Magnetic octupole strength in rare-earth nuclei: A sum rule approach

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We use linear energy-weighted sum rules within the proton-neutron interacting boson model to deduce a relationship between magnetic dipole and magnetic octupole transition probabilities. We then obtain a first estimate of the summed magnetic octupole strength to be expected in rare-earth nuclei.

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Recently, sum rules have proven useful in describing the low-lying summed magnetic dipole ($M1$) strength in rare-earth nuclei and establishing relations with other electromagnetic transition probabilities [1,2]. Unexpectedly, it was shown that the non-energy-weighted $M1$ sum rule turns out to be proportional to the electric monopole ($E0$) strength whereas the energy-weighted $M1$ sum rule becomes proportional to the non-energy-weighted electric quadrupole ($E2$) sum rule. These results lead to a number of interesting properties such as the dependence of $M1$ strength on nuclear deformation and its relation to the isotope shift [3,4]. The deformation dependence of the summed $M1$ strength was found experimentally for the Sm [5] and Nd [6] isotopes.

We now want to establish another relationship between magnetic octupole and magnetic dipole strength, starting from the linear energy-weighted sum rules in the interacting proton-neutron boson model (IBM-2) [1]:

$$\sum_f B(M1;0_1^+ \to 1_f^+)E_x(1_f^+) = \frac{9}{4\pi}(g_\pi - g_\nu)^2 \Big[-\kappa_{\pi\nu}\langle 0_1^+|\hat{Q}_\pi \cdot \hat{Q}_\nu|0_1^+\rangle + \lambda\langle 0_1^+|N_\pi \hat{n}_{d_\nu} + N_\nu \hat{n}_{d_\pi}|0_1^+\rangle + R\Big] \qquad (1)$$

and

$$\sum_f B(M3;0_1^+ \to 3_f^+)E_x(3_f^+) = \frac{49}{16\pi}(\Omega_\pi - \Omega_\nu)^2 \Big[-\kappa_{\pi\nu}\langle 0_1^+|\hat{Q}_\pi \cdot \hat{Q}_\nu|0_1^+\rangle + R' + \lambda\langle 0_1^+|N_\pi \hat{n}_{d_\nu} + N_\nu \hat{n}_{d_\pi}|0_1^+\rangle + R''\Big], \qquad (2)$$

where R, R'' are additional terms originating from the Majorana interaction and R' is an additional term in the quadrupole part. Following numerical calculations using the code NPBOS [7], one can neglect those terms since their contribution amounts to no more than 10% and 1%, respectively [8]. Hence, it shows that both the $M1$ and $M3$ energy-weighted sum rules are determined mainly by the quadrupole-quadrupole $\hat{Q}_\pi \cdot \hat{Q}_\nu$ term and the boson number contribution coming from the Majorana term. In the above expressions, $\kappa_{\pi\nu}$ and λ denote the strength of the quadrupole-quadrupole $\hat{Q}_\pi \cdot \hat{Q}_\nu$ and Majorana $\hat{M}_{\pi\nu}$ terms and g_ρ, Ω_ρ $(\rho = \pi, \nu)$ the gyromagnetic boson factors for the $M1$ and $M3$ operators, respectively.

For the ratio between both (approximate) sum rules one obtains the very simple result

$$\frac{\sum_f B(M1)E_x(1_f^+)}{\sum_f B(M3)E_x(3_f^+)} \simeq \left(\frac{6\,(g_\pi - g_\nu)}{7\,(\Omega_\pi - \Omega_\nu)}\right)^2. \qquad (3)$$

This relationship is interesting since it establishes a link between different parameters of the IBM-2, the gyromagnetic boson factors and octupole boson moments, and hence imposes a constraint on choosing them when fitting to spectroscopic data. One can also use this relationship the other way, using common values for these parameters $(g_\pi, g_\nu, \Omega_\pi, \Omega_\nu)$ as derived phenomenologically and/or microscopically, and deduce the summed magnetic octupole strength whenever experimental information on the magnetic dipole strength is available.

One should, however, bear in mind that the IBM-2 has proven to be useful only for the low-lying excitations in medium-heavy and heavy nuclei. We should therefore restrict applications to these mass and energy regions. For rare-earth nuclei the magnetic dipole strength calculated within the IBM-2 resides mainly in the first 1^+ state. Likewise the magnetic octupole strength is concentrated in the low-lying 3^+ states [9]. Clearly the restriction to the low-energy region is inherent to the sum-rule results. Any conclusion should therefore be confined to this energy region.

Hence it is meaningful only to introduce the experimental values of the summed $M1$ strength below $E_x \simeq 4$ MeV to deduce an estimate of $M3$ strength in the same energy region. We apply this method in the rare-earth region, where the model is known to work considerably well, and which is one such region where magnetic dipole transitions have been studied in much detail [10].

We thereby introduce some assumptions.

(i) For the boson gyromagnetic factors we use the common values $g_\pi = 1\mu_N, g_\nu = 0$, as derived by Sambataro

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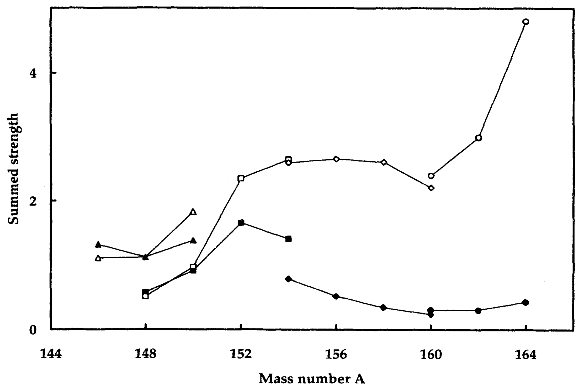

FIG. 1. Experimental values of summed magnetic dipole strength [5, 6, 12–14] (open symbols, units μ_N^2) and estimates of summed magnetic octupole strength following Eq. (3) (solid symbols, units $\mu_N^2 b^2$), using $\overline{E_x(1^+)} - \overline{E_x(3^+)} = 0.25$ MeV, $g_\pi = 1\mu_N, g_\nu = 0$, and Ω_π, Ω_ν values from Ref. [9], for Nd (Δ), Sm ($\Box$), Gd ($\Diamond$), and Dy ($\circ$).

et al. [11].

(ii) For the boson octupole moments Ω_π and Ω_ν we use the values derived by Scholten *et al.* [9]. Those vary from nucleus to nucleus and hence influence strongly the behavior of the predicted $M3$ strength throughout this mass region.

Here we stress the fact that the spin part contribution to both the $M1$ and $M3$ transition operators, expressed by the g_π, g_ν, Ω_π, and Ω_ν values within the boson model, is taken into account by mapping the full fermion $M1$ and $M3$ matrix elements into corresponding boson $M1$ and $M3$ matrix elements. This mapping procedure, called Otsuka-Arima-Iachello (OAI) mapping results in a very small and almost negligible spin contribution for the magnetic dipole operator [11]. For the $M3$ operator, the Ω_π and Ω_ν values are determined to a large fraction by the fermion intrinsic spin $M3$ contribution [9]. Therefore, the determination of the Ω_π and Ω_ν $M3$ quantities has been carried out taking the shell model structure of 3^+ states in the corresponding mass region into account in the best possible way.

(iii) To deduce the non-energy-weighted summed $M3$ strength one needs to eliminate the energy. We assume that the centroid energies are 3 MeV and 2.75 MeV for 1^+ and 3^+ excitations, respectively, based on the calculation for ^{154}Sm by Scholten *et al.* [9].

In Table I the ratio of energy-weighted sum rules is given for a range of parameter values covered by the calculations of Scholten *et al.* [9]. From these, one can conclude that the low-lying $M3$ strength should be comparable to the $M1$ strength for the lighter rare-earth nuclei, while it should be a lot smaller for the heavier ones. This of course goes back entirely to the determination of the octupole moments.

TABLE I. Ratio of the energy-weighted sum rules for $M1$ and $M3$ transition probabilities in the rare-earth region for different ranges of boson octupole moments following Ref. [9] and with boson gyromagnetic factors $g_\pi = 1\mu_N, g_\nu = 0$.

Z	Ω_π (μ_Nb)	Ω_ν (μ_Nb)	LEWSR($M1$)/ LEWSR($M3$)
60 – 62	0.50	-0.50	0.735
	0.50	-0.25	1.306
64 – 66	0.25	-0.25	2.939
	0.25	0.00	11.755

In Fig. 1 we show the summed $M1$ strength obtained experimentally [5, 6, 12–14], compared to the expected $M3$ strength for the low-energy region. Although one should realize that these results can be valued only as estimates, in view of the assumptions made and their inherent uncertainties, it is worth noting that for ^{164}Dy a summed $M3$ strength of $0.43\mu_N^2 b^2$ is predicted, in rather good agreement with the experimental value. In an (e,e') experiment [15] essentially one isoscalar excitation has been identified corresponding to the 3_γ^+ level with $B(M3)\uparrow = 0.3^{+0.1}_{-0.2}\mu_N^2 b^2$ with the possibility of isovector strength around 3 MeV, which is more fragmented and has an upper limit of $0.5\mu_N^2 b^2$. The sum rule, of course, cannot distinguish between different excitations but from calculations performed by Lo Iudice [16] using an isovector $M3$ operator within the two-rotor model and schematic random-phase approximation (RPA) approach, the isovector contribution to the $M3$ strength in ^{164}Dy appears to be small, about $0.2\mu_N^2 b^2$. Also a similar prediction for ^{154}Sm and ^{156}Gd [16, 17] gives much smaller values than the summed strengths obtained here, pointing toward an appreciable contribution to the lower-lying symmetric 3_γ^+ state. This is indeed found by Scholten *et al.* [9], who obtain for ^{154}Sm a value for the summed $M3$ strength of $1.1\mu_N^2 b^2$ fairly close to our estimate of $1.4\mu_N^2 b^2$.

In conclusion, we have obtained an estimate of the low-lying summed $M3$ strength in the rare-earth region starting from the linear energy-weighted sum rules in the IBM-2. Even taking into account the approximate character of the above relationship, one can use these estimates as a first guide for further experimental studies of $M3$ excitations in rare-earth nuclei. Indeed, in view of these and other theoretical predictions we expect that $M3$ strength can be detected in this mass region.

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