\theta). \tag{2}$$

Effectively the transformation maps the origin in EM space $r = 0$ to a cardioid $r' = a(1+\cos\theta)$ with its cusp located at the origin in physical space. The Jacobian transformation matrix from

EM to physical space is given by,

$$\Lambda = \begin{bmatrix} \frac{\partial r'}{\partial r} & \frac{\partial r'}{r\partial \theta} & 0 \\ 0 & \frac{r'}{r}\frac{\partial \theta'}{\partial \theta} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

The derivatives in Eq. 3 above are given by,

$$\begin{aligned} \frac{\partial r'}{\partial r} &= \frac{b-a(1+\cos\theta)}{b}, \\ \frac{\partial r'}{\partial \theta} &= \frac{a(r-b)}{b}\sin\theta. \end{aligned} \tag{4}$$

The permeability and permittivity tensors in physical space in cylindrical coordinates, following the formula

$$\varepsilon' = \mu' = \Lambda\Lambda^T/det(\Lambda), \tag{5}$$

are derived as

$$\varepsilon'_{r'r'} = \mu'_{r'r'} = \frac{(\frac{\partial r'}{\partial r})^2 + (\frac{\partial r'}{r\partial \theta})^2}{\frac{r'}{r}\frac{\partial r'}{\partial r}}, \tag{6}$$

$$\varepsilon'_{\theta' r'} = \varepsilon'_{r'\theta'} = \mu'_{\theta' r'} = \mu'_{r'\theta'} = \frac{\frac{\partial r'}{r\partial \theta}}{\frac{\partial r'}{\partial r}}, \tag{7}$$

$$\varepsilon'_{\theta'\theta'} = \mu'_{\theta'\theta'} = \frac{\frac{r'}{r}}{\frac{\partial r'}{\partial r}}, \tag{8}$$

$$\varepsilon'_{z'z'} = \mu'_{z'z'} = \frac{1}{\frac{r'}{r}\frac{\partial r'}{\partial r}}. \tag{9}$$

The above permeability and permittivity tensors in physical space can be re-cast in Cartesian coordinates by using the rotation matrix (R) from cylindrical coordinates to Cartesian coordinates [7], as

$$\varepsilon'_{cart} = \mu'_{cart} = R\varepsilon' R^T/det(R) = R\mu' R^T/det(R). \tag{10}$$

It is interesting to notice that along the angular direction $\theta' = \pi$, the mapping function in Eq. 1 reduces to $r = r'$. This means spaces along this direction undergo an identity transformation; therefore no difference in material parameters would occur there after the transformation. The above statement is also approximately valid for θ' values close to π. This is indeed observed in the plots of the material parameters for a cardioid cloak with $a = 1m$ and $b = 2.5m$, shown in Fig. 1. From the plot, we also notice that the zz-component of the material tensors have a finite value, while the remaining components exhibit an infinity at the inner boundary of the cloak. In the following we use COMSOL Multiphysics to verify that our designed cloak can indeed function as desired. For simplicity, we only deal with the 2D in-plane propagation scenario particularly with the transverse-electric (TE) polarization (with E_z, H_x, H_y field components)). Figure 2 shows the scattering pattern of the cardioid cloak (as characterized in Fig. 1) when a plane wave at 0.5 GHz is incident on it, both from the left and from the bottom. The interior surface of the cloak is coated with a perfect electric conductor (PEC). It is noticed that the plane wave passes through the cloak experiencing little perturbation.

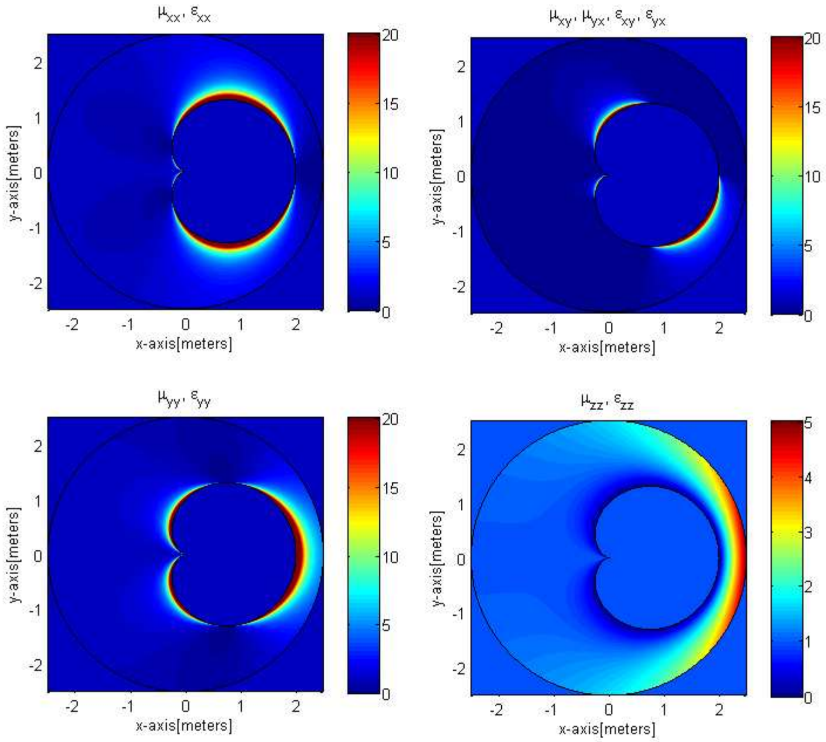
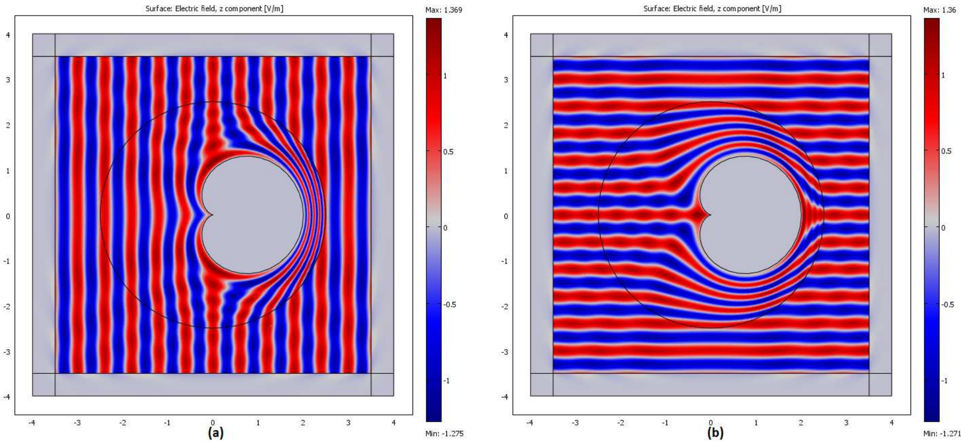


Fig. 1. Material parameters for the cardioid cloak; $a = 1m$ and $b = 2.5m$. It should be noticed that the *xy*- and *yx*-components of the tensors are equal. In the plot, we have only shown the material parameter values up to 20. Higher values are denoted with a saturated dark-red color.



(a)

(b)

Fig. 2. E_z field distribution for a plane wave interacting with a cardioid cloak. (a) left-to-right propagation; (b) bottom-to-top propagation.

3. Open cardioid cloak

Next, based on the perfect cardioid cloak, we construct the open cardioid cloak by removing a section of its annular body along the $\theta' = \pi$ direction. As such a tunnel connecting the interior and the exterior of the cloak can be formed for exchanging objects. We refer to such a cloak as an open cardioid cloak. As a specific example, an open cardioid cloak with a tunnel 2 m and 0.1 m wide at its outer and inner boundaries respectively is discussed in the following. The open cardioid cloak is constructed in COMSOL with its interior surface (except at the opening) coated with PEC. Figure 3 shows the results of the full-wave simulation for this open cloak, again operating at 0.5 GHz. By comparing the field plots in Fig. 3 with those in Fig. 2, one notices that the presence of the tunnel does not significantly affect the cloak's invisibility performance. Depending on application requirements, tunnels with other opening sizes can be used.

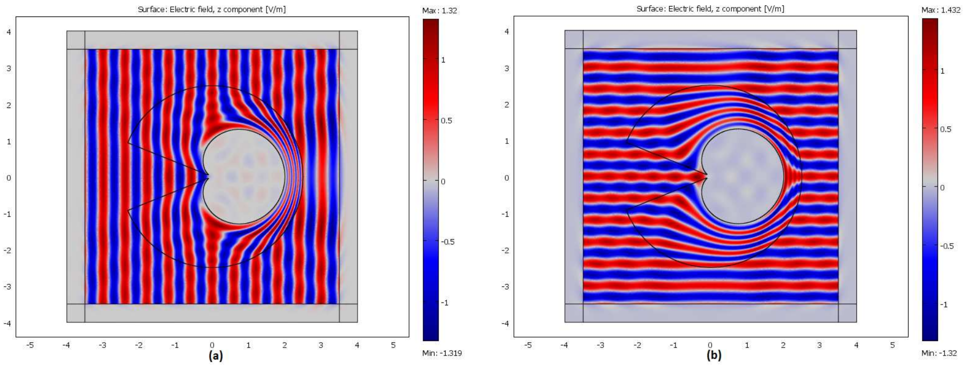


Fig. 3. E_z field distribution for a plane wave interacting with an open cardioid cloak. (a) left-to-right propagation; (b) bottom-to-top propagation.

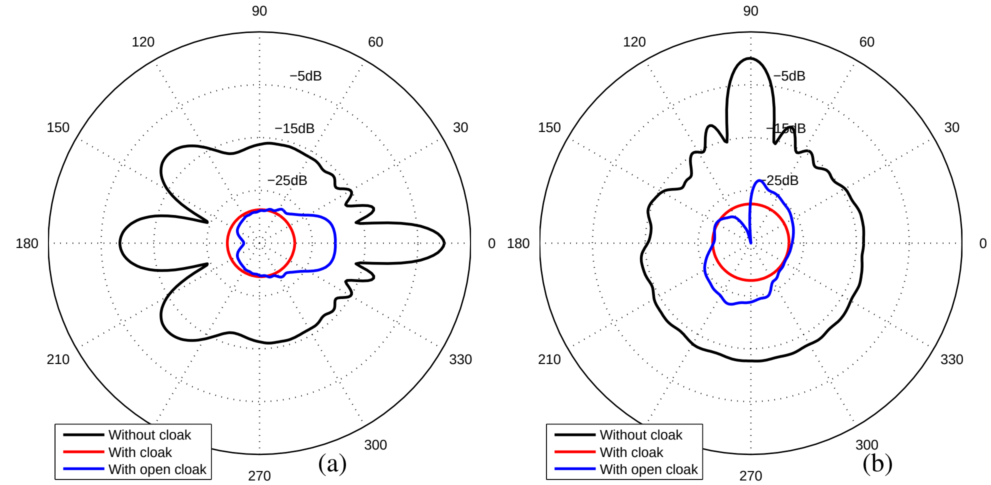


Fig. 4. Far field scattering patterns of a bare PEC cardioid, a PEC cardioid coated with a cloak, and a PEC cardioid (with an openning) coated with an open cloak, for plane waves incident from (a) left-to- right and (b) bottom-to-top. The maximum far field caused by the bare cardioid PEC is taken as reference, i.e. 0 dB.

To further assess the invisibility performance of the open cardioid cloak, we calculated its far field scattering patterns, as shown in Fig. 4 for plane waves incident from the left (a) and the bottom (b) of the cloak. In the same figure, we also superimposed the scattering patterns for a bare cardioid shaped PEC, and those for the full cardioid cloak studied in section 2. The full cardioid cloak can greatly reduce the scattering of the bare PEC cardioid shaped domain in all directions; the reduction in scattering is on average about 15dB, with a maximum reduction of ∼30dB for the forward scattering direction. Due to the presence of a tunnel, the open cardioid cloak has a slightly larger scattering cross-section than the cardioid cloak. For left-to-right plane wave incidence, the forward scattering is noticeably larger; however compared to the bare PEC, a reduction of ∼20 dB is still achieved. For bottom-to-top plane wave incidence, the open cardioid cloak exhibits slightly larger scattering along top-right and bottom-left directions; but still reductions around ∼10dB are achieved in those directions compared to the bare PEC scatterer case.

4. Reduced open cloak

We note that the ideal cloak as well as the open cloak designed above require infinitely large permeability parameters (Fig. 1), which can’t be achieved experimentally. Here we examine the performance of such a cloak at a reduced material parameter space. That is, instead of using unrestricted permeability values, we limit the parameters to a finite range of values. Such a study in part reveals the practicality of the designed cloaks. We refer to cloaks with a reduced material parameter space as *reduced cloaks*. The emphasis of our study is on the cloak design with an open channel.

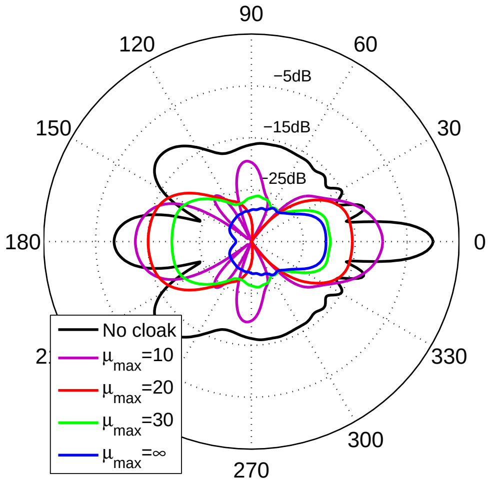


Fig. 5. Far-field scattering patterns of a bare PEC cardioid, an ideal open cardioid cloak, and reduced open cardioid cloaks designed with various μ_{max} values, for plane waves incident from left-to-right. The maximum far field caused by the bare PEC cardioid is taken as reference i.e. 0 dB.

Figure 5 shows the far-field scattering patterns of the same open cloak studied in the previous section, but with the maximum permeability set to various μ_{max} values. The simulations are carried out for the left-to-right propagation case. One sees that at $\mu_{\text{max}} = 10$ the reduced open cloak still reduces the overall scattering cross-section compared to the bare-PEC case by 5-10dB in most directions. The scattering becomes slightly more inhomogeneous with respect to certain directions, especially around +90° and -90° directions. The scattering cross-section further reduces when one sets $\mu_{\text{max}} = 20$. The reduction in forward scattering is as large as 15dB. When $\mu_{\text{max}} = 30$, the scattering pattern is approaching that of the open cloak studied in the previous section, i.e. with $\mu_{\text{max}} = \infty$, especially along the forward direction. Similar findings are observed with bottom-to-top incidence case (not shown). We therefore conclude that even with a limited material parameter space, one can achieve a reasonable level of reduction in scattering with the designed open cardioid cloak.

Though challenging at the optical wavelength regime, realization of such a cloaking device is possible at microwave frequencies with current metamaterial fabrication technology. At the microwave regime, the relatively large permeability value can be readily realized with metamaterials based on split-ring resonators [12]. Alternatively for the transverse-magnetic polarization (with H_z, E_x and E_y field components) scenario, the electric-circuit metamaterial proposed in [13] can be deployed to obtain an effective material with a maximum permittivity value around 20.

5. Conclusion

In conclusion, with transformation optics we have designed an invisibility cloak using a highly anisotropic transformation function. Such a cloak can have close-to-background material parameters along a certain angular direction, such that a section of the annular structure can be cut away without affecting its cloaking performance. This effectively opens a tunnel that allows objects to pass into or out of the cloak's interior freely. Despite the fact that the open cardioid cloak exhibits a slightly deteriorated invisibility performance compared to the cardioid cloak in certain scattering directions, it can still significantly reduce the scattering of, e.g. a bare PEC object.

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