

# On ghost condensation, mass generation and Abelian dominance in the Maximal Abelian Gauge

David Dudal\* and Henri Verschelde†

*Ghent University*

*Department of Mathematical Physics and Astronomy*

*Krijgslaan 281-S9*

*B-9000 GENT, BELGIUM*

Recent work claimed that the off-diagonal gluons (and ghosts) in pure Yang-Mills theories, with Maximal Abelian gauge fixing (MAG), attain a dynamical mass through an off-diagonal ghost condensate. This condensation takes place due to a quartic ghost interaction, unavoidably present in MAG for renormalizability purposes. The off-diagonal mass can be seen as evidence for Abelian dominance. We discuss why ghost condensation of the type discussed in those works cannot be the reason for the off-diagonal mass and Abelian dominance, since it results in a tachyonic mass. We also point out what the full mechanism behind the generation of a real mass might look like.

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## I. INTRODUCTION

As everybody knows, quarks are confined: nature as well as lattice simulations of nature are telling us that. Still, there is no rigorous proof of confinement. One proposal for the explanation of confinement is the idea of the dual superconductor: magnetic monopoles condense and induce a dual Meissner effect: color-electric flux between charges is squeezed and a string is created in between. The original work on this topic can be found in [1, 2, 3]. Abelian projection [4] is a way to reveal the relevant degrees of freedom (the monopoles). In a loose way of speaking, at points where the projection is ill-defined, singularities invoke (Abelian) monopoles. Abelian dominance means that low energy QCD is dominated by Abelian degrees of freedom. Some early work on this is presented in [5]. Numerical evidence can be found in e.g. [6, 7, 8] and more recently [9].

Can this Abelian dominance be founded on more theoretical grounds? In the light of renormalization à la Wilson, and assuming that the *off-diagonal* gluons (ghosts) attain a mass  $M$  while the *diagonal* ones remain massless, an effective theory in terms of the massless diagonal fields could be achieved at low energy ( $\ll M$ ), thereby realizing a kind of Abelian dominance. In the context of low energy theories, we like to refer to the Appelquist-Carazzone decoupling theorem [10], which states that heavy particle modes decouple at low energy. Notice that this decoupling does not mean "heavy terms" are simply removed by hand from the Lagrangian, their influence is still present through renormalization effects. As an illustration of this: a low energy, *Abelian* theory for Yang-Mills was derived in [11], but the corresponding  $\beta$ -function was shown to be the same as the full Yang-Mills one.

The aforementioned pathway has been followed in a series of papers by Kondo et al [11, 12, 13, 14, 15, 16, 17, 18] and more recently the technique of the exact renormalization group has been employed by Freire [19, 20] to construct effective low energy descriptions of Yang-Mills theory. The results have been used in order to construct a linearly rising potential between static quarks, a criterion for confinement. Their efforts were based on the dual superconductor picture, realized with MAG. Also the monopole condensation was discussed in their framework. An essential ingredient of their work is the mass scale of the off-diagonal fields. The monopole condensate is proportional to this mass squared [11]. The lattice reported a value of approximately 1.2 GeV for the off-diagonal gluon mass in MAG Yang-Mills [9]. Next to these numerical results, analytical information is needed how this mass raises. A few papers have been written on this issue [13, 21, 22, 23]. All these authors came to the same conclusion: *a dimension two ghost condensation gives an off-diagonal mass  $M$* . We already mentioned (but did not show explicitly) in a previous paper that we found the ghost condensation gives a *tachyonic* off-diagonal gluon mass [24]. In this paper, we will perform the calculations explicitly step by step. To make it self-contained, we will start from the beginning and in order to make comparison as transparent as possible, we will follow the

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\*Research Assistant of the Fund For Scientific Research-Flanders (Belgium); Electronic address: david.dudal@rug.ac.be

†Electronic address: henri.verschelde@rug.ac.be

(notational) conventions of [13]. For the sake of simplicity, we will restrict ourselves to the SU(2) case. We discuss the (in)completeness of presented work. We end with the path we intend to follow in the future to investigate dynamical mass generation in MAG.

## II. GHOST CONDENSATION IN THE MAXIMAL ABELIAN GAUGE

Consider the Yang-Mills Lagrangian in four-dimensional Minkowski space time

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}^A\mathcal{F}^{A\mu\nu} + \mathcal{L}_{GF+FP} \quad (1)$$

where  $\mathcal{L}_{GF+FP}$  is the gauge fixing and Faddeev-Popov part. We decompose the gauge field as

$$\mathcal{A}_\mu = \mathcal{A}_\mu^A T^A = a_\mu T^3 + A_\mu^a T^a \quad (2)$$

$$\mathcal{F}_{\mu\nu} = \mathcal{F}_{\mu\nu}^A T^A = \mathcal{D}_\mu \mathcal{A}_\nu - \mathcal{D}_\nu \mathcal{A}_\mu = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - ig[\mathcal{A}_\mu, \mathcal{A}_\nu] \quad (3)$$

The  $T^A$ 's are the Hermitian generators of SU(2) and obey the commutation relations  $[T^A, T^B] = if^{ABC}T^C$ .  $T^3$  is the diagonal generator. The capital index  $A$  runs from 1 to 3. Small indices like  $a, b, \dots$  run from 1 to 2 and label the off-diagonal components. We will drop the index 3 later on.

As a gauge fixing procedure, we use MAG. Introducing the functional

$$\mathcal{R}[A] = (VT)^{-1} \int d^4x \left( \frac{1}{2} A_\mu^a A^{\mu a} \right) \quad (4)$$

with  $VT$  the space time volume, MAG is defined as that gauge which minimizes  $\mathcal{R}$  under local gauge transformations. Since (4) is invariant under U(1) transformations w.r.t. the "photon"  $a_\mu$ , MAG is only a partial gauge fixing. We do not fix the residual U(1) gauge freedom, since it plays no role for what we are discussing here.

To implement the gauge fixing in the Lagrangian (1), we use the so-called modified MAG. This gauge is slightly different from the ordinary MAG, it possesses for instance some more symmetry (see [17] and references therein). Moreover, it generates the four-point ghost interaction, indispensable for the renormalizability of MAG, as was proven in [25].

Explicitly, we get

$$\mathcal{L}_{GF+FP} = i\delta_B \bar{\delta}_B \left( \frac{1}{2} A_\mu^a A^{\mu a} - \frac{\alpha}{2} i C^a \bar{C}^a \right) \quad (5)$$

where  $\alpha$  is a gauge parameter,  $C$  and  $\bar{C}$  denote the (off-diagonal) ghosts and anti-ghosts,  $\delta_B$  and  $\bar{\delta}_B$  are the BRST and anti-BRST transformation respectively, defined by [58]

$$\begin{aligned} \delta_B \mathcal{A}_\mu &= \mathcal{D}_\mu C = \partial_\mu C - ig[\mathcal{A}_\mu, C] \\ \delta_B C &= \frac{ig}{2} [C, C] \\ \delta_B \bar{C} &= i\mathcal{B} \\ \delta_B \mathcal{B} &= 0 \end{aligned} \quad (6)$$

$$\begin{aligned} \bar{\delta}_B \mathcal{A}_\mu &= \mathcal{D}_\mu \bar{C} = \partial_\mu \bar{C} - ig[\mathcal{A}_\mu, \bar{C}] \\ \bar{\delta}_B \bar{C} &= \frac{ig}{2} [\bar{C}, \bar{C}] \\ \bar{\delta}_B C &= i\bar{\mathcal{B}} \\ \bar{\delta}_B \bar{\mathcal{B}} &= 0 \\ \mathcal{B} + \bar{\mathcal{B}} &= g[C, \bar{C}] \end{aligned} \quad (7)$$

with the following properties

$$\begin{aligned} \delta_B^2 &= \bar{\delta}_B^2 = \{\delta_B, \bar{\delta}_B\} = 0 \\ \delta_B (XY) &= \delta_B(X)Y \pm X\delta_B(Y) \\ \bar{\delta}_B (XY) &= \bar{\delta}_B(X)Y \pm X\bar{\delta}_B(Y) \end{aligned} \quad (8)$$

where the upper sign is taken for bosonic  $X$ , and the lower sign for fermionic  $X$ .

Performing the BRST and anti-BRST transformations, yields

$$\begin{aligned} \mathcal{L}_{GF+FP} = & B^a D_\mu^{ab} A^{\mu b} + \frac{\alpha}{2} B^a B^a + i\bar{C}^a D_\mu^{ac} D^{\mu cb} C^b - ig^2 \epsilon^{ad} \epsilon^{cb} \bar{C}^a C^b A^{\mu c} A_\mu^d \\ & + i\bar{C}^a g \epsilon^{ab} C^3 D_\mu^{bc} A^{\mu c} - i\alpha g \epsilon^{ab} B^a \bar{C}^b C^3 + \frac{\alpha}{4} g^2 \epsilon^{ab} \epsilon^{cd} \bar{C}^a \bar{C}^b C^c C^d \end{aligned} \quad (9)$$

where

$$D_\mu^{ab} \equiv D_\mu^{ab}[a] = \partial_\mu \delta^{ab} - g \epsilon^{ab} a_\mu \quad (10)$$

is the covariant derivative w.r.t. the U(1) symmetry and

$$\epsilon^{12} = -\epsilon^{21} = 1 \quad (11)$$

$$\epsilon^{11} = \epsilon^{22} = 0 \quad (12)$$

When we integrate the multipliers  $B$  out, we finally obtain

$$\mathcal{L}_{GF+FP} = -\frac{1}{2\alpha} (D_\mu^{ab} A^{\mu b})^2 + i\bar{C}^a D_\mu^{ac} D^{\mu cb} C^b - ig^2 \epsilon^{ad} \epsilon^{cb} \bar{C}^a C^b A^{\mu c} A_\mu^d + \frac{\alpha}{4} g^2 \epsilon^{ab} \epsilon^{cd} \bar{C}^a \bar{C}^b C^c C^d \quad (13)$$

Notice that the diagonal ghost  $C^3$  has dropped out of (13).

For the (singular) choice  $\alpha = 0$ , the 4-ghost interaction cancels from the Lagrangian. However, radiative corrections due to the other, non-vanishing 4-point interactions, reintroduce this term. We further assume that  $\alpha \neq 0$ . Some more details concerning the properties for  $\alpha = 0$  can be found in [23].

To discuss the ghost condensation mechanism, we "Gaussianize" the 4-ghost interaction in the Lagrangian by means of the (U(1) invariant) auxiliary field  $\phi$

$$\frac{\alpha}{4} g^2 \epsilon^{ab} \epsilon^{cd} \bar{C}^a \bar{C}^b C^c C^d \rightarrow -\frac{1}{2\alpha g^2} \phi^2 - i\phi \epsilon^{ab} \bar{C}^a C^b \quad (14)$$

A useful identity to prove (14), reads

$$\epsilon^{ab} \epsilon^{cd} \bar{C}^a \bar{C}^b C^c C^d = 2 \left( i\epsilon^{ab} \bar{C}^a C^b \right)^2 \quad (15)$$

The part of the Lagrangian which concerns us for the moment is

$$\hat{\mathcal{L}} = i\bar{C}^a \partial_\mu \partial^\mu C^a - \frac{1}{2\alpha g^2} \phi^2 - i\phi \epsilon^{ab} \bar{C}^a C^b \quad (16)$$

Assuming constant  $\phi$ , we use the Coleman-Weinberg construction [26] of the effective potential  $V(\phi)$ . This means we are summing all 1-loop (off-diagonal) ghost bubbles with any number of  $\phi$ -insertions. This yields

$$(VT)V(\phi) = \int d^4x \frac{\phi^2}{2\alpha g^2} + i \ln \det (\partial_\mu \partial^\mu \delta^{ab} - \phi \epsilon^{ab}) \quad (17)$$

or

$$V(\phi) = \frac{\phi^2}{2\alpha g^2} - \frac{1}{i} \int \frac{d^4k}{(2\pi)^4} \ln (k^4 + \phi^2) \quad (18)$$

Employing the Wick rotation  $k_0 \rightarrow ik_0$  [59], and performing the integration in dimensional regularization within the  $\overline{MS}$  scheme, we arrive at

$$V(\phi) = \frac{\phi^2}{2\alpha g^2} + \frac{\phi^2}{32\pi^2} \left( \ln \frac{\phi^2}{\mu^4} - 3 \right) \quad (19)$$

This potential possesses a local maximum at  $\phi = 0$  (the usual vacuum), but has global minima at

$$\phi = \pm v = \pm \mu^2 e^{1 - \frac{8\pi^2}{\alpha g^2 (\pi^2)}} \quad (20)$$

We take  $\alpha > 0$  since  $v$  diverges for  $\bar{g}^2 \rightarrow 0$  if  $\alpha < 0$ .

Up to now, we find complete agreement with [13]. We proceed by calculating the ghost propagator in the non-zero vacuum ( $V(v) < 0$ ). Substituting  $\phi = v$  in (16), it is straightforward to determine the Feynman propagator

$$\langle C^a(x)\bar{C}^b(y) \rangle = \int \frac{d^4k}{(2\pi)^4} \frac{-k^2\delta^{ab} + v\epsilon^{ab}}{k^4 + v^2} e^{-ik(x-y)} \quad (21)$$

With the above propagator, we are ready to determine the 1-loop off-diagonal gauge boson polarization. Now, there exists a non-trivial contribution coming from the ghost bubble, originating in the interaction term  $-ig^2\epsilon^{ad}\epsilon^{cb}\bar{C}^a C^b A^{\mu c} A_\mu^d$ , resulting in a mass  $M$  for the off-diagonal gluons. Again Wick rotating  $k_0 \rightarrow ik_0$  to get an integral over Euclidean space time, one easily obtains

$$M^2 = g^2 \int \frac{d^4k}{(2\pi)^4} \frac{2k^2}{k^4 + v^2} \quad (22)$$

There is one remaining step, we still have to calculate the integral of (22). Using dimensional regularization, we find the finite result

$$M^2 = \frac{-g^2v}{16\pi} < 0 \quad (23)$$

where we have used that  $v > 0$ . Here we find a *different* result in comparison with the other references [13, 21, 22, 23]. To be more precise, we find the opposite sign. This sign difference is not meaningless, since the negative sign we find means that the off-diagonal fields have a *tachyonic* mass.

Hence, we state that a ghost condensation à la  $\langle \epsilon^{ab}\bar{C}^a C^b \rangle$  is *not* the mechanism behind the off-diagonal mass generation in MAG, and consequently does not give evidence for Abelian dominance.

Another important point is what happens with the diagonal gluon. Consider the term  $i\bar{C}^a D_\mu^{ac} D^{\mu cb} C^b$  of (13), it contains a part proportional to  $i\bar{C}^a C^a a_\mu a^\mu$ . Doing the same as for the off-diagonal gluons, the diagonal gluon  $a_\mu$  seems to get a (real) mass too, which is of the same order as the off-diagonal one (up to the sign). However, there are other 1-loop contributions coming from the terms proportional to  $\epsilon^{ab}(\partial_\mu \bar{C}^a) C^b a^\mu$  and  $\epsilon^{ab}\bar{C}^a(\partial_\mu C^b) a^\mu$ . These contributions cancel the one coming from the term proportional to  $i\bar{C}^a C^a a_\mu a^\mu$ . Consequently, the "photon"  $a_\mu$  remains massless, as could be expected by the residual U(1) invariance.

Another point of concern is the renormalizability of the "Gaussianized" Lagrangian. A completely analogous approach can be done in case of the 2-dimensional Gross-Neveu model [27], where the 4-fermion interaction can also be made Gaussian by the introduction of an auxiliary field  $\sigma$ . This works well at 1-loop order, but from 2 loops on, *ad hoc* counterterms have to be added in order to end up with finite results [28]. A successful formalism to deal with local composite operators in case of the Gross-Neveu model was developed in [29]. A similar approach should be used to investigate the ghost condensates.

One could wonder what the mechanism behind the mass generation might be, since the previous paragraphs showed that we didn't find a dynamically generated real mass for the (off-diagonal) particles. In order to find an answer to this question, we first give a very short overview of recent results in the Landau gauge, giving us a hint in which direction we should look for the mass generation.

### III. GLUON CONDENSATION VIA $A^2$ IN THE LANDAU GAUGE AND ITS NEPHEW $A^2$ IN THE MAXIMAL ABELIAN GAUGE

A well known condensate in QCD (or Yang-Mills) is the dimension four gluon condensate  $\langle \mathcal{F}_{\mu\nu}^A \mathcal{F}^{\mu\nu A} \rangle$ . This is the lowest dimensional gluonic condensate that can exist, since no local, gauge-invariant condensates with dimension lower than 4 exist. However, recently interest arised concerning a dimension 2 gluon condensate in Yang-Mills theory in the Landau gauge. One way it came to attention was the conclusion that there exists a non-negligible discrepancy between the lattice strong coupling constant  $\alpha_s$  (determined via the 3-point gluon interaction) and the perturbative one, into a relatively high energy region where this wouldn't be expected (up to 10 GeV). Also the propagator

showed a similar discrepancy. The  $\frac{1}{p^4}$  power correction due to  $\langle \mathcal{F}^2 \rangle$  is far too small to explain this. It was shown that a  $\frac{1}{p^2}$  power correction could solve the discrepancy. More precisely, the Operator Product Expansion (OPE) used in combination with the  $\langle \mathcal{A}^2 \rangle$  condensate was able to fit both predictions [30, 31]. An important question that naturally arises, sounds: has  $\langle \mathcal{A}^2 \rangle$  any physical meaning, or is it merely a gauge artefact? The point is that  $\mathcal{A}^2$  equals  $(VT)^{-1} \min_U \int d^4x \mathcal{A}_\mu^U \mathcal{A}^{\mu U}$  in the Landau gauge, and this latter operator is, although non-local, gauge-invariant. Hence,  $\mathcal{A}^2$  can be given some physical sense in the Landau gauge. Moreover, [32] discussed the relevance of  $\mathcal{A}^2$  in connection with topological structure (monopoles) of compact QED. The physical relevance of the Landau gauge, in the framework of geometrical monopoles, is explained in [33]. The authors of that paper also stress that the values found with an OPE calculation, only describe the soft (infrared) content of  $\langle \mathcal{A}^2 \rangle$ , while they argue that also hard (short range) contributions, inaccessible for OPE, may occur. In this context, we cite [34], where a formalism was constructed for the calculation of the vacuum expectation value of (local) *composite* operators. Since this is based on the effective action, it should in principle, give the "full" value of  $\langle \mathcal{A}^2 \rangle$ , i.e. soft and hard part. For example, one could assume an instanton background as a possible source of long range contributions. In fact, there is some preliminary evidence that instantons can explain the OPE values [35]. The conclusion that one can draw from all this is that the dimension 2 condensate  $\langle \mathcal{A}^2 \rangle$  may have some physical relevance in the Landau gauge.

Let us go back to MAG [60] now. In this particular gauge,  $(VT)^{-1} \min_U \int d^4x \mathcal{A}_\mu^U \mathcal{A}^{\mu U}$  no longer reduces to a local operator. It would be interesting to repeat e.g. the OPE calculations of [31] for the coupling constant and propagators in MAG, but which dimension 2 condensate(s) could take over the role of  $\langle \mathcal{A}^2 \rangle$  in the Landau gauge? To solve this, we draw attention to the striking similarity existing between the Landau gauge and MAG. The former one can be seen as that gauge minimizing  $(VT)^{-1} \int d^4x (\mathcal{A}_\mu^A \mathcal{A}^{\mu A})^U$ , while the latter one minimizes  $(VT)^{-1} \int d^4x (A_\mu^a A^{\mu a})^U$ . This operator reduces to the local one  $A^2$  in MAG and can be seen as the MAG version of  $\mathcal{A}^2$ . Due to the more complex nature of the (renormalizable) modified MAG, other dimension 2 condensates exist (the ghost condensates). Notice that all these condensates are U(1) invariants, hence the U(1) symmetry will be preserved.

The physics we see behind all these condensates is that they might have a common, deeper reason for existence. In this context, we quote [37, 38], where it was shown that the zero vacuum is instable (tachyonic) and a vacuum with lower energy is achieved through gluon pairing, and an accompanying gluon mass. The vacuum energy itself is a physical object. After choosing a certain gauge, the different types of dimension 2 condensates are just an expression of the fact that  $E = 0$  is a wrong vacuum state. In this sense, all these different condensates in different gauges are equivalent in a way, since they lower the vacuum energy to a stable  $E < 0$  vacuum. In [33], discussion can be found on the appearance of the soft part of  $\langle \mathcal{A}^2 \rangle$  in gauge-variant quantities like in an OPE improvement of the gluon propagator, while the hard part enters physical quantities. It is imaginable that the mechanism behind this hard part (see [33] for more details) is the same in different gauges, but reveals its importance with different condensates, depending on the specific gauge. This might justify its possible appearance in gauge-invariant quantities.

#### IV. FURTHER DISCUSSION ON THE GHOST CONDENSATION AND MASS GENERATION IN THE MODIFIED MAG

In [40], it was shown that it is possible to fix the residual abelian gauge freedom of MAG in such a way that the ghost condensate  $\langle \epsilon^{ab} \bar{C}^a C^b \rangle$  does not give rise to any mass term. This abelian gauge fixing (needed for a complete quantization of the theory) was based on the requirement that the fully gauge fixed Lagrangian has a SL(2,R) and anti-BRST invariance. A restricted [61] version of this SL(2,R) symmetry was originally observed in SU(2) MAG in [23], and later generalized to SU(N) MAG [41]. In [40], the symmetry was defined on all the fields (diagonal and off-diagonal). In fact, that SL(2,R) symmetry together with the (anti-) BRST symmetry form a larger algebra, the Nakanishi-Ojima (NO) algebra. This NO algebra is known to generate a symmetry of the Landau gauge and a certain class of generalized covariant gauges, more precisely the Curci-Ferrari gauges, given by the gauge fixing Lagrangian

$$\mathcal{L}_{GF+FP} = i\delta_B \bar{\delta}_B \left( \frac{1}{2} \mathcal{A}_\mu^A \mathcal{A}^{\mu A} - \frac{\alpha}{2} i C^A \bar{C}^A \right) \quad (24)$$

The Landau gauge corresponds to the gauge parameter choice  $\alpha = 0$ . For more details, see [42, 43, 44, 45, 46].

Yang-Mills theory with the gauge fixing (24) possesses a generalization to a massive SU(N) gauge model, the

so-called Curci-Ferrari model [47]. Although this model is non-unitary, it is known to be (anti-)BRST invariant and renormalizable, whereby the mass term is of the form

$$\mathcal{L}_{mass} = M^2 \left( \frac{1}{2} \mathcal{A}_\mu^A \mathcal{A}^{\mu A} - i\alpha \mathcal{C}^A \bar{\mathcal{C}}^A \right) \quad (25)$$

Keeping this in mind and recalling that in [34], a dynamically generated mass was found in case of the Landau gauge by coupling a source  $J$  to the operator  $\mathcal{A}^2$ , it becomes clear that in case of the Curci-Ferrari gauge, the same technique could be employed by coupling a source  $J$  to the composite operator

$$\mathcal{L}_{source} = J \left( \frac{1}{2} \mathcal{A}_\mu^A \mathcal{A}^{\mu A} - i\alpha \mathcal{C}^A \bar{\mathcal{C}}^A \right) \quad (26)$$

Returning to the case of MAG and comparing the gauge fixing Lagrangians (5) and (24), the equivalent of (26) reads

$$\mathcal{L}_{source} = J \left( \frac{1}{2} A_\mu^a A^{\mu a} - i\alpha C^a \bar{C}^a \right) \quad (27)$$

This idea to arrive at a dynamically generated mass in case of the Curci-Ferrari and Maximal Abelian gauge was already proposed in [36, 39]. There, it was explicitly shown that the operator coupled to the source  $J$  in the expressions (26) or (27), is on-shell BRST invariant.

We reserve the actual discussion of the aforementioned framework to get a dynamical mass for future publications, since it is quite involved and a clean treatment of it needs a combination of the local composite operator formalism [34] and the algebraic renormalization technique [48, 49].

Before turning to conclusions, we want to draw attention to the following. We decomposed the 4-ghost interaction with a real auxiliary field  $\phi$  whereby  $\phi \sim \epsilon^{ab} \bar{C}^a C^b$ . Let's make a small comparison with ordinary superconductivity. Usually, there is talked about BCS pairing, i.e. particle-particle and hole-hole pairing. The analogy of this in the ghost condensation case would be ghost-ghost pairing and antighost-antighost pairing. This can be achieved by an alternative decomposition of the 4-ghost interaction via a pair of auxiliary fields  $\sigma$  and  $\bar{\sigma}$  such that  $\sigma \sim \epsilon^{ab} C^a C^b$  and  $\bar{\sigma} \sim \epsilon^{ab} \bar{C}^a \bar{C}^b$ . This kind of pairing [62] was considered in [50]. A less known effect is the particle-hole pairing, the so-called Overhauser pairing [51]. This corresponds to the kind of condensation we and the papers [13, 21, 22, 23] considered. From the viewpoint of the  $SL(2, \mathbb{R})$  symmetry, the existence of different channels where the ghost condensation can take place should not be surprising. The different composite ghost operators are mutually changed into each other under the action of the symmetry. Here and in the other papers the *choice* was made to work with the Overhauser channel, but a complete treatment would need an analysis of all channels at once, and with the local composite operator technique. This analysis of the BCS versus Overhauser effect is nicely intertwined with the existence of the NO algebra and its (partial) breakdown, and it is very much alike for the MAG, Landau [52] and Curci-Ferrari gauge, just as in case of the mass generation mechanism. As an indication, it has been found recently that, although no 4-ghost interaction is present in the Landau gauge, the condensation à la  $f^{ABC} \bar{C}^A C^B$  etc. also occurs [53].

## V. CONCLUSION

We considered Yang-Mills theory in the Maximal Abelian Gauge. With this non-linear gauge choice, a 4-ghost interaction enters the Lagrangian. Such an interaction could allow a non-zero vacuum expectation value for (off-diagonal) dimension 2 ghost condensates. Consequently, it was expected that a mass generating mechanism for the off-diagonal gluons *and* the diagonal gluons due to 4-point interaction terms of the form gluon-gluon-ghost-anti-ghost was found.

We explained why this particular type of ghost condensation is not sufficient to construct a (off-diagonal) dynamical mass in  $SU(2)$  Yang-Mills theory in the Maximal Abelian Gauge, an indicator for Abelian dominance. We have restricted ourselves to the  $SU(2)$  case, but a similar conclusion will exist for general  $SU(N)$ . Explicit calculations showed that we ended up with a tachyonic off-diagonal mass  $M$  ( $M^2 < 0$ ). This result indicate something is missing. A comparison with Yang-Mills theory in the Landau gauge and the role played by the mass dimension 2 gluon condensate  $\langle \mathcal{A}^2 \rangle$ , shed some light on the route that should be followed.

We revealed certain shortcomings of the present available studies on the ghost condensation (renormalizability,

existence of more than one condensation channel).

The actual study of the mass generation and the ghost condensation with its symmetry breaking pattern will be discussed elsewhere. We will follow the local composite operator formalism of [34], where a source is coupled to each operator and the effective action can be treated consistently. This effective potential formalism allows a clean treatment of the role played by the dimension 2 operators. We remark that with essentially perturbative techniques one can obtain at least qualitatively trustworthy results [63] on the stability of the condensates and their relevance for e.g. mass generation and symmetry breakdown, without making it directly necessary to go to (or extrapolating to) strong coupling.

We conclude by mentioning that the dimension 2 condensates and the accompanying mass generation in Yang-Mills are not only of theoretical importance (the role of  $\langle \mathcal{A}^2 \rangle$  for OPE corrections [30, 31], monopoles [32, 33], *short range linear correction* to the Coulomb-like potential [30], low energy effective theories [54],...) but also have their importance for automated Feynmandiagram calculations [55, 56, 57] where a gluon mass serves as a infrared regulator. If this mass is generated in massless Yang-Mills, it does not have to be implemented by hand.

## VI. ACKNOWLEDGMENTS

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- [58]  $\mathcal{C} = (C^a, C^3)$  with  $C^3$  the diagonal ghost. Analogously for  $\bar{\mathcal{C}}$ .
- [59] If one would like to avoid Wick rotations, one could start immediately from the Euclidean version of Yang-Mills.
- [60] Here, with MAG we mean the gauge minimizing the functional (4), and not the modified MAG.
- [61] By restricted, we mean that the symmetry only acts non-trivially on the off-diagonal fields.
- [62] Our conclusion about the tachyonic mass is unaltered by this alternative decomposition of the 4-ghost interaction.
- [63] In case of the Gross-Neveu model, very accurate results were obtained [29]. In case of the  $\langle A^2 \rangle$  condensate in the Landau gauge, the relevant coupling constant was quite small, making the expansion acceptable [34].